SOME CLASSES OF STATISTICALLY CONVERGENT SEQUENCES OF FUZZY NUMBERS GENERATED BY A MODULUS FUNCTION

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Abstract. The purpose of this paper is to generalize the concepts of statistical convergence of sequences of fuzzy numbers defined by a modulus function using difference operator Δ and give some inclusion relations.

1. **Introduction**

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [23] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [13] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties. Matloka has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by Nanda [15], Nuray and Savas [17], Kwon [12], Basarır and Mursaleen ([4],[14]),Tripathy [22], Altinok et al. [1] and many others.

The notion of statistical convergence was introduced by Fast [9] and Schoenberg [20], independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Later on it was further investigated from the sequence space point of view and linked with summability theory by Fridy [10], Šalát [18], Connor [5] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Čech compactification of the natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability.

In the present paper, we extend the notions of statistical convergence of sequences of fuzzy numbers defined by a modulus function using difference operator Δ and give some relation theorems so as to fill up the existing gaps in the theory of statistical convergence of fuzzy numbers.

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2. Definitions and Preliminaries

In this section we recall some basic definitions and notations which will be used throughout this paper.

The idea of statistical convergence depends on the density of subsets of the set $\mathbb{N}$ of natural numbers. A subset E of $\mathbb{N}$ is said to have density positive integer denoted by $\delta(E)$, if $\delta(E) = \lim\limits_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \chi_E(k)$ exists, where χ_E is the characteristic function of E. It is clear that any finite subset of $\mathbb{N}$ has zero natural density and $\delta(E^c) = 1 - \delta(E)$.

If a property $P(k)$ holds for all $k \in E$ with $\delta(E) = 1$, we say that P holds for almost all k, and we abbreviate this by "a.a.k."

A sequence (x_k) is said to be statistically convergent to L if for every $\varepsilon > 0$, $\delta(\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}) = 0$. In this case we write $S - \lim x_k = L$.

Let $C(\mathbb{R}^n) = \{A \subset \mathbb{R}^n : A \text{ is compact and convex}\}$. The space $C(\mathbb{R}^n)$ has a linear structure induced by the operations $A + B = \{a + b : a \in A,\ b \in B\}$ and $\mu A = \{\mu a : a \in A\}$ for $A, B \in C(\mathbb{R}^n)$ and $\mu \in \mathbb{R}$. The Hausdorff distance between A and B in $C(\mathbb{R}^n)$ is defined as

$$\delta_\infty(A, B) = \max\left\{\sup_{a\in A}\inf_{b\in B}\|a - b\|, \sup_{b\in B}\inf_{a\in A}\|a - b\|\right\}.$$

It is well known that $(C(\mathbb{R}^n), \delta_\infty)$ is a complete metric space.

A fuzzy number is a function X from $\mathbb{R}^n$ to $[0, 1]$ which is normal, fuzzy convex, upper semi-continuous and the closure of $\{x \in \mathbb{R}^n : X(x) > 0\}$ is compact. These properties imply that for each $0 \leq \alpha \leq 1$, the $\alpha-$level set $[X]^\alpha = \{x \in \mathbb{R}^n : X(x) \geq \alpha\}$ is a non-empty compact convex subset of $\mathbb{R}^n$, with support $X^0 = \{x \in \mathbb{R}^n : X(x) > 0\}$. Let $L(\mathbb{R}^n)$ denote the set of all fuzzy numbers. The linear structure of $L(\mathbb{R}^n)$ induces the addition $X + Y$ and the scalar multiplication μX $(\mu \in \mathbb{R})$, in terms of $\alpha-$level sets, by

$$[X + Y]^\alpha = [X]^\alpha + [Y]^\alpha, \qquad [\mu X]^\alpha = \mu [X]^\alpha$$

for each $0 \leq \alpha \leq 1$.

