

## DEVELOPING NEW METHODS TO MONITOR PHASE II FUZZY LINEAR PROFILES

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**ABSTRACT.** In some quality control applications, the quality of a process or a product is described by the relationship between a response variable and one or more explanatory variables, called a profile. Moreover, in most practical applications, the qualitative characteristic of a product/service is vague, uncertain and linguistic and cannot be precisely stated. The purpose of this paper is to propose a method for monitoring simple linear profiles with a fuzzy and ambiguous response. To this end, fuzzy EWMA and fuzzy Hotelling's  $T^2$  statistics are developed using the extension principle. To monitor phase II of fuzzy linear profiles, two methods using fuzzy hypothesis testing, are presented based on these statistics. A case study in ceramic and tile industry, is provided. A simulation study to evaluate the performance of the proposed methods in terms of average run length (ARL) criterion showed that the proposed methods are very efficient in detecting various sized shifts in process profiles.

### 1. Introduction

The quality of services and products has a key role in the success and competitiveness of organizations. Several definitions have been proposed for quality; for example, in [11], the author defined quality of service or product as inversely proportional to variability of the process. Such a definition of quality is based on the belief that decreasing variability of key characteristics of a product or service increases its quality. Statistical quality control techniques such as Statistical Process Control (SPC) are effective tools for reducing process variability and improving quality. Although it is assumed, in most cases, that process quality can be defined using one or more quality characteristics, in some practical applications, the quality of a product or process is better represented and evaluated by a functional relationship between a dependent variable and one or more independent variables, called a profile. Monitoring profiles is carried out in two phases. The main goal of Phase I is to analyze a historical set of data collected over time to study the process variations, determine the stability of the process, and remove anomalous samples. The in-control values of the process parameters are then estimated and used in designing control charts for the Phase II analysis. The performance of a Phase I control chart method is usually measured according to the probability of obtaining at least one charted statistic outside the control limits, also known as the probability of signal. The purpose of Phase II is to quickly detect parameter

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changes from the in-control parameter values. The performance of control chart methods in Phase II is usually measured in terms of the average run length (ARL), in which run length is the number of taken samples until the chart shows an out-of-control signal. Several control charts have been proposed for the purpose of monitoring profiles in phases I and II. In practice, an appropriate chart is selected depending on the type of the functional structure (linear, non-linear, multiple) of the profile. However, in many real applications, the qualitative characteristics of a product or service cannot be measured and expressed in precise numbers. Many types of data are stated in terms of linguistic definitions such as high temperature or low flexibility; such data are called imprecise or fuzzy. Also, the data that are the result of precise measurement of a continuous variable are not precise numbers; they are more or less fuzzy [18]. In such circumstances, there are two types of uncertainty: one related to the variability that is modeled by probabilistic distributions and models and another that is modeled by theory of fuzzy sets. To date, in all studies published on phase II of monitoring fuzzy profiles, probabilistic models have been discussed without considering the second type of uncertainty. In this paper, we investigate monitoring fuzzy simple linear profiles in phase II and propose new methods based on fuzzy EWMA ( $\widetilde{FEWMA}$ ) and fuzzy Hotelling's  $T^2$  ( $\widetilde{FT}^2$ ) statistics for monitoring these profiles. The performance of the proposed models are evaluated and compared by simulation study using ARL criterion.

The organization of the paper is as follows: in section 2, previous researches for phase II monitoring of simple linear profiles are reviewed. Some preliminary concepts of fuzzy set theory are briefly recalled in section 3. In section 4, fuzzy linear profiles, their assumptions and fuzzy regression model for identifying fuzzy profiles of samples are described. Section 5, introduces fuzzy EWMA and fuzzy Hotelling's  $T^2$  statistics and briefly reviews fuzzy hypothesis testing. The proposed method for analyzing these statistics is also discussed in section 5. Section 6 demonstrates an application in the tile and ceramic industry. In section 7, a comprehensive comparison of the two methods is carried out on the basis of average run length (ARL) for various out-of-control scenarios. Finally, conclusions and possible future research themes are provided in section 8.

## 2. Literature Review

Relatively extensive studies have been carried out for monitoring crisp profiles in phase II. This section briefly reviews previous research for simple linear profiles, which is the issue in this article. The approaches proposed for phase II monitoring of simple linear profiles can be divided into two main groups [14]: omnibus control charts and individual control charts. Both groups of methods follow the general goal of rapidly detecting changes in process profile, with the only difference being that the first group of methods monitors changes in all profile parameters simultaneously, while the methods of the second group use individual control charts for monitoring each of the profile parameters separately.

