RESIDUAL ANALYSIS USING FOURIER SERIES TRANSFORM IN FUZZY TIME SERIES MODEL

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Abstract. In this paper, we propose a new residual analysis method using Fourier series transform into fuzzy time series model for improving the forecasting performance. This hybrid model takes advantage of the high predictable power of fuzzy time series model and Fourier series transform to fit the estimated residuals into frequency spectra, select the low-frequency terms, filter out high-frequency terms, and then have well forecasting performance. Two numerical examples are given to show that our proposed model can be applied with the best forecasting performance than the other models.

1. **Introduction**

Forecasting methodology is the most important and relevant in the field of management, including that for financial forecasting, production demand and supply forecasting, technology forecasting, and so on. The time series model is a popular method that requires at least 50 and preferably 100 observations. However, it is sometimes impossible to collect 50 data or more for forecasting a new product demand or a new system developing in today's rapidly changing socio-economic situations. Therefore, other forecasting models have been developed to cope with the problem when collected data is limited and violates the basic assumption of normal distribution of standard statistical models. Fuzzy time series model proposed by Song and Chissom [14, 15, 16] have been developed and applied to business forecasting [17, 18, 19, 20] as if the given datum is in linguistic terms or smaller than fifty data. Thereafter, fuzzy time series model had drawn much attention to the researchers. For model modifications, Chen [1] focused on the operator used in the model and simplified the arithmetic calculations to improve the composition operations and fuzzy logical groups to improve the forecast; Huarng [5] made a study on the effective length of intervals to improve the forecasting in fuzzy time series; Kou et al. [8] proposed a new hybrid forecasting model which combined particle swarm optimization to improve the forecasted accuracy; Cheng et al. [4] introduced fuzzy clustering in which fuzzy clustering are integrated in the processes of fuzzy time series to partition datasets objectively and enable processing of multiple attributes. For forecasting with applications, Yu [20] proposed a weighted method to

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forecast the TAIEX to tackle two issues, recurrence and weighting, in fuzzy time-series forecasting; Huarng and Yu [6] applied back propagation neural network to handle nonlinear forecasting problems in stock price forecasting; Chen et al. [3] presented high-order fuzzy time series based on multi-period adaptation model for stock markets forecasting. Furthermore, Chen et al. [2], Wang and Chen [18], Lee et al. [10], and Kou et al. [9] proposed different methods for Taiwan stock market forecasting.

Clearly, fuzzy time series model has been applied to business and engineering; however, fuzy time series model is still necessary to overcome its drawbacks. Khashei et al. [7] improved the forecasting performance by hybriding artificial neural networks and fuzzy model using autoregressive integrated moving average models. Besides, Moller and Reuter [12] proposed a new fuzzy ARMA model for the analysis and forecasting of time series with fuzzy data in the model-free and of model-based forecasting. Theoretically, if we applied the collected data in a Group A to construct a forecasting model for extrapolation, then, because of its similar structure, and when all conditions remain the same, we could use Group A for forecasting. However, once the trend of future changes in Group B is determined, the derived model cannot be used for forecasting because fuzzy time series model using fuzzy logical relationship to derive the forecasting values. If the derive fuzzy time series model cannot fit the data trend, then we may obtain worse forecasting results. Besides, when the collected time series data is limited with little information, each data is equally important for forecasting, and then only part of system structure could be fully captured which also contributs worse forecasting. In order to cope with mis-trend and limited data drawbacks in the fuzzy time series model, residual analysis becomes quite important in order to reuse some possile useful information [11]. The idea of Fourier transform is to fit the estimated residuals in to frequency spectra and then select low-frequency terms. On the other hand, Fourier series transform can filter out high-frequency terms, which are supposed to be noise, and then forecasting performance can be improved.

In Section 2, the basic concept of fuzzy time series model is introduced. Then, the residual analysis based on Fourier series transform is performed in Section 3. Besides, the forecasting performance between the proposed method and the other methods are evaluated and discussed with illustrated examples in Section 4. Finally, conclusion is drawn in Section 5.

