

Electromagnetic Design and Modeling of One Coreless Stator Axial Flux Permanent Magnet Machine for Wind Turbine Applications


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Article Info	ABSTRACT
Article type: Research Article Article history: Received: ***** Received in revised form: ***** Accepted: ***** Published online: ***** Keywords: [3 to 5 Keywords] Analytical model, Back-EMF, Fourier-Bessel series, Permanent Magnet (PM), Skew.	This paper presents a fast technique for electromagnetic design and modeling the coreless stator axial flux permanent magnet (CS-AFPM) machines. The proposed technique can be implemented in 3 stages. In the first stage, the initial design of CS-AFPM machine with trapezoidal PMs is done by using the sizing equations. In the second stage, one proposed 3D analytical model based on Fourier-Bessel series is used to study the effect of main parameters in initial design such as the inner and outer diameters, PM cover ratio, PM height, and the number of turns per phase on the phase back-EMF of CS-AFPM machine. In the third stage, to more improve the quality of phase back-EMF, the effects of different PM shapes including the circular PMs, the rectangular PMs with axial and halbach magnetization, the skewed and un-skewed trapezoid PMs are studied by using the proposed 3D analytical technique based on Fourier-Bessel series. The proposed model can accurately consider the edge effect, the curvature effect, the fringing effect and all leakage fluxes. The final optimal design of analyzed CS-AFPM machine is verified through comparing with the corresponding results obtained through 3D finite element method (3D FEM). The capability of proposed 3D analytical technique for on-load analysis is also verified by comparing with corresponding results obtained through MEC and 3D FEM.

I. Introduction

Axial flux permanent magnet (AFPM) machines have been frequently used in portable and wind power generation systems due to their structure with high aspect ratio [1-3]. Among them, the coreless stator AFPM (CS-AFPM) machines with dual rotors and single stator are attracting more and more attentions owing to having the advantages of low starting torque, zero cogging torque, lighter mass, and strong overload capacity, which are suitable at low wind speed [4]. The coreless design of AFPM machines can reduce the attractive force between rotor and stator, and can remove the related core losses [5]. Therefore, the CS-AFPM machine is a suitable choice for small wind turbine applications [6].

For CS-AFPM machines with a proper coil-pole combination, the concentrated winding is preferred to the overlapping winding due to lower copper loss, simple structure and higher output power [4, 7-9]. The constraint of

$1 < \frac{Q}{2p} < 2$ (ratio of coil per pole) was also considered in optimal design of CS-AFPM machine [10-11]. In [12], the stator and rotor design for CS-AFPM machine was done without taking into account the effect of slot-pole combination. Three-layer concentrated windings (each layer for one phase) have been used to provide the high number of coils, the high value of winding factor and the low torque ripple [4, 13]. The overlap coil was bent in [4] to even the air-gap for three phases, and to remove the problem of imbalanced back-EMF [12]. The PCB windings have been proposed for CS-AFPM machines [14-16]. The ceramic substrate windings have been proposed in [17] to more reduce the mass and volume the CS-AFPM machines. However, PCBs requires more turns in multi-layers to produce the needed ampere-turn, with complexity in manufacturing and the increased cost. A multi-disc CS-AFPM machine has been designed in [5,18] to provide high fault tolerance and high-power density. It has been proved

that the optimal value of pole embrace for CS-AFPM machines is in the range of 0.7 to 0.8 [19]. This range has been also validated to increase the average torque in radial flux surface mounted PM motors [20].

The accurate analytical techniques are necessary to reach the optimal design of CS-AFPM machines [14-15, 21-26]. So far, the analytical techniques have been frequently used for accurate modeling the radial flux permanent magnet synchronous machines [27-29]. In [21, 26], an analytical technique based on solving the Laplace equation in air-gap region has been proposed which can consider the curvature effect in radial direction by using the correction function. This technique has been extended in [22] to consider the curvature effect in radial direction. However, the extended technique is not suitable to consider different PM shapes. An improved magnetic equivalent circuit (MEC) was introduced in [23] for CS-AFPM machines which can consider all components of leakage flux, approximately. In [24], this improved MEC model was combined with the proposed technique in [21] to accurately calculate the air-gap flux density for CS-AFPM machines without considering the magnetic saturation. 2D analytical technique based on Hague's solution was proposed in [25] to analytically design one CS-AFPM machine while replacing the PMs with equivalent virtual currents.

In this paper, after obtaining the values for main parameters of initial designed CS-AFPM machine, a new 3D analytical technique based on Fourier-Bessel series is introduced which can consider the curvature and edge effects, all leakage fluxes and different PM shapes. This paper is organized as follows:

In section II, the sizing equations are used to initially design the CS-AFPM machine. In section III, a new 3D analytical model is introduced to study the effect of main parameters and some PM shapes in initial design on the performance of CS-AFPM machine. The optimal PM shape is found in section IV by using the proposed 3D analytical model. To verify the accuracy and capability of proposed 3D analytical technique, the MEC model is proposed in section V and is used for on-load analysis in section VI. The conclusions of work are presented in section VII.

II. Initial Design

The input parameters of the CS-AFPM machine are chosen as:

Number of phases $m=3$ with star connection

Synchronous speed $n_s=150$ (rpm)

Rated power $P_n=1200$ (W)

Rated voltage $V=100$ (volt)

Rated frequency $f=15$ (Hz)

Number of pole-pair $p=6$

The outer diameter of CS-AFPM machine can be calculated by using the power equation as follows:

$$D_o = \sqrt[3]{\frac{8P_n}{C(1+\lambda)^2(1-\lambda)n_s}} \quad (1)$$

where C is the machine constant, and λ is the ratio of inner diameter to outer diameter ($\lambda = \frac{D_i}{D_o}$).

An optimized value of $\lambda = \frac{1}{\sqrt{3}}$ has been introduced to maximize the power [25]. The machine constant C is related to the magnetic and electric loadings. It is estimated to $C = 1000 \left(\frac{VA}{m^3 rpm} \right)$ to obtain the rated voltage. Considering $B_g = 0.5$ (T) for magnetic loading, number of turns per phase is calculated as follows:

$$\begin{cases} \phi_p = B_g \times \alpha_p \times \pi \times \frac{(R_o^2 - R_i^2)}{2p} \\ N_{ph} = \frac{V}{\sqrt{3} \times 4.44 \times f \times \phi_p \times k_w} \rightarrow \\ N_{ph} \cong 360 \text{ (turn)} \end{cases} \quad (2)$$

where R_i and R_o are the inner and outer radii of PM poles, k_w is the winding factor, and α_p is the pole embrace, which is considered to be 0.7 to minimize the total harmonic distortion (THD) of air-gap flux density [19].

The non-overlapping concentrated winding is selected to realize a compact structure for the coreless stator [30]. The number of coils (N_c) is determined to be 18 while considering the following constraints [9-10]:

- $\frac{N_c}{3}$ should be an integer (symmetrical winding)
- $1 < \frac{N_c}{2p} < 2$
- $\frac{N_c}{2p} = \frac{3}{4}k$, (k is integer)

The rated phase current for the studied CS-AFPM machine is about 7 (A). For air-cooled CS-AFPM machine, the stator current density (J) can be considered in the range of 5-10 ($\frac{A}{mm^2}$). Some specifications of stator coil are calculated as follows:

$$\begin{cases} n = \frac{N_c}{6} \rightarrow n \cong 60 \text{ (turn)} \\ J = 7 \left(\frac{A}{mm^2} \right) \rightarrow a = 1 \text{ (mm}^2) \rightarrow d \cong 1 \text{ (mm)} \\ h_s = 5 \times 1.1 + 2 \times 0.5 \rightarrow h_s = 6.5 \text{ (mm)} \end{cases} \quad (3)$$

where n is the number of turns per coil, a is the area of the wire cross section, d is the rated diameter of wire, and h_s is the axial thickness of coils. $d = 1.1$ (mm) is the diameter of insulated wire. The arrangement of wires in the coil cross section is 5 wires (in axial direction) by 12 wires (in tangential direction). The axial increment of 0.5 (mm) is considered on both sides of stator disk due to the resin. The height of PM poles can be determined as follows [25]:

$$\begin{cases} \delta_m = h_s + 2 \times g \xrightarrow{g=1.5(mm)} \delta_m \cong 9.5 (mm) \\ h_m = \frac{k_F \times B_g \times \delta_m}{2\mu_0 \times H_c \times \left(1 - \frac{B_m}{B_r}\right)} \xrightarrow{B_m=0.6B_r} h_m \cong 8(mm) \end{cases} \quad (4)$$

where g is the physical air-gap length, $k_F \cong 1$ when neglecting the magnetic saturation, δ_m is the length of magnetic air-gap, H_c and B_r are respectively the coercivity and remanent flux density of used PM, and B_m is the operating flux density of PMs. The axial length of rotor core is calculated as follows:

$$\begin{cases} h_r = \frac{\Phi_p}{2 \times l_r \times B_{rotor}} \xrightarrow{B_{rotor}=1.8(T)} \\ h_r \cong 8 (mm) \\ l_r = R_o - R_i \end{cases} \quad (5)$$

where B_{rotor} is the rated operating flux density of rotor core.

