

MODELING OF EPISTEMIC UNCERTAINTY IN RELIABILITY ANALYSIS OF STRUCTURES USING A ROBUST GENETIC ALGORITHM

M. BAGHERI, M. MIRI AND N. SHABAKHTY

ABSTRACT. In this paper the fuzzy structural reliability index was determined through modeling epistemic uncertainty arising from ambiguity in statistical parameters of random variables. The First Order Reliability Method (FORM) has been used and a robust genetic algorithm in the alpha level optimization method has been proposed for the determination of the fuzzy reliability index. The sensitivity level of fuzzy response due to the introduced epistemic uncertainty was also measured using the modified criterion of Shannon entropy. By introducing bounds of uncertainty, the fuzzy response obtained from the proposed method presented more realistic estimation of the structure reliability compared to classic methods. This uncertainty interval is of special importance in concrete structures since the quality of production and implementation of concrete varies in different cross sections in reality. The proposed method is implementable in reliability problems in which most of random variables are fuzzy sets and in problems containing non-linear limit state functions and provides a precise acceptable response. The capabilities of the proposed method were demonstrated using different examples. The results indicated the accuracy of the proposed method and showed that classical methods like FORM cover only special case of the proposed method.

1. Introduction

A typical structural reliability analysis deals with models which are mathematical idealizations of the physical events. The idealization requires the definition of the basic random variables describing the geometry, loads and material properties. The mathematical model (G) establishes a relationship between the resistance (R) and the load (S) of structural components. This function is known as the Limit State Function (LSF) or performance function ($G = R - S$). In LSF, we encounter parameters which do not possess constant values and are of random and vague identity in nature. Hence, we surely face a sort of uncertainty in the safety assessment of structures. The nature of uncertainties and the methods of their modeling in assessing the safety of structures have been of researchers' interest for many years [29, 34, 27, 22, 21].

A popular classification of uncertainty, with respect to its sources, distinguishes between aleatory and epistemic uncertainty [5, 8]. Aleatory uncertainty is the

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representative of unknowns that differ each time we run the same experiment. It is random in nature and generally related to the uncertainty of the outcome of an event or experiment. Aleatory uncertainty is primarily associated with objectivity and cannot be reduced [42]. This type of uncertainty can be modeled with pure probabilistic methods. Using probabilistic methods employing random variables in assessing the safety of structures, has been of interest for many years and various analytical and simulation methods such as FORM [17, 30, 24, 23], SORM [7, 2, 47], Monte Carlo Simulation (MCS) [10, 36] and Importance Sampling (IS) [14, 45] have been developed.

Epistemic uncertainty is due to things that we could know in principle but couldn't in practice. This may be because we have not measured its quantity accurately, or because our model neglects certain effects or that particular data are deliberately hidden. Epistemic uncertainty may be comprised of substantial amounts of both objectivity and subjectivity simultaneously. This type of uncertainty is constituted on a non-probabilistic or a mixed of probabilistic and non-probabilistic mathematical basis. Epistemic uncertainty is a result of lack of adequate knowledge and information regarding the identity of the problem under investigation and may be caused by ambiguity in defining statistical parameters of random variables. The more knowledge about the respective problem, results in less of this uncertainty [42].

Due to lack of access to precise statistical properties of random variables in structural reliability problems, we generally encounter a combination of the two types of uncertainties. This statistical ambiguity may be caused by the small number of statistical samples, multiplicity of test equipment for determination of mechanical properties of materials and human errors in the sampling and measurement process of these properties. Hence, the random variables should simultaneously comprise both aleatory and epistemic uncertainty in order to have an exact mathematical model of LSF and consequently a reliable safety index.

Epistemic uncertainty may be modeled using fuzzy sets [48, 40] intervals [1, 41] convex sets [6, 39] and fuzzy random variables [25, 26, 31]. Furthermore, probabilistic methods do not provide an accurate realistic estimation of the safety level of structures, because these methods do not integrate epistemic uncertainty in the process of analyzing structural reliability [18].

A variety of mathematical approaches have been formulated, besides pure probabilistic, to take account of the available information as naturally as possible using fuzzy sets. These approaches combined aleatory and epistemic uncertainties in the process of analysis and safety assessment of structures. Fuzzy analysis [32, 16], fuzzy finite element [28, 35, 11], fuzzy optimization [38], and fuzzy reliability [33, 46, 12, 15], may be pointed out from these approaches.

In fuzzy structural reliability, random variables are modeled as fuzzy sets and fuzzy reliability index can be determined using methods such as the extension principle [44], sampling [20], vertex [9], function approximation [19], and alpha level optimization [32]. Sampling method is not computationally efficient in high dimensional problems because an appropriate sampling requires a large number of samples [20]. Vertex method is limited to monotonic and continuous functions

and does not yield an accurate response in problems with discontinuous and non-monotonic functions. Function approximation method substitutes the objective function with an approximate function called response surface. Thus, the accuracy of the response depends on the substituted function.

