

Adaptive Robust Dynamic Surface Control for Continuous Bioreactor Systems Incorporating DC Motor Actuator Dynamics

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Article Info	ABSTRACT
Article type: Research Article Article history: Received: ***** Received in revised form: ***** Accepted: ***** Published online: ***** Keywords: Dynamic surface control, Adaptive robust control, Bioreactor, Actuator dynamics, Nonlinear systems	Advanced control of bioreactor systems is essential for enhancing efficiency and reliability in industrial bioprocesses. This paper introduces an adaptive robust dynamic surface control (DSC) approach for continuous stirred bioreactors, characterized by nonlinear microbial growth and significant model uncertainties. Unlike conventional bioreactor control formulations that treat the dilution rate as an instantaneously available input, the proposed framework considers the complete bioreactor–actuator cascade and explicitly incorporates the actuator dynamics into both controller synthesis and closed-loop stability analysis. Its distinguishing feature is the introduction of a second dynamic surface on the armature current, which couples the electrical actuator subsystem with the nonlinear bioreactor dynamics while avoiding repeated analytical differentiation of virtual control laws. The proposed DSC framework effectively addresses both parametric and non-parametric uncertainties through adaptive estimation and robust control laws. Lyapunov-based stability analysis proves that all tracking errors are uniformly ultimately bounded (UUB) within a compact set. MATLAB-based numerical simulations confirm that the substrate concentration tracks the desired trajectory with a settling time of 2.2 hours and an Integral of Absolute Error (IAE) of 9.07 (g/L).h. Moreover, the computational time per control step is only 0.0089 ms, indicating low algorithmic computational demand. Furthermore, the adaptive mechanism ensures that the adaptive estimates remain stable throughout operation, confirming the robustness and computational feasibility of the proposed framework. Experimental validation remains necessary before industrial implementation.

I. Introduction

In the rapidly evolving field of bioprocess engineering, advanced bioreactor control is key to sustainable production of biofuels, pharmaceuticals, and biochemicals. However, these systems face specific challenges such as inherent nonlinearities, model uncertainties, and fluctuating environmental conditions [1]. Traditional control methods, such as Proportional-Integral-Derivative (PID) controllers, often struggle to remain stable amid unpredictable microbial behavior, substrate changes, and nonstationary

environments. To overcome these issues, advanced control strategies—focused on adaptability and resilience—have become essential solutions.

Fuzzy control systems excel in handling imprecise data through rule-based frameworks that emulate human reasoning [2-5]. For instance, fuzzy logic controllers have improved anaerobic hydrogen production by 95% over open-loop systems [3] and enabled precise temperature regulation in axon stretch growth bioreactors [5]. Apart from fuzzy methods, nonlinear control techniques—such as feedback linearization and backstepping—are widely used to manage

bioprocess dynamics [6, 7]. A notable example is the multi-input multi-output (MIMO) nonlinear control structure developed in [6] for non-isothermal bioreactors, which integrates three key components: (1) feedback linearization for precise glucose concentration control, (2) backstepping control for robust temperature regulation, and (3) a biomass estimation algorithm to maintain system stability against input disturbances.

The success of such methodologies has spurred further developments in nonlinear bioprocess control, including the application of sliding mode control (SMC), which ensures stability and robust performance despite uncertainties and disturbances [8-10]. For instance, a robust sliding mode controller was introduced for fed-batch bioreactors, utilizing only biomass and volume signals to effectively manage system complexities under perturbations [8].

Model predictive control (MPC), a well-established optimization-based control framework with strong theoretical foundations [11] further enhances bioprocess management by predicting future states and adjusting control inputs while explicitly handling system constraints. Advances in explicit MPC formulations have further enabled efficient real-time implementation by replacing online optimization with precomputed control laws [12].

Numerous studies have demonstrated the effectiveness of MPC in bioprocess control under different operating conditions [11-17]. For example, [13] successfully applied MPC to continuous ethanol fermentation systems, reducing oscillatory dynamics and increasing productivity by 18–22% through a serial fermenter configuration with cell recycling. Furthermore, [17] implemented an advanced MPC strategy for anaerobic digestion start-up, achieving precise multivariable regulation of feed flow, alkali dosing, and water addition. Collectively, these studies demonstrate the capability of MPC to achieve high-performance control of complex biochemical processes while effectively handling multivariable interactions and process constraints.

Observer-based estimation has also become an important component of modern nonlinear control systems when some state variables are unavailable or difficult to measure directly. Depending on the application requirements, various observer structures—including Luenberger observers, disturbance observers, extended state observers (ESO), adaptive observers, and model-free observers—have been developed to estimate unavailable states or lumped uncertainties while improving robustness against modeling errors and external disturbances. Recent studies have successfully integrated observer-based estimation with advanced nonlinear control strategies. For example,

observer-based techniques have been employed for state estimation in distributed-parameter bioreactors [18], while adaptive and model-free observers have been incorporated into robust nonlinear control frameworks for robotic systems to reduce sensing requirements and enhance robustness [19, 20]. These developments demonstrate the effectiveness of observer-based estimation in improving closed-loop performance, although they generally introduce additional design and implementation complexity.

Finally, adaptive and robust-adaptive strategies combine real-time adaptation with stability guarantees, offering resilience against system uncertainties [21-23]. These methods further exploit adaptive estimation mechanisms to compensate for kinetic uncertainties and process variability in bioreactor systems while preserving closed-loop stability [21]. Although these approaches address specific challenges, they often involve trade-offs among robustness, implementation complexity, computational requirements, and controller tuning, depending on the particular application and control formulation. This issue becomes especially evident when considering actuator dynamics in bioreactor systems, where neither high computational demand nor robustness sacrifices are acceptable.

Recent studies have also investigated adaptive nonlinear control under practical constraints. [24] combined adaptive fuzzy control with an anti-windup compensator to address parametric uncertainty and input saturation in a nonlinear biological system. Although this formulation explicitly accommodates saturation, it requires additional fuzzy-approximation and anti-windup components and does not incorporate actuator dynamics into a bioreactor-actuator cascade. [25] proposed a robust adaptive backstepping strategy for a nonlinear biological model subject to model uncertainties, external disturbances, and input saturation. While the method provides systematic robustness against these effects, it combines backstepping, sliding-mode control, Nussbaum gains, and an auxiliary system and is not formulated for continuous bioreactor control with explicit actuator dynamics. These studies demonstrate the effectiveness of adaptive nonlinear control while highlighting the need for a computationally simple actuator-aware framework for bioprocess applications.

Orthogonal-polynomial approximators have also been investigated as computationally efficient tools for uncertainty estimation in adaptive nonlinear control. In [26], Legendre polynomials were employed to estimate the uncertainty bound in robust task-space control of electrically driven robot manipulators, thereby reducing the conservatism associated with assigning a fixed uncertainty bound. More recently, [27] used Legendre polynomials

together with time-varying gains to estimate model uncertainties and disturbances in voltage-controlled non-holonomic mobile robots, reporting fewer tuning parameters than radial-basis-function neural networks and reduced initial motor-voltage demand. Although these studies concern robotic rather than bioreactor systems, they demonstrate the potential of Legendre-polynomial bases for uncertainty compensation in nonlinear systems with actuator dynamics. In contrast, the present framework estimates uncertainty bounds through adaptive laws embedded within an actuator-aware DSC architecture, without employing a polynomial function approximator.

Practical bioprocess systems inherently rely on actuators such as pumps and control valves, and several studies have incorporated actuator hardware into experimental or simulation-based bioreactor platforms. For example, microfluidic bioreactor systems have employed pneumatically actuated valves and pumps for substrate feeding and pH regulation [28], while recent plant-wide biomanufacturing models have emphasized more realistic integration of sensor and actuator behavior [29]. Nevertheless, these studies have primarily focused on process operation, hardware integration, or plant-level simulation rather than explicitly incorporating the full electromechanical dynamics of the actuator into nonlinear controller synthesis. In other engineering domains, particularly robotics, actuator dynamics have been shown to influence closed-loop performance and have therefore been explicitly included in adaptive and nonlinear control designs [30,31]. These observations indicate that actuator-aware control is well established in related fields, whereas its integration into adaptive DSC design for continuous bioreactors remains comparatively underexplored.

Despite this progress, two important research gaps remain. First, although practical actuator hardware has been considered in bioprocess applications, the explicit integration of electromechanical actuator dynamics into nonlinear bioreactor control design remains comparatively underexplored. In particular, electrical and mechanical effects such as armature inductance, back electromotive force (back-EMF), and mechanical inertia are typically not incorporated directly into the controller formulation or the associated closed-loop stability analysis. Consequently, the interaction between the bioreactor and actuator dynamics is often omitted during controller design. This constitutes the first gap addressed in the present work.

Second, a methodological gap remains because some existing control strategies achieve robustness at the expense of increased implementation complexity, for example through repeated online optimization in MPC or high-frequency switching in SMC. DSC has emerged as a

particularly promising control strategy that effectively combines the theoretical rigor of the traditional backstepping method with significantly improved computational efficiency. Unlike conventional backstepping, which often entails heavy computational loads due to the analytical differentiation of virtual control laws, DSC provides similar stability guarantees while significantly reducing complexity through first-order filters, making it potentially suitable for low computational demand.

The practical motivation for this study is to prevent the performance degradation that may result from neglecting actuator dynamics while retaining robustness and low online computational complexity. DSC is selected for this purpose because it offers clear advantages over other advanced nonlinear control methods. Unlike conventional SMC, it avoids the discontinuous switching responsible for chattering [32] while, unlike nonlinear MPC, it does not require repeated online optimization [33]. As a result, DSC provides an attractive balance between robustness, computational efficiency, and ease of implementation [34]. Motivated by these advantages, to the best of the authors' knowledge, the proposed framework is among the few adaptive robust DSC approaches for continuous bioreactors that explicitly incorporate actuator dynamics into both controller synthesis and closed-loop stability analysis. The proposed architecture provides three key contributions: (I) hardware-aware control through explicit integration of DC motor actuator dynamics; (II) computationally efficient adaptive control via first-order dynamic-surface filtering and online estimation of uncertainty bounds; and (III) Lyapunov-based stability analysis and comparative numerical evaluation without the repeated online optimization required by nonlinear MPC.

The proposed framework addresses the identified research gaps through a dual-layer structure. Explicit actuator dynamics are incorporated by introducing a second dynamic surface on the armature current, which couples the electrical actuator subsystem with the bioreactor dynamics. Furthermore, first-order filters eliminate the need for repeated analytical differentiation of virtual control laws, thereby simplifying the controller design and reducing computational complexity relative to conventional backstepping-based methods. Together with the proposed adaptive estimation laws, this architecture enables real-time compensation of model uncertainties while maintaining low computational cost. Consequently, the proposed controller achieves effective tracking performance with an average computation time of only 0.0089 ms per controller update. Since it avoids repeated online optimization, it provides a computationally efficient alternative to the benchmark nonlinear MPC implementation while avoiding the chattering issue typically associated with conventional SMC.

This paper is organized as follows: Section II presents fundamental concepts and preliminary materials. Section III details the proposed control design methodology and analyzes closed-loop stability. Section IV evaluates the approach through comprehensive numerical simulations, with concluding remarks provided in Section V.