2. **Concept of Fuzzy Time Series**

Let the universe of discourse $U = \{u_1, u_2, \ldots, u_n\}$ in which fuzzy set $\widetilde{A}_i$ ($i = 1, 2, \ldots, n$) is defined by

$$\widetilde{A}_i = \frac{f_{\widetilde{A}_i}(u_1)}{u_1} + \frac{f_{\widetilde{A}_i}(u_2)}{u_2} + \cdots + \frac{f_{\widetilde{A}_i}(u_n)}{u_n} \tag{1}$$

where $f_{\widetilde{A}_i}$ is the membership function of the fuzzy set $\widetilde{A}_i$, u_k is an element of fuzzy set $\widetilde{A}_i$, and $f_{\widetilde{A}_i}(u_k)$ is the membership degree of u_k belonging to $\widetilde{A}_i$, $\forall\ k = 1, 2, \ldots, n$.

Definition 2.1. Let the universe of discourse $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots, n, \ldots)$ be a subset of $\mathbf{R}$ defined by the fuzzy set $\widetilde{A}_i$. If $F(t)$ consists of $\widetilde{A}_i$ $(i = 1, 2, \ldots, n)$, $F(t)$ is defined as a fuzzy time series on $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots, n, \ldots)$.

Definition 2.2. Suppose $F(t)$ is caused by $F(t-1)$, then the relation of the *first-order model* of $F(t)$ is expressed as $F(t) = F(t-1) \circ R(t, t-1)$ where $R(t, t-1)$ is the relation matrix described by the fuzzy relationship between $F(t-1)$ and $F(t)$, and $\circ$ is max-min operator.

Let the relationship between $F(t)$ and $F(t-1)$ be denoted by $F(t-1) \to F(t)$, $(t = 0, 1, 2, \ldots, n)$. Then, the fuzzy logical relationship between $F(t)$ and $F(t-1)$ can be defined as below:

Definition 2.3. Suppose $F(t) = \widetilde{A}_j$ is caused by $F(t-1) = \widetilde{A}_i$, then the fuzzy logical relationship is defined as $\widetilde{A}_i \to \widetilde{A}_j$.

If there are fuzzy logical relationship obtained from state $\widetilde{A}_2$ and make a transition to another state $\widetilde{A}_j$, $\forall\ j = 1, 2, \ldots, n$, as $\widetilde{A}_2 \to \widetilde{A}_3$, $\widetilde{A}_2 \to \widetilde{A}_2$, …, $\widetilde{A}_2 \to \widetilde{A}_1$, then the fuzzy logical relationships can be grouped as follows:

$$\widetilde{A}_2 \to \widetilde{A}_1, \widetilde{A}_2, \widetilde{A}_3, \ldots \tag{2}$$

Although, various models have been proposed to establish fuzzy relationships, Chen's fuzzy logical relationship group approach [1] is easily used in proposed model. Without loss of generality, we describe the fuzzy time series model proposed by Chen's method [1] which is described as following steps.

Step 1: Define the universal discourse U for the historical data. When defining the universe, the minimum data $D_{\min}$ and the maximum data $D_{\max}$ of given historical data are first defined. Based on $D_{\min}$ and $D_{\max}$, we define the universal discourse U as $[D_{\min} - D_1, D_{\max} + D_2]$, where D_1 and D_2 are two proper positive numbers.

Step 2: Partition universal discourse U into several equal intervals. If U is partitioned into n equal intervals; the difference between two successive intervals can be defined as follows:

$$\ell = [(D_{\max} + D_2) - (D_{\min} - D_1)]/n, \tag{3}$$

then each interval could be obtained as $u_1 = [D_{\min} - D_1, D_{\min} - D_1 + \ell]$, $u_2 = [D_{\min} - D_1 + \ell, D_{\min} - D_1 + 2\ell]$, …, $u_n = [D_{\min} - D_1 + (n-1)\ell, D_{\max} + D_2]$.

Step 3: Define fuzzy sets on universal discourse U. In S & C's model, there is no restriction on determining how many linguistic variables to be fuzzy sets. Therefore, the fuzzy sets are defiend as $\widetilde{A}_1$ =(not many), $\widetilde{A}_2$ =(not too many), $\widetilde{A}_3$ =(many), $\widetilde{A}_4$ =(many many), $\widetilde{A}_5$ =(very many), $\widetilde{A}_6$ =(too many), $\widetilde{A}_7$ =(too many many). Each fuzzy set $\widetilde{A}_i$ $(i = 1, 2, \ldots, 7)$ is defined below by $\widetilde{A}_i = \{(\mu_{\widetilde{A}_i}(u_j)/u_j) | \mu_{\widetilde{A}_i}(u_j) \in$