From the above methods, it is only optimization method which is not limited to a certain type of functions and results the accurate response [20]. However, existing optimizations methods are predominantly gradient-based, complicated and computationally expensive [32, 33]. In this paper an alternative method has been proposed in order to arrive at a precise response and decrease the computational burden. Through introducing bounds of uncertainty, the obtained fuzzy reliability index provides a more realistic estimation of structural reliability compared to pure probabilistic methods.

2. Basic First Order Reliability Method

The main problem of structural reliability analysis is to estimate the failure probability of a structural component by evaluating the following integral [34]:

$$P_f = \int_{G(X) \leq 0} f_X(X_1, \dots, X_n) dX_1 \dots dX_n \quad (1)$$

where P_f is the failure probability, X_1, \dots, X_n are the basic random variables, X is the n -dimensional vector of random variables, $G(X)$ is the LSF ($G(X) \leq 0$ defines the failure domain of structural components) and f_X is the Joint Probability Density Function (JPDF) for the basic random variables. The failure and safe regions are shown in Figure 1. The integration of equation (1) is highly complex. Since there is no analytical solution for general cases available, approximation methods are generally used for these problems [34, 27, 21]. Evaluating the failure probability of the performance function is feasible with the simulation or sampling methods, such as MCS and IS.

In MCS the standard space is randomly sampled with numerous independent samples. These samples are then transformed to the original space and an estimate of failure probability is finally obtained from the sample mean. It is necessary to note that MCS requires large samples for small failure probabilities. To meet this disadvantage, reliability analysis based on simulation methods in combination with an adaptive low order polynomial response surface are extended using neural networks and splines [10]. This methodology makes use of the capability of an ANN to approximate a function for reproducing structural behavior, allowing the computation of performance measures at a much lower cost.

Another technique in this regard is IS [14, 45]. For an IS analysis, the samples are produced around the design point coming from a previous FORM analysis. Thus, IS requires previous information about the failure regions to be useful, moreover it faces challenges with high-dimension problems. However these methods are extremely computer intensive for complex physical simulations, such as dynamic problems, finite element methods, low failure probability and determination of fuzzy structural reliability analysis [4].

Since in this study the large number of reliability analysis has to be done, using simulation methods such as MCS and even improved ones entails a huge computational burden as it is too time consuming even for a simple structure. Therefore FORM as an iterative method has been used.

FORM aims at using a first-order approximation of the LSF in the standard normal space at the so-called design point U^* , which is the limit-state surface closest point to the origin [17]. The coordinate of design point can be found using the following constrained optimization problem.

$$U^* = \arg \min\{\|U\| \mid G(U) = 0\} \quad (2)$$

where $G(U)$ is the LSF in the standard normal space. By using the design point, the Hasofer-Lind reliability index (β) is computed as follows:

$$\beta = \alpha^T U^* \quad (3)$$

where $\alpha = \frac{-\nabla_U G(U^*)}{\|\nabla_U G(U^*)\|}$ is the negative normalized gradient vector which is called sensitivity factor and ∇_U is the gradient operator (Figure 1). It represents the distance from the origin to the design point in the standard space. The first order approximation of failure probability is defined as follows: $P_f = \Phi(-\beta)$ where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

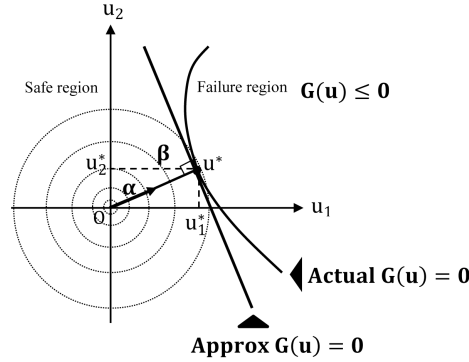


FIGURE 1. First Order Reliability Method

2.1. The Hasofer-Lind Iteration Procedure. Hasofer and Lind [17] proposed a general method for computing the reliability index which was defined on the basis of the shortest distance of LSF from the origin of the standard normal space where the basic random variables are uncorrelated. The correlated variables should be transformed into the uncorrelated ones by Nataf or Rosenblatt transformation [27]. The iterative process is as follows:

- a) Formulate the LSF for all uncorrelated random variables $X_i (i = 1, \dots, n)$ in the form of $G(X) = 0$.
- b) Transform the original design space X to the standard normal space U , using the Rackwitz-Fiessler transformation method [34]: $\frac{X - \mu(X)}{\sigma(X)}$. where $\mu(X)$ and $\sigma(X)$ are mean and standard deviation of random variable respectively.

- c) Express the LSF in terms of α_i and β noting that $U_i = \alpha_i\beta$.
- d) Calculate β and each α_i as a function of α_i and β .
- e) Obtain an initial design point U^* by assuming numerical values for β and α_i , noting that the α_i values should satisfy $\sum_{i=1}^n (\alpha_i)^2 = 1$.
- f) Calculate numerical values of α_i and β using function derived in step (d).
- g) If $|\beta_i - \beta_{i-1}| \leq \varepsilon$ ($\varepsilon = 10^{-5}$ in this paper), then stop the iteration and calculate the failure probability $P_f = \Phi(-\beta)$. Otherwise, let $\beta_{i-1} = \beta_i$ and go to step (f).