Define, for each $1 \leq q < \infty$,

$$d_q(X, Y) = \left(\int_0^1 \delta_\infty(X^\alpha, Y^\alpha)^q \, d\alpha\right)^{1/q}$$

and $d_\infty(X, Y) = \sup\limits_{0\leq\alpha\leq 1} \delta_\infty(X^\alpha, Y^\alpha)$. Clearly $d_\infty(X, Y) = \lim\limits_{q\to\infty} d_q(X, Y)$ with $d_q(X, Y) \leq d_s(X, Y)$ if $q \leq s$. Moreover $(C(\mathbb{R}^n), d_q)$ is a complete, separable and locally compact metric space [6]. Throughout the paper, d will denote d_q with $1 \leq q \leq \infty$.

A sequence $X = (X_k)$ of fuzzy numbers is a function X from the set $\mathbb{N}$ of all positive integers into $L(\mathbb{R})$. Thus, a sequence of fuzzy numbers (X_k) is a correspondence from the set of positive integers to a set of fuzzy numbers.

Nuray and Savas [17] defined the notion of statistical convergence for sequences of fuzzy numbers.

Let $X=(X_k)$ be a sequence of fuzzy numbers. Then (X_k) is said to be statistically convergent to the fuzzy number X_0, if

$$\lim_{n\to\infty}\frac{1}{n}\left|\{k\leq n : d\left(X_k,X_0\right)\geq\varepsilon\}\right|=0$$

for every $\varepsilon>0$, where the vertical bars indicate the number of elements in the enclosed set. In this case, we write $S_F-\lim X_k=X_0$.

The difference sequence spaces $\ell_\infty\left(\Delta\right)$, $c\left(\Delta\right)$ and $c_0\left(\Delta\right)$, consisting of all real valued sequences $x=(x_k)$ such that $\Delta^1 x=(x_k-x_{k+1})$ in the sequence spaces ℓ_∞, c and c_0, were defined by Kızmaz [11]. The idea of difference sequences was generalized by Et and Çolak [7], Et et al. [8], Altınok and Mursaleen [3] and Altınok and Çolak [2].

Let w be the set of all sequences of fuzzy numbers. The operators $\Delta^r,\Sigma^r : w\to w$ are defined by

$$\begin{aligned}\left(\Delta^1 X\right)_k &= \Delta^1 X_k = X_k - X_{k+1},\\ \left(\Sigma^1 X\right)_k &= \sum_{j=1}^{k-1} X_j,\ \left(k=0,1,\ldots\right),\\ \Delta^r &= \Delta^1\circ\Delta^{r-1},\\ \Sigma^r &= \Sigma^1\circ\Sigma^{r-1},\ \left(r\geq 2\right)\end{aligned}$$

and $\Sigma^r\circ\Delta^r=\Delta^r\circ\Sigma^r=id$, the identity on w.

It is trivial that the generalized difference operator Δ^r is a linear operator.

The notion of modulus function was introduced by Nakano [16] we recall that a modulus f is a function from $[0,\infty)$ to $[0,\infty)$ such that *i*) $f\left(x\right)=0$ if and only if $x=0$, *ii*) $f\left(x+y\right)\leq f\left(x\right)+f\left(y\right)$ for $x,y\geq 0$, *iii*) f is incerasing, *iv*) f is continuous from the right at 0. Since then, the concept of modulus function in sequences of fuzzy numbers has been studying by many mathematicians (See [19],[21])

3. **Proposed Theorems**

Definition 3.1. Let f be a modulus function and $L(\mathbb{R})$ be class of all fuzzy numbers defined on $\mathbb{R}$. A sequence $X=(X_k)\subset L(\mathbb{R})$ is said to be statistically convergent by f modulus function if for every $\varepsilon>0$ there is a fuzzy number $X_0\in L\left(\mathbb{R}\right)$ such that

$$\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\leq n : f\left[\overline{d}(X_k,X_0)\right]\geq\varepsilon\right\}\right|=0.$$

Where $\overline{d}$ defined as

$$\overline{d}(u,v)=\sup_{0\leq\alpha\leq 1} d([u]^\alpha,[v]^\alpha)$$

In this case we write $X_k\to X_0(S_F\left(f\right))$ or $S_F(f)-\lim X_k=X_0$.