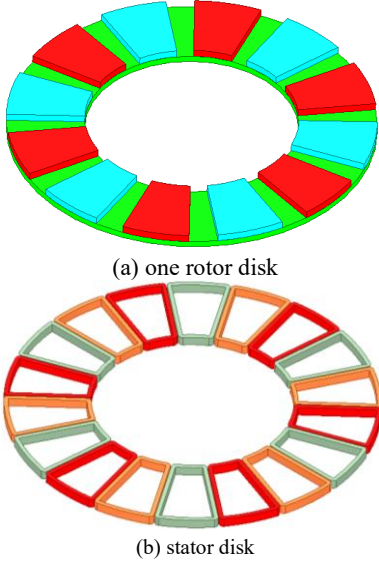


Fig.1. 3D geometry of CS-AFPM machine

$$\nabla^2 \psi_i(r, \alpha, z) = 0 \rightarrow$$

$$\psi_i(r, \alpha, z) = \sum_k \sum_l \left[\left(L_{i,(k,l)} e^{\frac{j(k,l)}{r_a} z} + M_{i,(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) J_l \left(\frac{j(k,l)}{r_a} r \right) \cos(l\alpha) \right. \\ \left. + \left(N_{i,(k,l)} e^{\frac{j(k,l)}{r_a} z} + O_{i,(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) J_l \left(\frac{j(k,l)}{r_a} r \right) \sin(l\alpha) \right] \quad (6)$$

$$B_r(r, \alpha, z) = -\mu_0 \sum_k \sum_l \left(\frac{j(k,l)}{2r_a} \right) \left[\left(L_{(k,l)} e^{\frac{j(k,l)}{r_a} z} + M_{(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) F \left(\frac{j(k,l)}{r_a} r \right) \cos(l\alpha) \right. \\ \left. + \left(N_{(k,l)} e^{\frac{j(k,l)}{r_a} z} + O_{(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) F \left(\frac{j(k,l)}{r_a} r \right) \sin(l\alpha) \right] \quad (8)$$

$$F \left(\frac{j(k,l)}{r_a} r \right) = J_{l-1} \left(\frac{j(k,l)}{r_a} r \right) - J_{l+1} \left(\frac{j(k,l)}{r_a} r \right) \\ B_t(r, \alpha, z) = \mu_0 \sum_k \sum_l \left(\frac{l}{r} \right) \left[\left(L_{(k,l)} e^{\frac{j(k,l)}{r_a} z} + M_{(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) J_l \left(\frac{j(k,l)}{r_a} r \right) \sin(l\alpha) \right. \\ \left. - \left(N_{(k,l)} e^{\frac{j(k,l)}{r_a} z} + O_{(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) J_l \left(\frac{j(k,l)}{r_a} r \right) \cos(l\alpha) \right] \quad (9)$$

$$B_z(r, \alpha, z) = -\mu_0 \sum_k \sum_l \left(\frac{j(k,l)}{r_a} \right) \left[\left(L_{(k,l)} e^{\frac{j(k,l)}{r_a} z} - M_{(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) J_l \left(\frac{j(k,l)}{r_a} r \right) \cos(l\alpha) \right. \\ \left. + \left(N_{(k,l)} e^{\frac{j(k,l)}{r_a} z} - O_{(k,l)} e^{-\frac{j(k,l)}{r_a} z} \right) J_l \left(\frac{j(k,l)}{r_a} r \right) \sin(l\alpha) \right] \quad (10)$$

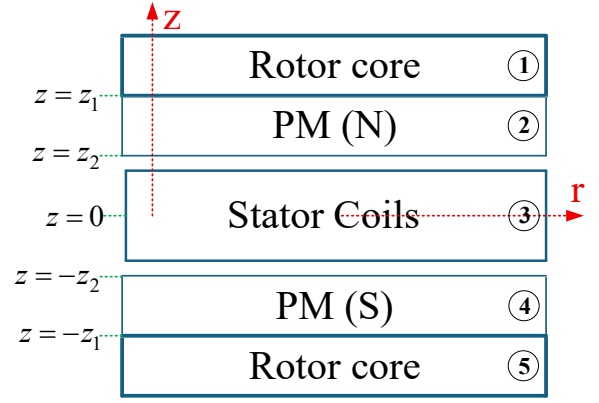


Fig. 2. General geometry in r-z plane

III. Analytical Modeling

Fig. 1 shows the stator and rotor disks of designed CS-AFPM machine. Different domains of this machine in r-z plane are shown in Fig. 2. A new 3D analytical technique which acts based on dividing the geometry into several domains (Fig. 2) is used to model and analyze the designed CS-AFPM machine.

Under no-load condition, which the stator currents are zero, Laplace equation can be written in i^{th} region in terms of the scalar magnetic potential (ψ) as (6).

The general solution in (6) has been obtained through using the method of separation of variables and Fourier-Bessel series. In (6), r_a is the radial length of the domain of solution,

and $j_{(k,l)}$ is the k^{th} root of the first kind Bessel function ($J_l(r)$).

$$M_z(r, \alpha) = -0.5 \sum_k A_{(k,0)} J_l \left(\frac{j_{(k,0)}}{r_a} r \right) + \sum_k \sum_l \left[A_{(k,l)} J_l \left(\frac{j_{(k,l)}}{r_a} r \right) \cos(l\alpha) + B_{(k,l)} J_l \left(\frac{j_{(k,l)}}{r_a} r \right) \sin(l\alpha) \right]$$

$$\begin{cases} A_{(k,l)} = \frac{2}{\pi r_a^2 [J_{l+1}(j_{(k,l)})]^2} \int_0^{2\pi} \int_0^{r_a} r M_z J_l \left(\frac{j_{(k,l)}}{r_a} r \right) \cos(l\alpha) dr d\alpha \\ B_{(k,l)} = \frac{2}{\pi r_a^2 [J_{l+1}(j_{(k,l)})]^2} \int_0^{2\pi} \int_0^{r_a} r M_z J_l \left(\frac{j_{(k,l)}}{r_a} r \right) \sin(l\alpha) dr d\alpha \end{cases} \quad (12)$$