3. Review of Genetic Operators

The fundamental concepts of genetic algorithm were developed in 1989 [13]. In the genetic algorithm, each individual is introduced by a string called chromosomes. Each chromosome is formed of a number of genes [13]. The fitness of each individual corresponds to the value of the objective function at that point. Genetic algorithm initiates the search with a random sample population and random operators. These random operators namely fitness, selection, crossover, and mutation were explained in detail in [43]. Selection is the process of selecting two parents for crossover. Selection aims at selecting fitter parents leading to reproduction of children with higher fitness. Three different types of selection methods were used in this paper: Roulette wheel, Tournament selection, and the hybrid combinations of these methods [3]. Chromosomes which are selected from the initial population for reproduction are called parents [13]. The initial population was produced randomly and the numbers of generated populations were selected between 10 to 50.

The probability of a chromosome taking part in the subsequent reproduction is proportional to its fitness level. Thus, a fitter chromosome is more likely to participate in reproduction. Crossover acts upon two chromosomes which are randomly selected and generates two new chromosomes through replacing their genes from a point which is also selected randomly [13].

The main operator in creating a new generation in the multiplication stage is crossover. Its frequency is controlled by cross-over probability P_c . Basically this probability should have a large value. In this paper the probability was considered as $P_c = 0.8$. Also a hybrid combination of Arithmetic cross-over (AMXO) and Average Convex cross-over (ACXO) methods was used [3]. Each of the children resulted from this operator possesses a part of the parent's characteristics. Mutation is a process in which a part of a gene randomly changes. This operator should be used with low probability, generally P_m varies from 0.01 to 0.1.

4. Fundamental Concepts of Fuzzy Analysis

In this section, basic concepts of fuzzy analysis which have been used in this paper are briefly described.

4.1. Fuzzy Set. A fuzzy set is defined via its membership function. The membership function assigns the elements of the universe set U to the fuzzy set \tilde{A} by assigning a value from the interval $[0,1]$ which is written as : $\mu_A(x) : U \rightarrow [0,1]$.

By means of membership function the properties of crisp sets such as subset, complement and union may be generalized to fuzzy sets [40].

4.2. Triangular Fuzzy Number. Triangular fuzzy numbers have always been of interest for engineering problems mainly owing to their computationally efficient. These numbers are typically presented as $\tilde{X} = \langle a, b, c \rangle$, where a is the lower bound, b is the middle value and c is the upper bound of the fuzzy triangular number [40], (Figure 2-a). Owing to their membership function, modeling the epistemic uncertainty of structural parameters by triangular fuzzy numbers seems to be more realistic in safety assessment process.

4.3. The α -Cut of Fuzzy Number. The α -cut of the fuzzy number \tilde{X} , is a crisp set and encompasses elements which corresponding membership degree is more than α [31], (Figure 2-b). In the other words:

$$X_\alpha = \{x \in \tilde{X} | \mu_X(x) \geq \alpha\} \quad (4)$$

For $\alpha \in (0, 1]$.

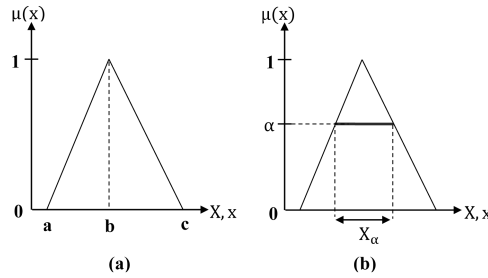


FIGURE 2. a-Triangular Fuzzy Number, b- α -Cut of the Fuzzy Number

4.4. The Extension Principle. The extension principle is one of the fundamental concepts in fuzzy sets. It explains the way, functions and mathematical operators can be extended to fuzzy sets. Assume f as a function from X to Y , that is $f : X \rightarrow Y$ and $y = f(x_1, x_2, \dots, x_n)$ [31]. When f operates on fuzzy sets $\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n$, the result would be a fuzzy subset B in Y space which is determined by equation (5):

$$B = f(A_1, A_2, \dots, A_n) = \{(y, \mu_B(y)) | y = f(x_1, x_2, \dots, x_n)\} \quad (5)$$

where

$$\mu_B(y) = \begin{cases} \max[\min(\mu_{A_1}(x_1), \mu_{A_2}(x_2), \dots, \mu_{A_n}(x_n))] & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases} \quad (6)$$

where, $f^{-1}(y)$ is the inverse function of y and $\mu_B(y)$ is the membership function of $y = f(x_1, x_2, \dots, x_n)$. In order to employ the extension principle, the fuzzy numbers should be discretized to their α -cuts. The purpose of this discretization is the approximation of a fuzzy set to a number of crisp sets, in a way that mathematical operators are carried out separately for each element of the X_α sets and the final result is determined by equation (6).