$[0,1]$, $u_j \in R$, $j = 1, \ldots, 7\}$ with the membership degree $\mu_{\widetilde{A}_i}(u_j)$ of u_j as folows:

$$\begin{aligned}
\widetilde{A}_1 &= \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7}, \\
\widetilde{A}_2 &= \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7}, \\
\widetilde{A}_3 &= \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7}, \\
\widetilde{A}_4 &= \frac{0}{u_1} + \frac{0}{u_2} + \frac{0.5}{u_3} + \frac{1}{u_4} + \frac{0.5}{u_5} + \frac{0}{u_6} + \frac{0}{u_7}, \\
\widetilde{A}_5 &= \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0.5}{u_4} + \frac{1}{u_5} + \frac{0.5}{u_6} + \frac{0}{u_7}, \\
\widetilde{A}_6 &= \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0.5}{u_5} + \frac{1}{u_6} + \frac{0.5}{u_7}, \\
\widetilde{A}_7 &= \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0.5}{u_6} + \frac{1}{u_7},
\end{aligned}$$

Step 4: Fuzzify the historical data. This is to find an equivalent fuzzy set for each input data. The commonly used method is to define a cut set for each $\widetilde{A}_i$ $(i = 1, \ldots, 7)$.

Step 5: Determine fuzzy logical relationship group. By the **Definition 3**, the fuzzy logical relationship group can be easily obtained.

Step 6: Calculate the forecasted outputs. If $F(t-1) = \widetilde{A}_j$, the forecasting of $F(t)$ is conducted based on the following rules. *Rule* 1: If the fuzzy logical relationship group of $\widetilde{A}_j$ is one-to-one (i.e. $\widetilde{A}_j \to \widetilde{A}_k$, $\forall\, j$, $k = 1, 2, \ldots, 7$), then the forecasting of $F(t)$ is m_k, the midpoint of u_k:

$$F(t) = m_k \tag{4}$$

Rule 2: If the fuzzy logical relationship group of $\widetilde{A}_j$ is one-to-many (i.e. $\widetilde{A}_j \to \widetilde{A}_1, \widetilde{A}_3, \widetilde{A}_5$, $\forall\, j = 1, 2, \ldots, 7$), then the forecasting of $F(t)$ is equal to the arithmetic average of m_1, m_3, m_5, the midpoint of u_1, u_3, u_5:

$$F(t) = \frac{m_1 + m_3 + m_5}{3} \tag{5}$$

Although, fuzzy time series models have been applied and designed to forecast linguistic or small a fifty data, it is still a developing method, where any innovation in improving forecasting performance is important. The more the informations related to the system dynamics are considered, the better the prediction will be. Therefore, esidual analysis is incorporated with the fuzzy time series model to further enhance the predicted accuracy.

3. **Fuzzy Time Series for Residual Analysis**

Residual analysis is important in regression analysis, where a well-defined model always derives a random distributed estimated error. Fuzzy time series model uses the fuzzy logic method to forecast without considering the data type, whereas some large forecasting error terms usually reduce its forecasting performance. Therefore, any adjustment for these large forecasting errors to be lower error terms should be considered in order to promote forecasting performance. In order to cope with such problem, the idea of Fourier series transform is selected to transform the residuals into frequency spectra, select the low-frequency terms, filter out high-frequency terms which are supported to be noise, and then have well forecasting performance. The analysis steps are described as following steps.

Step 1: Define the universe of discourse U for the historical data. First, we find the minimum data $D_{\min}$ and the maximum data $D_{\max}$ in the historical time series data, then U is defined as $D_{\min} - D_1, D_{\max} + D_2]$, where D_1 and D_2 are two proper positive numbers.

Step 2: Define the fuzzy sets $\widetilde{A}_i$ on universe of discourse U. Partition the collected data into equal length of intervals $u_1, u_2, \ldots, u_n$, then the fuzzy sets $\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_n$ are defined as $\widetilde{A}_i = \mu_{\widetilde{A}_i}(u_1)/u_1 + \mu_{\widetilde{A}_i}(u_2)/u_2 + \cdots + \mu_{\widetilde{A}_i}(u_n)/u_n$, $\forall\, i = 1, 2, \ldots, n$.

Step 3: Establish the fuzzy logical relationships. Fuzzify the historical data and fit to their fuzzy sets $\widetilde{A}_i$ $(i = 1, 2, \ldots, n)$ then we use the second-order fuzzy relationship described as $\widetilde{A}_i$, $\widetilde{A}_j \to \widetilde{A}_k$ which means that times $t-2$ and $t-1$ are in states $\widetilde{A}_i$ and $\widetilde{A}_j$, respectively and time t is in the next state $\widetilde{A}_k$. Finally, we can establish fuzzy relationship groups for all fuzzy relationships and group them.