$$\begin{cases} e^{\frac{j_{(k,l)}}{r_a} z_1} \times L_{2,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_1} \times M_{2,(k,l)} = 0 \\ e^{\frac{j_{(k,l)}}{r_a} z_1} \times N_{2,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_1} \times O_{2,(k,l)} = 0 \\ e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{2,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{2,(k,l)} - e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{3,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{3,(k,l)} = 0 \\ e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{2,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{2,(k,l)} - e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{3,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{3,(k,l)} = 0 \\ \left(e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{2,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{2,(k,l)} - e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{3,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{3,(k,l)} \right) = \left(\frac{r_a}{j_{(k,l)}} \right) A_{(k,l)} \\ \left(e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{2,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{2,(k,l)} - e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{3,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{3,(k,l)} \right) = \left(\frac{r_a}{j_{(k,l)}} \right) B_{(k,l)} \\ e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{3,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{3,(k,l)} - e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{4,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{4,(k,l)} = 0 \\ e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{3,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{3,(k,l)} - e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{4,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{4,(k,l)} = 0 \\ \left(-e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{3,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{3,(k,l)} \right) + \left(e^{\frac{j_{(k,l)}}{r_a} z_2} \times L_{4,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times M_{4,(k,l)} \right) = \left(\frac{r_a}{j_{(k,l)}} \right) A_{(k,l)} \\ \left(-e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{3,(k,l)} + e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{3,(k,l)} \right) + \left(e^{\frac{j_{(k,l)}}{r_a} z_2} \times N_{4,(k,l)} - e^{-\frac{j_{(k,l)}}{r_a} z_2} \times O_{4,(k,l)} \right) = \left(\frac{r_a}{j_{(k,l)}} \right) B_{(k,l)} \\ e^{-\frac{j_{(k,l)}}{r_a} z_1} \times L_{4,(k,l)} + e^{\frac{j_{(k,l)}}{r_a} z_1} \times M_{4,(k,l)} = 0 \\ e^{-\frac{j_{(k,l)}}{r_a} z_1} \times N_{4,(k,l)} + e^{\frac{j_{(k,l)}}{r_a} z_1} \times O_{4,(k,l)} = 0 \end{cases} \quad (17)$$

The magnetic field intensity ($\vec{H}$) and air-gap flux density ($\vec{B}$) can be calculated while having ψ as follows:

$$\begin{aligned} \vec{H}(r, \alpha, z) &= -\vec{\nabla}\psi = H_r \vec{a}_r + H_t \vec{a}_t + H_z \vec{a}_z \\ \vec{B}(r, \alpha, z) &= \mu_0 \times \vec{H}(r, \alpha, z) \end{aligned} \quad (7)$$

Three components of air-gap flux density are then calculated as shown in (8)-(10). The vector of flux density in the region of PMs is defined as follows:

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) \quad (11)$$

In (11), $\vec{M} = M_z \vec{a}_z$ is the magnetization vector of PMs, and is defined as (12). To determine the unknown constants $L_{(k,l)}$, $M_{(k,l)}$, $N_{(k,l)}$ and $O_{(k,l)}$ in (8)-(10), the following boundary conditions should be applied on Fig. 2.

$$z = z_1 \rightarrow H_{t,2} = 0 \quad (13)$$

$$z = z_2 \rightarrow \begin{cases} H_{t,2} = H_{t,3} \\ B_{z,2} = B_{z,3} \rightarrow (H_{z,2} + M_z) = H_{z,3} \end{cases} \quad (14)$$

$$z = -z_2 \rightarrow \begin{cases} H_{t,3} = H_{t,4} \\ B_{z,3} = B_{z,4} \rightarrow H_{z,3} = (H_{z,4} + M_z) \end{cases} \quad (15)$$

$$z = -z_1 \rightarrow H_{t,4} = 0 \quad (16)$$

One system of algebraic equations is obtained as shown in (17), which should be solved in all rotor positions to determine the unknown constants L_{2-4} , M_{2-4} , N_{2-4} and O_{2-4} for every value of (k, l) . For initial designed CS-AFPM machine with trapezoid PM poles, the approximated distribution of PM magnetization obtained through Fourier-Bessel series is shown in Fig.3.

The distribution of three components of air-gap flux density on the middle plane of air-gap obtained through (17) is shown in Fig. 4. As shown, the edge effect in inner and outer radii can be considered by using the proposed 3D analytical technique. As seen in Fig. 4b-c, the magnitude of radial and tangential components of air-gap flux density is negligible in the middle of air-gap of CS-AFPM machines due to the large magnetic air-gap. Fig. 5 shows the tangential and radial components of air-gap flux density on the surface of PM poles is considerable unlike the middle of air-gap.

Here, in Fourier-Bessel series, the values of $\begin{cases} k = 1: 50 \\ l = 0: 49 \end{cases}$ are considered to reach the convergence behavior of the model.

In real, 50 harmonics have been considered for calculating the air-gap flux density. For studied CS-AFPM machine with trapezoid PM poles, Fig. 6 shows the zoomed view of axial component of air-gap flux density (B_z) in the middle of air-gap obtained through 3D analytical technique while

considering different number of harmonics in Fourier-Bessel series. As shown, there are additional oscillations in B_z when the number of harmonics is considered less than 50.

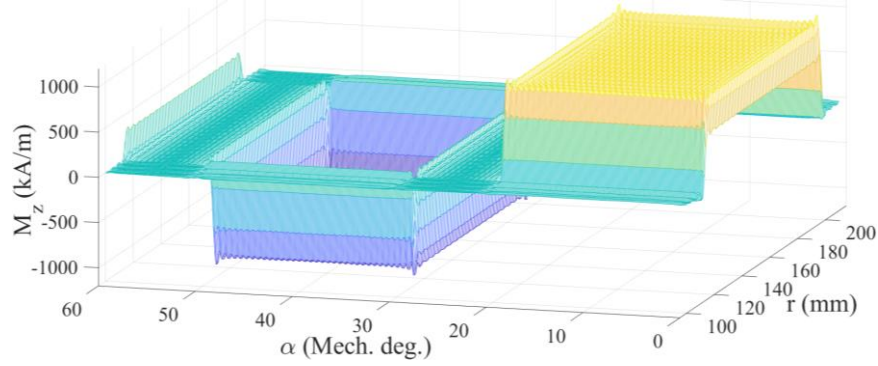
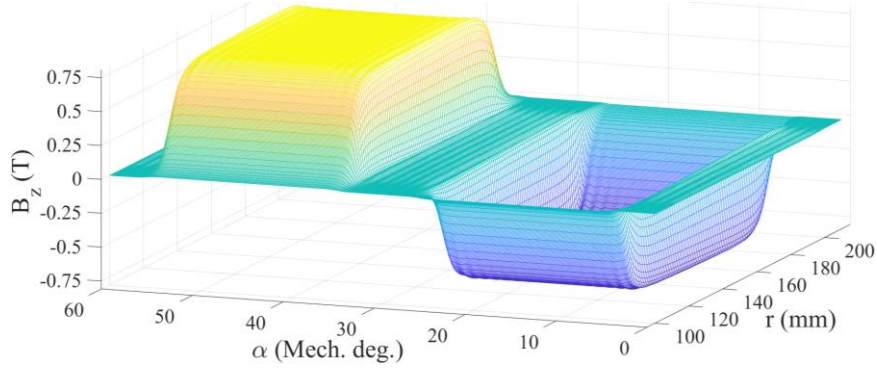
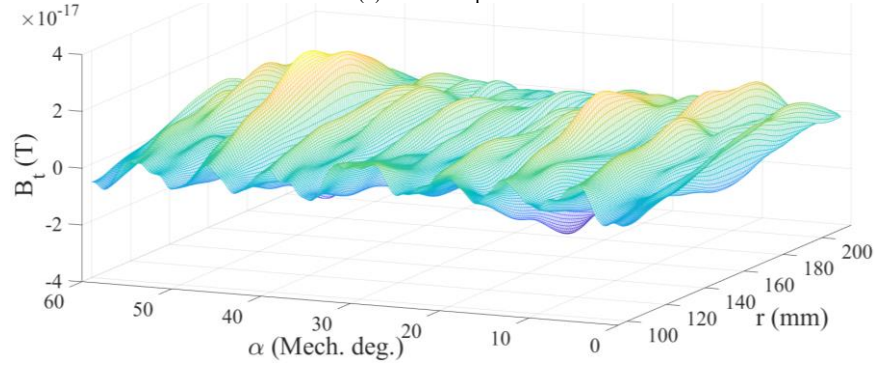