4.5. The α -Level Optimization Method. Using the extension principle in applied engineering problems with complex non-linear functions entails a large computational burden. Thus as previously noted, optimization methods based on the discretization of fuzzy variables have been developed as a substitute for the extension principle [20]. If the analysis algorithm is defined as $y = f(x_1, x_2, \dots, x_n)$, the same α -cuts of fuzzy input variables, constitute a spatial shape which is called the crisp subspace X_{α_k} . The elements of α -cut set Y_{α_k} corresponding to the fuzzy output can be obtained if the input variables are convex [32].

in other words, the above interval knowing the smallest and largest elements would be specified [31, 32]. Typically there are two optimum points x_{opt} in X_{α_k} which yield the smallest and largest elements of the Y_{α_k} interval. Thus, determining these points turns into a constraint optimization problem that aims at finding optimums in the crisp subspace, Figure 3. In which $x_{1_{opt}}$ and $x_{2_{opt}}$ yields the minimum and maximum elements of the Y_{α_k} interval respectively. The objective functions of the above optimization problem are defined as equations (7) and (8), while the $(x_1, x_2, \dots, x_n) \in X_{\alpha_k}$ is considered as a constrain.

$$y = f(x_1, x_2, \dots, x_n) \rightarrow Max (x_1, x_2, \dots, x_n) \in X_{\alpha_k} \tag{7}$$

$$y = f(x_1, x_2, \dots, x_n) \rightarrow Min (x_1, x_2, \dots, x_n) \in X_{\alpha_k} \tag{8}$$

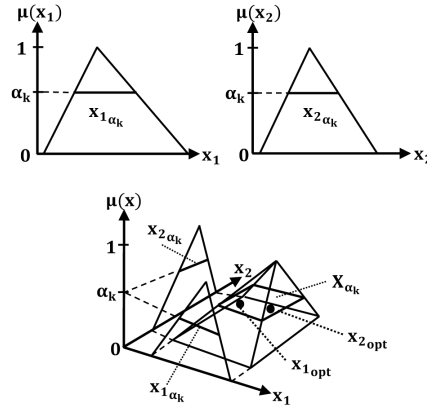


FIGURE 3. Crisp Subspace and Optimum Points

However, the existing optimization methods in this regard are complicated and computationally expensive [32]. Hence in this paper a robust genetic algorithm is adopted in order to overcome these challenges. While in classic optimization methods a complicated directional optimization procedure is applied. In the proposed method the crisp subspace related to each α -cut is formed and the inside points are then located exactly. Whilst in the classic α -level optimization methods, points inside the crisp subspace are produced randomly.

5. The Proposed Method

In the proposed method, the form of LSF was initially determined in the standard normal space. Then the mean and standard deviation of random variables which were modeled as fuzzy triangular numbers were discretized to their α -cuts. If the respective reliability problem consists of two, three, and n fuzzy random variables, the crisp subspace would take the shape of a rectangle, cube, and an n -dimensional hyper cuboid respectively. The entire points inside this subspace were used as inputs to the analysis algorithm in order to determine the corresponding interval of the respective fuzzy responses for the same α -cuts, that is Y_{α_k} , Figure 4. Assembling these intervals for the values $\alpha_k \in (0, 1]$ yields the membership function of fuzzy outputs.

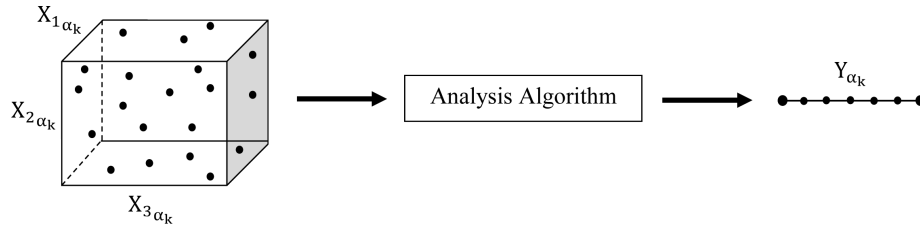


FIGURE 4. Three Dimensional Crisp Subspace, Analysis Algorithm, Interval Y_{α_k}

The $X_{i_{\alpha_k}}$ sets were discretized into m equal parts in order to determine the points inside the X_{α_k} , thus $(m + 1)$ points were specified on each set. When the problem has n fuzzy random variables, $n(m + 1)$ points on $X_{i_{\alpha_k}}$ sets are obtained. All the cases of $(m + 1)^n$ distinct combinations were specified in order to increase the accuracy of the proposed method and they were located inside the crisp subspace, Figure 5. Consequently, the crisp subspace was discretized into a number of rectangular cubes in the three-dimensional case whose corner points were the inputs of the FORM algorithm. For each of these points, the output of the reliability analysis problem would be one of the elements of the interval Y_{α_k} .