Step 4: Calculate the forecasted outputs. Based on the second-order fuzzy logical relationships, if $F(t-2) = \widetilde{A}_i$ and $F(t-1) = \widetilde{A}_j$, then the forecasting $F(t)$ is obtained by the following rules.

Rule 1: If the fuzzy logical relationship group is $\widetilde{A}_i, \widetilde{A}_j \to \widetilde{A}_y$, then the forecasting of $F(t)$ is

$$F(t) = m_y \tag{6}$$

where m_y is the midpoint of u_y, $\forall\, y = 1, 2, \ldots, n$.

Rule 2: If the fuzzy logical relationship groups are $\widetilde{A}_i, \widetilde{A}_j \to \widetilde{A}_1$, $\widetilde{A}_i, \widetilde{A}_j \to \widetilde{A}_2$, …, $\widetilde{A}_i, \widetilde{A}_j \to \widetilde{A}_p$ then the forecasting of $F(t)$ is

$$F(t) = \frac{m_1 + m_2 + \cdots + m_p}{p}, \qquad p \le n \tag{7}$$

Step 5: Adjusted forecasting result. The aim of residual analysis is to transform the residuals into frequency spectra, select the low-frequency terms, filter out high-frequency terms, and then have well forecasting performance. The analysis process for the residual series are derived as following:

$$\varepsilon(k) = Y(k) - F(k), \qquad \forall\, k = 1, 2, \ldots, n, \tag{8}$$

where $Y(k)$ and $F(k)$ are the actual value and forecasting value at time k, respectively, and the matrix of the estimated residual series could be derived as below

$$\boldsymbol{\varepsilon(k)} = [\varepsilon(1), \varepsilon(2), \ldots, \varepsilon(n-1), \varepsilon(n)]^T \tag{9}$$

Next, Fourier series trnsform can be used to catch the implied periodic phenomenon in the residual series. Then, using Fourier correction technique in residual analysis, we can increase prediction capability from the considered input data set. The estimated residual series can be modeled by Fourier series transform as

$$\hat{\varepsilon}(k) = \frac{1}{2} + \sum_{i=1}^{z}\left[a_i \cos\left(\frac{2\pi i}{T}k\right) + b_i \sin\left(\frac{2\pi i}{T}k\right)\right], \quad \text{for } k = 1, 2, \ldots, n \tag{10}$$

where T is the length of the residual series which is equal to N, and z is the minimum deployment frequency of Fourier series transform which is suggested to the integer portion of $[(n-1)/2]$. The parameters a_0, a_i and b_i for $i = 1, 2, \ldots, z$ are estimated using the least square method and obtained as

$$\boldsymbol{C} = (\boldsymbol{P}^T\boldsymbol{P})^{-1}\boldsymbol{\varepsilon}(\boldsymbol{k}) \tag{11}$$

where $\boldsymbol{C} = [a_0, a_1, b_1, a_2, b_2, \ldots, a_z, b_z]^T$ and matrix $\boldsymbol{P}$ is shown as below

$$\boldsymbol{P} = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi}{T}\right) & \sin\left(\frac{2\pi}{T}\right) & \cos\left(\frac{2\pi 2}{T}\right) & \sin\left(\frac{2\pi 2}{T}\right) & \cdots & \cos\left(\frac{2\pi z}{T}\right) & \sin\left(\frac{2\pi z}{T}\right) \\ \frac{1}{2} & \cos\left(2\frac{2\pi}{T}\right) & \sin\left(2\frac{2\pi}{T}\right) & \cos\left(2\frac{2\pi 2}{T}\right) & \sin\left(2\frac{2\pi 2}{T}\right) & \cdots & \cos\left(2\frac{2\pi z}{T}\right) & \sin\left(2\frac{2\pi z}{T}\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{2} & \cos\left(N\frac{2\pi}{T}\right) & \sin\left(N\frac{2\pi}{T}\right) & \cos\left(N\frac{2\pi 2}{T}\right) & \sin\left(N\frac{2\pi 2}{T}\right) & \cdots & \cos\left(N\frac{2\pi z}{T}\right) & \sin\left(N\frac{2\pi z}{T}\right) \end{bmatrix} \tag{12}$$

substituting the estimated parameters solved by equation (11) into equation (10), we obtain the estimated residual series as follows:

$$\hat{\boldsymbol{\varepsilon}}(\boldsymbol{k}) = [\hat{\varepsilon}(1), \hat{\varepsilon}(2), \ldots, \hat{\varepsilon}(n), \hat{\varepsilon}(n+1)]^T \tag{13}$$

The revised forecasting value of this step can be obtained as

$$\hat{Y}'(t+1) = F(t+1) + \hat{\varepsilon}(t+1), \tag{14}$$

where $F(t+1)$ and $\hat{\varepsilon}(t+1)$ are forecasting and estimated residual values at time $t+1$.

