ADMISSIBILITY ANALYSIS FOR DISCRETE-TIME SINGULAR SYSTEMS WITH TIME-VARYING DELAYS BY ADOPTING THE STATE-SPACE TAKAGI-SUGENO FUZZY MODEL

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Abstract. This paper is pertained with the problem of admissibility analysis of uncertain discrete-time nonlinear singular systems by adopting the state-space Takagi-Sugeno fuzzy model with time-delays and norm-bounded parameter uncertainties. Lyapunov Krasovskii functionals are constructed to obtain delay-dependent stability condition in terms of linear matrix inequalities, which is dependent on the lower and upper delay bounds. Finally, numerical examples are provided to substantiate the theoretical results.

1. Introduction

In the past decades, Takagi-Sugeno (T-S) fuzzy model has been widely used in the study of system performance of a class of nonlinear systems due to the fact that it provides a general framework to represent a nonlinear plant by using a set of local linear models which are smoothly connected through nonlinear fuzzy membership functions [17, 18]. A great number of stability results for T-S fuzzy systems in both the continuous and discrete contexts have been reported in the literature by various approaches; see, e.g., [1, 22].

Singular systems have received much attention since singular model can preserve the structure of practical systems better than regular ones. Singular systems are also referred as descriptor, semi-state and generalized systems arise naturally as a linear approximation of systems models, or linear system models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large-scale systems, interconnected systems, economics, optimization problems, feedback systems, robotics, biology, etc for details reader may refer [3, 5, 16] and references therein. Since singular systems include not only dynamic equations but also static equations, the study of such systems is much more complicated than that for standard state-space systems. The existence and uniqueness of a solution to a given singular time-delay system is not always guaranteed and the system can also have undesired impulsive behavior. Therefore, for a singular time-delay system, it is important to develop conditions which guarantee that the given singular system is not only stable but also regular and impulse free [27].

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Singular system with time-varying delay is more general and natural in describing the practical dynamical systems than standard state-space systems, such as large-scale systems, power systems and constrained control systems. The results on stability analysis and stabilization for singular time-delay systems can also be classified into two categories, that is, delay-independent criteria and delay-dependent ones. In general, delay-dependent criteria are less conservative than delay-independent ones. Also, there is always some error between the mathematical model and the physical system while modeling a dynamical system due to the external perturbations and parameter fluctuations, which can cause the system to be unstable [14, 26]. The parameter uncertainties and time delays constitute significant challenges to the dynamic analysis of singular systems [4]. Any model parameter uncertainties would possibly degrade the model quality significantly, or even make the modeling meaningless [25]. Moreover, theoretical models without consideration of these factors may even provide wrong predictions.

Recently, several interesting results have been reported that have focused on the stability and stabilization of both continuous-time and discrete-time linear singular systems, see [10, 23, 21, 12]. In terms of the LMI method, many approaches have been addressed for singular time-delay systems. In [10], the robust stability and stabilization problems for discrete singular systems with interval time-varying delay and linear fractional uncertainty have been discussed. The problem of admissibility and dissipativity has been studied for discrete-time singular systems with mixed time-varying delays via delay partitioning technique in [21]. In [8], robust reliable dissipative filtering for uncertain discrete-time singular system with interval time-varying delay and sensor failures have been investigated. Robust stabilization for uncertain discrete singular systems with time-varying delay with delay-distribution-dependent Lyapunov function has been proposed in [13]. But in the existing literature, compared to continuous-time singular systems, there are only few results available on stability of nonlinear singular discrete systems [29]. In [20], a robust fuzzy controller has been designed to stabilize a class of solvable continuous-time nonlinear descriptor systems with time-varying delay. It is necessary to note that not only stability but also regularity must be considered, while the latter does not occur in standard state space systems. To overcome this difficulty, a special Lyapunov criterion for stability has been introduced in [28]. This method is rather suitable for singular discrete systems. The same method has been utilized to derive the stability of nonlinear singular discrete systems in [29]. Fuzzy descriptor systems have been addressed in [19]. The stability problem of discrete singular T-S fuzzy model has been investigated in [9] and the problem of robust stability analysis of uncertain discrete-time singular fuzzy systems has been studied in [27]. The time delays were not considered in both the approaches [27, 9], which limits the scope of the applications of these results. To the best of the authors' knowledge, the stability analysis for discrete- time nonlinear singular systems via T-S fuzzy approach with time-varying delay has received little attention, which motivates the work of this paper.

This paper is concerned with the robust stability analysis of nonlinear and linear discrete-time singular systems with time-varying delays. Sufficient condition is derived to ensure the stability of discrete-time singular systems in terms of LMIs, which can be easily solved by using available software packages. Moreover, as an example of linear time-varying singular system, DC motor has been considered in numerical example which has most distributed applications in automotive systems such as engine cooling fan, electronic throttle plate, fuel pump, engine management, steering system, active suspensions etc.

This paper is organized as follows. Problem formulation and preliminaries are given in Section 2. Section 3 gives the admissibility criterion for the uncertain discrete-time nonlinear singular systems with time-varying delay. For the linear singular systems, the robust admissibility conditions are presented in Section 4. In Section 5, numerical examples are demonstrated to illustrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 6.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices respectively. The superscript T and (-1) denote the matrix transposition and matrix inverse respectively. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. I is an identity matrix with appropriate dimension. The notation $*$ always denotes the symmetric block in one symmetric matrix.