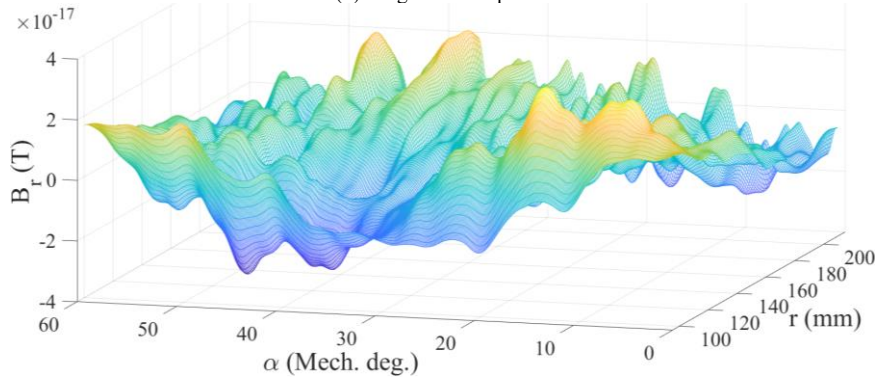
Fig. 3. Approximated distribution of M_z for trapezoid PM



(a) Axial component



(b) Tangential component



(c) Radial component

Fig. 4. Air-gap flux density due to trapezoid PMs

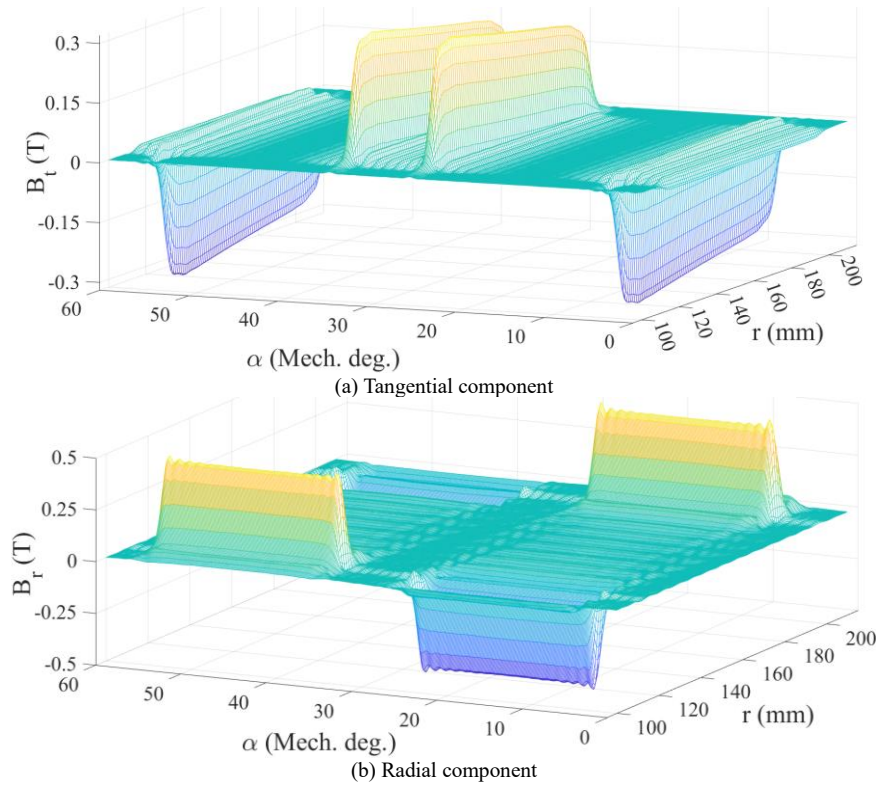


Fig. 5. Air-gap flux density on the surface of PMs

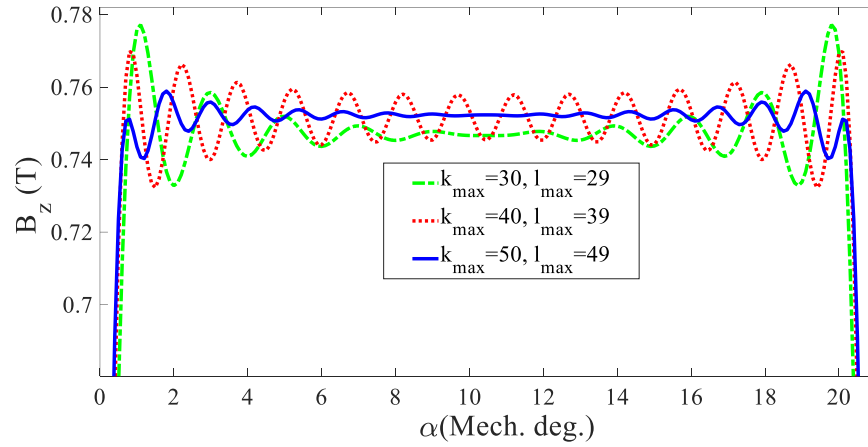


Fig. 6. Model accuracy in terms of the number of considered harmonics

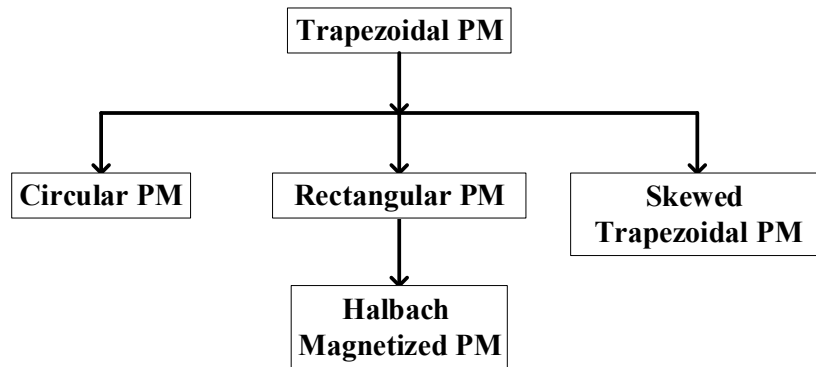


Fig. 7. Different PM shapes for studied CS-AFPM machine

IV. PM optimization

To obtain the more sinusoidal phase back-EMF for initial designed CS-AFPM machine, different PM shapes including trapezoidal, rectangular (with axial and halbach magnetization), circular, and skewed trapezoidal are studied while maintaining a constant area and height for PM poles (Fig. 7). The main goal in optimization process is to reduce the THD while maintaining the RMS value of phase back-EMF. In this paper, the optimal model is obtained while considering the maximum allowable value of THD equal to 5%. To reduce the axial length of designed CS-AFPM machine, the volume of PM poles is designed to be shallower and with a low height (Area=4727.6 mm², Height=8 mm). For this reason, it is not possible to implement the technique of skewing on the rectangular PM shapes of studied CS-AFPM machine.

The trapezoid PM poles in initial designed CS-AFPM machine can be replaced with circular PM poles with diameter of 77.6 mm and rectangular PM poles with dimension of 60.256 mm × 78 mm while keeping the volume of PM pole constant. Fig. 8 shows the distribution of axial component of air-gap flux density due to the circular and rectangular PM poles with axial magnetization. To study the effect of PM shape on the phase back-EMF of designed CS-AFPM machine, it can be calculated as follows:

$$\left\{ \begin{array}{l} \lambda_{c,i}(\theta_r) = \int_0^{2\pi} \int_{r_{si}}^{r_{so}} n_{c,i}(r, \alpha) \times B_z(r, \alpha) \times r dr d\alpha \\ \lambda_{ph} = \sum_{i=1}^6 \lambda_{c,i} \\ E_{ph} = \frac{d\lambda_{ph}}{dt} \end{array} \right. \quad (18)$$