4. **Illustrations**

In this section, two illustrations are used for showing the forecasting performance in our proposed method. Example 1 is an enrollment forecasting comparisons among revised fuzzy time series models; and example 2 is the Japanese tourists forecasting between our proposed method and some statistcal models.

Example 4.1. An enrollment forecasting adopted from Song & Chissom [14] is illustrated for experiment. In this experiment, we use the data of student enrollments at University of Alabama from 1971-1992 for predicting and showing the proposed model is better than others.

Step 1: Define the universe of discourse U for the historical data. In Table 1, the minimum enrollment $D_{\min} = 13055$ and the maximum enrollment $D_{\max} = 19337$ with $D_1 = 55$ and $D_2 = 663$. Therefore, the universe of discourse $U = [13000, 20000]$.

Step 2: Define the fuzzy sets $\widetilde{A}_i$ on universe of discourse U. U is dividend into seven intervals as $u_1 = [13000, 14000]$, $u_2 = [14000, 15000]$, $u_3 = [15000, 16000]$, $u_4 = [16000, 17000]$, $u_5 = [17000, 18000]$, $u_6 = [18000, 19000]$, $u_7 = [19000, 20000]$. The linguistic variables are fuzzy sets defined as $\widetilde{A}_1$, $\widetilde{A}_2$, …, $\widetilde{A}_7$.

Step 3: Establish the fuzzy logical relationships. The fuzzy sets for each enrollment is listed in Table 2 whose second-order fuzzy logical relationships are derived as $\widetilde{A}_1, \widetilde{A}_1 \to \widetilde{A}_1$, $\widetilde{A}_1, \widetilde{A}_1 \to \widetilde{A}_2$, $\widetilde{A}_1, \widetilde{A}_1 \to \widetilde{A}_3$, $\widetilde{A}_2, \widetilde{A}_3 \to \widetilde{A}_3$, $\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_3$, $\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_3$, $\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_4$, $\widetilde{A}_3, \widetilde{A}_4 \to \widetilde{A}_4$, $\widetilde{A}_4, \widetilde{A}_4 \to \widetilde{A}_4$, $\widetilde{A}_4, \widetilde{A}_4 \to \widetilde{A}_3$, $\widetilde{A}_4, \widetilde{A}_3 \to \widetilde{A}_3$, $\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_3$, $\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_3$, $\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_3$, $\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_4$, $\widetilde{A}_3, \widetilde{A}_4 \to \widetilde{A}_6$, $\widetilde{A}_4, \widetilde{A}_6 \to \widetilde{A}_6$, $\widetilde{A}_6, \widetilde{A}_6 \to \widetilde{A}_7$, $\widetilde{A}_6, \widetilde{A}_7 \to \widetilde{A}_7$, $\widetilde{A}_7, \widetilde{A}_7 \to \widetilde{A}_6$. Next, we use the above relationships to establish fuzzy relationship groups and show in Table 2.

Step 4: Calculate the forecasted outputs. In the column 3 of Table 2, applying the rules described in equations (6) and (7), we can obtain these forecasting values.

Step 5: Adjusted the forecasting results. By equation (8), the residual terms between actual value and forecasted value can be derived abd shown in 4^{th} column of Table 3. Next, in order to obtain more precise forecasting values, we use the collected data to solve the parameters of Fourier series by equation (11) which is used to obtain the estimated residuals as equation (13), filter out the noise terms and obtain the adjusted residual terms as shown in the fifth column of Table 3.

After operating the above steps, the forecasting performance is measured in terms of the mean absolute percentage error (MAPE). In order to compare the forecasting accuracy of our proposed model to other models, the MAPE and RMSE have been computed for all the methods which are shown in Table 4 and their comparison for the forecasting values are shown in Figure 1. Clearly, our proposed model has a smaller MAPE than others, which is much better.