II. Bioreactor Modeling and Control-Oriented Problem Formulation

A. Bioreactor model

For biological systems, unstructured models represent the simplest approach to describing biological processes. These models treat cell mass as a single chemical species and disregard any intracellular reactions. Typically, unstructured modeling focuses on growth dynamics based on a single limiting substrate, incorporating only substrate consumption, biomass production, and product formation into the modeling framework [23]. Throughout this paper, $\mathbb{R}$ denotes the set of real numbers. Let us consider a continuous fermenter system with the scalar state variables: substrate concentration ($S \in \mathbb{R}$) ($\frac{g}{L}$), biomass concentration ($X \in \mathbb{R}$) ($\frac{g}{L}$), and ethanol concentration ($P \in \mathbb{R}$) ($\frac{g}{L}$), as outlined in [35].

$$\begin{aligned}\frac{dS}{dt} &= D(S_{in} - S) - \frac{1}{Y_{XS}} \mu(S)X \\ \frac{dX}{dt} &= -DX + \mu(S)X \\ \frac{dP}{dt} &= -DP + \frac{1}{Y_{XP}} \mu(S)X + \delta X\end{aligned}\quad (1)$$

Here, $S_{in} \in \mathbb{R}$ ($\frac{g}{L}$) represents the mass concentration of the inlet substrate, $D \in \mathbb{R}$ (h^{-1}) denotes the dilution rate, which is related to the input substrate concentration and the biomass yield coefficient, and Y_{XS} ($\frac{g}{g}$), Y_{XP} ($\frac{g}{g}$), δ (h^{-1}) are constants. The cell growth rate, $\mu(S)$, is determined using Monod's equation [36].

$$\mu(S) = \mu_{max} \frac{S}{K_S + S} \quad (2)$$

where μ_{max} (h^{-1}) is the maximum specific growth rate and $K_S \in \mathbb{R}^+$ ($\frac{g}{L}$) is the substrate saturation constant.

B. Dilution rate control via actuation

The dilution rate, D (h^{-1}), is regulated by a control valve whose open area depends on the torque (τ_a) of a DC motor (Nm). For controller design, two assumptions are adopted around the nominal operating condition: (I) The valve

opening area varies approximately linearly with applied motor torque, (II) variations in the pressure drop, ΔP (Pa), and fluid density, χ (kg/m^3), remain sufficiently small around their nominal values. Under these assumptions, the dilution rate relates to motor torque as:

$$D = \frac{F}{U} = \frac{C_v K_c \tau_a}{K_{ss} U} \sqrt{\frac{\Delta P}{\chi}} = K_v \tau_a \quad (3)$$

where F ($\frac{L}{h}$) is the inlet flow rate, C_v (m^2) is the valve coefficient, K_c is the geometric constant, K_{ss} ($N \cdot m/rad$) is the stem spring constant, U (L) is the bioreactor volume, ΔP is the pressure drop across the valve, χ is the fluid density, and $K_v = \frac{C_v K_c}{K_{ss} U} \sqrt{\frac{\Delta P}{\chi}}$ ($h^{-1} \cdot (N \cdot m)^{-1}$) is the torque-to-flow sensitivity. The present actuator model does not explicitly account for practical valve nonlinearities such as hysteresis, dead-band, and saturation; their explicit compensation is left for future extension of the proposed framework.

Substituting (3) into (1) yields

$$\begin{aligned}\frac{dS}{dt} &= K_v \tau_a (S_{in} - S) - \frac{1}{Y_{XS}} \mu(S)X \\ \frac{dX}{dt} &= -K_v \tau_a X + \mu(S)X \\ \frac{dP}{dt} &= -K_v \tau_a P + \frac{1}{Y_{XP}} \mu(S)X + \delta X\end{aligned}\quad (4)$$

Remark 1. The linear relationship in Eq. (3) is a standard first-order approximation for proportional valves operating near a nominal operating point [37], which is selected based on the desired steady-state region of the bioreactor. While real valves exhibit inherent nonlinearities, their installed behavior can be reasonably approximated as linear when pressure drop and fluid density remain nearly constant. This approximation is valid for moderate torque variations around this nominal point, and is justified by the simulation results in Section 4 (see Figs. 1 and 5), which demonstrate successful set-point regulation and small-signal tracking within the intended operating range. For wide-range operation, the nonlinear valve characteristic can be represented by a set of local affine models, with K'_v and, if required, the controller gains scheduled according to the measured operating point and smoothly interpolated between adjacent regions. Significant pressure or density variations are acknowledged as limitations and deferred to future work (Section V).

C. Actuator Dynamics

To incorporate actuator dynamics, the bioreactor is modeled as being actuated by a brushed DC motor coupled with a

mechanical gearbox. The electrical dynamics of the motor armature are governed by the following relation [38]:

$$\begin{aligned} V_m &= L_a \dot{I}_a + R_a I_a + k_b \dot{\omega}_m + v_d \\ \tau_m &= k_t I_a \end{aligned} \quad (5)$$

where V_m (V) denotes the applied armature voltage, L_a and R_a are the armature inductance (H) and resistance (Ω), respectively, and I_a (A) is the armature current. The term $k_b \dot{\omega}_m$ represents the back electromotive force (EMF), with k_b (V·s/rad) being the back EMF constant and $\dot{\omega}_m$ ($\frac{rad}{s}$) the motor's angular velocity. The lumped effect of unstructured disturbances—such as measurement noise and unmodeled high-frequency dynamics—is captured by the disturbance term v_d . The electromagnetic torque produced by the motor is given by $\tau_m = k_t I_a$ (Nm), where k_t (N·m/A) is the torque constant. Accounting for the gear transmission with ratio n , the torque and angular velocity at the output shaft are related to those at the motor side by $\tau_a = n\tau_m$ (N·m) and $\dot{\omega}_m = n\dot{\omega}_v$, respectively. Based on these relationships, the overall actuator dynamics are formulated as follows:

$$\begin{aligned} V_m &= L_a \dot{I}_a + R_a I_a + nk_b \dot{\omega}_v + v_d \\ \tau_a &= nk_t I_a \end{aligned} \quad (6)$$

The motor's torque directly affects the valve's rotational speed. This effect is determined by the valve's dynamic equations of motion. Assume that the valve has a friction coefficient (B_v) (N·m·s/rad). The dynamic equation of motion is written as follows:

$$\tau_a = B_v \dot{\omega}_v \quad (7)$$

According to (7), the actuator dynamics are rewritten as:

$$\begin{aligned} V_m &= L_a \dot{I}_a + R_a I_a + k_a \dot{\omega}_v + v_d \\ \tau_a &= nk_t I_a \end{aligned} \quad (8)$$

where $k_a = \frac{nk_t k_b}{B_v}$.

D. Integrated Bioreactor-Actuator System Model

Considering (6) and (4), the following model is derived

$$\begin{aligned} \frac{dS}{dt} &= K_v' I_a (S_{in} - S) - \frac{1}{Y_{XS}} \mu(S) X \\ \frac{dX}{dt} &= -K_v' I_a X + \mu(S) X \\ \frac{dP}{dt} &= -K_v' I_a P + \frac{1}{Y_{XP}} \mu(S) X + \delta X \end{aligned} \quad (9)$$

where $K_v' = nK_v k_t$. By defining the new variables $x_1 = S$, $x_2 = X$, and $x_3 = P$, the system represented by equation (9) can be reformulated as follows:

$$\begin{aligned} \dot{x}_1 &= K_v' I_a (x_{1in} - x_1) - \frac{1}{Y_{XS}} \mu(x_1) x_2 \\ \dot{x}_2 &= -K_v' I_a x_2 + \mu(x_1) x_2 \\ \dot{x}_3 &= -K_v' I_a x_3 + \frac{1}{Y_{XP}} \mu(x_1) x_2 + \delta x_2. \end{aligned} \quad (10)$$

Model (10) offers a modified representation of the bioreactor system, as outlined by (4). The physical configuration of the integrated bioreactor-actuator system is illustrated in Fig. 1. Table I summarizes all model parameters used in the integrated bioreactor-actuator system, including their physical meanings and units. This table aids reproducibility and provides context for the modeling assumptions.

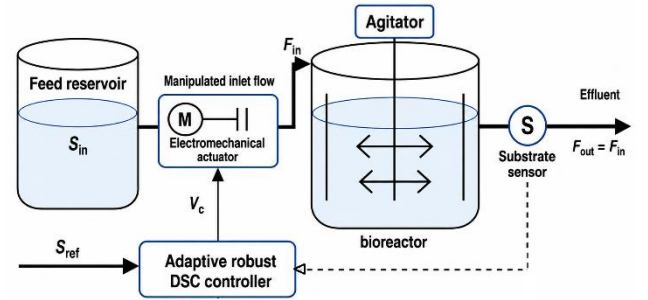


Fig. 1. Schematic representation of the integrated continuous stirred bioreactor-actuator system and its feedback-control configuration.

Based on this coupled dynamical model, the proposed adaptive robust DSC scheme is developed in the following section.

TABLE I Nomenclature and physical units of model parameters for the integrated bioreactor-actuator system.

Symbol	Description	Unit
S_{in}	Inlet substrate concentration	g/L
Y_{XS}	Biomass yield coefficient	g/g
μ_{max}	Maximum specific growth rate	h ⁻¹
K_S	Monod saturation constant	g/L
Y_{XP}	Product yield coefficient	g/g

δ	Ethanol inhibition rate	h^{-1}
K_v	Valve sensitivity constant (Effective torque-to-dilution gain)	$\text{h}^{-1} \cdot (\text{N} \cdot \text{m})^{-1}$
B_v	Valve sensitivity (torque $\rightarrow$ flow)	$\text{N} \cdot \text{m} \cdot \text{s} / \text{rad}$
L_a	Armature inductance	H
R_a	Armature resistance	Ω
k_b	Back-EMF constant	$\text{V} \cdot \text{s} / \text{rad}$
k_t	Torque constant	$\text{N} \cdot \text{m} / \text{A}$
n	Gear ratio	—
U	Bioreactor volume	L
χ	Fluid density	kg / m^3
x_1	Substrate concentration	g / L
x_2	Biomass concentration	g / L
x_3	Ethanol concentration	g / L
ΔP	Pressure drop across valve	Pa
D	Dilution rate	h^{-1}
C_v	Valve flow coefficient	m^2
K_{ss}	Stem spring constant	$\text{N} \cdot \text{m} / \text{rad}$
K_c	Valve geometric constant	—
τ_a	Actuator torque	$\text{N} \cdot \text{m}$
S	Substrate concentration	g / L
X	Biomass concentration	g / L
P	Ethanol concentration	g / L
μ	Specific growth rate	h^{-1}
V_m	Applied armature voltage	V
I_a	Armature current	A
$\dot{\omega}_m$	Motor angular velocity	rad / s
$\dot{\omega}_v$	Gear output angular velocity	rad / s
τ_m	Motor torque	$\text{N} \cdot \text{m}$
B_v	Friction coefficient	$\text{N} \cdot \text{m} \cdot \text{s} / \text{rad}$
v_d	Lumped actuator disturbance	—

III. Controller Design and Stability Analysis

This section describes the design of a set-point regulation controller for bioreactor systems that use electrically actuated control valves. The controller is developed using

the DSC methodology to tackle challenges related to actuator dynamics and model uncertainties. A Lyapunov-based stability analysis is then performed to demonstrate that the regulation error remains ultimately uniformly bounded (UUB).

A. Control Objective

The control objective is twofold: first, to force the substrate concentration $x_1(t)$ to track a desired bounded trajectory $x_d(t)$ with guaranteed transient and steady-state performance despite parametric uncertainties and non-parametric disturbances. Second, to ensure that all closed-loop signals, particularly the tracking error Z_1 , the actuator current error Z_2 , and the parameter estimation errors $\tilde{\alpha}$ and $\tilde{\beta}$, remain uniformly ultimately bounded with explicitly computable bounds, while maintaining computational efficiency suitable for real-time implementation on standard industrial controllers.

The following assumptions are essential for the design and development of the control algorithm:

Assumption 1. Real-time measurement of the current I_a is accessible.

Assumption 2. The desired trajectory and its first derivative are bounded.

Assumption 3. $0 < h(x_1, x_2) \leq \Gamma < \infty$ [39].

Assumption 4. $0 < f(x_1) \leq M < \infty$ [39].