2. Problem Description and Preliminaries

Consider an uncertain nonlinear discrete-time singular system with time-varying delay,

$$\begin{cases} Ex(k+1) & = f(k,x(k),x(k-\tau(k)) \\ x(k) & = \phi(k),\ k\in[-\tau_M,\ 0] \end{cases} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state variable, $f\in\mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n$ is the nonlinear smooth function and $\phi(k)$ is the initial condition. The above nonlinear uncertain discrete-time singular system can be represented by an extended T-S fuzzy dynamic model, in which the i-th rule is described as follows:

Plant Rule i : IF $s_1(k)$ is μ_{i1} and $s_2(k)$ is μ_{i2} and ... and $s_p(k)$ is μ_{ip} THEN

$$\begin{cases} Ex(k+1) & = (A_i+\Delta A_i(k))x(k)+(A_{di}+\Delta A_{di}(k))x(k-\tau(k)) \\ x(k) & = \phi(k),\ k\in[-\tau_M,\ 0] \end{cases} \tag{2}$$

where $\mu_{i\alpha}$ is a fuzzy term of rule i corresponding to the function $s_\alpha(k),\ \alpha = 1,2,...,p,\ p$ is a positive integer, $i=1,2,...,m,\ \tau(k)$ is the time-varying delay satisfying $\tau_m\le\tau(k)\le\tau_M$ and $\phi(k)$ is the initial condition at time instant k. The matrix $E\in\mathbb{R}^{n\times n}$ is singular and $rank\ E=r<n,\ A_i\in\mathbb{R}^{n\times n},\ A_{di}\in\mathbb{R}^{n\times n}$ are the real constant matrices, $\Delta A_i(k),\ \Delta A_{di}(k)$ are norm bounded uncertain parameter matrices assumed to be of the form

$$[\Delta A_i(k)\ \Delta A_{di}(k)] = M_iF_i(k)[N_{1i}\ N_{2i}] \tag{3}$$

where $F_i(k):\mathbb{N}\longrightarrow\mathbb{R}^{l_1\times l_2}$ is an unknown parameter matrix satisfying

$$F_i^T(k)F_i(k)\le I,\ \forall\ k \tag{4}$$

$M_i,\ N_{1i},\ N_{2i}$ are known constant matrices with appropriate dimensions. The parameter uncertainties $\Delta A_i(k),\ \Delta A_{di}(k)$ are said to be admissible if both (3) and (4) hold. The final output of the uncertain discrete-time singular fuzzy system is inferred as follows:

$$Ex(k+1) = \sum_{i=1}^{m} h_i(s(k))[(A_i + \Delta A_i(k))x(k) + (A_{di} + \Delta A_{di}(k))x(k-\tau(k))], \quad (5)$$

where

$$h_i(s(k)) = \frac{w_i(s(k))}{\sum_{j=1}^{m} w_j(s(k))},\ w_i(s(k)) = \prod_{j=1}^{p} \Omega_{ij}(s_j(k)),$$
$$s(k) = [s_1(k)\ s_2(k)\ ...\ s_p(k)],$$

in which $\Omega_{ij}(s_j(k))$ is the grade of membership of $s_j(k)$ in fuzzy set μ_{ij}. We assume

$$w_i(s(k)) \geq 0,\ i = 1, 2, ..., m,$$
$$\sum_{j=1}^{m} w_j(s(k)) > 0,\ \forall\ k.$$

Then it is easy to see that for all k,

$$h_i(s(k)) \geq 0,\ i = 1, 2, ..., m,$$
$$\sum_{j=1}^{m} h_j(s(k)) = 1.$$

The discrete-time singular fuzzy system is given by

$$Ex(k+1) = \bar{A}(k)x(k) + \bar{A}_d(k)x(k-\tau(k)) \quad (6)$$

where

$$\bar{A}(k) = \sum_{i=1}^{m} h_i(s(k))[A_i + \Delta A_i(k)],\ \bar{A}_d(k) = \sum_{i=1}^{m} h_i(s(k))[A_{di} + \Delta A_{di}(k)].$$

The following definitions and lemmas are needed to conclude the main results.

Definition 2.1. [5],[11]
1. The pair $(E,\ A_i)$ is regular if $det(zE - A_i) \neq 0$.
2. The pair $(E,\ A_i)$ is said to be causal if it is regular and $deg(det(zE - A_i)) = rank\ E$.
3. The discrete-time singular system is said to be admissible, if it is regular, causal and stable.

Definition 2.2. [5],[26] For given integers $\tau_m > 0,\ \tau_M > 0$, the discrete singular time-delay system (6) is said to be regular and causal for any time delay $\tau(k)$ satisfying $\tau_m \leq \tau(k) \leq \tau_M$, if the pair $(E,\ A_i)$ is regular and causal.

Definition 2.3. [5],[26] The discrete singular time-delay system (6) is said to be stable if for any scalar $\epsilon > 0$, there exists a scalar $\delta(\epsilon) > 0$ such that, for any compatible initial conditions $\phi(k)$ satisfying $\sup_{-\tau_M \leq k \leq 0} \parallel \phi(k) \parallel \leq \delta(\epsilon)$, the solution $x(k)$ of system (6) satisfies $\parallel x(k) \parallel \leq \epsilon$ for any $k \geq 0$, moreover $\lim_{k\to\infty} x(k) = 0$.