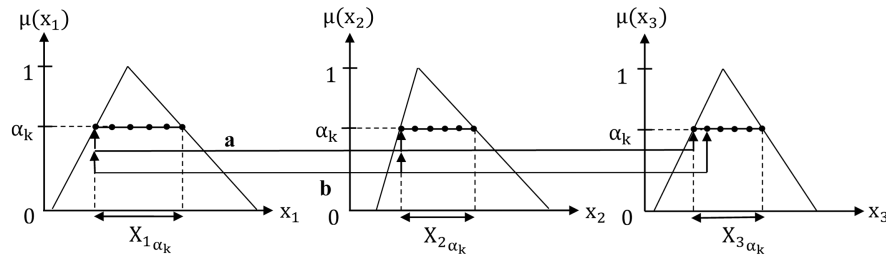


FIGURE 5. Discretization of Random Variables, Distinct Cases a and b

As mentioned before, the interval Y_{α_k} knowing the smallest and largest elements would be specified. Two optimum points $x_{opt_{\alpha_k l}}$ and $x_{opt_{\alpha_k r}}$ in the crisp subspace which eventuate the smallest and largest elements of the interval are presented in Figure 6. If all of the fuzzy random variables \tilde{X}_i are convex and uncorrelated, the search for optimum points encompasses all of the regions within the crisp subspace. Otherwise, this search would be limited to certain regions of the subspace such as boundary regions. In the proposed method, the search for the optimum points was conducted via a robust genetic algorithm in order to decrease the computational cost.

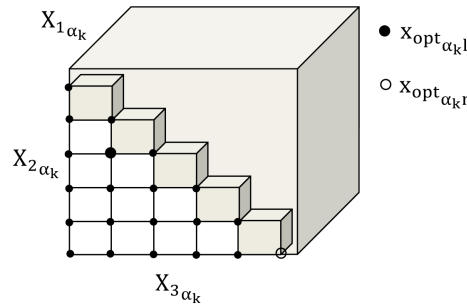


FIGURE 6. Optimum Points in the Three Dimensional Crisp Subspace

The initial population was generated by randomly selecting some points from the crisp subspace. In the three dimensional case, each of the selected cubes played the role of a chromosome comprising of three genes in the optimization process of the genetic algorithm. An initial fitness value was assigned to each chromosome considering the reliability index which that chromosome yields. Fitter parents for reproducing the next generation were selected through comparing the fitness value of chromosomes, and the subsequent generation was produced using crossover and mutation random operators and the fitness level of the new generation was determined, Figure 7. By comparing the fitness of chromosomes, fitter parents were selected for reproducing the next generation and through employing crossover and mutation operators, the subsequent generation was reproduced. This process carries on until the convergence conditions were satisfied and the search process stopped. The above procedure was repeated for all cuts $\alpha_k \in (0, 1]$ of the input random variables and the membership degree function of fuzzy reliability index was determined. In the proposed method, due to the uncertainty bounds defined for random variables, the numeric value of the fuzzy output for membership degree equal to one ($\alpha_k = 1$), indicated the results obtained from the classic method of first order reliability method.

The proposed method can be implemented in the reliability problems where most of variables are modeled as fuzzy sets and in problems having non-linear limit state functions. In summary, the proposed method comprises the following steps in order to determine the membership function of the fuzzy reliability index.

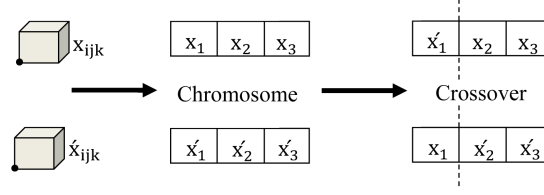


FIGURE 7. Schematic of Chromosomes and Crossover Operator

- a) Formulation of the fuzzy limit state function in the standard normal space, $\tilde{G}(u) = 0$.
- b) Formation of the corresponding crisp subspace X_{α_k} , by creating identical α -cuts on all fuzzy random variables.
- c) Random selection of the initial population and assigning an initial fitness value to each chromosome of the population.
- d) Selection of parents and reproduction of the new generation using crossover and mutation operators.
- e) Determining fitness values for each chromosome of the population.
- f) Iteration of steps (d) and (e) until the convergence criterion is fulfilled.
- g) Saving the minimum ($\beta_{\alpha_k l}$) and maximum ($\beta_{\alpha_k r}$) values of interval (β_{α_k}).
- h) Increasing the value of α_k and iteration from step (b) while $\alpha_k \in [0, 1)$.
- i) Assembling the results obtained from step (g) and the formation of the membership function of fuzzy response.

Schematic flowchart of the proposed method is shown in Figure 8.

5.1. Verification of the Proposed Method. In order to verify the proposed method an illustrative example was taken from [31]. It was proposed to determine the fuzzy reliability index of the simple steel beam under the concentrated load shown in Figure 9-a. Failure mode was considered according to the first order plastic joint theory. Figure 9-b shows the corresponding failure mechanism. The limit state function was defined according to equation (9).

$$G(P, F_Y) = F_Y - \frac{L_1 L_2}{W_{PL}(L_1 + L_2)} P \quad (9)$$

where P and F_Y are concentrated load and yield stress of steel respectively. The values for L_1 and L_2 are listed in Figure 9-a. The plastic moment of resistance is $W_{PL} = 3.66 \times 10^{-4} m^3$. The structural resistance characterized via the fully plastic moment M_{PL} was determined by $M_{PL} = F_Y W_{PL}$. The random variable P follows a type-I extreme value distribution while random variable F_Y is lognormally distributed. In this example, random variables are statistically independent and their fuzzy distribution properties are presented in Table 1. The mean value of F_Y was considered to be $\mu_{F_Y} = 28.8 \times 10^4 \frac{kN}{m^2}$. Fuzzy reliability indices obtained from the proposed method and reference [31] were plotted in Figure 10. The results showed the accuracy of the proposed method.