B. First Dynamic Surface and Virtual Control

Define the first dynamic surface as:

$$Z_1 = x_1 - x_d, \quad (11)$$

where Z_1 represents the first dynamic surface and x_d denotes the desired trajectory. Taking (10) into account and applying (11) results in:

$$\dot{Z}_1 = I_a f(x_1) - h(x_1, x_2) Z_1 + \varsigma_1 \quad (12)$$

where $f(x_1) = K_v'(x_{1in} - x_1)$, $h(x_1, x_2) = \frac{\mu_{2max}}{\gamma_{XS}(K_S + x_1)}$, and $\varsigma_1 = h(x_1, x_2)x_d - \dot{x}_d$ represents the uncertain nonlinearities, which are bounded as $\|\varsigma_1\| \leq g(x_d, \dot{x}_d)$. A detailed proof is provided in Appendix A. Throughout this paper, $\|\cdot\|$ denotes the standard Euclidean norm for vectors and the absolute value for scalar quantities.

Remark 2. The existence of the bound $\|\varsigma_1\| \leq g(x_d, \dot{x}_d)$ is formally established by Assumption 3. In practice, this

bound is physically justified as follows. For the bioreactor subsystem, the state variables S and X are inherently bounded due to mass conservation: $0 \leq S \leq S_{in}$ and $X \leq Y_{XS}S_{in}$. Consequently, any lumped uncertainty arising from parametric variations in the growth kinetics remains bounded. An explicit invariance-based proof establishing the boundedness of S and X , and consequently the finite bounds on ς_1 is provided in Appendix A.

Now, the following adaptive robust virtual tracking control law is introduced:

$$I_v = f^{-1}(x_1)(-K_1 Z_1 - g_c(\hat{g}(x_d, \dot{x}_d) \text{sgn}(Z_1))) \quad (13)$$

where K_1 is a positive-definite diagonal gain. The control term $g_c \in R$ represents a continuous approximation of the Signum function and satisfies the following conditions [40]:

$$\begin{aligned} Z_1 g_c(\hat{g} \text{sgn}(Z_1)) &\geq 0 \\ \hat{g} \|Z_1\| - \|Z_1\| g_c(\hat{g} \text{sgn}(Z_1)) &\leq \delta_1(t). \end{aligned} \quad (14)$$

In the control law (13), $\hat{g}(x_d, \dot{x}_d) = \hat{\alpha} G(x_d, \dot{x}_d)$ is the estimated upper-bound function for $g(x_d, \dot{x}_d)$, where $\hat{\alpha}$ is the estimate of the unknown positive parameter vector α and $G(x_d, \dot{x}_d) = [\|x_d\| \quad \|\dot{x}_d\|]$ is the regressor vector. Note that α represents the unknown bounds on the uncertainties; its estimate $\hat{\alpha}$ is updated using the following adaptation law:

$$\dot{\hat{\alpha}} = \Lambda_1 G(x_d, \dot{x}_d) \|Z_1\| - \Lambda_1 \sigma_1 (\hat{\alpha} - \alpha_0), \quad (15)$$

where $\Lambda_1 = \lambda_1 I_{2 \times 2}$ represents the adaptation gain where $I_{2 \times 2}$ denotes the 2×2 identity matrix, σ_1 is a small positive number, and $\alpha_0 \in R^2$ denotes the a priori estimate of the parameters. The virtual control signal I_v is then processed through the following first-order filter:

$$\kappa \dot{I}_f + I_f = I_v, \quad I_f(0) = I_v(0), \quad (16)$$

yielding the filtered signal I_f , which serves as the reference for the next subsystem. The introduction of this filter is the key feature of DSC: it eliminates the need for analytical differentiation of the virtual control law, thereby avoiding the explosion of complexity inherent in backstepping.

C. Second Dynamic Surface and Actual Control

The second dynamic surface is then defined as:

$$Z_2 = I_a - I_f. \quad (17)$$

Remark 3. In the conventional DSC framework, the second dynamic surface is typically defined on the system state variable. However, because our system explicitly incorporates actuator dynamics, we define the second surface on the armature current I_a , which serves as the coupling variable between the actuator and the bioreactor dynamics. This modification is not arbitrary but rather a principled extension of DSC to cascade systems where the actuator dynamics are of relative degree one with respect to the control input. This definition enables direct compensation of electrical uncertainties (R_a , L_a , k_a) while maintaining the structural advantages of DSC, including the elimination of analytical differentiation.

By taking the derivative of Z_2 and substituting (8), we can write:

$$L_a \dot{Z}_2 = L_a \dot{I}_a - L_a \dot{I}_f = V_m - (R_a + k_a) Z_2 + \varsigma_2, \quad (18)$$

Here, $\varsigma_2 = -(R_a + k_a) I_f - v_d - L_a \dot{I}_f$ represents the uncertain nonlinearities. This term is assumed to be bounded as $\|\varsigma_2\| \leq q(I_f, \dot{I}_f) = Q(I_f, \dot{I}_f) \beta$, where β is a vector of uncertain parameters and $Q(I_f, \dot{I}_f) = [1 \quad \|I_f\| \quad \|\dot{I}_f\|]$.

Remark 4. For the actuator subsystem, the armature current I_a is limited by the motor's rated capacity and the back-EMF is limited by the maximum achievable speed, ensuring boundedness of ς_2 . For the formal analysis, the applied motor voltage and the lumped actuator uncertainty are bounded. An explicit proof that the electrical dynamics yield bounded I_a , while the stable first-order filter yields bounded I_f , $\dot{I}_f$, and consequently ς_2 , is provided in Appendix B.

Based on this, the actual control input is proposed as follows:

$$V_m = -K_2 Z_2 - q_c(\hat{q} \text{sgn}(Z_2)), \quad (19)$$

where $q_c \in R$ represents the continuous approximation of the Signum function, which also fulfills the following conditions [40]:

$$\begin{aligned} Z_2 q_c(\hat{q} \text{sgn}(Z_2)) &\geq 0 \\ \hat{q} \|Z_2\| - Z_2 q_c(\hat{q} \text{sgn}(Z_2)) &\leq \delta_2(t), \end{aligned} \quad (20)$$

where $\delta_2(t)$ is a bounded time-varying positive scalar, $\hat{q} = Q \hat{\beta}$ represents the estimated upper-bounding function of q , and $\hat{\beta}$ is updated by:

$$\dot{\hat{\beta}} = \Lambda_2 Q^T \|Z_2\| - \Lambda_2 \sigma_2 (\hat{\beta} - \beta_0), \quad (21)$$

where $\Lambda_2 = \lambda I_{3 \times 3}$ with $\lambda > 0$ is the diagonal adaptation gain matrix, σ_2 is a small design parameter introduced to prevent

parameter drift, and $\beta_0 \in R^3$ represents the a priori initial estimate of the parameter vector.

D. Boundary Layer Error Dynamics

Let the boundary layer error be defined as:

$$e = I_f - I_v \quad (22)$$

By incorporating (13), (16), and (22), the time derivative of e is obtained as:

$$\dot{e} = \dot{I}_f - \dot{I}_v = -\frac{e}{\kappa} + (\dot{x}_1 \frac{\partial f(x_1)}{\partial x_1} f(x_1) I_v + f^{-1}(x_1)(\dot{K}_1 Z_1 - \dot{g}_c) \quad (23)$$

For a comprehensive discussion on continuously differentiable robust control g_c , the reader is directed to [41]. Here, $\dot{g}_c$ denotes the total time derivative of g_c . For the specific robust-control realization adopted in (40), its explicit derivation and boundedness analysis are provided in Appendix C. It can be shown that under the assumption that the desired trajectory x_d and its time derivative $\dot{x}_d$, all terms in (23) are dominated by a continuous function of the dynamic surfaces Z_1, Z_2 , the boundary layer error e , and the parameter estimate $\hat{\alpha}$. Consequently, the following bound holds:

$$\|\dot{e} + \frac{e}{\kappa}\| \leq \gamma(Z_1, Z_2, x_d, \dot{x}_d, e, \hat{\alpha}) \quad (24)$$

where γ is a continuous function. Multiplying both sides of (24) by e^T yields:

$$e\dot{e} + \frac{1}{\kappa}e^2 \leq \|e\dot{e} + \frac{1}{\kappa}e^2\| \leq \gamma\|e\| \quad (25)$$

As a result, the following holds:

$$e\dot{e} \leq -\frac{1}{\kappa}e^2 + \gamma\|e\| \leq -\frac{1}{\kappa}e^2 + e^2 + \frac{1}{4}\gamma^2 \quad (26)$$

E. Closed-Loop System Dynamics

Substituting (17) and (22) into (12), the resulting dynamics of the closed-loop system can be derived as:

$$\begin{aligned} \dot{Z}_1 &= -K_1 Z_1 - h(x_1, x_2) Z_1 + \varsigma_1 \\ &- g_c(\hat{g}(x_d, \dot{x}_d) \text{sgn}(Z_1)) + (Z_2 + e)f(x_1) \end{aligned} \quad (27)$$

Substituting (19) into (18) gives:

$$L_a \dot{Z}_2 = -K_2 Z_2 - q_c(\hat{q} \text{sgn}(Z_2)) - (R_a + k_a) Z_2 + \varsigma_2 \quad (28)$$

Figure 2 summarizes the overall closed-loop architecture of the proposed adaptive robust DSC scheme, including the two control layers, the first-order dynamic filter, the adaptive laws, the actuator dynamics, and the nonlinear bioreactor.

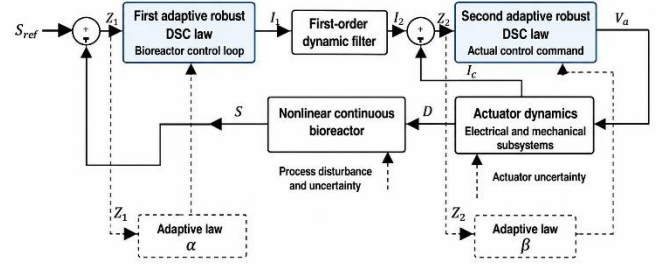


Fig. 2. Block diagram of the proposed adaptive robust DSC architecture for the coupled bioreactor-actuator system.

F. Stability Analysis

The stability of the closed-loop dynamics achieved through the proposed tracking controller is summarized in the following theorem.

Theorem 1. Consider the bioreactor system governed by the dynamics in (8) and (10). For a bounded and continuous desired trajectory, and under Assumptions 1–4, let the σ -modification coefficients σ_1 and σ_2 be selected as strictly positive constants, and let the auxiliary Young-inequality parameter ϖ satisfy $\varpi > \frac{1}{\sqrt{2}}$. Then, the adaptive robust tracking controller defined by (13) and (19), along with the signum approximations satisfying (14) and (20) and the parameter adaptation laws (15) and (21), ensures that all signals in the closed-loop system remain UUB. Moreover, the dynamic surfaces Z_1, Z_2 converge exponentially to an arbitrarily small neighborhood of the origin, with the size of the neighborhood adjustable via the design parameters $K_1, K_2, \kappa, \sigma_1$, and σ_2 .