Studies in the first group are as follows: In [1], authors proposed a method based on the principle of inverse calibration. Two types of control charts for phase II monitoring, one based on the bivariate  $T^2$  statistic and another based on exponentially

weighted moving average (EWMA) statistic, along with  $R$  are suggested in [6]; In [21], authors used change point method while assuming unknown model parameters and available controlled historical data for estimating the model parameters and used the likelihood ratio statistic for investigating change in these parameters. The use of Generalized Linear Test (GLT) along with  $R$  control charts is suggested in [12]. A self-starting control chart approach to monitor phase II profiles is proposed in [22]. In [19], the authors proposed the use of control charts based on the likelihood ratio for monitoring simple linear profiles. This chart integrates EWMA procedure for detecting changes in slope, intercept and standard deviation. The Variable Sampling Interval (VSI) EWMA method is proposed in [9]. In [20], authors proposed methods for monitoring the slope of linear profiles in phases I and II. The use of neural networks for monitoring simple linear profiles is suggested in [5] and the ANN1 and ANN-a methods, which belong to the group of omnibus control charts, and 3ANN method, which is related to the other group, are also presented. Authors in [2] propose using adaptive neuro-fuzzy systems for monitoring profiles in phase II. Methods of the second category use individual control charts in order to monitor profile parameters. All these methods are based on the method proposed in [7]. The authors in [7] showed that coding values of independent variables in order to change the averages to zero, lead to independent estimators for profile parameters. Then they proposed separate EWMA control charts for monitoring each of model parameters. In [15], the use of CUSUM statistic instead of the proposed EWMA in [7] is suggested. A fuzzy exponentially weighted moving average control chart is discussed in [16] for univariate data. A fuzzy exponentially weighted moving average control chart is discussed in [16] for univariate data.

Few studies have already been published in the field of fuzzy linear profiles the subject of which are phase I fuzzy profile monitoring. In [13], a univariate approach for monitoring phase I fuzzy quality profiles is developed and in [3] a multivariate approach for monitoring process/product fuzzy quality profiles in phase I is proposed. This multivariate approach includes three fuzzy multivariate control charts (fuzzy  $T^2$ , fuzzy MEWMA and fuzzy MCUSUM). All the studies published in phase II of simple linear profiles have been performed in crisp and deterministic environments.

### 3. Preliminary Concepts

**Definition 3.1.** The fuzzy number  $\tilde{A} = (a, \lambda, \beta)$  is called an asymmetrical triangular fuzzy number with center  $a$ , left spread  $\lambda$  and right spread  $\beta$  if its membership function is as follows:

$$\mu_{\tilde{A}(x)} = \begin{cases} \frac{x - (a - \lambda)}{\lambda} & a - \lambda < x \leq a \\ \frac{(a + \beta) - x}{\beta} & a \leq x \leq a + \beta \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

If right and left spreads of a triangular fuzzy number are equal ( $\lambda = \beta = s$ ), the triangular fuzzy number is called symmetric and denoted by  $(a, s)$ .

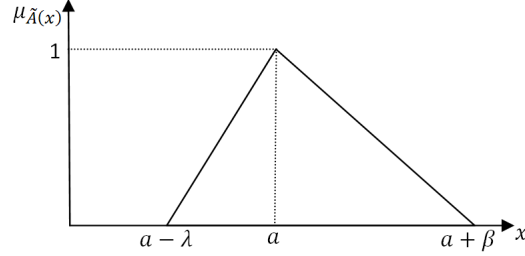


FIGURE 1. Triangular Fuzzy Number

**Definition 3.2.** The  $\alpha$ -cut of fuzzy set  $\tilde{A}$  is a crisp set including elements from reference set  $X$  with membership degree in fuzzy set  $\tilde{A}$  of at least as big as  $\alpha$  ( $0 < \alpha \leq 1$ ). We denoted the  $\alpha$ -cut of fuzzy set  $\tilde{A}$  by  $C_\alpha(\tilde{A})$ ,

$$C_\alpha(\tilde{A}) = \{x \in X \mid \mu_{\tilde{A}(x)} \geq \alpha\} \quad (2)$$

If  $\tilde{A}$  is an asymmetrical triangular fuzzy number,  $C_\alpha(\tilde{A})$  is:

$$C_\alpha(\tilde{A}) = [(\alpha - 1)\lambda + a, (1 - \alpha)\beta + a] = [a^L(\alpha), a^R(\alpha)] \quad (3)$$