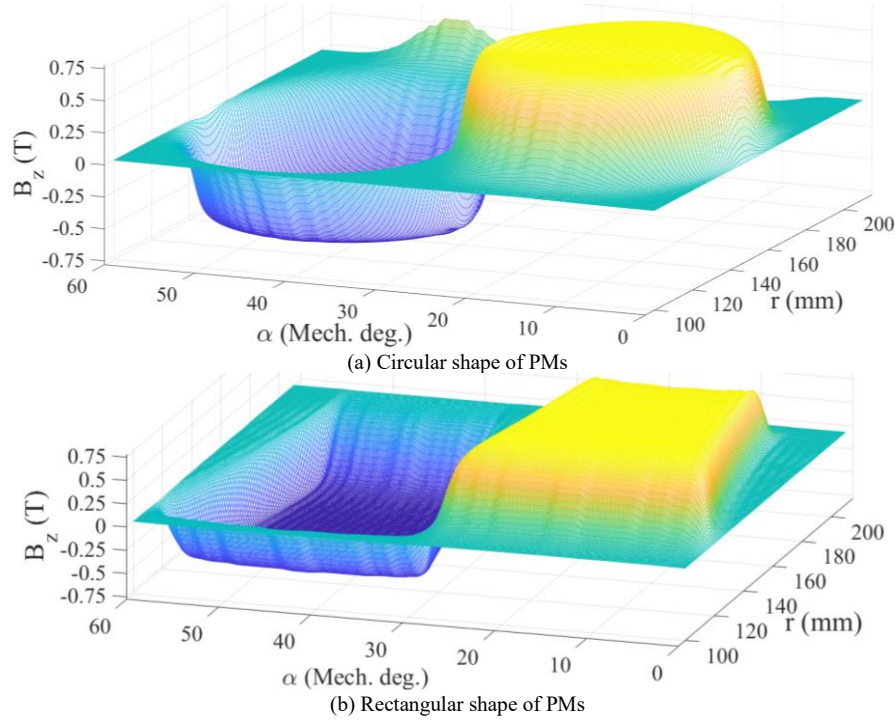


Fig. 8. Effect of other PM shapes on the air-gap flux density

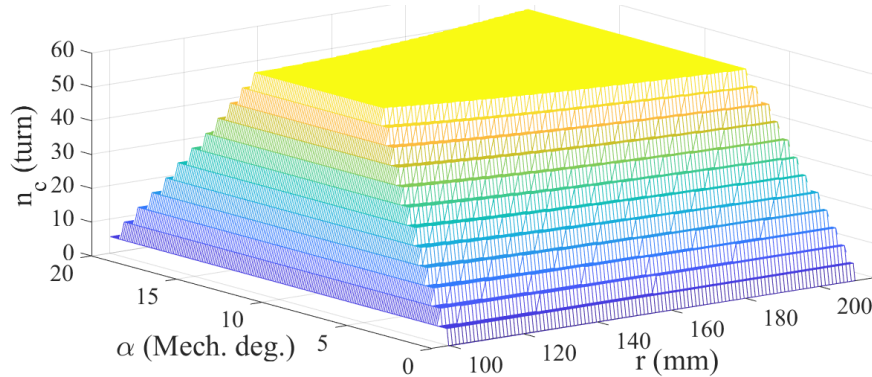


Fig. 9. Turn distribution of one coil

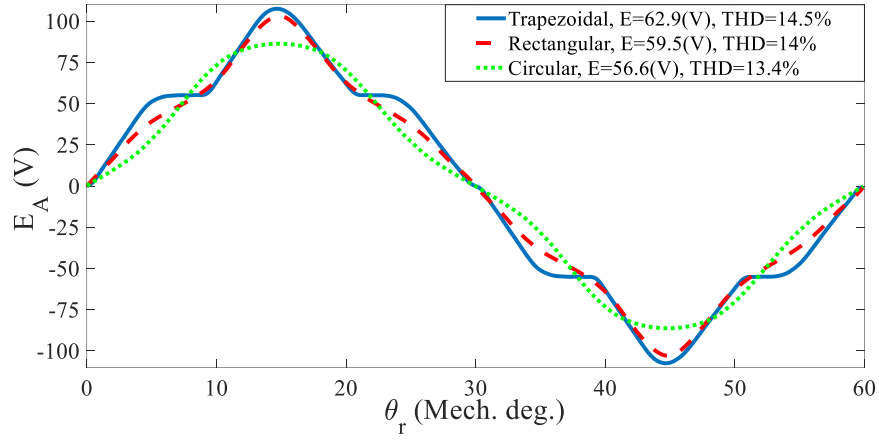


Fig. 10. Phase back-EMF due to conventional PM shapes

In (18), θ_r is the rotor position, $n_{c,i}(r, \alpha)$ is the turn distribution of i^{th} coil (Fig. 9), and $\lambda_{c,i}$ is the flux-linkage with i^{th} coil of stator. Fig. 10 shows the phase back-EMF of studied CS-AFPM machine with trapezoidal, rectangular and circular shapes of PM poles. As seen, the root mean square (RMS) value of phase back-EMF is very close to the design goal whereas the THD value is high for all three PM shapes. As shown in Fig. 7, the rectangular PM poles can be replaced with the segmented halbach magnetized PM poles to reduce the THD value of phase back-EMF. To this end,

each rectangular PM pole is divided into three segments with relevant magnetization angle, as shown in Fig. 11. The response surface methodology [31] has been used to find the optimal value of α and β in order to reduce the THD of phase back-EMF while maintaining the RMS value of it about 57 (V). The optimum values of α and β are obtained 45° and 0.25, respectively. The back-EMFs of stator phase A of the analyzed CS-AFPM machine obtained through optimal model with three-segmented halbach magnetized PM poles and the conventional model with rectangular axial magnetized PM poles are compared in Fig. 12.