Lemma 2.4. [21] *For any matrix* $\begin{bmatrix} M & S \\ * & M \end{bmatrix} \geq 0$, *integers* $\tau_m,\ \tau_M,\ \tau(k)$ *satisfying* $\tau_m \leq \tau(k) \leq \tau_M$, *and vector function* $x(k + \cdot) : \mathbb{N} \longrightarrow \mathbb{R}^n$, *such that the sums concerned are well defined, then*

$$-\tau_1 \sum_{\alpha=k-\tau_M}^{k-\tau_m-1} \eta^T(\alpha) M \eta(\alpha) \leq \varpi^T(k) \Omega \varpi(k),$$

where $\tau_1 = \tau_M - \tau_m,\ \eta(\alpha) = x(\alpha+1) - x(\alpha)$ *and*

$$\begin{aligned} \varpi(k) &= [x^T(k-\tau_m)\ x^T(k-\tau(k))\ x^T(k-\tau_M)]^T, \\ \Omega &= \begin{bmatrix} -M & M-S & S \\ * & -2M+S+S^T & -S+M \\ * & * & -M \end{bmatrix}. \end{aligned}$$

Lemma 2.5. [15] *Let* X, Y *and* H *be real matrices of appropriate dimensions with* $H^T H \leq I$. *Then for any scalar* $\epsilon > 0$,

$$XHY + (XHY)^T \leq \epsilon X^T X + \epsilon^{-1} Y^T Y.$$

Lemma 2.6. [2] *(Schur Complement) Given constant matrices* Ω_1, Ω_2 *and* Ω_3 *with appropriate dimensions, where* $\Omega_1^T = \Omega_1$ *and* $\Omega_2^T = \Omega_2 > 0$, *then*

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0,$$

if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ * & -\Omega_2 \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ * & \Omega_1 \end{bmatrix} < 0.$$

3. Delay-dependent Stability Analysis for Discrete-time Nonlinear Singular System

In this section, we aim to establish a criterion that guarantees the admissibility of the delay-dependent discrete-time uncertain singular fuzzy system (6).

Theorem 3.1. *The system (6) is said to be admissible if there exist symmetric positive definite matrices* $P_i,\ R_{1i},\ R_{2i},\ R_{3i},\ Z_{1i},\ Z_{2i},\ Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \\ * & Q_{3i} \end{bmatrix}$, *real matrices* T_i, S_i *with appropriate dimensions, positive scalars* ϵ_i *such that the following LMIs hold:*

$$\begin{bmatrix} \Xi & \Psi_{1i} M_i & \epsilon_i \Psi_{2i} \\ * & -\epsilon_i I & 0 \\ * & * & -\epsilon_i I \end{bmatrix} < 0,\ i = 1, 2, ..., m, \tag{7}$$

where $R \in \mathbb{R}^{n\times(n-r)}$ *is any matrix satisfies* $E^T R = 0$ *with rank* $R = n - r$,

$$\Psi_{1i} = \begin{bmatrix} S_i R^T \\ T_i^T \\ 0 \\ 0 \\ 0 \\ P_i \end{bmatrix},\ \Psi_{2i} = \begin{bmatrix} N_{1i}^T \\ 0 \\ 0 \\ N_{2i}^T \\ 0 \\ 0 \end{bmatrix},\ \Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & A_i^T P_i \\ * & \Xi_{22} & 0 & \Xi_{24} & 0 & 0 \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & A_{di}^T P_i \\ * & * & * & * & \Xi_{55} & 0 \\ * & * & * & * & * & -P_i \end{bmatrix}, \tag{8}$$

with

$$\begin{aligned}
\Xi_{11} &= -E^TP_iE+\tau_mE^TZ_{1i}E+\tau_1^2E^TZ_{2i}E-E^TZ_{1i}E+\tau_1Q_{1i}+R_{1i}+R_{2i}\\
&\quad+(\tau_1+1)R_{3i}+S_iR^TA_i+A_i^TRS_i^T,\\
\Xi_{12} &= -\tau_mE^TZ_{1i}^T-\tau_1^2E^TZ_{2i}^T+\tau_1Q_{2i}+A_i^TT_i,\ \Xi_{13}=E^TZ_{1i}E,\ \Xi_{14}=S_iR^TA_{di},\\
\Xi_{22} &= \tau_m^2Z_{1i}+\tau_1^2Z_{2i}+\tau_1Q_{3i}-T_i-T_i^T,\ \Xi_{24}=T_i^TA_{di},\\
\Xi_{33} &= -E^TZ_{1i}E-E^TZ_{2i}E-R_{1i},\ \Xi_{34}=E^TZ_{2i}E-E^TY_iE,\\
\Xi_{35} &= E^TY_iE,\ \Xi_{44}=-2E^TZ_{2i}E+E^TY_iE+EY_i^TE^T-R_{3i},\\
\Xi_{45} &= -E^TY_iE+E^TZ_{2i}E,\ \Xi_{55}=-E^TZ_{2i}E-R_{2i}.
\end{aligned}$$

Proof. To prove regular and causal, we consider

$$\begin{bmatrix} \Omega_{1i} & \Omega_{2i} & \Omega_{3i}\\ * & \Omega_{4i} & \Omega_{5i}\\ * & * & \Omega_{6i}\end{bmatrix}<0, \tag{9}$$

where

$$\begin{aligned}
\Omega_{1i} &= \Xi_{11}-\tau_mE^TZ_{1i}E-\tau_1^2E^TZ_{2i}E-\tau_1Q_{1i}-R_{1i}-R_{2i}-(\tau_1+1)R_{3i},\\
\Omega_{2i} &= \Xi_{12}-\tau_1Q_{2i},\ \Omega_{3i}=\Xi_{14},\ \Omega_{4i}=\Xi_{22}-\tau_mZ_{1i}-\tau_1^2Z_{2i}-\tau_1Q_{3i},\ \Omega_{5i}=\Xi_{24},\\
\Omega_{6i} &= \Xi_{44}-E^TY_iE-EY_i^TE^T.
\end{aligned}$$