**Definition 3.3.** The distance between two asymmetrical triangular fuzzy numbers  $\tilde{A}, \tilde{B}$  ( $d(\tilde{A}, \tilde{B})$ ) is defined by:

$$d(\tilde{A}, \tilde{B}) = \left( \int_0^1 d^2(C_\alpha(\tilde{A}), C_\alpha(\tilde{B})) d\alpha \right)^{\frac{1}{2}} \quad (4)$$

where

$$d^2(C_\alpha(\tilde{A}), C_\alpha(\tilde{B})) = [a^R(\alpha) - b^R(\alpha)]^2 + [a^L(\alpha) - b^L(\alpha)]^2$$

If we write  $C_\alpha(\tilde{A}) = [a^L(\alpha), a^R(\alpha)]$  and  $C_\alpha(\tilde{B}) = [b^L(\alpha), b^R(\alpha)]$ , the square of distance between two asymmetrical triangular fuzzy numbers  $\tilde{A} = (a, \lambda_a, \beta_a)$  and  $\tilde{B} = (b, \lambda_b, \beta_b)$  is:

$$d^2(\tilde{A}, \tilde{B}) = 2(a - b)^2 + \frac{1}{3}[(\lambda_a - \lambda_b)^2 + (\beta_a - \beta_b)^2] + (a - b)((\beta_a - \lambda_a) - (\beta_b - \lambda_b)) \quad (5)$$

**Definition 3.4.** (Extension principle): If  $f : X \rightarrow Y$  is an arbitrary function from  $X$  to  $Y$  and  $\tilde{A}(x)$  is a fuzzy set with reference set  $X$  and membership function  $\mu_{\tilde{A}(x)}$ , then  $\tilde{B} = f(\tilde{A})$  is a fuzzy set with reference set  $Y$  and the following membership function:

$$\mu_{\tilde{B}(y)} = \begin{cases} \sup_{x \in X; f(x)=y} \mu_{\tilde{A}(x)} & \text{If } \exists x \mid f(x) = y \\ 0 & \text{If } \nexists x \mid f(x) = y \end{cases} \quad (6)$$

**Definition 3.5.** A fuzzy number is called a fuzzy interval if all its  $\alpha$ -cuts are non-empty, closed bounded intervals.

**Definition 3.6.** If  $\tilde{A} = (a, \lambda_a, \beta_a)$  and  $\tilde{B} = (b, \lambda_b, \beta_b)$ , then the sum and difference of these two asymmetrical triangular fuzzy numbers and multiplication of a fuzzy number by a scalar are calculated as follows:

$$\tilde{A} \oplus \tilde{B} = (a + b, \lambda_a + \lambda_b, \beta_a + \beta_b) \quad (7)$$

$$\tilde{A} \ominus \tilde{B} = (a - b, \lambda_a + \beta_b, \beta_a + \lambda_b) \quad (8)$$

$$k \otimes \tilde{A} = \begin{cases} (ka, k\lambda_a, k\beta_a) & \text{If } k \succ 0 \\ (ka, -k\beta_a, -k\lambda_a) & \text{If } k \prec 0 \end{cases} \quad (9)$$

**Definition 3.7.** The support of a fuzzy set  $\tilde{A}$ ,  $\text{supp}(\tilde{A})$ , is the crisp set of all  $x \in X$  such that  $\mu_{\tilde{A}(x)} > 0$ .

**Definition 3.8.** A  $k$ -dimensional fuzzy vector  $\underline{x}^*$  is determined by its so-called vector characterizing function  $\mu_{\underline{x}^*}(\dots)$  which is a real function of  $k$  real variables  $x_1, x_2, \dots, x_k$  obeying the following:

- $\mu_{\underline{x}^*} : R^k \rightarrow [0, 1]$
- The support of  $\mu_{\underline{x}^*}(\dots)$  is a bounded set.
- $\forall \alpha \in (0, 1]$  the  $\alpha$ -cut  $C_\alpha(\underline{x}^*) = \{\underline{x} \in R^k : \mu_{\underline{x}^*}(\underline{x}) \geq \alpha\}$  is non-empty, bounded, and a finite union of simply connected and closed bounded sets.

**Theorem 3.9.** Suppose that  $f : R^n \rightarrow R$  is a continuous function and  $\underline{x}^*$  a fuzzy  $n$ -dimensional interval. Then:

- $f(\underline{x}^*)$  defined by the extension principle is a fuzzy interval.

$$b) \quad C_\alpha[f(\underline{x}^*)] = \left[ \begin{array}{cc} \text{Min}f(\underline{x}) & , \quad \text{Max}f(\underline{x}) \\ \underline{x} \in C_\alpha(\underline{x}^*) & \quad \underline{x} \in C_\alpha(\underline{x}^*) \end{array} \right] \quad (10)$$

Proved in [18].