Proof: Let us consider the following Lyapunov function candidate:

$$V_L = \frac{1}{2} Z_1^2 + \frac{1}{2} L_a Z_2^2 + \frac{1}{2} e^2 + \frac{1}{2} \tilde{\alpha}^T \Lambda_1^{-1} \tilde{\alpha} + \frac{1}{2} \tilde{\beta}^T \Lambda_2^{-1} \tilde{\beta} \quad (29)$$

Taking the derivative of (29) along (11), (17), (27), (28), (15), and (21), and noting that $\dot{\tilde{\alpha}} = -\dot{\hat{\alpha}}$ and $\dot{\tilde{\beta}} = -\dot{\hat{\beta}}$, we obtain:

$$\begin{aligned} \dot{V}_L &= -K_1 Z_1^2 - h(x_1, x_2) Z_1^2 + Z_1 \varsigma_1 - Z_1 g_c + Z_1 (Z_2 + e) f(x_1) - K_2 Z_2^2 - Z_2 q_c - (R_a + k_a) Z_2^2 + Z_2 \varsigma_2 + \\ &e\dot{e} - \tilde{\alpha} G^T \|Z_1\| + \tilde{\alpha} \sigma_1 (\hat{\alpha} - \alpha_0) - \tilde{\beta} Q^T \|Z_2\| \\ &+ \tilde{\beta} \sigma_2 (\hat{\beta} - \beta_0) \leq -K_1 Z_1^2 - \|h\| \|Z_1\| + \|Z_1\| \|\varsigma_1\| \\ &- Z_1 g_c + \|Z_1\| \|Z_2\| \|f\| + \|Z_1\| \|e\| \|f\| - K_2 Z_2^2 - \\ &Z_2 q_c - (R_a + k_a) Z_2^2 \end{aligned}$$

$$+ \|Z_2\| \|\zeta_2\| + e\dot{e} - \tilde{\alpha}G^T \|Z_1\| + \tilde{\alpha}\sigma_1(\hat{\alpha} - \alpha_0) - \tilde{\beta}Q^T \|Z_2\| + \tilde{\beta}\sigma_2(\hat{\beta} - \beta_0) \quad (30)$$

Considering $\|\zeta_1\| \leq \alpha G$, $\|\zeta_2\| \leq \beta Q$, assumption 3 and 4, the inequality (30) can be rewritten as:

$$\begin{aligned} \dot{V}_L \leq & -K_1 Z_1^2 - \Gamma Z_1^2 + \|Z_1\| \alpha G - Z_1 g_c + M \|Z_1\| \|Z_2\| \\ & + M \|Z_1\| \|e\| - K_2 Z_2^2 - Z_2 q_c - (R_a + k_a) Z_2^2 + \|Z_2\| Q \\ & + e\dot{e} - \tilde{\alpha}G^T \|Z_1\| + \tilde{\alpha}\sigma_1(\hat{\alpha} - \alpha_0) - \tilde{\beta}Q^T \|Z_2\| \\ & + \tilde{\beta}\sigma_2(\hat{\beta} - \beta_0) \end{aligned} \quad (31)$$

Since the sets $\mathcal{E}_1 = \{(x_d, \dot{x}_d): x_d^2 + \dot{x}_d^2 \leq l_1\}$, $\mathcal{E}_2 = \{(Z_1, Z_2, e, \tilde{\alpha}, \tilde{\beta}): V_L(t) \leq l_2\}$ are compact in $\mathbb{R}^2$ and $\mathbb{R}^8$, respectively, for any $l_1 > 0$, $l_2 > 0$. The set $\mathcal{E}_1$ is closed and bounded and therefore compact. Moreover, the Lyapunov function in (29) is continuous, positive definite, and radially unbounded with respect to $(Z_1, Z_2, e, \tilde{\alpha}, \tilde{\beta})$. Consequently, its sublevel set $\mathcal{E}_2$ is also closed and bounded and hence compact. The symbol $\times$ denotes the Cartesian product; therefore, $\mathcal{E}_1 \times \mathcal{E}_2$ is compact. Since γ in (24) is continuous, the extreme-value theorem guarantees that it attains a finite maximum γ_m on $\mathcal{E}_1 \times \mathcal{E}_2$, i.e., $\gamma_m = \max_{\mathcal{E}_1 \times \mathcal{E}_2} \gamma < \infty$. Therefore, $\gamma \leq \gamma_m$ for all $(x_d, \dot{x}_d, Z_1, Z_2, e, \tilde{\alpha}, \tilde{\beta}) \in \mathcal{E}_1 \times \mathcal{E}_2$. Taking this bound into account, revisiting inequality (26), and using the following inequalities:

$$K \|x_1\| \|x_2\| \leq \frac{K}{2} x_1^T x_1 + \frac{K}{2} x_2^T x_2, \quad \forall x_1, x_2 \in \mathbb{R}^n, \forall K > 0 \quad (32)$$

One can express (31) in the following form:

$$\begin{aligned} \dot{V}_L \leq & -K_1 Z_1^2 - \Gamma Z_1^2 + |Z_1| \alpha G - Z_1 g_c + \frac{M}{2} \|Z_1\|^2 \\ & + \frac{M}{2} \|Z_2\|^2 + \frac{M}{2} \|Z_2\|^2 + \frac{M}{2} \|e\|^2 - K_2 Z_2^2 \\ & - Z_2 q_c - (R_a + k_a) Z_2^2 + \|Z_2\| \beta Q - \frac{1}{\kappa} e^2 \\ & + e^2 + \frac{1}{4} \gamma_m^2 - \tilde{\alpha}G^T \|Z_1\| + \tilde{\alpha}\sigma_1(\hat{\alpha} - \alpha_0) \\ & - \tilde{\beta}Q^T \|Z_2\| + \tilde{\beta}\sigma_2(\hat{\beta} - \beta_0) \end{aligned} \quad (33)$$

Recalling that $\hat{\alpha} = \alpha - \tilde{\alpha}$, $\hat{\beta} = \beta - \tilde{\beta}$, the uncertainty-bound terms are combined as $\|Z_1\| \alpha G - \tilde{\alpha}G^T \|Z_1\| = \|Z_1\| \hat{\alpha} G$ and $\|Z_2\| \beta Q - \tilde{\beta}Q^T \|Z_2\| = \|Z_2\| \hat{\beta} Q$. For the leakage terms, completing the square gives $-\mu_{\sigma_1} \left(1 - \frac{1}{2\bar{\omega}^2}\right) \|\tilde{\alpha}\|^2 - \frac{\mu_{\sigma_1}}{2\bar{\omega}^2} \|\tilde{\alpha} - \bar{\omega}^2(\alpha - \alpha_0)\|^2 + \frac{\bar{\omega}^2 \mu_{\sigma_1}}{2} \|\alpha - \alpha_0\|^2 \leq -\mu_{\sigma_1} \left(1 - \frac{1}{2\bar{\omega}^2}\right) \|\tilde{\alpha}\|^2 + \frac{\bar{\omega}^2 \mu_{\sigma_1}}{2} \|\alpha - \alpha_0\|^2$, with an analogous expression for $\tilde{\beta}$. Therefore, (33) can be rewritten as:

$$\begin{aligned} \dot{V}_L \leq & -K_1 Z_1^2 - \Gamma Z_1^2 + |Z_1| \hat{\alpha} G - Z_1 g_c + M \|Z_1\|^2 \\ & + \frac{M}{2} \|Z_2\|^2 + \frac{M}{2} \|e\|^2 - K_2 Z_2^2 - Z_2 q_c \\ & - (R_a + k_a) Z_2^2 + \|Z_2\| \hat{\beta} Q - \frac{1}{\kappa} e^2 \\ & + e^2 + \frac{1}{4} \gamma_m^2 - \mu_{\sigma_1} \left(1 - \frac{1}{2\bar{\omega}^2}\right) \|\tilde{\alpha}\|^2 \\ & + \frac{1}{2} \bar{\omega}^2 \mu_{\sigma_1} \|\alpha - \alpha_0\|^2 - \mu_{\sigma_2} \left(1 - \frac{1}{2\bar{\omega}^2}\right) \|\tilde{\beta}\|^2 + \\ & \frac{1}{2} \bar{\omega}^2 \mu_{\sigma_2} \|\beta - \beta_0\|^2, \end{aligned} \quad (34)$$

where $\mu_{\sigma i} = \sigma_i > 0$, $i = 1, 2$. By imposing the conditions (14) and (20) on (34), we obtain:

$$\begin{aligned} \dot{V}_L \leq & -A \|Z_1\|^2 - B \|Z_2\|^2 - C \|e\|^2 \\ & - \varpi_1 \|\tilde{\alpha}\|^2 - \varpi_2 \|\tilde{\beta}\|^2 + \iota(t) \end{aligned} \quad (35)$$

where

$$A = K_1 + \Gamma - M$$

$$B = K_2 + R_a + k_a - \frac{M}{2}$$

$$C = \frac{1}{\kappa} - \frac{M}{2} - 1$$

$$\bar{\omega}_1 = \mu_{\sigma_1} \left(1 - \frac{1}{2\bar{\omega}^2}\right)$$

$$\bar{\omega}_2 = \mu_{\sigma_2} \left(1 - \frac{1}{2\bar{\omega}^2}\right)$$

$$\begin{aligned} \iota(t) = & \frac{1}{2} \bar{\omega}^2 \mu_{\sigma_1} \|\alpha - \alpha_0\|^2 + \frac{1}{2} \bar{\omega}^2 \mu_{\sigma_2} \|\beta - \beta_0\|^2 + \\ & \delta_1(t) + \delta_2(t) + \frac{1}{4} \gamma_m^2, \end{aligned} \quad (36)$$

The controller parameters K_1 , K_2 , and κ can be selected such that the coefficients A , B , and C are all strictly positive, i.e., $K_1 + \Gamma > M$, $K_2 + R_a + k_a > \frac{M}{2}$, and $\frac{1}{\kappa} > \frac{M}{2} + 1$. The σ -modification coefficients σ_1 and σ_2 are selected as strictly positive constants. Furthermore, the condition $\bar{\omega} > \frac{1}{\sqrt{2}}$ specified in Theorem 1, together with $\mu_{\sigma_1} > 0$ and $\mu_{\sigma_2} > 0$, the coefficients ϖ_1 and ϖ_2 defined in (36) are strictly positive. Consequently, inequality (35) can be rewritten as:

$$\dot{V}_L \leq -\phi V_L(t) + \iota(t). \quad (37)$$

where $\phi = \min\{A, B, C, \varpi_1, \varpi_2\}$.

By solving the differential inequality (37), we obtain:

$$\dot{V}_L \leq V_L(0) e^{-\phi t} + \frac{\iota}{\phi} (1 - e^{-\phi t}), \quad \forall t \in [0, \infty]. \quad (38)$$

Consequently, provided that the coefficients A, B, and C are strictly positive, the Lyapunov function V_L decreases monotonically and ultimately enters the compact set

$$\psi = \left\{ (Z_1, Z_2, e, \tilde{\alpha}, \tilde{\beta}) \in R^7 \times R: \right. \\ \left. V_L(Z_1, Z_2, e, \tilde{\alpha}, \tilde{\beta}) \leq \max(V(t_0), \frac{\iota}{\phi}) \right\}. \quad (39)$$

This implies that $(Z_1, Z_2, e, \tilde{\alpha}, \tilde{\beta})$ are UUB and converge exponentially to a neighborhood of the origin. The ratio $\frac{\iota}{\phi}$ directly governs the size of this neighborhood, which can be made arbitrarily small by appropriately tuning the controller parameters.

The following remarks provide practical guidelines for tuning the controller parameters to achieve desired performance.

Remark 5. From equations (36), increasing the values of K_1, K_2 , while decreasing κ , enhances the convergence rate ϕ and reduces the ultimate bound $\frac{\iota}{\phi}$.

Remark 6. Smaller values of σ_1 and σ_2 reduce the perturbation term $\iota(t)$, which in turn decreases the ultimate bound $\frac{\iota}{\phi}$.

Remark 7. The time-dependent functions $\delta_1(t), \delta_2(t)$ in the saturation-type controllers g_c and q_c (as defined in (13) and (19)) can be adjusted to balance tracking accuracy and control signal smoothness.

Remark 8. Larger values of $\delta_1(t), \delta_2(t)$ produce smoother control signals but may increase the perturbation term $\iota(t)$ in (36), potentially enlarging the ultimate bound $\frac{\iota}{\phi}$.