Let $V=\begin{bmatrix} I & A_i^T & 0\\ 0 & A_{di}^T & I\end{bmatrix}$. Pre and post multiplying (9) by V and V^T, respectively yields,

$$\Lambda_i=\begin{bmatrix} \Lambda_{1i} & \Lambda_{2i}\\ * & \Lambda_{3i}\end{bmatrix} \tag{10}$$

where

$$\begin{aligned}
\Lambda_{1i} &= -E^TP_iE-E^TZ_{1i}E+S_iR^TA_i+A_i^TRS_i^T-\tau_mA_i^TZ_{1i}E-\tau_1^2A_i^TZ_{2i}E\\
&\quad-\tau_mE^TZ_{1i}^TA_i-\tau_1^2E^TZ_{2i}^TA_i,\\
\Lambda_{2i} &= -\tau_mE^TZ_{1i}^TA_{di}-\tau_1^2E^TZ_{2i}^TA_{di}+S_iR^TA_{di},\ \Lambda_{3i}=-2E^TZ_{2i}E-R_{3i}.
\end{aligned}$$

Since $rank\ E=r<n$, there must exist two invertible matrices $\hat{G}$ and $\hat{H}\in\mathbb{R}^{n\times m}$ such that $\hat{E}=\hat{G}E\hat{H}=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix}$. Then R can be parameterized as $R=\hat{G}\begin{bmatrix} 0\\ \Phi\end{bmatrix}$, $\Phi\in\mathbb{R}^{(n-r)\times(n-r)}$ is any non-singular matrix. Similarly we define,

$$\begin{aligned}
\hat{A}_i &= \hat{G}A_i\hat{H}=\begin{bmatrix} \hat{A}_{11i} & \hat{A}_{12i}\\ \hat{A}_{21i} & \hat{A}_{22i}\end{bmatrix},\ \hat{A}_{di}=\hat{G}A_{di}\hat{H}=\begin{bmatrix} \hat{A}_{d11i} & \hat{A}_{d12i}\\ \hat{A}_{d21i} & \hat{A}_{d22i}\end{bmatrix},\\
\hat{P}_i &= \hat{G}^{-T}P_i\hat{G}^{-1}=\begin{bmatrix} \hat{P}_{11i} & \hat{P}_{12i}\\ \hat{P}_{21i} & \hat{P}_{22i}\end{bmatrix},\ \hat{Z}_{1i}=\hat{G}^{-T}Z_{1i}\hat{G}^{-1}=\begin{bmatrix} \hat{Z}_{111i} & \hat{Z}_{112i}\\ \hat{Z}_{121i} & \hat{Z}_{122i}\end{bmatrix},\\
\hat{Z}_{2i} &= \hat{G}^{-T}Z_{2i}\hat{G}^{-1}=\begin{bmatrix} \hat{Z}_{211i} & \hat{Z}_{212i}\\ \hat{Z}_{221i} & \hat{Z}_{222i}\end{bmatrix},\ \hat{R}_{3i}=\hat{G}^{-T}R_{3i}\hat{G}^{-1}=\begin{bmatrix} \hat{R}_{311i} & \hat{R}_{312i}\\ \hat{R}_{321i} & \hat{R}_{322i}\end{bmatrix},\\
\hat{S}_i &= \hat{H}^TS_i=\begin{bmatrix} \hat{S}_{11i}\\ \hat{S}_{21i}\end{bmatrix}.
\end{aligned}$$

Pre and post multiplying Λ_{1i} by $\hat{H}^T$ and $\hat{H}$, we can obtain the following

$$\hat{\Lambda}_{1i} = \hat{H}^T \Lambda_{1i} \hat{H} = \begin{bmatrix} \hat{\Lambda}_{11i} & \hat{\Lambda}_{12i} \\ * & \hat{A}_{22i}^T \Phi \hat{S}_{21i}^T + \hat{S}_{21i} \Phi \hat{A}_{22i} \end{bmatrix}. \tag{11}$$

From (11), it is easy to see that

$$\hat{A}_{22i}^T \Phi \hat{S}_{21i}^T + \hat{S}_{21i} \Phi \hat{A}_{22i} < 0 \tag{12}$$

and thus $\hat{A}_{22i}$ is non-singular. Suppose that $\hat{A}_{22i}$ is singular, there must exists a non-zero vector $\kappa \in \mathbb{R}^{n-r}$, which ensures that $\hat{A}_{22i}\kappa = 0$. Then one can conclude that $\kappa^T(\hat{A}_{22i}^T \Phi \hat{S}_{21i}^T + \hat{S}_{21i} \Phi \hat{A}_{22i})\kappa = 0$, this contradicts (12). It is seen that $\hat{A}_{22i}$ is non-singular. Thus, $(E,\ A_i)$ is regular and causal. By Definition 2.2, system (6) is regular and causal.

Next we prove the stability of the system (6). Choose the Lyapunov-Krasovskii functional for system (6) as

$$V(k) = \sum_{i=1}^{5} V_i(k), \tag{13}$$

where

$$\begin{aligned}
V_1(k) &= x^T(k) E^T \bar{P} E x(k) \\
V_2(k) &= \sum_{s=k-\tau_m}^{k-1} x^T(s) \bar{R}_1 x(s) + \sum_{s=k-\tau_M}^{k-1} x^T(s) \bar{R}_2 x(s), \\
V_3(k) &= \sum_{s=k-\tau(k)}^{k-1} x^T(s) \bar{R}_3 x(s) + \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{s=k+j}^{k-1} x^T(s) \bar{R}_3 x(s), \\
V_4(k) &= \tau_m \sum_{j=-\tau_m}^{-1} \sum_{s=k+j}^{k-1} \eta^T(s) E^T \bar{Z}_1 E \eta(s) + \tau_1 \sum_{j=-\tau_M}^{-\tau_m-1} \sum_{s=k+j}^{k-1} \eta^T(s) E^T \bar{Z}_2 E \eta(s), \\
V_5(k) &= \sum_{s=k-\tau(k)}^{k-1} \zeta^T(s) \bar{Q} \zeta(s) + \sum_{j=-\tau_M+1}^{-\tau_m-1} \sum_{s=k+j}^{k-1} \zeta^T(s) \bar{Q} \zeta(s),
\end{aligned}$$