**Remark 3.10.** Based on [18] and [8], for a random sample of LR fuzzy numbers,  $\tilde{X}_1^*, \tilde{X}_2^*, \dots, \tilde{X}_n^*$ , where  $\tilde{X}_i^* = (x_i, \lambda_i, \beta_i)$ ,  $\sigma_{\tilde{X}_i^*}^2$  will be:

$$S_{\tilde{X}_i^*}^2 = S_{x_i}^2 + \frac{1}{6}(S_{\lambda_i}^2 + S_{\beta_i}^2) + \frac{1}{2}(S_{x_i, \beta_i} - S_{x_i, \lambda_i}) \quad (11)$$

#### 4. Simple Fuzzy Linear Profile

If  $X$  and  $Y$  respectively denote a single explanatory variable and a response, the simple linear profile model is of the form:

$$Y_{ij} = B_0 + B_1 X_j + \varepsilon_{ij} \quad i = 1, 2, \dots, m, \quad j = 1, \dots, n \quad (12)$$

It is assumed that the independent variable  $X_j$  has fixed values from one sample to another and that the  $\varepsilon_{ij}$ s, the error components, are independent and follow a normal distribution with mean zero and variance  $\sigma^2$ . In some processes where precise measurement is impossible, we deal with samples in the form of  $(x_j, \tilde{y}_{ij})$ ,  $i = 1, 2, \dots, m$   $j = 1, \dots, n$ , where  $\tilde{y}_{ij}$  is a fuzzy number. Assume that  $\tilde{y}_{ij}$  is an asymmetrical triangular fuzzy number of the form  $(y_{ij}, l_{ij}, r_{ij})$  and represents the

fuzzy perception of  $j$ th value of the dependent variable in the  $i$ th sample. Then fuzzy linear profile under control mode is as follows:

$$\tilde{Y}_{ij} = \tilde{B}_0 + \tilde{B}_1 X_j \quad (13)$$

In phase II, the in-control values of the parameters  $\tilde{B}_0, \tilde{B}_1$  are assumed to be known or estimated from a data set reflecting expected in-control performance where  $\tilde{B}_r = (b_r, \lambda_r, \beta_r)$ ,  $r = 0, 1$  are asymmetrical triangular fuzzy numbers.

#### 4.1. Determining the Profile of Samples Using Fuzzy Linear Regression.

We use the fuzzy regression method proposed in [4], to estimate parameters of the sample profile. The authors in [4] used the deviation of estimated value of the dependent variable ( $\hat{y}_{ij}$ ) from its actual value ( $\tilde{y}_{ij}$ ) as a criterion and suggested a method based on goal programming for determining coefficients of the following model:

$$\tilde{Y} = \tilde{B}_0 + \tilde{B}_1 X_1 + \cdots + \tilde{B}_p X_p = \sum_{r=0}^p \tilde{B}_r X_r \quad (14)$$

Here, the observed responses and regression coefficients of the  $i$ th profile, are the asymmetrical triangular fuzzy numbers  $\tilde{y}_j = (y_j, l_j, r_j)$ ,  $j = 1, 2, \dots, n$  and  $\tilde{B}_r = (b_r, \lambda_r, \beta_r)$ ,  $r = 0, 1, \dots, p$ , respectively. Furthermore, we assume, without loss of generality, that the observed values of the independent variables ( $x_{rj}$ ) are greater than zero. Then, the estimated values of the dependent variable ( $\hat{y}_j$ ) can be calculated using the arithmetic of fuzzy numbers as follows:

$$\hat{y}_j = \left( \sum_{r=0}^p b_r x_{rj}, \sum_{r=0}^p \lambda_r x_{rj}, \sum_{r=0}^p \beta_r x_{rj} \right), \quad j = 1, \dots, n \quad (15)$$

To obtain estimates of the parameters  $\lambda_r$ ,  $\beta_r$  and  $b_r$  for  $r = 0, \dots, p$ , we use the following linear programming model [4]. In this model (GP1), the objective is to minimize the deviation of centers and the left and right spread of  $\hat{y}_j$  and  $\tilde{y}_j$ .

$$\text{Min } z = \sum_{j=1}^n (n_{jL} + p_{jL} + n_{jC} + p_{jC} + n_{jR} + p_{jR}) \quad (GP1) \quad (16)$$

$$\text{S.t. : } \sum_{r=0}^p b_r x_{rj} + n_{jC} - p_{jC} = y_j, \quad j = 1, 2, \dots, n \quad (17)$$

$$\sum_{r=0}^p \lambda_r x_{rj} + n_{jL} - p_{jL} = l_j, \quad j = 1, 2, \dots, n \quad (18)$$

$$\sum_{r=0}^p \beta_r x_{rj} + n_{jR} - p_{jR} = r_j, \quad j = 1, 2, \dots, n \quad (19)$$

$$n_{jk} p_{jk} = 0, \quad j = 1, 2, \dots, n, \quad k = L, C, R \quad (20)$$

$$b_r \in R, \lambda_r, \beta_r \geq 0, \quad r = 0, 1, \dots, p, \quad (21)$$

$$n_{jk}, p_{jk} \geq 0, \quad j = 1, 2, \dots, n, \quad k = L, C, R \quad (22)$$