Year	Actual enrollment	Fuzzified enrollment	Year	Actual enrollment	Fuzzified enrollment
1971	13055	$\widetilde{A}_1$	1982	15433	$\widetilde{A}_3$
1972	13563	$\widetilde{A}_1$	1983	15497	$\widetilde{A}_3$
1973	13867	$\widetilde{A}_1$	1984	15145	$\widetilde{A}_3$
1974	14696	$\widetilde{A}_2$	1985	15163	$\widetilde{A}_3$
1975	15460	$\widetilde{A}_3$	1986	15984	$\widetilde{A}_3$
1976	15311	$\widetilde{A}_3$	1987	16859	$\widetilde{A}_4$
1977	15603	$\widetilde{A}_3$	1988	18150	$\widetilde{A}_6$
1978	15861	$\widetilde{A}_3$	1989	18970	$\widetilde{A}_6$
1979	16807	$\widetilde{A}_4$	1990	19328	$\widetilde{A}_7$
1980	16919	$\widetilde{A}_4$	1991	19337	$\widetilde{A}_7$
1981	16388	$\widetilde{A}_4$	1992	18876	$\widetilde{A}_6$

TABLE 1. Historical Actual Enrollments and Fuzzy Enrollments

Example 4.2. Over the past few decades, the Japanese tourists are Taiwan's major visitors in the tourism market, because most Japanese are conscious of the closely historical and geographic relationship between Japan and Taiwan, in this

Fuzzy relationship groups			Forecasting value
Group 1	$\widetilde{A}_1, \widetilde{A}_1 \to \widetilde{A}_1$	$\widetilde{A}_1, \widetilde{A}_1 \to \widetilde{A}_2$	$14000(= m_1 + m_2)/2)$
Group 2	$\widetilde{A}_1, \widetilde{A}_2 \to \widetilde{A}_3$		$15500(= m_3)$
Group 3	$\widetilde{A}_2, \widetilde{A}_3 \to \widetilde{A}_3$		$15500(= m_3)$
Group 4	$\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_3$	$\widetilde{A}_3, \widetilde{A}_3 \to \widetilde{A}_4$	$16000(= m_3 + m_4)/2)$
Group 5	$\widetilde{A}_3, \widetilde{A}_4 \to \widetilde{A}_4$	$\widetilde{A}_3, \widetilde{A}_4 \to \widetilde{A}_6$	$17500(= m_4 + m_6)/2)$
Group 6	$\widetilde{A}_4, \widetilde{A}_4 \to \widetilde{A}_4$	$\widetilde{A}_4, \widetilde{A}_4 \to \widetilde{A}_3$	$16000(= m_4 + m_3)/2)$
Group 7	$\widetilde{A}_4, \widetilde{A}_3 \to \widetilde{A}_3$		$15500(= m_3)$
Group 8	$\widetilde{A}_4, \widetilde{A}_6 \to \widetilde{A}_6$		$18500(= m_6)$
Group 9	$\widetilde{A}_6, \widetilde{A}_6 \to \widetilde{A}_7$		$19500(= m_7)$
Group 10	$\widetilde{A}_6, \widetilde{A}_7 \to \widetilde{A}_7$		$19500(= m_7)$
Group 11	$\widetilde{A}_7, \widetilde{A}_7 \to \widetilde{A}_6$		$18500(= m_6)$

Table 2. Fuzzy Logical Relationship and Forecasted Values

Year	Actual enrollment	Forecasting value	Residual	Adjusted residual	Adjusted forecasting
1971	13055				
1972	13563				
1973	13867	14000	-133	-133	13867
1974	14696	14000	696	882.388	14882.39
1975	15460	15500	-40	-171.57	15328.43
1976	15311	15500	-189	-301.895	15198.11
1977	15603	16000	-397	-310.17	15689.83
1978	15861	16000	-139	-163.561	15836.44
1979	16807	16000	807	896.049	16896.05
1980	16919	17500	-581	-527.066	16972.93
1981	16388	16000	388	265.29	16265.29
1982	15433	16000	-567	-546.807	15453.19
1983	15497	15500	-3	-14.455	15485.55
1984	15145	16000	-855	-853.119	15146.88
1985	15163	16000	-837	-724.818	15275.18
1986	15984	16000	-16	-75.28	15924.72
1987	16859	16000	859	772.452	16772.45
1988	18150	17500	650	682.677	18182.68
1989	18970	18500	470	363.633	18863.63
1990	19328	19500	-172	-18.616	19481.38
1991	19337	19500	-163	-74.004	19426
1992	18876	18500	376	205.869	18705.87