with $\eta(k) = x(k+1) - x(k),\ \zeta^T(k) = [x^T(k)\ (Ex(k+1))^T],\ \tau_1 = \tau_M - \tau_m$ and

$$\begin{aligned}
\bar{P} &= \sum_{i=1}^{m} h_i(s(k)) P_i,\ \bar{R}_u = \sum_{i=1}^{m} h_i(s(k)) R_{ui},\ u = 1,\ 2,\ 3, \\
\bar{Z}_v &= \sum_{i=1}^{m} h_i(s(k)) Z_{vi},\ v = 1,\ 2,\ \bar{Q} = \sum_{i=1}^{m} h_i(s(k)) Q_i.
\end{aligned}$$

Calculating the difference of $V_i(k)$ by defining $\Delta V_i(k) = V_i(k+1) - V_i(k)$, which yield,

$$\begin{aligned}
\Delta V_1(k) &= x^T(k+1)E^T\bar{P}Ex(k+1) - x^T(k)E^T\bar{P}Ex(k),\\
\Delta V_2(k) &= x^T(k)[\bar{R}_1+\bar{R}_2]x(k) - x^T(k-\tau_m)\bar{R}_1x(k-\tau_m)\\
&\quad -x(k-\tau_M)\bar{R}_2x(k-\tau_M),\\
\Delta V_3(k) &\le (\tau_1+1)x^T(k)\bar{R}_3x(k) - x^T(k-\tau(k))\bar{R}_3x(k-\tau(k)),\\
\Delta V_4(k) &= \tau_m^2\eta^T(k)E^T\bar{Z}_1E\eta(k) - \tau_m\sum_{s=k-\tau_m}^{k-1}\eta^T(s)E^T\bar{Z}_1E\eta(s)\\
&\quad +\tau_1^2\eta^T(k)E^T\bar{Z}_2E\eta(k) - \tau_1\sum_{s=k-\tau_M}^{k-\tau_m-1}\eta^T(s)E^T\bar{Z}_2E\eta(s),\\
\Delta V_5(k) &= \zeta^T(k)\bar{Q}\zeta(k) + \Big(\sum_{s=k+1-\tau(k+1)}^{k-\tau(k)-1} + \sum_{s=k-\tau(k)}^{k-1} - \sum_{s=k-\tau(k)}^{k-1}\Big)\zeta^T(k)\bar{Q}\zeta(k)\\
&\quad +(\tau_1-1)\zeta^T(k)\bar{Q}\zeta(k) - \sum_{j=-\tau_M+1}^{-\tau_m-1}\zeta^T(k+j)\bar{Q}\zeta(k+j),\\
&\le \tau_1\zeta^T(k)\bar{Q}\zeta(k).
\end{aligned}$$

Define

$$\bar{Y} = \sum_{i=1}^{m}h_i(s(k))Y_i,\ \bar{T} = \sum_{i=1}^{m}h_i(s(k))T_i,\ \bar{S} = \sum_{i=1}^{m}h_i(s(k))S_i.$$

Furthermore, applying Lemma 2.4, we obtain

$$\begin{aligned}
-\tau_m\sum_{s=k-\tau_m}^{k-1}\eta^T(s)E^T\bar{Z}_1E\eta(s) &\le -\Big(\sum_{s=k-\tau_m}^{k-1}\eta^T(s)E^T\Big)\bar{Z}_1\Big(\sum_{s=k-\tau_m}^{k-1}E\eta^T(s)\Big),\\
&\le \begin{bmatrix} x(k)\\ x(k-\tau_m)\end{bmatrix}^T\Gamma_1\begin{bmatrix} x(k)\\ x(k-\tau_m)\end{bmatrix}
\end{aligned} \tag{14}$$

$$-\tau_1\sum_{s=k-\tau_M}^{k-\tau_m-1}\eta^T(s)E^T\bar{Z}_2E\eta(s) \le \begin{bmatrix} x(k-\tau_m)\\ x(k-\tau(k))\\ x(k-\tau_M)\end{bmatrix}^T\Gamma_2\begin{bmatrix} x(k-\tau_m)\\ x(k-\tau(k))\\ x(k-\tau_M)\end{bmatrix}, \tag{15}$$

where

$$\begin{aligned}
\Gamma_1 &= \begin{bmatrix} -E^T\bar{Z}_1E & E^T\bar{Z}_1E\\ * & -E^T\bar{Z}_1E\end{bmatrix},\\
\Gamma_2 &= \begin{bmatrix} -E^T\bar{Z}_2E & E^T\bar{Z}_2E - E^T\bar{Y}E & E^T\bar{Y}E\\ * & -2E^T\bar{Z}_2E + E^T\bar{Y}E + E\bar{Y}^TE^T & -E^T\bar{Y}E + E^T\bar{Z}_2E\\ * & * & -E^T\bar{Z}_2E\end{bmatrix}.
\end{aligned}$$

Using the free weighting matrix method, notice that

$$0 = 2x^T(k+1)E^T\bar{T}^T[-Ex(k+1) + \bar{A}(k)x(k) + \bar{A}_d(k)x(k-\tau(k))]. \tag{16}$$