Here  $p_{jC}, n_{jC}$  are the positive and negative deviations from center of estimated  $\hat{y}_j$  and actual  $\tilde{y}_j$  response variables respectively and  $p_{jL}, n_{jL}$  ( $P_{jR}, n_{jR}$ ) are the positive and negative deviations between their left (right) spreads. Since the variables in constraints (17), (18) and (19) are separate, the problem GP1 can be transformed into three separate problems as follows:

$$\text{Min } z = \sum_{j=1}^n (n_{jC} + p_{jC}) \quad (GP2) \quad (23)$$

$$\text{S.t. : } \sum_{r=0}^p b_r x_{rj} + n_{jC} - p_{jC} = y_j, \quad j = 1, 2, \dots, n \quad (24)$$

$$n_{jC} p_{jC} = 0, \quad j = 1, 2, \dots, n, \quad (25)$$

$$b_r \in R, \quad r = 0, \dots, p, \quad (26)$$

$$n_{jC}, p_{jC} \geq 0, \quad j = 1, 2, \dots, n \quad (27)$$

$$\text{Min } z = \sum_{j=1}^n (n_{jL} + p_{jL}) \quad (GP3) \quad (28)$$

$$\text{S.t. : } \sum_{r=0}^p \lambda_r x_{rj} + n_{jL} - p_{jL} = l_j, \quad j = 1, 2, \dots, n \quad (29)$$

$$n_{jL} p_{jL} = 0, \quad j = 1, 2, \dots, n, \quad (30)$$

$$\lambda_r \geq 0, \quad r = 0, \dots, p, \quad (31)$$

$$n_{jL}, p_{jL} \geq 0, \quad j = 1, 2, \dots, n \quad (32)$$

$$\text{Min } z = \sum_{j=1}^n (n_{jR} + p_{jR}) \quad (GP4) \quad (33)$$

$$\text{S.t. : } \sum_{r=0}^p \beta_r x_{rj} + n_{jR} - p_{jR} = r_j, \quad j = 1, 2, \dots, n \quad (34)$$

$$n_{jR} p_{jR} = 0, \quad j = 1, 2, \dots, n, \quad (35)$$

$$\beta_r \geq 0, \quad r = 0, \dots, p, \quad (36)$$

$$n_{jR}, p_{jR} \geq 0, \quad j = 1, 2, \dots, n \quad (37)$$

In problems GP1 to GP4, the constraints (20), (25), (30) and (35) are removable. Therefore, the models can be solved using linear programming methods. Note that if both positive and negative  $x_{rj}$  are considered, the above decomposition is no longer valid. In this case, it can be decomposed into two models, one of which finds the centers  $b_r$  and the other one finds the spreads  $\lambda_r$  and  $\beta_r$ . Given that in a simple fuzzy linear profile ( $r = 0, 1$ ), the problems GP1 (or GP2 to GP4) must be solved assuming  $p = 2$ ,  $X_{0j} = 1$  and  $X_{1j} = X_j$  in order to determine the  $i$ th ( $i = 1, \dots, m$ ) profile parameters.

## 5. Monitoring of Fuzzy Linear Profile in Phase II

After calculating the profile parameters of the samples, it is time to decide on performance mode of the process: whether the process profile in correspondence

with the under control mode or ensuring the quality of products depends on applying changes and new settings in the process. In this section, in order to monitor the fuzzy parameters of the profile, we introduce two fuzzy statistics, fuzzy EWMA (FEWMA) and fuzzy Hotelling's  $T^2(\widetilde{FT}^2)$ , based on the extension principle. Then, we discuss fuzzy hypothesis testing to determine the mode of the process profile (under control or out of control). Figure 2 summarizes the steps taken for monitoring simple fuzzy linear profiles.