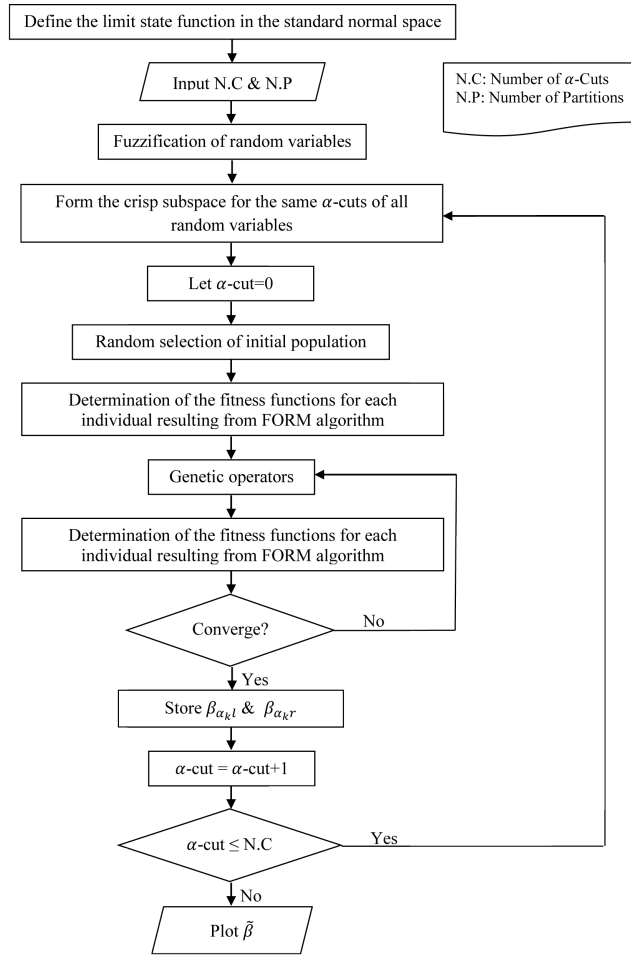


FIGURE 8. Flowchart of the Proposed Method

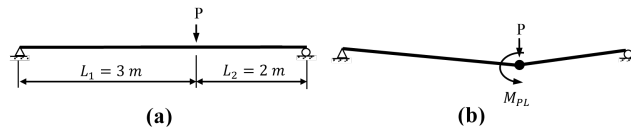


FIGURE 9. a-Simple Steel Beam, b-Failure Mechanism of Beam

5.2. Uncertainty Measure of Fuzzy Output. The obtained fuzzy output indicated the effect of taking epistemic uncertainty in the safety assessment process into consideration. In other words, this response revealed the sensitivity rate of safety index to the epistemic uncertainty introduced in the structural reliability assessment process. In this paper, the modified Shannon entropy criterion [31] for

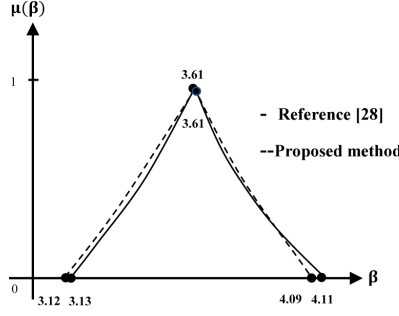


FIGURE 10. Comparison of Fuzzy Reliability Indices

Fuzzy Random Variable	Fuzzy Mean	Fuzzy S.D
$\tilde{P}(kN)$	$\langle 47, 50, 52 \rangle$	$\langle 4.5, 5, 6 \rangle$
$\tilde{F}_Y (\times 10^4 \frac{kN}{m^2})$	-	$\langle 2.2, 2.64, 2.8 \rangle$

TABLE 1. Fuzzy Distribution Properties of the Random Variables

measuring uncertainty was used in order to estimate the rate of this sensitivity. Using the criterion, according to equation (10) the uncertainty level of the reliability index is measured.

$$H_u(\tilde{\beta}) = -k \int_{\beta_{\alpha_0 l}}^{\beta_{\alpha_0 r}} [\mu(\beta) \log(\mu(\beta)) + (1 - \mu(\beta)) \log(1 - \mu(\beta))] d\beta \quad (10)$$

where $\beta_{\alpha_0 l}$ and $\beta_{\alpha_0 r}$ are the smallest and the largest value of fuzzy reliability index in the support α -cut respectively, and k is a constant value. The more the value of $H_u(\tilde{\beta})$, the larger interval of uncertainty is covered by the fuzzy output and the more its sensitivity to epistemic uncertainty in the process of structure reliability analysis and vice versa.

6. Numerical Examples

In this section, one mathematical and two engineering examples are presented and fuzzy reliability indices were determined in order to indicate the efficiency of the proposed method. In addition, the sensitivity level of fuzzy response due to the epistemic uncertainty introduced in the safety assessment process was evaluated.