On the other hand, it is clear from $E^T R = 0$ that

$$\begin{aligned} 0 &= 2x^T(k)\bar{S}R^T Ex(k+1), \\ 0 &= 2x^T(k)\bar{S}R^T \bar{A}(k)x(k) + 2x^T(k)\bar{S}R^T \bar{A}_d(k)x(k-\tau(k)). \end{aligned} \tag{17}$$

Combining the above equations and using Lemma 2.5 we have

$$\Delta V(k) \leq \sum_{i=1}^{m} h_i(s(k)) \sum_{j=1}^{m} h_j(s(k))\xi^T(k)\aleph\, \xi(k), \tag{18}$$

where

$$\begin{aligned} \xi(k) &= \begin{bmatrix} x^T(k) & (Ex(k+1))^T & x^T(k-\tau_m) & x^T(k-\tau(k)) & x^T(k-\tau_M) \end{bmatrix}^T, \\ \aleph &= \Xi + \epsilon_i \begin{bmatrix} S_i R^T M_i \\ T_i^T M_i \\ 0 \\ 0 \\ 0 \\ P_i M_i \end{bmatrix} \begin{bmatrix} S_i R^T M_i \\ T_i^T M_i \\ 0 \\ 0 \\ 0 \\ P_i M_i \end{bmatrix}^T + \epsilon_i^{-1} \begin{bmatrix} N_{1i}^T \\ 0 \\ 0 \\ N_{2i}^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} N_{1i}^T \\ 0 \\ 0 \\ N_{2i}^T \\ 0 \\ 0 \end{bmatrix}^T. \end{aligned}$$

According to Lemma 2.6 (Schur Complement), we get $\aleph < 0$. Therefore,

$$\Delta V(k) \leq -\lambda \parallel x(k) \parallel^2 \tag{19}$$

where $\lambda = -\lambda_{max}(\aleph) > 0$. Thus we obtain,

$$\sum_{i=0}^{k} \parallel x(i) \parallel^2 \leq \frac{1}{\lambda} V(0) < \infty \tag{20}$$

which shows that the system (6) is stable. This completes the proof. □

When the uncertainties are removed, system (6) reduces to the following

$$Ex(k+1) = \sum_{i=1}^{m} h_i(s(k))[A_i x(k) + A_{di} x(k-\tau(k))]. \tag{21}$$

Then we have the following corollary.

Corollary 3.2. *The system (21) is said to be admissible if there exist symmetric positive definite matrices P_i, R_{1i}, R_{2i}, R_{3i}, Z_{1i}, Z_{2i}, $Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \\ * & Q_{3i} \end{bmatrix}$ and real matrices T_i, S_i with appropriate dimensions such that the following LMI holds:*

$$\Xi < 0, \tag{22}$$

where Ξ is defined as in Theorem 3.1 and $R \in \mathbb{R}^{n\times(n-r)}$ is any matrix satisfies $E^T R = 0$ with rank $R = n - r$.

Remark 3.3. Recently for continuous case, authors in [6, 24] have been discussed the problem of stability for a class of nonlinear singular systems with time-delays. Only very few results are focused on discrete-time nonlinear singular systems [29, 30] compared to continuous systems. To the best of the authors' knowledge, T-S fuzzy approach to the discrete-time nonlinear singular systems with time-varying delay is new to the literature.

Remark 3.4. In this paper, augmented type Lyapunov-Krasovskii functional candidate $V_5(k)$ is utilized. The augmented vector $\zeta(k)$ consists of two terms namely the state vector $x(k)$ and the system $Ex(k+1)$. So, the derived stability and causality conditions take the full advantage of the information of the whole system. Thus, results in this investigation are expected to be less conservative.

Remark 3.5. Theorem 3.1 provides the admissibility criterion of the system (6) in terms of LMIs. By solving the LMIs for τ_M with respect to fixed τ_m, one can find the maximum upper bound of time-delay guaranteeing the stability of the system (6).

4. Delay-dependent Stability Analysis for Linear Singular Time-delay Systems

Consider the linear discrete-time singular system with time-varying delay,

$$Ex(k+1) = (A + \Delta A(k))x(k) + (A_d + \Delta A_d(k))x(k - \tau(k)) \tag{23}$$

the admissibility criterion is given in the following theorem.

Theorem 4.1. *The system (23) is said to be admissible if there exist symmetric positive definite matrices* $P,\ R_1,\ R_2,\ R_3,\ Z_1,\ Z_2,\ Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}$, *real matrices* $T,\ S$ *with appropriate dimensions, a positive scalar* ϵ *such that the following LMI holds:*

$$\begin{bmatrix} \Sigma & \Psi_1 M & \epsilon\Psi_2 \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0, \tag{24}$$

where $R \in \mathbb{R}^{n\times(n-r)}$ *is any matrix satisfying* $E^T R = 0$ *with rank* $R = n - r$,

$$\Psi_1 = [(SR^T)^T \quad T \quad 0 \quad 0 \quad 0 \quad P]^T,\ \Psi_2 = [N_1 \quad 0 \quad 0 \quad N_2 \quad 0 \quad 0]^T,$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & 0 & A^T P \\ * & \Sigma_{22} & 0 & \Sigma_{24} & 0 & 0 \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & 0 \\ * & * & * & \Sigma_{44} & \Sigma_{45} & A_d^T P \\ * & * & * & * & \Sigma_{55} & 0 \\ * & * & * & * & * & -P \end{bmatrix},$$