**5.1. FEWMA Statistics.** It is shown in [7] that by coding independent variable values ( $X$ ) in a way that the correlation coefficient between the profile parameters become zero ( $X'_j = X_j - \bar{X}$ ), separate control charts can be used to monitor the profile parameters. In [13], authors generalized this method to fuzzy profile parameters. Having transformed the  $X$ -values, an alternative form of the underlying model in Equation (13) is obtained as:

$$\tilde{Y}_{ij} = \tilde{A}_0 + \tilde{A}_1 X'_j, \quad (38)$$

where  $\tilde{A}_0 = \tilde{B}_0 + \tilde{B}_1 \bar{X}$ ,  $\tilde{A}_1 = \tilde{B}_1$  and  $X'_j = (X_j - \bar{X})$ .

**5.1.1. FEWMA<sub>I</sub> Statistic for Monitoring Intercept of Fuzzy Linear Profile.** When the data set is crisp, assuming that  $A_0$  is the value of intercept in the under control mode, the EWMA<sub>I(i)</sub> statistic is calculated as follows:

$$\text{EWMA}_{I(i)} = \theta \left( \frac{a_{0i} - A_0}{\sigma_{a_{0i}}} \right) + (1 - \theta) \text{EWMA}_{I(i-1)} \quad (39)$$

In the above equation,  $0 < \theta \leq 1$  is a smoothing constant,  $a_{0i}$  is the estimated value of the intercept from the  $i$ th profile and  $\text{EWMA}_{I(0)} = 0$ . The parameter  $\theta$  is selected so that the control chart produces a particular ARL in control mode.

If  $\tilde{a}_{0i}$ ,  $\tilde{A}_0$  are fuzzy,  $\widetilde{\text{FEWMA}}_{I(i)} = (\text{FEWMA}_{I(i)}, \lambda_{I(i)}, \beta_{I(i)})$  is defined using Definition 6, as follows:

$$\widetilde{\text{FEWMA}}_{I(i)} = \theta \tilde{a}'_i + (1 - \theta) \widetilde{\text{FEWMA}}_{I(i-1)}, \quad (40)$$

where

$$\begin{aligned} \tilde{a}'_i &= (a'_i, \lambda'_i, \beta'_i) = \frac{\tilde{a}_{0i} - \tilde{A}_0}{\sigma_{\tilde{a}_{0i}}} = \left( \frac{1}{\sigma_{\tilde{a}_{0i}}} \right) (a_{0i} - a_0, \lambda_{0i} + \beta_0, \beta_{0i} + \lambda_0) \\ \text{FEWMA}_{I(i)} &= \theta a'_i + (1 - \theta) \text{FEWMA}_{I(i-1)}, \lambda_{I(i)} = \theta \lambda'_i + (1 - \theta) \lambda_{I(i-1)}, \\ \beta_{I(i)} &= \theta \beta'_i + (1 - \theta) \beta_{I(i-1)} \end{aligned}$$

and  $\widetilde{\text{FEWMA}}_{I(0)} = \left( \frac{1}{\sigma_{\tilde{a}_{0i}}} \right) (0, \lambda_0 + \beta_0, \beta_0 + \lambda_0) \cdot \sigma_{\tilde{a}_{0i}}$  is the standard deviation of the intercept ( $\tilde{a}_{0i}$ ), which is assumed to be known or estimated from phase I in control data using equation (11) based on [8], [18].



### 5.1.2. FEWMA<sub>S(i)</sub> Statistic for Monitoring Slope of Fuzzy Linear Profile.

If  $A_1$  is the value of the slope in the under control mode, when the data set is crisp, the EWMA<sub>S(i)</sub> statistic can be calculated from the following equation:

$$\text{EWMA}_{S(i)} = \theta \left( \frac{a_{1i} - A_1}{\sigma_{a_{1i}}} \right) + (1 - \theta)\text{EWMA}_{S(i-1)} \quad (41)$$

In the above equation,  $0 < \theta \leq 1$  is a smoothing constant, is the estimated slope of the  $i$ th profile and  $\text{EWMA}_{S(0)} = 0$ . The parameter  $\theta$  is selected so that the control chart produces a particular in control ARL.

Assuming that  $\tilde{a}_{1i}$ ,  $\tilde{A}_1$  are fuzzy,  $\widetilde{\text{FEWMA}}_{S(i)} = (\text{FEWMA}_{S(i)}, \lambda_{S(i)}, \beta_{S(i)})$  is defined using Definition 6, as follows:

$$\widetilde{\text{FEWMA}}_{S(i)} = \theta \tilde{a}_i'' + (1 - \theta)\widetilde{\text{FEWMA}}_{S(i-1)}, \quad (42)$$