Example 6.1. A 3-span continuous steel beam with 5 meters length span, under uniform distributed load w was investigated according to the following LSF [34], Figure 11.

$$G(L, w, E, I) = \frac{L}{360} - 0.0069 \frac{wL^4}{EI} \quad (11)$$

where L is the span length, w is the uniform distributed load, E is the modulus of elasticity, and I is the moment of inertia of the beam section. The term $\frac{L}{360}$ is the

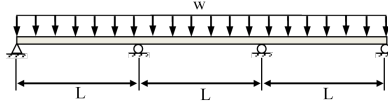


FIGURE 11. Three Span Continuous Steel Beam

allowable deflection of the beam under dead load and the value $0.0069 \frac{wL^4}{EI}$ is the maximum deflection of the beam caused by the distributed load. The fuzzy mean and fuzzy standard deviation of all random variables were modeled via fuzzy triangular numbers in order to determine the membership function of fuzzy reliability index. The random variables were normal and statistically independent. Fuzzy distribution properties of random variables are presented in Table 2. In this example, the crisp subspace was a six-dimensional hyper cuboid. In the analysis process, each $\alpha_k \in (0, 1]$ was divided into five equal intervals. Figure 12 shows the membership function of the obtained fuzzy response. As shown, the smallest and largest elements of interval $Y_{i_{\alpha_k}}$ were determined for five α -cuts of $\alpha_k = 0, 0.2, 0.4, 0.6, 0.8$. As shown in the Figure 12 the value of the obtained fuzzy response for the membership degree $\alpha_k = 1$ is exactly the same as the result presented in [34]. It can be seen from the Figure 12 that the results of the FORM method are only a special case of those obtained by the proposed method. As shown in Figure 12, the reliability index comprises the interval $[2.257, 3.937]$ considering the epistemic uncertainty which is more realistic than 3.173 obtained from classic methods. It would then be easy to derive a design reliability index using standard defuzzification procedures [31]. In this paper fuzzy reliability indices have been defuzzified using centroid method which yields the crisp value as the center of the area below the membership function of the fuzzy response. The target reliability index of this example was $\beta_T = 3.11$. The result for the uncertainty measure of the fuzzy output using equation (10) was equal to $H_u(\tilde{\beta}) = 0.839k$.

Fuzzy Random Variable	Fuzzy Mean	Fuzzy S.D
$\tilde{W}(\frac{kN}{m})$	$\langle 8.5, 10, 11 \rangle$	$\langle 0.36, 0.4, 0.46 \rangle$
$\tilde{E}(\times 10^7 \frac{kN}{m^2})$	$\langle 1.7, 2, 2.2 \rangle$	$\langle 0.45, 0.5, 0.575 \rangle$
$\tilde{I}(\times 10^{-4} m^4)$	$\langle 6.8, 8, 8.8 \rangle$	$\langle 1.35, 1.5, 1.735 \rangle$

TABLE 2. Fuzzy Distribution Properties of the Random Variables of Example 6.1

Example 6.2. In this example, the fuzzy reliability analysis of a reinforced concrete beam was determined. The LSF of the beam was demonstrated by equation (12), [34]:

$$G(A_S, F_Y, F_C, Q) = A_S F_Y d - 0.59 \frac{(A_S F_Y)^2}{F_c b} - Q \quad (12)$$

where A_S , F_Y , F_C , and Q are steel reinforcement area, yield stress of the steel, compressive strength of concrete and the total moment respectively. Parameters of b and d are the width and height of the section which are considered to be 12 and 19

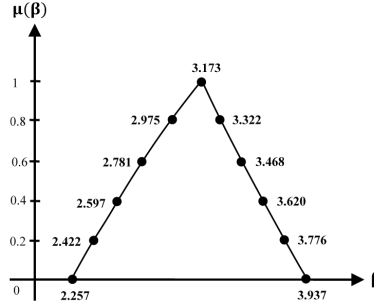


FIGURE 12. Membership Function of the Fuzzy Reliability Index of Example 6.1

inches respectively. The random variables are normal, statistically independent and their statistical properties are presented in Table 3. The crisp subspace was an eight-dimensional hyper cuboid in this example. Each $\alpha_k \in (0, 1]$ was divided into five equal intervals in the analysis process. Figure 13 shows the membership function of the fuzzy response obtained from the proposed method and the lower and upper bounds for each of the five α -cuts. As shown in the Figure 13, the numeric value of the obtained fuzzy reliability index for the membership degree $\alpha_k = 1$ was exactly the same as with the result presented in [34]. As shown in Figure 13, the reliability index contains the support interval $[0.863, 3.846]$, whereas that of the classic method FORM equals 2.35. The fuzzy reliability index of the proposed method offers more acceptable estimation of structural safety by introducing bounds of uncertainty. This uncertainty is of great importance particularly in concrete structures owing to the variation in the production quality and implementation of concrete in different cross sections. The target reliability index of this example was $\beta_T = 2.328$. The uncertainty measure of fuzzy output using equation (10) was $H_u(\tilde{\beta}) = 1.490k$, where indicated high sensitivity of the reliability index to the epistemic uncertainty. This relatively large interval of uncertainty for the fuzzy reliability index can be seen in Figure 13.