where

$$\begin{aligned}
\Sigma_{11} &= -E^T PE + \tau_m E^T Z_1 E + \tau_1^2 E^T Z_2 E - E^T Z_1 E + \tau_1 Q_1 + R_1 + R_2 \\
&\quad +(\tau_1 + 1)R_3 + SR^T A + A^T RS^T, \\
\Sigma_{12} &= -\tau_m E^T Z_1^T - \tau_1^2 E^T Z_2^T + \tau_1 Q_2 + A^T T,\ \Sigma_{13} = E^T Z_1 E,\ \Sigma_{14} = SR^T A_d, \\
\Sigma_{22} &= \tau_m^2 Z_1 + \tau_1^2 Z_2 + \tau_1 Q_3 - T - T^T,\ \Sigma_{24} = T^T A_d, \\
\Sigma_{33} &= -E^T Z_1 E - E^T Z_2 E - R_1,\ \Sigma_{34} = E^T Z_2 E - E^T Y E,\ \Sigma_{35} = E^T Y E, \\
\Sigma_{44} &= -2E^T Z_2 E + E^T Y E + EY^T E^T - R_3,\ \Sigma_{45} = -E^T Y E + E^T Z_2 E, \\
\Sigma_{55} &= -E^T Z_2 E - R_2.
\end{aligned}$$

Proof. The proof follows immediately by the same way of Theorem 3.1. Hence it is omitted here. □

When the parameter uncertainties are removed, the linear discrete-time linear singular system with time-varying delay is described as

$$Ex(k+1) = Ax(k) + A_d x(k-\tau(k)). \tag{25}$$

The following corollary affords the admissibility criterion of the above system.

Corollary 4.2. *The system (25) is said to be admissible if there exist symmetric positive definite matrices* $P,\ R_1,\ R_2,\ R_3,\ Z_1,\ Z_2,\ Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}$ *and real matrices* T, S *with appropriate dimensions such that the following LMI holds:*

$$\Sigma < \ 0, \tag{26}$$

where Σ *is defined as in Theorem 4.1 and* $R \in \mathbb{R}^{n\times(n-r)}$ *is any matrix satisfying* $E^T R = 0$ *with rank* $R = n - r$.

5. Illustrative Examples

In this section numerical examples are provided to substantiate the theoretical results.

Example 5.1. Consider the following simple discrete-time nonlinear singular system with time-varying delay.

$$\begin{aligned} x_1(k+1) &= -x_1^2(k) + 0.3x_2(k) + 0.1x_1^2(k-\tau(k)) - 0.2x_1(k-\tau(k))x_2(k-\tau(k)), \\ 0 &= 0.1x_1(k) + x_2(k). \end{aligned} \tag{27}$$

We assume that $x_1(k) \in [-0.5, 0.5]$, and select the membership functions as

$$\mu_1(x_1(k)) = \frac{1}{2}(1 - 2x_1(k)),\ \mu_2(x_1(k)) = \frac{1}{2}(1 + 2x_1(k)).$$

Then, the nonlinear time-delay singular system can be represented by the T-S fuzzy model. The 2-rules fuzzy model is given as follows:

$$\begin{aligned} \textit{Plant Rule } 1 \quad &: \quad \textit{IF } x_1(k) \textit{ is } \mu_{11}\ (x_1(k))\ \textit{THEN} \\ Ex(k+1) &= (A_1 + \Delta A_1(k))\ x(k) + (A_{d1} + \Delta A_{d1}(k))x(k-\tau(k)) \\ \textit{Plant Rule } 2 \quad &: \quad \textit{IF } x_1(k) \textit{ is } \mu_{21}\ (x_1(k))\ \textit{THEN} \\ Ex(k+1) &= (A_2 + \Delta A_2(k))x(k) + (A_{d2} + \Delta A_{d2}(k))x(k-\tau(k)) \end{aligned}$$

Here, the system matrices are assumed to have uncertainties. The system matrices are given as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.5 & 0.3 \\ 0.1 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.5 & 0.3 \\ 0.1 & 1 \end{bmatrix},\ A_{d1} = \begin{bmatrix} -0.05 & 0.1 \\ 0 & 0 \end{bmatrix},\ R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ A_{d2} &= \begin{bmatrix} 0.05 & -0.1 \\ 0 & 0 \end{bmatrix},\ F_i(k) = \begin{bmatrix} sin(k) & 0 \\ 0 & cos(k) \end{bmatrix}, E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\ N_{11} &= N_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix},\ N_{21} = N_{22} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix},\ M_1 = M_2 = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

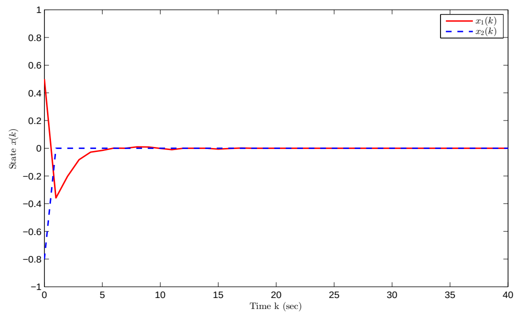

FIGURE 1. State Trajectories of the System (27) with Uncertainties

$\tau(k)$ represents the time-varying state delay, which is given by $\tau(k) = round(8sin(k)+9)$. The delay bounds are $\tau_m = 1$ and $\tau_M = 17$. By solving the LMIs (7) given in Theorem 3.1, we obtain a set of solution matrices as

$$P_1 = \begin{bmatrix} 62.9869 & -8.4482 \\ -8.4482 & 12.2263 \end{bmatrix}, \; P_2 = \begin{bmatrix} 53.3100 & -5.3138 \\ -5.3138 & 10.9802 \end{bmatrix},$$
$$\epsilon_1 = 21.1622, \; \epsilon_2 = 18.6569.$$

The state responses of the system are given in Figure 1, which shows that the considered system achieves stability.