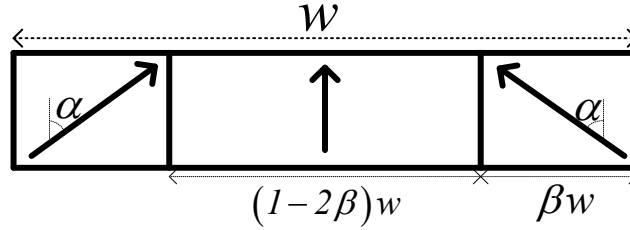


Fig. 11. Segmented halbach magnetized PM pole

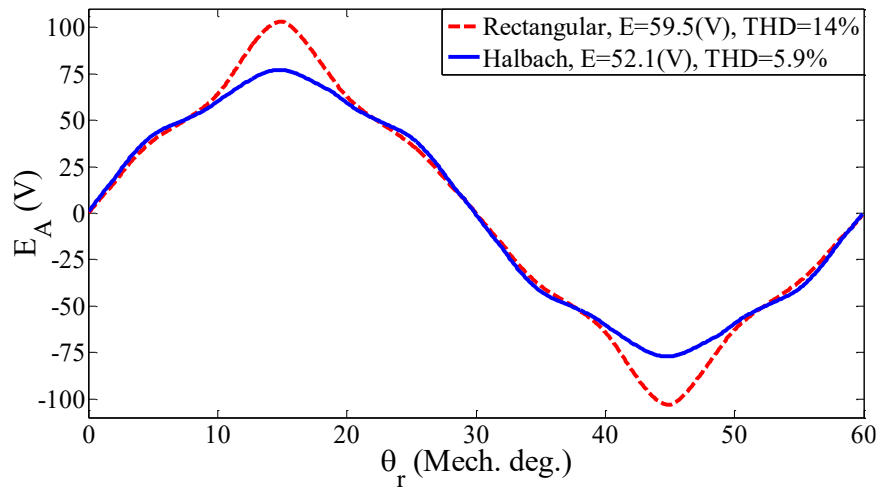


Fig. 12. Phase back-EMF due to rectangular PM poles with different magnetization

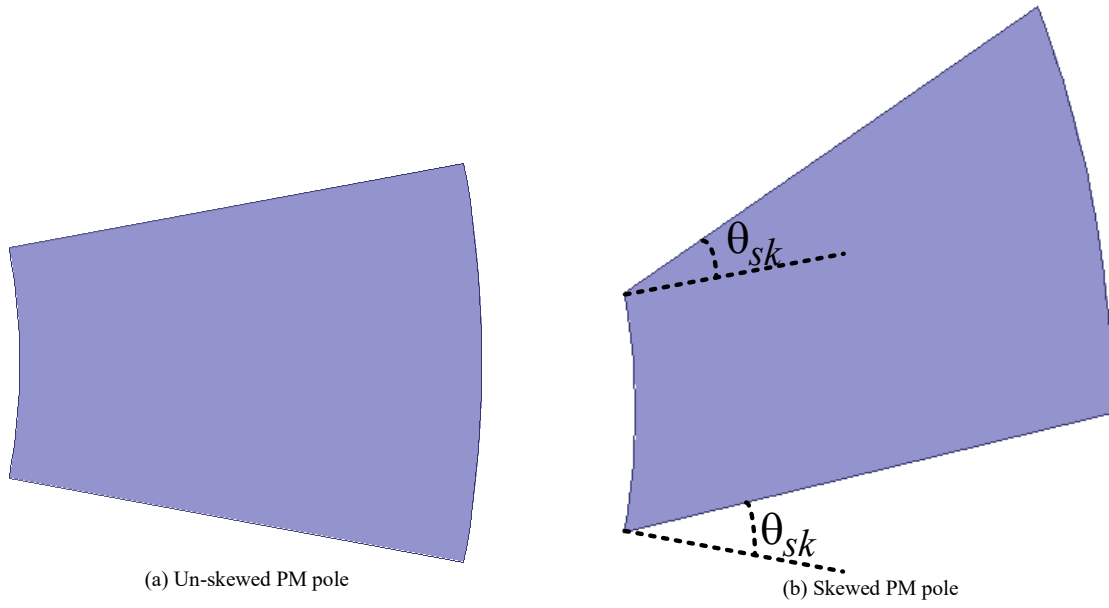


Fig. 13. 2D view of trapezoid PM pole

As seen in Fig. 12, 6% reduction in THD has been achieved at the cost of 12% reduction in the RMS value of phase back-EMF. To reach the lower THD with higher RMS value of phase back-EMF, the technique of skewing is implemented

on the trapezoidal PM poles of analyzed CS-AFPM machine. Fig. 13 shows the 2D view of un-skewed and skewed trapezoid PM poles with the skew angle of θ_{sk} .

TABLE I. Fourier results of E_A

θ_{sk}	RMS value of E_A	THD
0°	62.9 (V)	14.55%
5°	62.74 (V)	13.96%
10°	62.28 (V)	12.37%
15°	61.58 (V)	10.3%
20°	60.7 (V)	8.34%
25°	59.66 (V)	6.95%
30°	58.45 (V)	6.13%
35°	57.05 (V)	5.59%
40°	55.45 (V)	5.12%
45°	53.71 (V)	4.58%

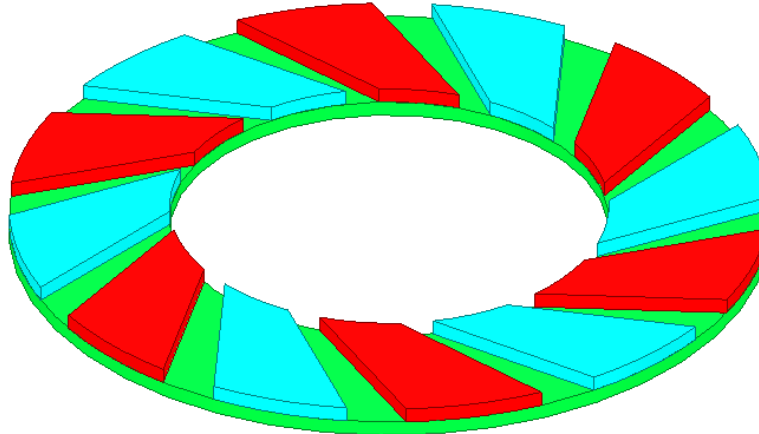


Fig. 14. Rotor disk with skewed PMs

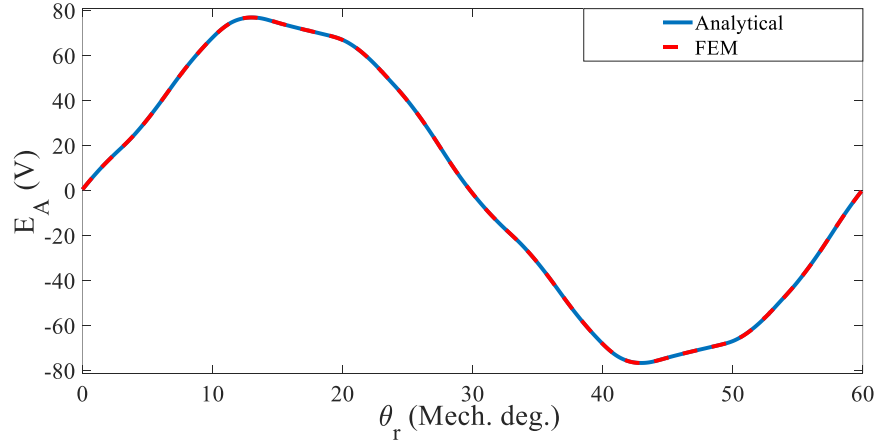


Fig. 15. Verification of optimal result

The Fourier analysis results of phase back-EMF of studied CS-AFPM machine are presented in Table I. The maximum allowable skew angle (θ_{sk}) is 45° while considering the geometrical limitations of trapezoid PM poles. It can be said that the results obtained through skewed trapezoid PM poles with $\theta_{sk} = 45^\circ$ is more optimal than the three-segmented halbach magnetized PM poles. Fig. 14 shows the rotor disk of analyzed CS-AFPM machine with skewed trapezoid PM poles ($\theta_{sk} = 45^\circ$). The optimal result of E_A obtained through CS-AFPM machine with the rotor disk shown in Fig. 14 has

been verified by comparing with corresponding result obtained from 3D-FEM, as shown in Fig. 15.

V. MEC model

The MEC model presented in [23] for CS-AFPM machines is used to compare with the corresponding results obtained through proposed 3D analytical technique. This MEC model is illustrated in Fig. 16 under no-load condition. The elements of MEC model are calculated as (19-34).

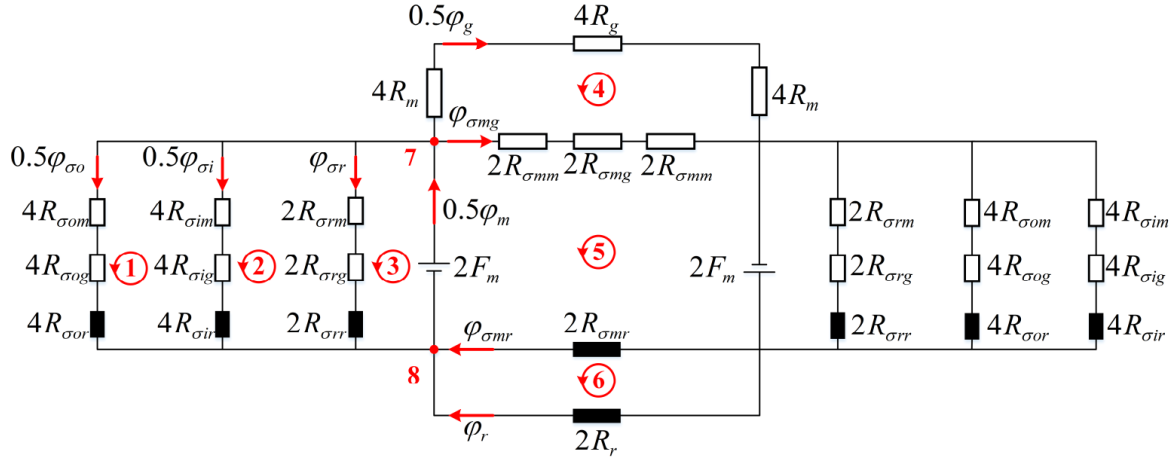


Fig. 16. MEC model of CS-AFPM machine [23]