Table 3. Enrollment Forecasting

example, for simplicity, the research period is limited on 1993-2008. Because the importance of the tourists from Japan, an accurate tourism demand forecasting can support much information to improve the tourism envrionment, including the linkage of scenic sites, developing new tourism spots, facility of transportation vehicles, development, training for the tourist local guides, and more supply of hotel graduates. The forecasting procedure for the the proposed method is described as the following steps. The minimum enrollment $D_{\min}$ is 323375 and the maximum enrollment $D_{\max}$ is 721351, then arbitrary chosen $D_1 = 375$ and $D_2 = 649$ derived $U = [323000, 722000]$. Next, we cut the intervals per 1000 in order to obtain more precise forecasting result, then the universe of discourse U is dividend into 399 intervals as $u_1 = [323000, 324000]$, $u_2 = [324000, 325000]$, ..., $u_{399} = [721000, 722000]$. Thus, the corresponding fuzzy sets are obtained as $\widetilde{A}_1$, $\widetilde{A}_2, \ldots, \widetilde{A}_{399}$.

By analyzing the fuzzy logical relationships, we can obtain fuzzy logical relationship groups as shown in Table 5, then their forecasting values are obtained in 3-th and 6-th columns of table 5. We take the residuals from 1995 to 2008 in the 5-th column of Table 6 into equation (10), the parameters a_0, a_i and b_i in Fourier transform for $i = 1, 2, \ldots, 5$, are solved. Then, we can obtain the adjusted residual shown in 6-th column of Table 6, where the adjusted forecasting values are obtained in 7-th column. At the end of the illustration, performance comparisons among the forecasting models are shown in Table 7, including exponential smoothing model (ESM), quadratic regression, cubic regression, ARIMA(1, 0, 1), ARIMA(1, 0, 2). Clearly, our proposed method makes good forecasts.

Year	enrollment	Proposed method	Song & Chissom [14]	S.R. Singh [13]	Cheng et al. [3]	Chen [1]
1971	13055					
1972	13563		14000		13680.5	14000
1973	13867	13867	14000		13731.3	14000
1974	14696	14882.39	14000	14500	13761.7	14000
1975	15460	15328.43	15500	15358	15194.6	15500
1976	15311	15198.11	16000	15500	15374.8	16000
1977	15603	15689.83	16000	15500	15359.9	16000
1978	15861	15836.44	16000	15500	16410.3	16000
1979	16807	16896.05	16000	16500	16436.1	16000
1980	16919	16972.93	16813	16500	17130.7	16833
1981	16388	16265.29	16813	16500	17141.9	16833
1982	15433	15453.19	16789	15581	15363.8	16833
1983	15497	15485.55	16000	15500	15372.1	16000
1984	15145	15146.88	16000	15500	15378.5	16000
1985	15163	15275.18	16000	15500	15343.3	16000
1986	15984	15924.72	16000	15500	15345.1	16000
1987	16859	16772.45	16000	16402	16448.4	16000
1988	18150	18182.68	16813	18500	17135.9	16833
1989	18970	18863.63	19000	18500	189157	19000
1990	19328	19481.38	19000	19471	18997	19000
1991	19337	19426	19000	19500	19032.8	19000
1992	18876	18705.87		19651	19033.7	19000
MAPE(%)		**0.49512**	**3.22376**	**1.71204**	**2.08722**	**3.11005**