Example 5.2. Consider the above nonlinear system without uncertainties. The 2−rules fuzzy model is given as follows:

$$\begin{aligned} \textit{Plant Rule } 1 &: \; \textit{IF } x_1(k) \textit{ is } \mu_{11} \; (x_1(k)) \; \textit{THEN} \\ Ex(k+1) &= A_1 \; x(k) + A_{d1}x(k-\tau(k)) \\ \textit{Plant Rule } 2 &: \; \textit{IF } x_1(k) \textit{ is } \mu_{21} \; (x_1(k)) \; \textit{THEN} \\ Ex(k+1) &= A_2x(k) + A_{d2}x(k-\tau(k)) \end{aligned}$$

with the parameters defined as in Example 5.1. Solving the LMIs (22) given in Corollary 3.2, we get the solution matrices as

$$P_1 = \begin{bmatrix} 60.5394 & -7.6229 \\ -7.6229 & 11.0781 \end{bmatrix}, \; P_2 = \begin{bmatrix} 46.5163 & -4.0643 \\ -4.0643 & 10.4512 \end{bmatrix}$$

Figure 2 shows the state trajectories of the nonlinear singular system (27) without uncertainties. Thus, the simulation results affirm our theoretical results.

Example 5.3. For the practical applicability of the result stated in Theorem 4.1, consider the nominal discrete-time model with time-varying delay for a DC motor in a hydraulic system [4] described by,

$$\begin{aligned} x_1(k+1) &= 0.4121x_1(k) + 0.8113x_2(k) \\ 0 &= -0.345x_1(k) - x_2(k) + 0.1x_1(k-\tau(k)) + 0.1x_2(k-\tau(k)) \end{aligned} \quad (28)$$

where $x_1(k)$ is the axis speed and $x_2(k)$ is the armature current. Because of viscous-friction coefficient B, torque constantK_t and the motor back-EMF constant K_b may

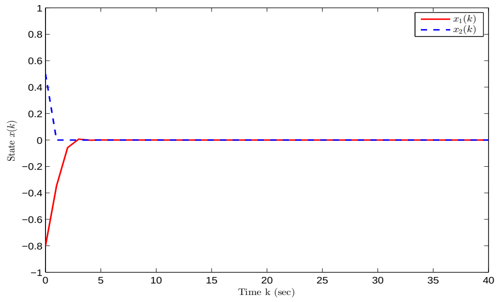

FIGURE 2. State Trajectories of the System (27) without Uncertainties

not be measured exactly, that results in the parametric uncertainties in the system model described by

$$Ex(k+1) = (A + \Delta A(k))x(k) + A_d x(k - \tau(k)).$$

Comparing the above model with system (23), the parameters are assumed to be

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ A = \begin{bmatrix} 0.4121 & 0.8113 \\ 0.1 & 1 \end{bmatrix}, \ A_d = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0.0345 & 0 \end{bmatrix}, \ N_2 = 0, \ M = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}.$$

The lower and upper bounds of the time-varying delay $\tau(k)$ are assumed as $\tau_m = 1$ and $\tau_M = 6$ respectively. By solving the LMIs in (24), we obtain

$$P = \begin{bmatrix} 5.9574 & 1.9922 \\ 1.9922 & 1.9986 \end{bmatrix}, \ \epsilon = 2.8664$$

which ensures the stability of DC motor in a hydraulic system (28).

Example 5.4. Consider system (25) with

$$A = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \ A_d = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}, \ E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Here we choose $\tau_m = 1$ and $\tau_M = 10$, by solving the LMI (26), one can obtain

$$P = \begin{bmatrix} 490.3260 & 3.9895 \\ 3.9895 & 76.0967 \end{bmatrix}.$$

By assuming $\tau_m = 3$ and applying the methods of [10, 21, 8, 7] and Corollary 4.2 of this paper, the allowable maximum bounds of τ_M, ensuring the admissibility of the system are listed in Table 1. It is clear from Table 1 that the proposed delay-dependent sufficient stability conditions are less conservative than the results available in the literature. Further, the performance improvement of the proposed results with respect to driving time is shown in Figure 3. It is obvious from the figure that, our method reduces the time complexity greatly with maximum delay upper bound.

Methods	Maximum delay bound τ_M	Number of variables
Theorem 2 of [7]	16	$6n(n+1)/2+2n^2$
Theorem 1 of [10]	18	$17n(n+1)/2+5n^2$
Corollary 1 of [21]	18	$9n(n+1)/2+n^2$
Corollary 1 of [8]	18	$6n(n+1)/2+2n^2$
Theorem 1 of [13]	19	$7n(n+1)/2$
Corollary 4.2	82	$9n(n+1)/2+2n^2$

Table 1. Comparison of the Allowable Upper Bound τ_M

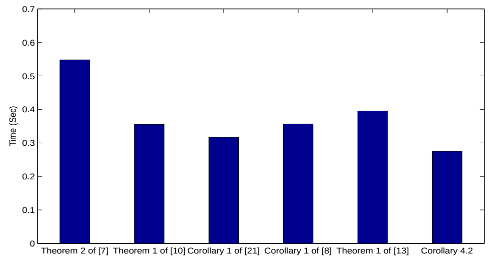

Figure 3. Performance Improvement with Respect to Driving Time

6. Conclusion

In this paper, we have investigated the admissibility conditions for discrete-time singular systems with time-varying delays by adopting T-S fuzzy model. Based on the Lyapunov-Krasovskii functional method and by using free-weighting method, robust admissibility conditions have been derived in terms of LMIs. Finally, numerical examples have been exploited to illustrate the applicability and usefulness of the theoretical results. By comparing the proposed results with the recent results, it has been shown that the derived stability criteria for linear singular systems are less conservative than the recent results available in the literature.

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