IV. Simulation outcomes

In this section, two numerical simulations were carried out using MATLAB R2023a on an ASUS Vivobook X1504VA equipped with an Intel Core i7-1355U processor and 40 GB of RAM under 64-bit Windows 10 Enterprise to assess the tracking performance and robustness of the proposed controller for a continuous bioreactor system. The system is exposed to both parametric and non-parametric uncertainties. For Scenario 1, a set-point regulation problem is considered where the desired trajectory is set to a constant value. For Scenario 2, a sinusoidal reference tracking problem is considered to evaluate the controller's performance under time-varying conditions. The proposed closed-loop system was numerically propagated using the

explicit Euler method with a fixed sampling interval of 0.01h (36 s).

A. Simulation Setup

The values of the model parameters are $x_{1in} = 10gL^{-1}$, $Y_{XS} = 0.7gg^{-1}$, $Y_{XP} = 0.2$, $\delta = 16h^{-1}$, $\mu_{max} = 0.4h^{-1}$, $K_S = 10gL^{-1}$, $K_v = 0.02kg^{\frac{-3}{2}}m^{-1}s$. The initial condition for bioreactor model is $x = [x_1, x_2, x_3] = [0, 0.25, 0.05]^T$. For simulation purposes, the actuator parameters are selected as follows [42]: $B_v = 0.1$, $L_a = 0.1\Omega s$, $R_a = 5\Omega$, $n = 48$, $k_b = 0.02 \frac{V}{rad}$ and $k_t = 0.2 \frac{Nm}{A}$. However, these parameters are assumed to be unknown to the controller, reflecting a realistic model-uncertainty scenario. For the actuator states, the initial conditions are $I_a(0) = 0A$. The non-parametric uncertainties in the actuator dynamics are modeled as: $v_d = \sin(\frac{t}{20})$.

B. Controller Tuning

A systematic tuning procedure was adopted based on the stability conditions derived in Section III and the performance guidelines summarized in Remarks 5–8. First, K_1 , K_2 , and κ are selected such that the scalar coefficients (A), (B), and (C) defined in Eq. (36) are strictly positive, thereby satisfying the sufficient conditions used in the Lyapunov stability analysis. Starting from values that satisfy these inequalities with a positive stability margin, K_1 and K_2 are increased if faster convergence is required, whereas κ is decreased to reduce the ultimate bound, as indicated by Remark 5. Second, the adaptation gains Λ_1 and Λ_2 are gradually increased until sufficiently fast uncertainty estimation is obtained without introducing excessive transient variation in the parameter estimates. Third, the small positive parameters σ_1 and σ_2 are selected to reduce the perturbation term in the Lyapunov bound while maintaining numerically well-behaved adaptation. Finally, the boundary-layer functions $\delta_1(t)$ and $\delta_2(t)$ are adjusted to achieve a compromise between tracking accuracy and smooth control action. The tuning process is terminated when further gain modification produces no meaningful improvement in tracking performance while all stability conditions remain satisfied. The final selected values are: $K_1 = 10.5$, $K_2 = 9$, $\kappa = 0.025$. The adaptation gains are set to: $\Lambda_1 = 0.005$, $\Lambda_2 = 10^{-3}diag[1, 1, 1]$. The other control parameters are configured as $\sigma_1 = 0.009$, $\sigma_2 = 0.00005$, $\alpha_0 = 0$, and $\beta_0 = 0$. Since $\hat{\alpha}(0) = \alpha_0$ and $\hat{\beta}(0) = \beta_0$, these choices determine the initial parameter-estimation errors and can therefore influence the adaptation transient. As shown by the residual terms involving and in the stability analysis, any finite selection preserves the UUB conclusion,

whereas a smaller prior mismatch reduces the theoretical estimate of the ultimate bound. Finally, the robust control laws in Equations (13) and (19) are implemented as follows:

$$\begin{aligned} g_c(\hat{g} \operatorname{sgn}(Z_1)) &= \frac{Z_1 \hat{g}^2}{\hat{g} \|Z_1\| + \delta_1(t)} \\ q_c(\hat{q} \operatorname{sgn}(Z_2)) &= \frac{Z_2 \hat{q}^2}{\hat{q} \|Z_2\| + \delta_2(t)} \end{aligned} \quad (40)$$

These control laws satisfy the conditions outlined in (14) and (20). The continuous differentiability of (g_c) and (q_c) with respect to (Z_1) and (Z_2) , including at the origin, is verified in Appendix C, where the explicit total derivative and boundedness of (g_c) required in (24) are also established. Additionally, the boundary layer thickness in the control laws (40) is selected as:

$$\delta_1(t) = \begin{cases} 5000 & \text{if } 0 \leq t \leq 5 \\ 1 + 4999e^{-0.2(t-5)} & \text{if } t > 5 \end{cases}, \quad (41)$$

and $\delta_2(t) = 100$.

C. Scenario 1: Set-Point Regulation

In the first simulation, the desired trajectory is set to $x_d = 10 \frac{g}{L}$. This scenario corresponds to maintaining the substrate concentration at a constant desired value despite uncertainties and disturbances.

Figure 3 illustrates the closed-loop response of the substrate concentration. The system exhibits rapid convergence to the desired set-point with a short settling time, even under significant parametric and nonparametric uncertainties, demonstrating the controller's high tracking fidelity and robust performance. Figure 4 shows the control input applied to the bioreactor actuator. The control signal remains smooth throughout operation, indicating the absence of undesirable chattering or abrupt changes—a crucial advantage in practical applications. Meanwhile, Figures 5 and 6 display the estimated values of the unknown parameters associated with the upper-bounding functions for the bioreactor model and actuator dynamics, respectively. As observed, these estimates remain bounded and stable during operation, confirming the effectiveness of the adaptive mechanisms. Furthermore, the user can influence key performance metrics—such as the size of the ultimate bound, the rate of convergence, and the smoothness of the control effort—by tuning the design parameter $K_1, K_2, \kappa, \sigma_1, \sigma_2$.

The influence of the controller parameters on closed-loop performance was quantitatively examined through a simulation-based one-at-a-time sensitivity analysis. The simulations were repeated by independently varying K_1, K_2

and κ by $\pm 20\%$ around their nominal values, while all other controller parameters and simulation conditions were kept unchanged. Stable closed-loop responses were obtained for all investigated variations, although the degree of sensitivity differed among the three parameters. The transient performance was more sensitive to K_1 and κ . A 20% decrease in K_1 increased the IAE from 0.2698 to 0.4888 and the 2% settling time from 0.11 to 0.17 h, whereas a 20% increase reduced them to 0.1548 and 0.08 h, respectively. Similarly, decreasing κ by 20% reduced the IAE to 0.1531 and the settling time to 0.08 h, whereas increasing it by 20% increased these values to 0.3758 and 0.13 h, respectively. In contrast, the response was considerably less sensitive to K_2 ; its $\pm 20\%$ variation resulted in IAE values between 0.2649 and 0.2750, while the 2% settling time remained unchanged at 0.11 h. These results indicate that K_1 and κ have a more pronounced influence on the transient response, whereas moderate variations in K_2 have a comparatively limited effect within the investigated range.

To further evaluate the robustness of the proposed controller against stochastic measurement noise, an additional simulation was performed by adding zero-mean Gaussian noise with a standard deviation of 0.15 g/L to the measured substrate concentration while retaining the original deterministic disturbance. The controller operates using the noise-corrupted measurement, whereas the tracking performance is evaluated based on the actual substrate concentration. As shown in Figure 7(a), the proposed DSC maintains stable closed-loop operation and accurately converges to the desired set-point despite the presence of stochastic measurement noise. Figure 7(b) illustrates the Gaussian measurement noise applied to the substrate concentration measurement.

To evaluate the capability of the proposed controller to attenuate an external process disturbance, an additional simulation was performed under the constant reference. The inlet substrate concentration was temporarily increased from $10 \frac{g}{L}$ to $11 \frac{g}{L}$ at $t = 10$ h and was returned to its nominal value at $t = 15$. Figure 8 presents the applied disturbance profile. As shown in Figure 8, during both the application and removal of the disturbance, the substrate concentration remained very close to the reference value of $10 \frac{g}{L}$, with no visually appreciable deviation at the scale shown. These results demonstrate effective attenuation of the tested inlet-feed disturbance without controller retuning, while preserving closed-loop stability and reference tracking.

D. Scenario 2: Sinusoidal Reference Tracking

To further evaluate the controller under dynamic operating conditions, a composite reference trajectory is considered in

the second simulation scenario, as defined in (42). During ($t < 25$ h), the controller tracks a sinusoidally varying reference. At ($t = 25$ h), the reference is abruptly changed to the constant set point of (10) g/L, thereby also evaluating the closed-loop response to a step-like set-point transition. The external disturbance ($v_d = \sin(\frac{t}{20})$) remains active throughout both phases of this scenario.

$$x_d = \begin{cases} 1 + \sin(2t) & t \leq 25 \\ 10 & t > 25 \end{cases} \quad (42)$$

The controller parameters $K_1 = 10$, $K_2 = 40$ and adaptive gains $\Lambda_1 = 0.5 \text{diag}[1,1]$, $\Lambda_2 = 10^{-5} \text{diag}[1,1,1]$ yield effective trajectory tracking performance. The remaining parameters are kept consistent with the initial simulation. For this scenario, the initial condition for the bioreactor model is defined as in the previous scenario.

Figure 9 presents the tracking performance of the substrate concentration for the sinusoidal reference. The controller successfully tracks the varying set-point with minimal error, validating its robustness and adaptability under dynamic conditions. Figure 10 shows the corresponding computed control efforts. Figures 11 and 12 illustrate the estimated upper-bounding parameters for the bioreactor and actuator uncertainties during the sinusoidal tracking task. The estimates remain well within acceptable limits, further confirming the stability and reliability of the adaptive robust control strategy.

Remark 9. The negative values observed in Figures 4 and 10 for the control signal $V_m(t)$ require practical clarification: Negative motor voltages are physically realizable in industrial DC motor drives equipped with four-quadrant operation capability (e.g., H-bridge configurations). These drives allow bidirectional current flow, enabling both motoring and regenerative braking modes. The negative voltage transients correspond to temporary reverse torque application, which may occur during rapid set-point changes or disturbance rejection. In practice, commercial drives support voltage ranges of ± 10 V or ± 24 V, well within our simulation ranges.

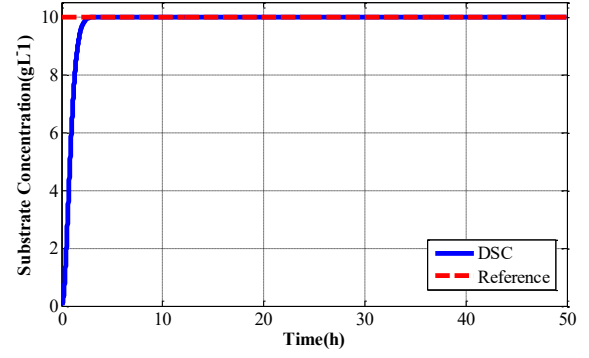


Fig. 3. Closed-loop response of substrate concentration $x_1 \frac{g}{L}$ (solid blue) tracking constant reference $x_d = 10 \frac{g}{L}$ (dashed red) under dynamic surface control.

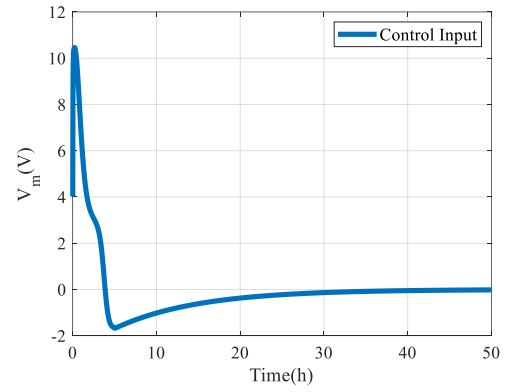


Fig. 4. Control input voltage V_m (in Volts) applied to the DC motor actuator during constant reference tracking.