We shall use $S_F(f)$ to denote the set of all statistically convergent by a modulus function sequences of fuzzy numbers. That is

$$S_F\left(f\right)=\left\{X=(X_k)\in w\left(F\right) : X_k\to X_0(S_F(f)),X_0\in L\left(\mathbb{R}\right)\right\}.$$

$(S_F(f), d_f\ )$ is a metric space where

$$d_f(X,Y) = \sup_{k\in\mathbb{N}} f\left[\overline{d}(X_k, Y_k)\right].$$

We can easily show that d_f satisfies metric axioms for $X, Y, Z \in S_F(f)$

$$\begin{aligned} i)\ d_f(X,Y) = 0 & \\ &\Leftrightarrow \sup_{k\in\mathbb{N}} f\left[\overline{d}(X_k, Y_k)\right] = 0 \\ &\Leftrightarrow \sup_{k\in\mathbb{N}} f[\sup_{0\le\alpha\le1} \overline{d}([X_k]^\alpha, [Y_k]^\alpha) = 0 \\ &\Leftrightarrow f[\sup_{0\le\alpha\le1} \overline{d}([X_k]^\alpha, [Y_k]^\alpha) = 0 \\ &\Leftrightarrow \sup_{0\le\alpha\le1} \overline{d}([X_k]^\alpha, [Y_k]^\alpha) = 0 \\ &\Leftrightarrow \overline{d}([X_k]^\alpha, [Y_k]^\alpha) = 0 \\ &\Leftrightarrow [X_k]^\alpha = [Y_k]^\alpha \\ &\Leftrightarrow X = Y \end{aligned}$$

and taking into account definition of supremum, $d_f(X,Y) \ge 0$ is obvious.

$$\begin{aligned} ii)\ d_f(X,Y) &= \sup_{k\in\mathbb{N}} f\left[\overline{d}(X_k, Y_k)\right] \\ &= \sup_{k\in\mathbb{N}} f\left[\overline{d}(Y_k, X_k)\right] \\ &= d_f(Y,X). \end{aligned}$$

From properties of supremum, d and modulus function, following inequality holds

$$\begin{aligned} iii)\ d_f(X,Y) &= \sup_{k\in\mathbb{N}} f\left[\overline{d}(X_k, Y_k)\right] \\ &\le \sup_{k\in\mathbb{N}} f\left[\overline{d}(X_k, Z_k) + \overline{d}(Z_k, Y_k)\right] \\ &\le \sup_{k\in\mathbb{N}} f\left[\overline{d}(X_k, Z_k)\right] + \sup_{k} f\left[\overline{d}(Z_k, Y_k)\right] \\ &= d_f(X,Z) + d_f(Z,Y). \end{aligned}$$

Definition 3.2. Let $f : [0,\infty) \to [0,\infty)$ be a modulus function and $\Delta X_k = X_k - X_{k+1}$. A sequence $X = (X_k) \subset L(\mathbb{R})$ is said to be $\Delta-$statistically convergent by modulus function f if for every $\varepsilon > 0$ there is a fuzzy number $X_0 \in L(\mathbb{R})$ such that

$$\lim_{n\to\infty} \frac{1}{n} \left|\left\{k \le n : f\left[\overline{d}(\Delta X_k, X_0)\right] \ge \varepsilon\right\}\right| = 0.$$

In this case we write $X_k \to X_0(S_F(\Delta, f))$ or $S_F(\Delta, f) - \lim X_k = X_0$.