where

$$\begin{aligned} \tilde{a}_i'' &= (a_i'', \lambda_i'', \beta_i'') = \frac{\tilde{a}_{1i} - \tilde{A}_1}{\sigma_{\tilde{a}_{1i}}} = \left( \frac{1}{\sigma_{\tilde{a}_{1i}}} \right) (a_{1i} - a_1, \lambda_{1i} + \beta_1, \beta_{1i} + \lambda_1) \\ \text{FEWMA}_{S(i)} &= (\theta)a_i'' + (1 - \theta)\text{FEWMA}_{S(i-1)}, \lambda_{S(i)} = \theta\lambda_i'' + (1 - \theta)\lambda_{S(i-1)}, \\ &\beta_{S(i)} = \theta\beta_i'' + (1 - \theta)\beta_{S(i-1)} \end{aligned}$$

and  $\text{FEWMA}_{S(0)} = \left( \frac{1}{\sigma_{\tilde{a}_{1i}}} \right) (0, \lambda_1 + \beta_1, \beta_1 + \lambda_1) \cdot \sigma_{\tilde{a}_{1i}}$  is the standard deviation of the slop ( $\tilde{a}_{1i}$ ), which is assumed to be known or estimated from phase I in control data using equation (11) based on [8], [18].

### 5.1.3. EWMA<sub>E(i)</sub> Statistic for Monitoring Standard Deviation.

Since the square of the distance between estimated and actual dependent variables ( $d_{ij}^2(\tilde{Y}_{ij}, \hat{Y}_{ij})$ ) equals the square of the estimation error and  $SSE_i = \sum_{j=1}^n d_{ij}^2(\tilde{Y}_{ij}, \hat{Y}_{ij})$  equals the sum of squared errors (SSE), the mean of squared error, which is calculated using Equation (43), can be considered as an estimate of the error variance of the  $i$ th sample:

$$MSE_i = SSE_i / (n_i - 2) \quad (43)$$

Authors in [7] suggested a control chart for monitoring the process variance based on the EWMA<sub>E(i)</sub> statistic below:

$$\text{EWMA}_{E(i)} = \text{Max}\{\theta(\ln(MSE_i)) + (1 - \theta)\text{EWMA}_{E(i-1)}, \ln(MSE_0)\}, \quad (44)$$

where

$$\text{EWMA}_{E(0)} = \ln(MSE_0)$$

This chart is a one-sided EWMA scheme and only monitors increases in process variability. Thus, the upper limit of the control chart can be calculated as follows:

$$UCL = \ln(MSE_0) + L_E[\theta \text{Var}(\ln(MSE_i)) / (2 - \theta)] \quad (45)$$

The parameters  $L_E$  and  $\theta$  are selected so as to produce a particular in control ARL and  $MSE_0$  is the in-control value of the error variance. An estimate of  $Var(\ln(MSE_i))$  is:

$$Var(\ln(MSE_i)) \cong \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} + \frac{16}{15(n-2)^5} \quad (46)$$

**5.2. Fuzzy  $T^2(\widetilde{FT}^2)$  Statistic.** In [6], authors presented a method for monitoring linear profiles in phase II, based on the bivariate  $T^2$  statistic (Eq. (47)).

$$T_i^2 = (Z_i - \mu)^T \Sigma^{-1} (Z_i - \mu) \quad (47)$$

In the above equation,  $Z_i = (a_{0i}, a_{1i})^T$  is a vector of estimated parameters of the profile that are calculated using sample data.  $\mu = (A_0, A_1)^T$  and  $\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$  are the mean vector and the covariance matrix of  $Z$ , respectively. Therefore, when data set is crisp, the  $T_i^2$  statistic is calculated as follows:

$$T_i^2 = \frac{1}{\sigma_1^2 \sigma_2^2 - \sigma_{12}^2} (\sigma_2^2 (a_{0i} - A_0)^2 + \sigma_1^2 (a_{1i} - A_1)^2 - 2\sigma_{12} (a_{0i} - A_0)(a_{1i} - A_1)) \quad (48)$$

In phase II, if  $\mu$  and  $\Sigma$  are known, the  $T_i^2$  statistic follows a  $\chi^2$  distribution with two degrees of freedom. Otherwise,  $\mu$  and  $\Sigma$  are estimated based on the in-control data set of phase I and the  $T_i^2$  statistic has the  $\frac{2m}{m+1} F_{2,m(n-2)}$  distribution. However, if  $\mu$  and  $\Sigma$  are estimated from a large number of preliminary samples of phase I, it is customary to use  $\chi_2^2$  as the approximate distribution of the  $T_i^2$  statistic in both phase I and phase II [11], [6].