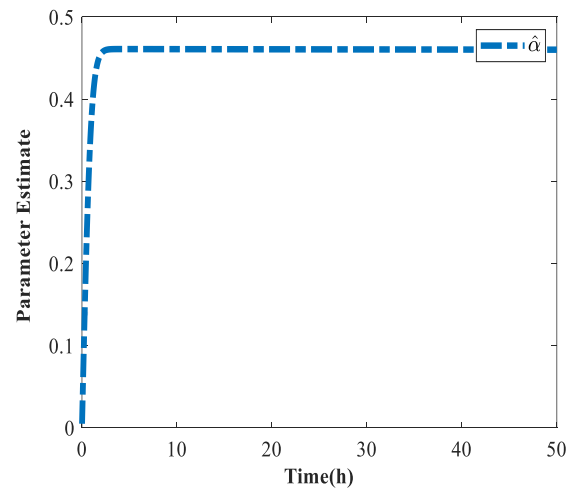


Fig. 5. Estimated parameter $\hat{\alpha}$ defining the upper bounds of bioreactor uncertainties.

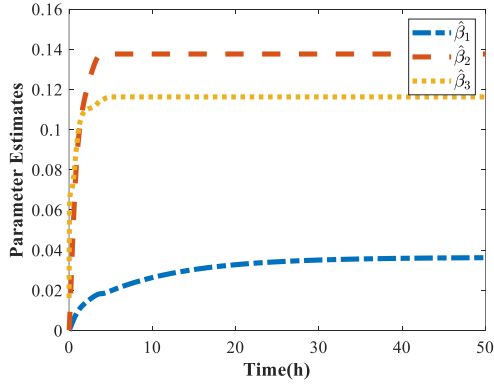


Fig. 6. Estimated parameters $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ for actuator uncertainty bounds.

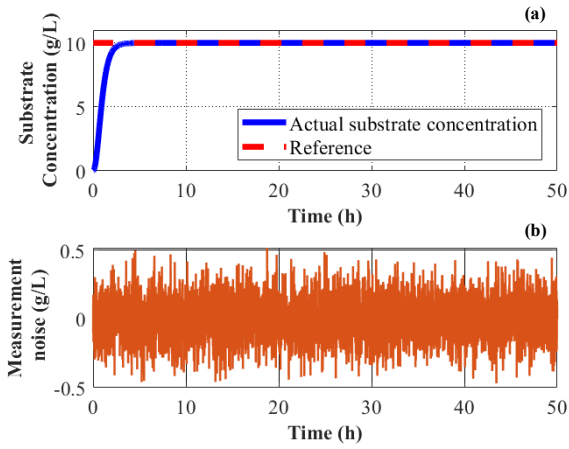


Fig. 7. Robustness of the proposed DSC against measurement noise. (a) Closed-loop response under zero-mean Gaussian measurement noise (g/L). (b) Gaussian measurement noise added to the substrate concentration measurement.

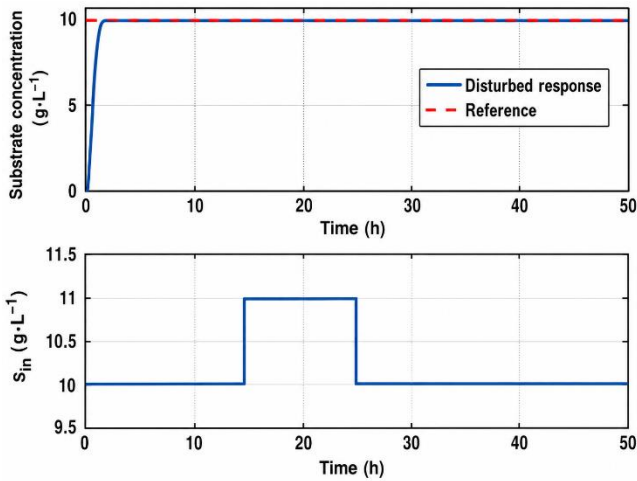


Fig. 8. Response of the proposed DSC to a temporary 10% step disturbance in the inlet substrate concentration:

substrate-concentration tracking response and applied disturbance profile.

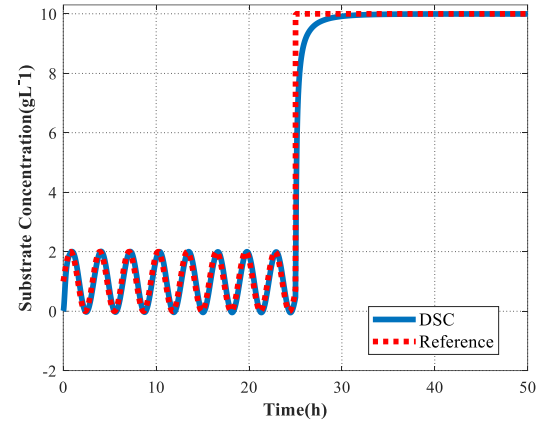


Fig. 9. Tracking performance under a time-varying reference: $x_d = 1 + \sin(2t)$ for $t \leq 25h$, then $x_d = 10(\frac{g}{L})$. Actual x_1 (solid blue) closely follows reference (solid red) under dynamic surface control.

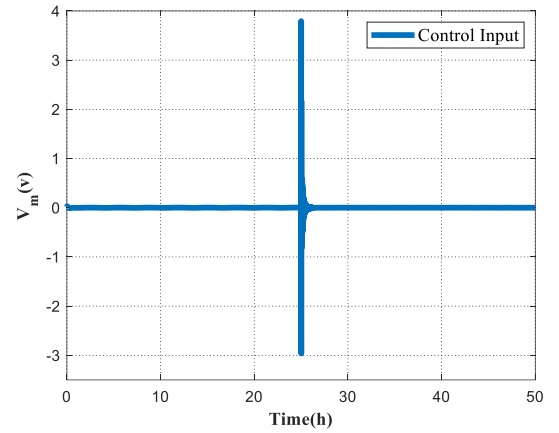


Fig. 10. Control input voltage V_m (in Volts) applied to the DC motor actuator during sinusoidal tracking.

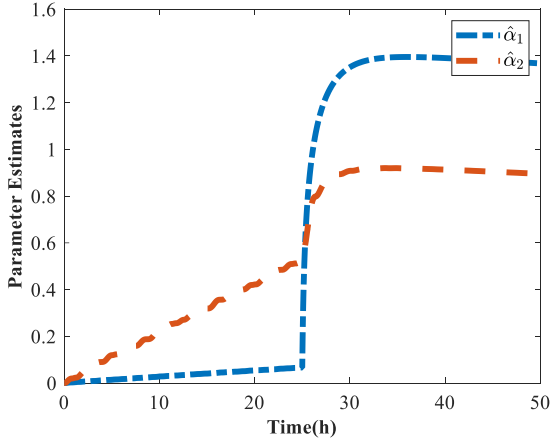


Fig. 11. Estimated parameters $\hat{\alpha}_1, \hat{\alpha}_2$ defining the upper bounds of bioreactor uncertainties.

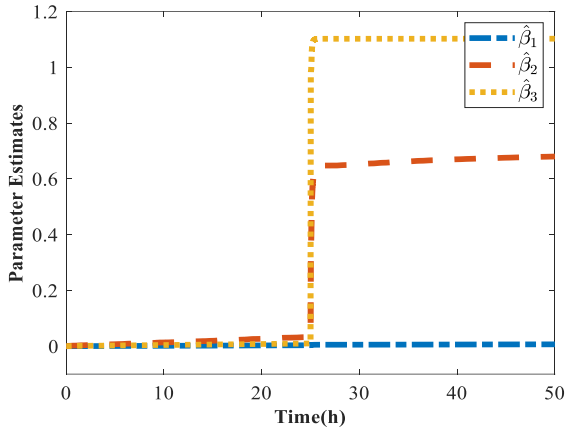


Fig. 12. Estimated actuator uncertainty bounds $\hat{\beta}_1, \hat{\beta}_2$, and $\hat{\beta}_3$ during sinusoidal tracking.

It should be emphasized that the sinusoidal reference adopted in this scenario is not intended to represent a realistic operating objective in continuous bioreactors, where the primary control task is to maintain the substrate concentration at a desired constant set-point. Instead, this time-varying reference is intentionally introduced as a challenging test signal to demonstrate the trajectory-tracking capability and robustness of the proposed DSC under dynamic reference conditions. Consequently, this scenario is presented solely to verify the tracking capability of the proposed controller under a time-varying reference, rather than to provide an additional quantitative benchmark against the SMC and MPC controllers.

E. Comparative simulation framework

Along with evaluating the performance of the proposed DSC controller, the study also conducted comparative simulations against two well-established control strategies from the literature: (i) the SMC controller [43], and (ii) the MPC controller [44]. These benchmarks were chosen as representative examples of robust control (SMC) and optimization-based predictive control (MPC), respectively. All simulations were carried out under the same conditions—including the same bioreactor model (Eq. 1), initial states, and reference trajectory—to ensure a fair and meaningful comparison.

Comparative study with SMC: The sliding mode controller proposed by Tham et al. [43] is implemented as a benchmark for robust control of continuous bioreactors. As explicitly stated in [43], the control objective is the regulation of substrate concentration (S) by manipulating the dilution rate D , which aligns precisely with the control goal of the proposed DSC. The SMC law is given by:

$$\frac{du}{dt} = -\eta_1 \operatorname{sgn}(v) - \eta_2 e_1 - \eta_3 e_2 - \eta_4 S_{ref} \quad (43)$$

where $e_1 = S - S_{ref}$, $e_2 = \dot{e}_1$, and $v = e_2 + ce_1$ is the sliding surface. The controller parameters $c = 3$, $\eta_1 = 50$, $\eta_2 = 8$, $\eta_3 = 5$, and $\eta_4 = 48$ are selected. These numerical gains are specific to the present implementation and were not directly adopted from [43] or obtained through global performance-based optimization.

Figures 13 and 14 depict the substrate concentration tracking performance and the corresponding control input of the fixed-gain sliding mode controller under a constant set-point. Although the system reaches the desired steady state, the control input exhibits persistent high-frequency switching (chattering) even after convergence.

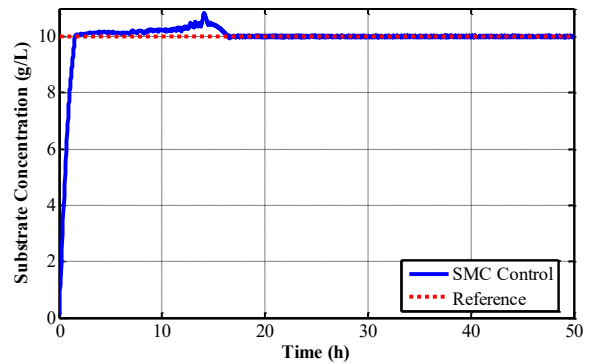


Fig. 13. Closed-loop response of substrate concentration (solid blue) tracking a constant reference (dashed red) under sliding mode control.

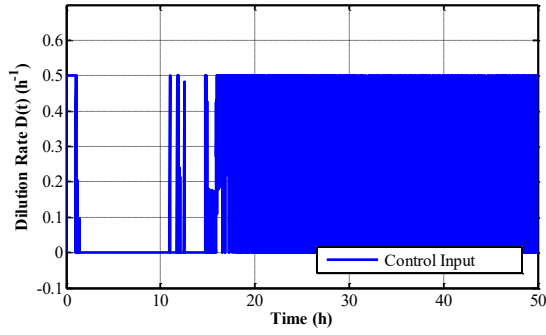


Fig. 14. Control input signal under sliding mode control.