We shall use $S_F(\Delta, f)$ to denote the set of all $\Delta-$statistically convergent by a modulus function f sequences of fuzzy numbers. That is

$$S_F(\Delta, f) = \{X = (X_k) \in w(F) : X_k \to X_0(S_F(\Delta, f)), X_0 \in L(\mathbb{R})\}.$$

Theorem 3.3. *Let f_1 and f_2 be two any modulus functions. Then*

$$S_F(f_1) \cap S_F(f_2) \subseteq S_F(f_1 + f_2).$$

Proof. Suppose that $X = (X_k) \in S_F(f_1) \cap S_F(f_2)$. Then

$$\lim_{n\to\infty} \frac{1}{n} \left|\left\{k \le n : f_1 \left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| = 0$$

and

$$\lim_{n\to\infty} \frac{1}{n} \left|\left\{k \le n : f_2 \left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| = 0$$

hold, for every $\varepsilon > 0$. Since

$$(f_1 + f_2)\left[\overline{d}(X_k, X_0)\right] = f_1\left[\overline{d}(X_k, X_0)\right] + f_2\left[\overline{d}(X_k, X_0)\right],$$

we can write following equality:

$$\lim_{n\to\infty} \frac{1}{n} \left|\left\{k \le n : (f_1 + f_2)\left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| = 0.$$

Hence $X = (X_k) \in S_F(f_1 + f_2)$. Since $X = (X_k)$ is an arbitrary sequence we have $S_F(f_1) \cap S_F(f_2) \subseteq S_F(f_1 + f_2)$. □

Theorem 3.4. *Let f_1 and f_2 be two modulus functions. Then*

$$S_F(\Delta, f_1) \cap S_F(\Delta, f_2) \subseteq S_F(\Delta, f_1 + f_2).$$

Proof. Proof is obvious from Theorem 3.3. therefore we omitted it. □

Theorem 3.5. *Let f_1 and f_2 be two modulus functions such that $f_1(u) \le f_2(u)$ for every $u \in [0, \infty)$. Then*

$$S_F(f_2) \subseteq S_F(f_1).$$

Proof. Let $X = (X_k)$ be any element of $S_F(f_2)$. Then for every $\varepsilon > 0$ there is a fuzzy number X_0 such that

$$\lim_{n\to\infty} \frac{1}{n} \left|\left\{k \le n : f_2 \left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| = 0.$$

Since

$$\left|\left\{k \le n : f_1 \left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| \le \left|\left\{k \le n : f_2 \left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right|$$

holds for any $n \in \mathbb{N}$, we can write

$$\lim_{n\to\infty} \left|\left\{k \le n : f_1 \left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| = 0.$$

So $X = (X_k) \in S_F(f_1)$. Since $X = (X_k)$ is an arbitrary sequence, we have $S_F(f_2) \subseteq S_F(f_1)$. □

Theorem 3.6. *Let f_1 and f_2 be two modulus functions such that $f_1(u) \le f_2(u)$ for every $u \in [0, \infty)$. Then*

$$S_F(\Delta, f_2) \subseteq S_F(\Delta, f_1).$$

Proof. Since proof is obvious from Theorem 3.5. we omitted it. □

Theorem 3.7. *Let f_1 and f_2 be any two modulus functions. Then*

$$S_F(f_1) \subset S_F(f_2 \circ f_1).$$

Proof. Let $X=(X_k)$ be any element of $S_F(f_1)$. For every $\varepsilon>0$ there is an inverse image $f_2^{-1}(\varepsilon)$. On the other hand, there is a fuzzy number X_0 for $f_2^{-1}(\varepsilon)>0$ such that

$$\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n : f_1\left[\overline{d}\left(X_k,X_0\right)\right]\ge f_2^{-1}(\varepsilon)\right\}\right|=0.$$

Then we write for $k\le n$

$$\begin{aligned} f_2\left(f_1\left[\overline{d}\left(X_k,X_0\right)\right]\right)&\ge f_2(f_2^{-1}(\varepsilon)),\\ (f_2\circ f_1)\left[\overline{d}\left(X_k,X_0\right)\right]&\ge\varepsilon.\end{aligned}$$

Hence we have

$$\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n : (f_2\circ f_1)\left[\overline{d}\left(X_k,X_0\right)\right]\ge\varepsilon\right\}\right|=0$$

that is $X=(X_k)\in S_F\left(f_2\circ f_1\right)$. Since $X=(X_k)$ is arbitrary, we have $S_F\left(f_1\right)\subset S_F\left(f_2\circ f_1\right)$. □