TABLE 4. Comparisons Forecast Result with Different Models

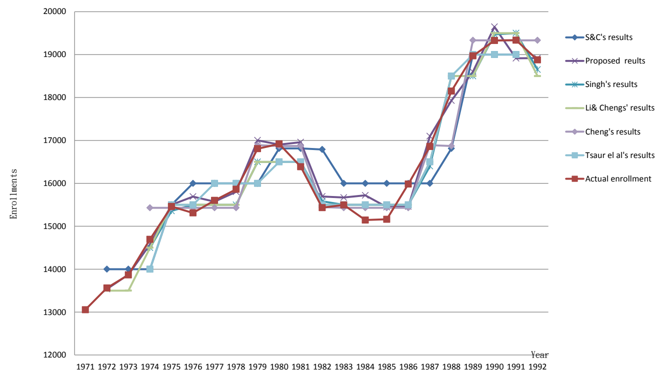

FIGURE 1. Comparisons Among the Fuzzy Time Series Methods

Fuzzy relationship groups		Forecasting value	Fuzzy relationship groups		Forecasting value
Group 1	$\widetilde{A}_{161}, \widetilde{A}_{252} \to \widetilde{A}_{304}$	626500	Group 7	$\widetilde{A}_{159}, \widetilde{A}_{205} \to \widetilde{A}_{269}$	591500
Group 2	$\widetilde{A}_{252}, \widetilde{A}_{304} \to \widetilde{A}_{283}$	605500	Group 8	$\widetilde{A}_{205}, \widetilde{A}_{269} \to \widetilde{A}_{265}$	587500
Group 3	$\widetilde{A}_{161}, \widetilde{A}_{283} \to \widetilde{A}_{253}$	575500	Group 9	$\widetilde{A}_{269}, \widetilde{A}_{265} \to \widetilde{A}_{1}$	323500
Group 4	$\widetilde{A}_{283}, \widetilde{A}_{253} \to \widetilde{A}_{175}$	497500	Group 10	$\widetilde{A}_{265}, \widetilde{A}_{1} \to \widetilde{A}_{138}$	460500
Group 5	$\widetilde{A}_{253}, \widetilde{A}_{175} \to \widetilde{A}_{159}$	481500	Group 11	$\widetilde{A}_{1}, \widetilde{A}_{138} \to \widetilde{A}_{355}$	677500
Group 6	$\widetilde{A}_{175}, \widetilde{A}_{159} \to \widetilde{A}_{205}$	527500	Group 12	$\widetilde{A}_{138}, \widetilde{A}_{355} \to \widetilde{A}_{399}$	721500

TABLE 5. Fuzzy Logical Relationship Groups and the Forecasting Values

Year	Actual tourist	Fuzzified tourist	Forecasting value	Residual	Adjusted residual	Adjusted forecasting value
1993	483,481	$\widetilde{A}_{161}$				
1994	574,323	$\widetilde{A}_{252}$				
1995	626,152	$\widetilde{A}_{304}$	626500	-348	-160.874	626339.126
1996	605,673	$\widetilde{A}_{283}$	605500	173	145.776	605645.776
1997	575,613	$\widetilde{A}_{253}$	575500	113	82.172	575582.172
1998	497,928	$\widetilde{A}_{175}$	497500	428	498.277	497998.277
1999	481,544	$\widetilde{A}_{159}$	481500	44	-14.765	481485.235
2000	527,074	$\widetilde{A}_{205}$	527500	-426	-309.605	527190.395
2001	591,081	$\widetilde{A}_{269}$	591500	-419	-491.602	591008.398
2002	587,170	$\widetilde{A}_{265}$	587500	-330	-344.256	587155.744
2003	323,375	$\widetilde{A}_{1}$	323500	-125	-168.927	323331.073
2004	460,231	$\widetilde{A}_{138}$	460500	-269	-223.418	460276.582
2005	677,937	$\widetilde{A}_{355}$	677500	437	352.581	677852.581
2006	721,351	$\widetilde{A}_{399}$	721500	-149	-83.918	721416.082
2007	737,638	$\widetilde{A}_{451}$	737500	138	-104.79	737395.21
2008	674,506	$\widetilde{A}_{352}$	674500	6	96.35	674596.35

TABLE 6. Adjusted Tourist Forecasting

	ESM	Quadratic regression	Cubic regression	ARIMA (1,0,2)	ARIMA (1,0,1)	Proposed method
MAPE	15.072%	13.972%	13.099%	11.204%	12.202%	1.387%

Table 7. Comparisons among the Forecasting Methods

5. Conclusion

Since the fuzzy time series model proposed, it has been successfully adopted in various fields. However, the precesion of fuzzy time series model may be unsatisfied when some large forecasting error terms usually reduce its forecasting performance. Hence, this paper has proposed a new residual analysis method which combines Fourier series transform into fuzzy time series model. This combined model takes the advantage of Fourier series transform to select the low-frequency terms and filter out high-frequency terms, and then obtain the adjusted forecasting values to promote the forecasting performance. Finally, the illustrated example indicated both the forecasting results and the analysis confirm the potential benefits of the new approach. Most importantly, the illustrated example is archived with smaller MAPE. If the Fourier series transform in the fuzzy time series model meets its expectations, then this approach will be easily applied to be an important tool in forecasting limited data.

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