Comparative study with MPC: The model predictive controller developed [44] is selected as the second benchmark to represent an optimization-based predictive control strategy. The MPC formulation follows [44] and solves a finite-horizon optimization problem at each sampling instant, where the objective function minimizes the weighted sum of tracking error and control-effort variation:

$$J = \sum_{j=0}^{P-1} (\|y(k+j|k) - y_{ref}\|^2 Q + \|\Delta u(k+j|k)\|^2 S) + \|y(k+P|k) - y_{ref}\|^2 Q \quad (44)$$

with $Q = 10, S = 0.001$ and prediction horizon $P = 8$. Here, $y(k+j|k)$ denotes the predicted controlled output and the term $\Delta u(k+j|k) = u(k+j|k) - u(k+j-1|k)$ penalizes abrupt changes in the control input to ensure smooth actuator operation. At each sampling instant, the nonlinear finite-horizon optimization problem was solved using MATLAB's `fmincon` solver with a maximum of 50 iterations. The MPC was updated using the same sampling interval of (0.01 h). The prediction and control horizons were set to ($P=8$) and ($M=1$), respectively. No state or upper-input constraints were imposed. Only the lower input constraint ($D \geq -1$) was enforced. As indicated by the control trajectory, this constraint became active after the initial transient.

Note: A sensitivity analysis of the MPC prediction horizon over $P = 4 - 20$ while keeping all other MPC parameters fixed, showed diminishing performance improvements beyond $P = 8$. For example, increasing the horizon from $P = 8$ to $P = 15$ reduced the IAE by only approximately 0.73%, while the 2% settling time remained unchanged at 0.10 h. Accordingly, $P = 8$ was retained as a reasonable compromise between prediction depth and closed-loop performance rather than as a globally optimal prediction horizon.

Figure 15 shows substrate concentration regulation performance under a constant set-point condition. The controller successfully drives the system to the desired steady-state value, demonstrating its ability to achieve stable regulation in nonlinear bioreactor systems. However, the response exhibits a noticeable overshoot and a relatively slow settling time compared to the proposed DSC. This behavior is typical of MPC implementations that prioritize constraint handling and smooth control effort over aggressive transient performance.

Figure 16 shows the control input under a constant set-point condition. The controller produces a smooth, chatter-free response with no high-frequency switching—a key benefit of MPC over discontinuous controllers like SMC. However, the control input shows an initial significant spike, followed by a rapid decay to a steady value, indicating that the MPC uses aggressive control actions during the transient phase to reduce tracking error. This behavior aligns with typical MPC designs, which prioritize rapid convergence at the expense of higher initial control effort.

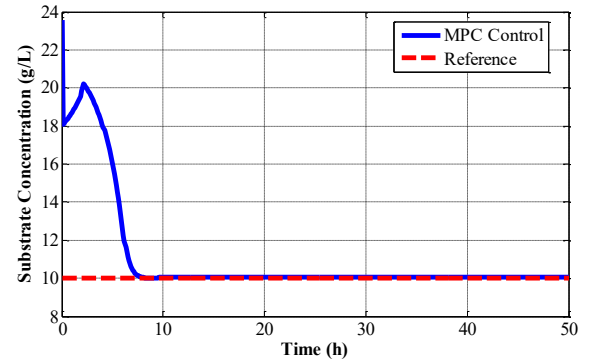


Fig. 15. Closed-loop response of substrate concentration (solid blue) tracking a constant reference (dashed red) under model predictive control.

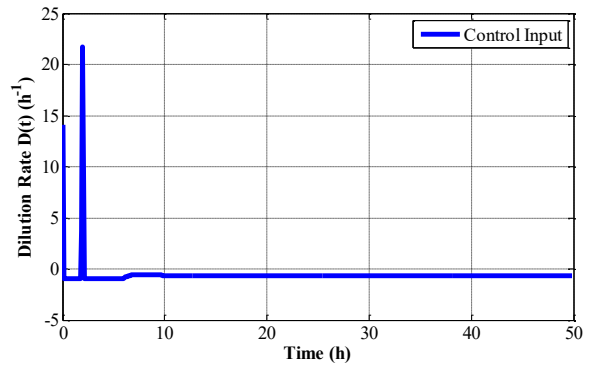


Fig. 16. Control input signal under model predictive control.

While the nonlinear MPC benchmark adopted from [44] requires solving a nonlinear optimization problem at each sampling instant, recent advances in optimization algorithms, including warm-start strategies and efficient active-set methods, have significantly reduced its computational burden, particularly for systems with relatively slow dynamics and moderate dimensions. Nevertheless, MPC still fundamentally relies on repeated online optimization, which increases its implementation complexity compared with analytical nonlinear control laws. In addition, as reported in [13] and also observed by [44], MPC performance remains sensitive to tuning parameters such as the prediction horizon. Inappropriate parameter selection may lead to degraded transient performance, oscillatory responses, or convergence to undesired periodic attractors rather than the desired steady-state operating point. In contrast, the proposed adaptive dynamic surface controller is derived in closed form and therefore completely avoids online optimization while achieving excellent tracking accuracy, smooth control action, and strong robustness against model uncertainties and actuator dynamics. These characteristics make the proposed controller an attractive alternative for practical industrial bioreactor applications where implementation simplicity, computational efficiency, and reliability are important considerations.

Performance evaluation and comparative metrics: To provide a rigorous, objective comparison of the proposed DSC with benchmark controllers, a set of standard performance metrics was evaluated under identical simulation conditions, including the same bioreactor model, initial states, and reference trajectory. The selected metrics include the Integral of Absolute Error (IAE), settling time (measured using a 2% tolerance band around the reference value), maximum control effort, computational complexity, and the Integral of Time-weighted Absolute Error (ITAE). While IAE measures the overall accumulated tracking error, ITAE assigns progressively greater weight to errors occurring at later times and therefore provides complementary information regarding the persistence of tracking errors and long-term convergence behavior. The results for constant set-point regulation are summarized in Table II. The controller computation time was measured using MATLAB's tic–toc commands around the controller update and was averaged over all 5000 iterations without excluding warm-up samples. The timed section included the control law, dynamic filter, adaptation laws, and associated in-memory updates, but excluded plant-model propagation and physical data acquisition, communication, and actuator-interface operations. Therefore, the reported value indicates low algorithmic computational demand rather than an end-to-end experimental real-time benchmark.

TABLE II. Performance comparison of the proposed DSC with SMC and MPC in terms of quantitative and qualitative results under the same simulation conditions.

Performance metric	Proposed DSC	SMC	MPC
IAE	9.0694	3.1689	38.7951
Settling time (h)	2.200	12.730	5.800
Max. control effort	10.4597	0.5	21.5128
Chattering	No	Yes	No
Computational time	0.0089 ms	5.9036 ms	81.9073 ms
ITAE ($\frac{g}{L} h^2$)	5.5078	41.7217	164.8625

As shown in Table II, the proposed DSC achieves the fastest settling time (2.2 h) while eliminating chattering, unlike SMC. Although the IAE of DSC (9.0694) is higher than that of SMC (3.1689), these two performance indices characterize different aspects of the transient response and therefore do not necessarily correlate. The IAE accumulates the absolute tracking error over the entire simulation horizon, whereas the settling time depends only on the time required for the response to enter and remain within the $\pm 2\%$ tolerance band. To provide further insight into the temporal distribution of the tracking error, the ITAE was also evaluated. Unlike IAE, ITAE assigns progressively greater weight to errors occurring at later times. The proposed DSC achieves an ITAE of 5.5078, which is substantially lower than those of SMC (41.7217) and MPC (164.8625). This result indicates that, although SMC accumulates less absolute error overall, its tracking error persists for a longer period, whereas the error under DSC is concentrated primarily in the early transient and decays rapidly. This observation is consistent with the corresponding settling times of 2.2 h for DSC and 12.73 h for SMC.

As observed in Figure 1, the DSC response exhibits a smooth convergence toward the reference without noticeable overshoot or sustained oscillation. Therefore, its higher IAE relative to SMC should not be interpreted as evidence of oscillatory behavior. Instead, it mainly reflects the accumulated tracking error during the initial transient resulting from the large initial condition mismatch ($x_1(0) = 0 \frac{g}{L}$, $x_d = 10 \frac{g}{L}$). In contrast, the longer settling time and higher ITAE of SMC are associated with the transient deviation around the set-point visible in Figure 10 before the response permanently remains inside the settling band. MPC exhibits both a substantially higher IAE (38.7951) and the highest ITAE (164.8625), together with a longer settling time (5.8 h) than DSC. Taken together, the IAE, ITAE, and settling-time results show that the proposed DSC provides rapid suppression of persistent tracking errors and faster convergence, despite the larger error accumulated during its initial transient compared with SMC. Moreover, DSC

requires substantially less computational time (0.0089 ms/step) than SMC (5.9036 ms/step) and MPC (81.9073 ms/step), supporting its suitability for real-time implementation.

The robustness of the proposed DSC against stochastic measurement noise was further evaluated using the simulation shown in Figure 7. Under noisy measurements, the proposed controller achieved an IAE of 9.0791 (g/L)h, an ITAE of 5.5334 (g/L)h², and a 2% settling time of 2.21 h. Compared with the nominal case (IAE = 9.0813 (g/L)h, ITAE = 5.5285 (g/L)h², and settling time = 2.21 h), the tracking performance remains essentially unchanged, indicating that the proposed DSC maintains robust closed-loop tracking under the considered level of stochastic measurement noise. Regarding the adaptation mechanism, sensor noise enters the first adaptation law through the measured substrate concentration and the corresponding dynamic-surface error, while its effect propagates indirectly to the second adaptation law through the virtual control and dynamic-surface filter. Consequently, measurement noise may induce fluctuations in the adaptive estimates. The leakage terms associated with σ_1 and σ_2 help suppress noise-induced parameter drift and provide a practical trade-off between adaptation capability and noise robustness. Increasing these leakage coefficients can improve the robustness of the adaptation mechanism to persistent measurement disturbances; however, excessively large values may increase estimation bias and weaken adaptive compensation.

To evaluate the effect of actuator dynamics on closed-loop performance, three cases were compared, as shown in Figure 17. The ideal-actuator case assumes instantaneous realization of the desired actuator command and is included as a theoretical reference case. In the second case, the full DC motor dynamics are retained in the plant but neglected in the controller design, whereas the third case corresponds to the proposed DSC, in which the actuator dynamics are explicitly incorporated through the second dynamic surface. The ideal-actuator reference case provides an IAE of 2.3646 (g/L)·h, an ITAE of 38.1983 (g/L)·h², and a 2% settling time of 1.08 h. When the actuator dynamics are present but neglected in the controller design, the IAE increases to 63.5499 (g/L)·h, the ITAE to 245.1411 (g/L)·h², and the settling time to 13.01 h. In contrast, the proposed actuator-aware DSC achieves an IAE of 9.0694 (g/L)·h, an ITAE of 5.5078 (g/L)·h², and a settling time of 2.20 h. Relative to the actuator-unaware controller, this corresponds to reductions of 85.73% in IAE, 97.75% in ITAE, and 83.09% in settling time.

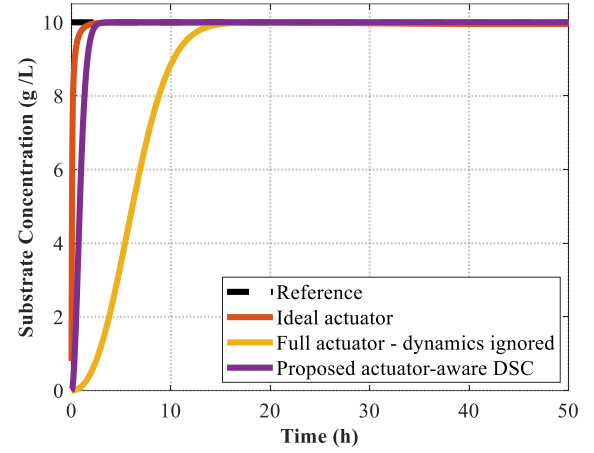


Fig. 17. Effect of actuator dynamics on closed-loop tracking performance: ideal-actuator benchmark, full actuator dynamics neglected in the controller design, and the proposed actuator-aware DSC.