Fuzzy Random Variable	Fuzzy Mean	Fuzzy S.D
$\bar{A}_S(in^2)$	$\langle 3.468, 4.08, 4.488 \rangle$	$\langle 0.072, 0.08, 0.092 \rangle$
$\bar{F}_Y(ksi)$	$\langle 37.4, 44, 48.4 \rangle$	$\langle 4.158, 4.62, 5.313 \rangle$
$\bar{F}_C(ksi)$	$\langle 2.652, 3.12, 3.432 \rangle$	$\langle 0.396, 0.44, 0.506 \rangle$
$\bar{Q}(k - in)$	$\langle 1744.2, 2052, 2257.2 \rangle$	$\langle 221.4, 246, 282.9 \rangle$

TABLE 3. Fuzzy Distribution Properties of the Random Variables of Example 6.2

Example 6.3. In order to assay the proposed method for non-normal random variables, the following mathematical LSF has been taken from [37]:

$$G(X_1, X_2, X_3) = X_1 X_2 - 78.12 X_3 \quad (13)$$

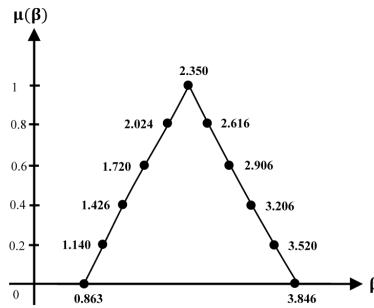


FIGURE 13. Membership Function of the Fuzzy Reliability Index of Example 6.2

In which X_1 and X_2 are lognormally distributed whereas X_3 follows a type-I extreme value distribution. The random variables are statistically independent and their fuzzy statistical properties are shown in Table 4. In this example the lower and upper bounds of the fuzzy reliability index for each of the five α -cuts were determined from within the six-dimensional crisp subspace. Figure 14 shows the membership function of the fuzzy reliability index for each of the five α -cuts. As shown in Figure 14 the membership degree of fuzzy reliability index for $\alpha_k = 1$ is the same as the result offered in [43]. The uncertainty measure of fuzzy output using equation (10) also resulted $H_u(\tilde{\beta}) = 0.376k$. Also the target reliability index of this example was $\beta_T = 4.614$.

Fuzzy Random Variable	Fuzzy Mean	Fuzzy S.D
$\tilde{X}_1 (\times 10^7)$	$\langle 1.7, 2, 2.2 \rangle$	$\langle 0.45, 0.5, 0.575 \rangle$
$\tilde{X}_2 (\times 10^{-4})$	$\langle 0.85, 1, 1.1 \rangle$	$\langle 0.18, 0.2, 0.23 \rangle$
\tilde{X}_3	$\langle 3.4, 4, 4.4 \rangle$	$\langle 0.9, 1, 1.15 \rangle$

TABLE 4. Fuzzy Distribution Properties of the Random Variables of Example 6.3

7. Conclusion

In this paper, modeling the epistemic uncertainty of random variables via fuzzy triangular numbers in the FORM reliability method has been brought into focus. A robust heuristic algorithm has been proposed to determine optimum points resulting from the corresponding crisp subspace of the entire variables. The implementation of the proposed method leads to the definition of certain bounds for the reliability index, resulting in more realistic values of the corresponding indices compared to pure probabilistic methods. This uncertainty interval is of special importance in concrete structures since the quality of the production and implementation of concrete varies in different cross sections in reality. Whilst it was not taken into account in classic methods of reliability.

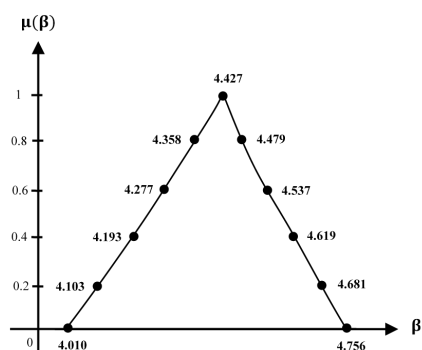


FIGURE 14. Membership Function of the Fuzzy Reliability Index of Example 6.3

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MANSOUR BAGHERI, DEPARTMENT OF CIVIL ENGINEERING, UNIVERSITY OF SISTAN AND BALUCHES-TAN, ZAHEDAN, IRAN
E-mail address: mnsrbagheri@gmail.com

MAHMOUD MIRI*, DEPARTMENT OF CIVIL ENGINEERING, UNIVERSITY OF SISTAN AND BALUCHES-TAN, ZAHEDAN, IRAN
E-mail address: mmiri@eng.usb.ac.ir

NASER SHABAKHTY, DEPARTMENT OF CIVIL ENGINEERING, UNIVERSITY OF SISTAN AND BALUCHES-TAN, ZAHEDAN, IRAN
E-mail address: shabakhty@eng.usb.ac.ir

*CORRESPONDING AUTHOR