Example 3.8. Let $f\left(u\right)=\frac{u}{u+1}$ be a modulus function and $X=(X_k)$ be a sequence of fuzzy numbers as follows:

$$X_k=\begin{cases}\left.\begin{array}{ll}\frac{k}{2k+3}x+\frac{-2k+3}{2k+3}, & \text{if } x\in\left[\frac{2k-3}{k},4\right]\\ -\frac{k}{2k+3}x+\frac{6k+3}{2k+3}, & \text{if } x\in\left[4,\frac{6k+3}{k}\right]\\ 0, & \text{otherwise}\end{array}\right\}, & \text{if } k=3^n,(n=1,2,3,...)\\ X_0\left(x\right), & \text{if } k\ne 3^n\end{cases}$$

$$X_0\left(x\right)=\begin{cases}x+1, & \text{if } x\in[-1,0]\\ -x+1, & \text{if } x\in[0,1]\\ 0, & \text{otherwise.}\end{cases}$$

Then

$$\left[X_k\left(x\right)\right]^{\alpha}=\begin{cases}\left[\frac{(2k+3)\alpha+2k-3}{k},\frac{-(2k+3)\alpha+6k+3}{k}\right], & \text{if } k=3^n\\ \left[\alpha-1,1-\alpha\right], & \text{if } k\ne 3^n\end{cases}$$

and we can write

$$\left[\Delta X_k\left(x\right)\right]^{\alpha}=\begin{cases}\left[\frac{(3+3k)\alpha+k-3}{k},\frac{-(3k+3)\alpha+7k+3}{k}\right], & \text{if } k=3^n\\ \left[\frac{(6+3k)\alpha-7k-10}{k+1},\frac{-(6+3k)\alpha-k+2}{k+1}\right], & \text{if } k+1=3^n\\ \left[2\alpha-2,-2\alpha+2\right], & \text{otherwise.}\end{cases}$$

So if $k\ne 3^n$ then $\overline{d}\left(X_k,X_0\right)=0$. If $k=3^n$ then

$$\begin{aligned}\overline{d}\left(X_k,X_0\right)&=\sup_{0\le\alpha\le1} d\left(\left[X_k\right]^{\alpha},\left[X_0\right]^{\alpha}\right)\\ &=d\left(\left[X_k\right]^{0},\left[X_0\right]^{0}\right)\\ &=\max\left\{\left|\underline{\left[X_k\right]^{0}-\left[X_0\right]^{0}}\right|,\left|\overline{\left[X_k\right]^{0}-\left[X_0\right]^{0}}\right|\right\}\\ &=\frac{5k+3}{k}.\end{aligned}$$

On the other hand, we have

$$f\left[\overline{d}\left(X_k, X_0\right)\right] = \begin{cases} \frac{5k+3}{6k+3}, & \text{if } k = 3^n, (n = 1, 2, 3, ...) \\ 0, & \text{if } k \neq 3^n. \end{cases}$$

Similarly we have

$$f\left[\overline{d}\left(\Delta X_k, L\right)\right] = \begin{cases} \frac{5k+3}{6k+3}, & \text{if } k = 3^n \\ \frac{5k+8}{6k+9}, & \text{if } k+1 = 3^n \\ 0, & \text{otherwise.} \end{cases}$$

Then for every $\varepsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} \left|\left\{k \leq n : f\left[\overline{d}\left(\Delta X_k, L\right)\right] \geq \varepsilon\right\}\right| = 0.$$

Hence $X = (X_k) \in S_F(\Delta, f)$. (See Figure 1)