V. Conclusions

In this study, an adaptive robust control strategy based on DSC is proposed for a continuous bioreactor system. To simplify the backstepping design, a single virtual controller is used, and the DSC technique eliminates the need to differentiate virtual control signals, thereby reducing computational complexity while mitigating sensitivity to measurement noise and model inaccuracies. A Lyapunov-based stability analysis ensures that all tracking errors are uniformly ultimately bounded and that the adaptive estimates remain bounded, with the ultimate bound adjustable through control gains to achieve desired performance trade-offs. Simulation results demonstrate that the proposed DSC provides competitive tracking performance with lower online computational requirements compared with the considered SMC and nonlinear MPC benchmarks, while avoiding chattering, which is a key advantage over conventional SMC. Although MPC remains a powerful and widely applicable strategy for bioprocess control, the proposed DSC framework offers a closed-form and optimization-free control solution with reduced implementation complexity, making it attractive for real-time industrial bioreactor applications where robustness, smooth actuation, and computational simplicity are important. Future work will focus on experimental validation of the proposed framework using laboratory- or pilot-scale bioreactor facilities when such facilities become available, as well as extending the framework to account for media-related parameter variations and more realistic actuator behavior. In particular, nonlinear valve effects, including hysteresis, dead-band, and saturation, will be incorporated into the torque-dilution model to further assess and enhance the practical applicability of the proposed control strategy.

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Appendix A. Boundedness of the Bioreactor and Actuator States

The bounds introduced in Assumptions 3 can be verified from the positive invariance of the bioreactor states. Consider the physically admissible conditions $D = K_v I_a(t) \geq 0$, $S(0) \geq 0$, $X(0) \geq 0$, and $S_{in} \geq 0$. At $S = 0$, and $\mu(0) = 0$ $\dot{S} = DS_{in} \geq 0$, whereas at $X = 0$, $\dot{X} = 0$. Therefore, the nonnegative orthant is invariant. Defining $W = S + \frac{X}{Y_{XS}}$, the first two equations in (10) give $\dot{W} = D(S_{in} - W)$. Consequently

$W(t) = S_{in} + [W(0) - S_{in}]e^{-\int_0^t D(\tau)d\tau}$ and hence $0 \leq W(t) \leq \bar{W} = \max\{W(0), S_{in}\}$. It follows that $0 \leq S(t) \leq \bar{W}$, $0 \leq X(t) \leq Y_{XS}\bar{W}$. Therefore, $0 \leq h(S, X) \leq \frac{\mu_{\max}\bar{W}}{K_s} =: \bar{h} < \infty$, and $|f(S)| \leq K'_v(S_{in} + \bar{W}) =: \bar{f} < \infty$. In the admissible operating region $S < S_{in}$, $f(S) > 0$, consistently with Assumption 4. Moreover, using Assumption 2, $|\zeta_1| \leq \bar{h}|x_d| + |\dot{x}_d| =: g(x_d, \dot{x}_d) < \infty$. Thus, the boundedness condition used in (12) follows explicitly from the invariant properties of the bioreactor model and is consistent with the conditions reported in [39].

Appendix B. Boundedness of the Armature Current and Actuator Uncertainty

The boundedness of the armature current follows directly from the electrical actuator dynamics. Let $a = R_a + k_a > 0$. Equation (8) can be written as $L_a \dot{I}_a = V_m - aI_a - v_d$. For a physical actuator, the applied voltage is limited by the finite voltage supply, such that $|V_m(t)| \leq \bar{V}_m$. The lumped actuator uncertainty is also assumed to satisfy $|v_d(t)| \leq \bar{v}_d$. The solution of the current dynamics

$$I_a(t) = e^{-\frac{at}{L_a}} I_a(0) + \frac{1}{L_a} \int_0^t e^{-\frac{a\tau}{L_a}} (V_m(\tau) - v_d(\tau)) d\tau. \quad (B.1)$$

Therefore,

$$\begin{aligned} |I_a(t)| &\leq e^{-\frac{at}{L_a}} |I_a(0)| + \frac{\bar{V}_m + \bar{v}_d}{a} \left(1 - e^{-\frac{at}{L_a}}\right) \\ &\leq \max\left\{|I_a(0)|, \frac{\bar{V}_m + \bar{v}_d}{a}\right\} < \infty \end{aligned} \quad (B.2)$$

Hence, I_a remains bounded for every bounded applied voltage and bounded actuator uncertainty. Furthermore, the stable first-order filter maps a bounded virtual control

command into bounded I_f and $\dot{I}_f$. Consequently $|\zeta_2| \leq a|I_f| + \bar{v}_d + L_a|\dot{I}_f| < \infty$, which justifies the boundedness condition employed after (18).

Appendix C. Differentiability and Boundedness of the Robust-Control Functions

Consider the robust term selected in (40):

$$\begin{aligned} g_c(\hat{g} \operatorname{sgn}(Z_1)) &= \frac{Z_1 \hat{g}^2}{\hat{g}\|Z_1\| + \delta_1(t)} \\ q_c(\hat{q} \operatorname{sgn}(Z_2)) &= \frac{Z_2 \hat{q}^2}{\hat{q}\|Z_2\| + \delta_2(t)} \end{aligned} \quad (C.1)$$

Define $d_1 = \hat{g}\|Z_1\| + \delta_1(t)$ and $d_2 = \hat{q}\|Z_2\| + \delta_2(t)$. According to (41), $\delta_1(t)$ and $\delta_2(t)$ are strictly positive. Therefore, positive constants δ_{1min} and δ_{2min} exist such that: $d_1 \geq \delta_1(t) \geq \delta_{1min}$ and $d_2 \geq \delta_2(t) \geq \delta_{2min}$. For $Z_1 \neq 0$ and $Z_2 \neq 0$, direct differentiation of (C.1) with respect to the corresponding dynamic surfaces gives:

$$\begin{aligned} \frac{\partial g_c}{\partial Z_1} &= \frac{\hat{g}^2 \delta_1(t)}{(\hat{g}\|Z_1\| + \delta_1(t))^2}, \quad Z_1 \neq 0 \\ \frac{\partial q_c}{\partial Z_2} &= \frac{\hat{q}^2 \delta_2(t)}{(\hat{q}\|Z_2\| + \delta_2(t))^2}, \quad Z_2 \neq 0 \end{aligned} \quad (C.2)$$

Although the absolute-value function itself is not differentiable at the origin, the complete robust-control functions in (C.1) are differentiable there. Because $g_c(0) = 0$ and $q_c(0) = 0$, their derivatives at the origin are obtained from the corresponding difference quotients:

$$\begin{aligned} \frac{\partial g_c}{\partial Z_1} \Big|_{Z_1=0} &= \lim_{h \rightarrow 0} \frac{\hat{g}^2}{\hat{g}\|h\| + \delta_1(t)} = \frac{\hat{g}^2}{\delta_1(t)} \\ \frac{\partial q_c}{\partial Z_2} \Big|_{Z_2=0} &= \lim_{h \rightarrow 0} \frac{\hat{q}^2}{\hat{q}\|h\| + \delta_2(t)} = \frac{\hat{q}^2}{\delta_2(t)} \end{aligned} \quad (C.3)$$

These values coincide with the limits of the corresponding derivatives as $Z_1 \rightarrow 0$ and $Z_2 \rightarrow 0$. Hence, the derivatives admit continuous extensions at the origin. Consequently, g_c and q_c are continuously differentiable with respect to Z_1 and Z_2 , respectively. This property is consistent with the continuously differentiable robust-control approximations discussed in [41].

The explicit total time derivative of g_c can now be obtained. For $Z_1 \neq 0$, applying the chain rule to (C.1) gives:

$$\dot{g}_c = \frac{\hat{g}^2 \delta_1}{d_1^2} \dot{Z}_1 + \frac{\partial Z_1 (\hat{g}\|Z_1\| + 2\delta_1)}{d_1^2} \dot{\hat{g}} - \frac{\hat{g}^2 Z_1}{d_1^2} \dot{\delta}_1 \quad (C.4)$$

According to (41), $\delta_1(t)$ is positive, continuous, and piecewise continuously differentiable, and its derivative is bounded wherever it exists. Moreover, because the adaptive estimate $\hat{g}$ is nonnegative, $d = \hat{g}\|Z_1\| + \delta_1(t) \geq \delta_1(t) \geq 1$. On the compact set considered in the stability analysis, Z_1 , $\dot{Z}_1$, $\hat{g}$, and $\dot{\hat{g}}$ are bounded. Therefore, every numerator appearing in (C.4) is bounded, while its denominator d_1^2 is



bounded away from zero. Consequently $\dot{g}_c$ is well defined and bounded almost everywhere. The isolated nondifferentiability of $\delta_1(t)$ at $t = 5$ does not affect the Lyapunov analysis, since the corresponding derivative inequality is required to hold almost everywhere along the closed-loop trajectories. Therefore, the right-hand side of (23) admits a finite continuous majorant, which justifies the bounding function γ introduced in (24).

Appendix D. Intermediate Derivation of Equation (30)

Differentiating the Lyapunov function in (29) gives:

$$\dot{V}_L = Z_1\dot{Z}_1 + L_a Z_2\dot{Z}_2 + e\dot{e} + \tilde{\alpha}^T \Lambda_1^{-1} \dot{\tilde{\alpha}} + \tilde{\beta}^T \Lambda_2^{-1} \dot{\tilde{\beta}} \quad (\text{D.1})$$

From (17) and the definition $e = I_f - I_v$, we have $I_a = Z_2 + I_f = Z_2 + e + I_v$. Using the first-surface dynamics and the virtual control law in (13), for which $I_v = f^{-1}(x_1)(-K_1 Z_1 - g_c(\hat{g}(x_d, \dot{x}_d) \text{sgn}(Z_1)))$ gives:

$$\begin{aligned} Z_1\dot{Z}_1 = & -K_1 Z_1^2 - h(x_1, x_2) Z_1^2 \\ & + \varsigma_1 Z_1 - g_c Z_1 + Z_1(Z_2 + e)f(x_1) \end{aligned} \quad (\text{D.2})$$

Similarly, substituting the actual control law $V_m = -K_2 Z_2 - q_c(\hat{q} \text{sgn}(Z_2))$ from (19) into the second-surface dynamics yields:

$$L_a Z_2\dot{Z}_2 = -K_2 Z_2^2 - Z_2 q_c - (R_a + k_a) Z_2^2 + \varsigma_2 Z_2 \quad (\text{D.3})$$

From the parameter-error definitions, $\dot{\tilde{\alpha}} = -\dot{\hat{\alpha}}$ and $\dot{\tilde{\beta}} = -\dot{\hat{\beta}}$. Using the adaptation laws (15) and (21), the corresponding terms become:

$$\begin{aligned} \tilde{\alpha}^T \Lambda_1^{-1} \dot{\tilde{\alpha}} = & -\tilde{\alpha}^T G^T \|Z_1\| + \tilde{\alpha} \sigma_1 (\hat{\alpha} - \alpha_0) \\ \tilde{\beta}^T \Lambda_2^{-1} \dot{\tilde{\beta}} = & -\tilde{\beta}^T Q^T \|Z_2\| + \tilde{\beta} \sigma_2 (\hat{\beta} - \beta_0) \end{aligned} \quad (\text{D.4})$$

Substituting (D.2)–(D.4) into (D.1) produces the equality in (30). The upper bound in (30) then follows from:

$$\varsigma_1 Z_1 \leq \|Z_1\| \varsigma_1,$$

$$Z_2 \varsigma_2 \leq \|Z_2\| \varsigma_2,$$

$$Z_1(Z_2 + e)f(x_1) \leq \|Z_1\| \|Z_2\| \|f\| + \|Z_1\| \|e\| \|f\|$$

together with $-h(x_1, x_2) Z_1^2 = -\|h\| Z_1^2$ because $h(x_1, x_2) > 0$. Therefore, direct substitution followed by these standard norm inequalities yields (30).

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