PM reluctance	$R_m = \frac{h_m}{\mu_o \times \mu_{rm} \times \frac{\pi \alpha_p}{2p} \times (R_o^2 - R_i^2)}$	(19)
Air-gap reluctance	$R_g = \frac{\delta_m}{\mu_o \times \frac{\pi \alpha_p}{2p} \times (R_o^2 - R_i^2)}$	(20)
Rotor yoke reluctance	$R_r = \frac{\frac{\pi}{2p} (R_o + R_i)}{\mu_o \times \mu_{rc} \times h_r \times (R_o - R_i)}$	(21)
MMF of PM	$F_m = \frac{B_R}{\mu_o} \times h_m$	(22)
PM-PM leakage reluctance (inside PM)	$R_{\sigma mm} = \frac{h_m}{\mu_o \times \mu_{rm} \times \frac{\delta_m}{2} \times (R_o - R_i)}$	(23)

PM-PM leakage reluctance (inside air-gap)	$\frac{1}{R_{\sigma mg}} = \mu_o \int_0^{\frac{\delta_m}{2}} \int_{R_i}^{R_o} \frac{dr dz}{\frac{\pi}{p} (1 - \alpha_p) r + \pi z}$	(24)
PM-PM leakage reluctance (inside rotor yoke)	$R_{\sigma mr} = \frac{\delta_m + \frac{\pi}{2p} (1 - \alpha_p) (R_o + R_i)}{\mu_o \times \mu_{rc} \times h_r \times (R_o - R_i)}$	(25)
PM-rotor yoke leakage reluctance at effective radius range (inside PM)	$\begin{cases} R_{\sigma rm} = \frac{h_m}{\mu_o \times \mu_{rm} \times \int_{R_i}^{R_o} L(r) dr} \\ L(r) = \min \left\{ 0.5\delta_m, \frac{\pi r (1 - \alpha_p)}{2p} \right\} \end{cases}$	(26)
PM-rotor yoke leakage reluctance at effective radius range (inside air-gap)	$\frac{1}{R_{\sigma rg}} = \mu_o \times \int_{R_i}^{R_o} \int_0^{L(r)} \frac{dz dr}{\pi z + h_m}$	(27)
PM-rotor yoke leakage reluctance at effective radius range (inside rotor yoke)	$\frac{1}{R_{\sigma rr}} = \mu_o \times \mu_{rc} \times \int_{R_i}^{R_o} \frac{h_r dr}{2L(r)}$	(28)
PM-rotor yoke leakage reluctance at inner radius of PM (inside PM)	$R_{\sigma im} = \frac{h_m}{\mu_o \times \mu_{rm} \times \frac{\delta_m}{2} \times R_i \times \frac{\pi \alpha_p}{p}}$	(29)
PM-rotor yoke leakage reluctance at inner radius of PM (inside air-gap)	$\frac{1}{R_{\sigma ig}} = \frac{2\mu_o \alpha_p}{3p} \left[0.5\delta_m + \left(R_i - \frac{2h_m}{3\pi} \right) \ln \left(1 + \frac{3\pi\delta_m}{4h_m} \right) \right]$	(30)
PM-rotor yoke leakage reluctance at inner radius of PM (inside rotor yoke)	$R_{\sigma ir} = \frac{0.5\delta_m}{\mu_o \times \mu_{rc} \times h_r \times R_i \times \frac{\pi \alpha_p}{p}}$	(31)
PM-rotor yoke leakage reluctance at outer radius of PM (inside PM)	$R_{\sigma om} = \frac{h_m}{\mu_o \times \mu_{rm} \times \frac{\delta_m}{2} \times R_o \times \frac{\pi \alpha_p}{p}}$	(32)
PM-rotor yoke leakage reluctance at outer radius of PM (inside air-gap)	$\frac{1}{R_{\sigma og}} = \frac{2\mu_o \alpha_p}{3p} \left[-0.5\delta_m + \left(R_o + \frac{2h_m}{3\pi} \right) \ln \left(1 + \frac{3\pi\delta_m}{4h_m} \right) \right]$	(33)
PM-rotor yoke leakage reluctance at outer radius of PM (inside rotor yoke)	$R_{\sigma or} = \frac{0.5\delta_m}{\mu_o \times \mu_{rc} \times h_r \times R_o \times \frac{\pi \alpha_p}{p}}$	(34)

The air-gap flux (ϕ_g) can be determined by applying the KVL and KCL laws to the MEC model in Fig. 16 as follows:

$$\phi_g = \frac{(2R_{\sigma r} - R_r R_{\sigma m})}{R_{\sigma r} (R_g + 2R_m)} F_m \quad (35)$$

where $R_{\sigma r} = R_{\sigma rm} + R_{\sigma rg} + R_{\sigma rr}$ and $R_{\sigma m} = 2R_{\sigma mm} + R_{\sigma mg}$.

The axial component of air-gap flux density is then calculated as follows:

$$B_z(r, \alpha) = \frac{2p\phi_g}{\pi\alpha_p} (R_o^2 - R_i^2) \lambda(r, \alpha) \delta(r) \quad (36)$$

where $\lambda(r, \alpha)$ and $\delta(r)$ are the correction functions to consider the fringing effects respectively in tangential and radial directions as follows:

$$\lambda(r, \alpha) = \begin{cases} \frac{1}{2} e^{\frac{r\alpha + \frac{r\pi\alpha_p}{2p}}{\sigma}} - \pi \leq \alpha \leq \frac{-\pi\alpha_p}{2p} \\ 1 - \frac{1}{2} e^{-\frac{r\alpha + \frac{r\pi\alpha_p}{2p}}{\sigma}} - \frac{-\pi\alpha_p}{2p} < \alpha \leq 0 \\ 1 - \frac{1}{2} e^{\frac{r\alpha - \frac{r\pi\alpha_p}{2p}}{\sigma}} 0 < \alpha \leq \frac{\pi\alpha_p}{2p} \\ \frac{1}{2} e^{-\frac{r\alpha - \frac{r\pi\alpha_p}{2p}}{\sigma}} \frac{\pi\alpha_p}{2p} < \alpha \leq \pi \end{cases} \quad (37)$$

$$\begin{aligned} \sigma &= \frac{1}{2} \sqrt{\frac{g}{2} \left(\frac{g}{2} + \frac{h_m}{\mu_{rm}} \right)} \\ \delta(r) &= \left(1 + \left| \frac{r - 0.5(R_o + R_i)}{\beta_1} \right|^{\beta_2} \right)^{-1} \\ \beta_1 &= 0.5(R_o - R_i) \\ \beta_2 &= \frac{6(R_o - R_i)}{\delta_m^{0.77}} \end{aligned} \quad (38)$$

The phase flux-linkage and phase back-EMF are then calculated by using (18).

VI. On-load analysis

The proposed 3D analytical technique can be used for magnetic field calculation due to the armature currents. To this end, the stator coils are replaced with the equivalent virtual PMs while maintaining the MMF of stator coils. The remanent flux density of equivalent virtual PMs (B_R) is calculated as follows:

$$n_c \times I_c = \frac{B_R}{\mu_0} \times h_s \rightarrow B_R = \frac{\mu_0 \times n_c \times I_c}{h_s} \quad (39)$$

where n_c is the turn number of coil, I_c is the current of coil and h_s is the axial thickness of coil.

Similar to no-load condition, the air-gap magnetic field due to the stator currents can be calculated. The distribution of axial component of air-gap magnetic flux density is shown in Fig. 17 while considering the excitations $I_A = 1(A)$, $I_B = I_C = 0(A)$ for phase currents.