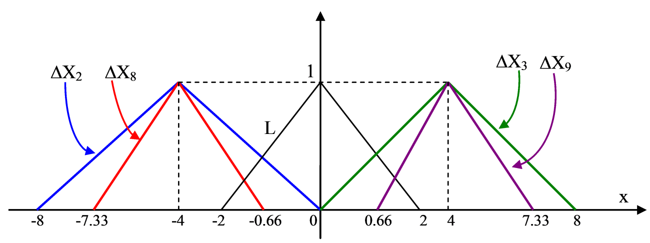

FIGURE 1

In general, we call the terms of a sequence preventing it converging to an element of a space as contradictory term. Therefore, a statistically convergent sequence can create a convergent sequence by excluding contradictory terms. For example in the sequence $(0,0,0,0,1,0,0,0,0,2,0,0,0,0,3,...)$, 5th, 10th, 15th, etc. terms are contradictory terms. These terms prevent the sequence to converge zero. For a sequence (x_k) if for each $\varepsilon > 0$, there exists x_0 such that $\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |x_k - x_0| \geq \varepsilon\}| = 0$, then we say that "$x_k$ is statistically convergent to x_0". Therefore, for a statistically convergent sequence, if we call the size of the speed of convergence as "the statistical convergence of the sequence is much more powerful", then the statistical convergence of a sequence becomes powerful when the amount of contradictory terms decreases. In this work, we see that the power of statistical convergence of a sequence is affected by its modulus function.

Corollary 3.9. *Let f_1 and f_2 be two modulus functions such that $f_1(u) \leq f_2(u)$ for every $u \in [0,\infty)$. Then we can say "statististical convergence of any sequence according to f_1 is stronger than according to f_2 ". Now we give an example of this corollary for sequence of fuzzy numbers.*

Example 3.10. Let $f_1(u) = \frac{u}{10u+1}$ and $f_2(u) = 10^{10}u$ be two modulus functions and $X = (X_k)$ a sequence of fuzzy numbers as follows;

$$X_k(x) = \begin{cases} \left.\begin{matrix} 1, & \text{if } k = \frac{1}{x} \\ 0, & \text{if } k \neq \frac{1}{x} \end{matrix}\right\}, & \text{if } x \neq 0 \\ \overline{0}, & \text{if } x = 0. \end{cases}$$

So we can write $[X_k]^\alpha = \left\{\frac{1}{k}\right\}$ and $\left[\overline{0}\right]^\alpha = \{0\}$ where $0 \le \alpha \le 1$. Also we get $\overline{d}\left(X_k, \overline{0}\right) = \frac{1}{k}$. We get following results

$$\begin{aligned} f_1\left[\overline{d}\left(X_1, \overline{0}\right)\right] &= \frac{1}{11},\ f_1\left[\overline{d}\left(X_2, \overline{0}\right)\right] = \frac{1}{12}, \\ f_1\left[\overline{d}\left(X_3, \overline{0}\right)\right] &= \frac{1}{13}, \cdots,\ f_1\left[\overline{d}\left(X_k, \overline{0}\right)\right] = \frac{1}{k}, \cdots \end{aligned}$$

and

$$\begin{aligned} f_2\left[\overline{d}\left(X_1, \overline{0}\right)\right] &= 10^{10}, f_2\left[\overline{d}\left(X_2, \overline{0}\right)\right] = \frac{10^{10}}{2}, \\ f_2\left[\overline{d}\left(X_3, \overline{0}\right)\right] &= \frac{10^{10}}{3}, \cdots,\ f_2\left[\overline{d}\left(X_k, \overline{0}\right)\right] = \frac{10^{10}}{k}, \cdots \end{aligned}$$

Since the following inequality holds

$$\left|\left\{k \le n : f_1\left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| \le \left|\left\{k \le n : f_2\left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right|,$$

statistical convergence of (X_k) to $\overline{0}$ to f_1 is stronger than one of (X_k) according to f_2. For example

$$\left|\left\{k \le n : f_1\left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| = 90$$

and

$$\left|\left\{k \le n : f_2\left[\overline{d}(X_k, X_0)\right] \ge \varepsilon\right\}\right| = 10^{12}$$

where $\varepsilon = 10^{-2}$ and $n \ge 10^{12}$. This means that number of contradictory terms of sequence (X_k) according to f_1 is 90 but according to f_2 is 10^{12}.

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