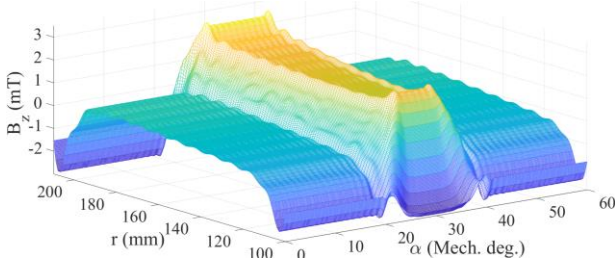


Fig. 17. Typical armature reaction air-gap magnetic field

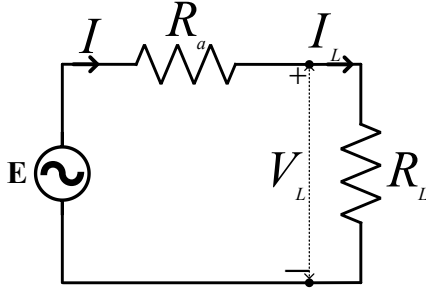


Fig. 18. Single-phase equivalent circuit of stator

The self and mutual inductances are then calculated as follows:

$$\begin{cases} L_{A-A} = \frac{\lambda_A}{I_A} \Big|_{I_B = I_C = 0(A)} = 3.45 \text{ (mH)} \\ L_{B-A} = \frac{\lambda_B}{I_A} \Big|_{I_B = I_C = 0(A)} = -0.85 \text{ (mH)} \end{cases} \quad (40)$$

where

$$\begin{cases} \lambda_A = \int_0^{2\pi} \int_{r_{si}}^{r_{so}} N_A(r, \alpha) \times B_z(r, \alpha) \times ds(r, \alpha) \\ \lambda_B = \int_0^{2\pi} \int_{r_{si}}^{r_{so}} N_B(r, \alpha) \times B_z(r, \alpha) \times ds(r, \alpha) \end{cases} \quad (41)$$

$N_A(r, \alpha)$ and $N_B(r, \alpha)$ are the turn distribution of stator phase A and phase B, respectively. Consequently, the inductance matrix of stator phases is obtained for every rotor position as follows:

$$L = \begin{bmatrix} 3.45 & -0.85 & -0.85 \\ -0.85 & 3.45 & -0.85 \\ -0.85 & -0.85 & 3.45 \end{bmatrix} \text{ (mH)}$$

The obtained values for self and mutual inductances are low due to the large magnetic air-gap. For this reason, as shown in Fig. 17, the armature reaction in CS-AFPM machines is weak. The analyzed CS-AFPM machine with un-skewed trapezoid PM poles is tested as an independent generator loaded by the load resistances R_L in star connection. The phase resistance (R_a) is equal to 2.5 (Ω). The on-load analysis can be done while having the inductance matrix. The single-phase electrical equivalent circuit of the stator of studied CS-AFPM machine is considered as Fig. 18. The result of applying KVL law on the circuit in Fig. 18 is as follows:

$$\begin{cases} [E] = \frac{d[\lambda]}{dt} \\ [\lambda] = [\lambda_m] + [L] \times [I] \end{cases} \rightarrow \begin{cases} \frac{d[\lambda_m]}{dt} + [L] \times \frac{d[I]}{dt} + [R] \times [I] = 0 \xrightarrow{E_m = \frac{d[\lambda_m]}{dt}} \\ [E_m] + [L] \times \frac{d[I]}{dt} + [R] \times [I] = 0 \end{cases} \quad (42)$$

where $[\lambda_m] = \begin{bmatrix} \lambda_{m,a} \\ \lambda_{m,b} \\ \lambda_{m,c} \end{bmatrix}$ is the phase flux-linkage due to the

rotor PMs, $[L]$ is the inductance matrix, $[I] = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$ is the matrix of phase currents, and $[R] = \begin{bmatrix} (R_a + R_L) & 0 & 0 \\ 0 & (R_a + R_L) & 0 \\ 0 & 0 & (R_a + R_L) \end{bmatrix}$ is the matrix of phase resistance including the load resistance R_L .

The electrical differential equation of analyzed CS-AFPM machine can be solved by using the finite difference method as follows:

$$[I(k+1)] = [1 - \Delta t \times [L]^{-1} \times [R]][I(k)] - \Delta t \times [L]^{-1} \times [E_m(k)] \quad (43)$$

where Δt is the time-step of simulation.

After obtaining the stator phase currents, the load voltage (V_L) and output power (P_{out}) are calculated as follows:

$$\begin{cases} \begin{bmatrix} V_{L,a} \\ V_{L,b} \\ V_{L,c} \end{bmatrix} = \begin{bmatrix} R_L & 0 & 0 \\ 0 & R_L & 0 \\ 0 & 0 & R_L \end{bmatrix} \times \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\ P_{out} = 3 \times V_{L,rms} \times I_{L,rms} \end{cases} \quad (44)$$

The results of output power (P_{out}) for different resistance loads (R_L) obtained through different techniques are illustrated in Fig. 19 while considering non-skewed trapezoid PM poles on the rotor disks. As shown, the maximum P_{out} is obtained when $R_L \cong R_a$, and there is a good agreement between the results obtained through 3D-FEM and proposed 3D-Analytical technique. Under the loading condition with maximum P_{out} , the phase flux-linkage obtained through different techniques are shown and compared in Fig. 20.

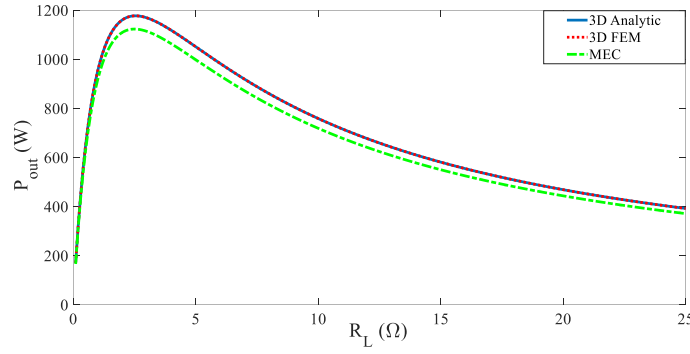


Fig. 19. Output power

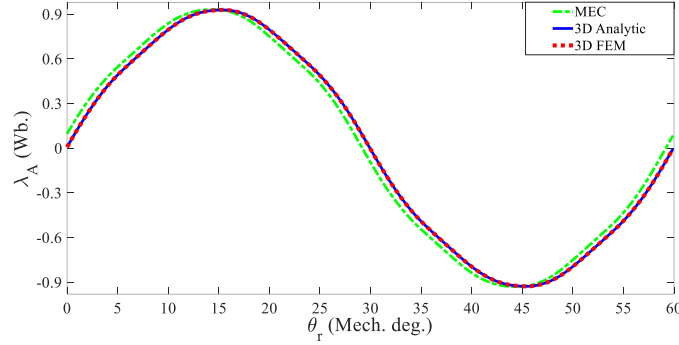


Fig. 20. Flux-linkage of phase A

TABLE II. Comparison of time computation

Technique	Computational time of one operating point
3D FEM	About 5 minutes
3D Analytic	Fast (About 0.1 second)
MEC	Fast (About 0.1 second)

The error of MEC result in Figs. 19-20 is due to its inability to take into account the effect of armature reaction. The presented MEC model also cannot consider the skewing effect and other shapes of PM poles. The computation time of different techniques are quantitatively compared in Table II. As seen, the proposed 3D analytical technique acts very fast, which is suitable for optimization problems.

VII. Conclusions

In this paper, a new optimal design of CS-AFPM machines was presented with low THD and high RMS value of phase back-EMF. The initial design of CS-AFPM machine was done by using the sizing equation while taking into account the design considerations such as the maximum power, the minimum THD of phase back-EMF and the small axial length. A new 3D analytical model was proposed to model and analyze the initial design with different PM shapes at a low computational time and with high accuracy while keeping the area and height of PM poles constant. The proposed 3D analytical model acts based on Fourier-Bessel series to consider the all components of air-gap flux while regarding the edge and curvature effects. Three-segmented halbach magnetized PM poles were used to improve the performance of studied CS-AFPM machine with rectangular PM poles. To more optimize the initial design of CS-AFPM machine, the technique of skewing was implemented on the

trapezoidal PM poles, and the best results were obtained with the skew angle of 45° . However, the manufacturability and implementation complexity of segmented Halbach and skewed PMs, assembly difficulty and magnetization requirements should be considered in final selection of optimal model. In final, the capability of proposed 3D analytical technique was proved for the on-load analysis. The more analysis of on-load condition is considered as a direction for future research.

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