Assume that vectors  $\tilde{\mu} = (\tilde{A}_0, \tilde{A}_1)^T$  and  $\tilde{Z}_i = (\tilde{a}_{0i}, \tilde{a}_{1i})^T$  denote the fuzzy values of known and estimated profile parameters respectively and  $\Sigma = \begin{bmatrix} \sigma_{\tilde{a}_{0i}}^2 & \sigma_{\tilde{a}_{0i}, \tilde{a}_{1i}} \\ \sigma_{\tilde{a}_{0i}, \tilde{a}_{1i}} & \sigma_{\tilde{a}_{1i}}^2 \end{bmatrix}$  is the covariance matrix of  $\tilde{Z}$ . The elements of  $\Sigma$  are variances and covariance of  $\tilde{a}_{0i}$  and  $\tilde{a}_{1i}$ . These elements are assumed to be known or estimated from in-control data set of phase I. Based on [8] and [18], it can be assumed that these elements are crisp numbers. Hence, for simplicity and avoiding computational complexity, we assume that the in-control value of these parameters are computed using equation (11) from in-control data of phase I. If the data are coded as in Section 5.1, the elements on the minor diagonal of variance-covariance matrix will be equal to zero ( $\sigma_{\tilde{a}_{0i}, \tilde{a}_{1i}} = 0$ ). Then the  $\widetilde{FT}_i^2$  statistic is calculated as follows:

$$\widetilde{FT}_i^2 = \frac{1}{\sigma_{\tilde{a}_{0i}}^2} (\tilde{a}_{0i} - \tilde{A}_0)^2 + \frac{1}{\sigma_{\tilde{a}_{1i}}^2} (\tilde{a}_{1i} - \tilde{A}_1)^2, \quad (49)$$

If  $\tilde{a}_{0i} = (a_{0i}, \lambda_{0i}, \beta_{0i})$ ,  $\tilde{a}_{1i} = (a_{1i}, \lambda_{1i}, \beta_{1i})$ ,  $\tilde{A}_0 = (a_0, \lambda_0, \beta_0)$ ,  $\tilde{A}_1 = (a_1, \lambda_1, \beta_1)$ , by Definition 6, we obtain:

$$\tilde{D}_{a_{0i}} = \tilde{a}_{0i} - \tilde{A}_0 = (a_{0i} - a_0, \lambda_{0i} + \beta_0, \beta_{0i} + \lambda_0) \quad (50)$$

$$\tilde{D}_{a_{1i}} = \tilde{a}_{1i} - \tilde{A}_1 = (a_{1i} - a_1, \lambda_{1i} + \beta_1, \beta_{1i} + \lambda_1) \quad (51)$$

Therefore, by the extension principle and Theorem 1, we have:

$$C_\alpha(\widetilde{FT}_i^2) = [FT_i^{2L}(\alpha), FT_i^{2R}(\alpha)] = \left[ \begin{array}{l} \text{Min } T_i^2(\underline{x}'') \quad \text{Max } T_i^2(\underline{x}'') \\ \text{S.t :} \\ \underline{x}'' \in C_\alpha(\underline{x}''^*) \end{array} , \begin{array}{l} \text{S.t} \\ \underline{x}'' \in C_\alpha(\underline{x}''^*) \end{array} \right], \quad (52)$$

where  $\underline{x}''^* = (\tilde{D}_{a_{0i}}, \tilde{D}_{a_{1i}})$ .

Writing  $C_\alpha(\tilde{D}_{a_{0i}}) = [D_{a_{0i}}^L(\alpha), D_{a_{0i}}^R(\alpha)]$ ,  $C_\alpha(\tilde{D}_{a_{1i}}) = [D_{a_{1i}}^L(\alpha), D_{a_{1i}}^R(\alpha)]$ , the solution of the optimization problems in (52) is obtained as:

$$\text{Min } FT_i^2(\underline{x}'') = FT_i^{2L}(\alpha) \quad (53)$$

S.t :

$$\underline{x}'' \in C_\alpha(\underline{x}''^*) = \begin{cases} 0 & \text{If } 0 \in C_\alpha(\tilde{D}_{a_{0i}}) \text{ and } 0 \in C_\alpha(\tilde{D}_{a_{1i}}) \\ \frac{1}{\sigma_{\tilde{a}_{0i}}^2} \times \text{Min}\{(D_{a_{0i}}^L(\alpha))^2, (D_{a_{0i}}^R(\alpha))^2\} & \text{If } 0 \notin C_\alpha(\tilde{D}_{a_{0i}}) \text{ and } 0 \in C_\alpha(\tilde{D}_{a_{1i}}) \\ \frac{1}{\sigma_{\tilde{a}_{1i}}^2} \times \text{Min}\{(D_{a_{1i}}^L(\alpha))^2, (D_{a_{1i}}^R(\alpha))^2\} & \text{If } 0 \in C_\alpha(\tilde{D}_{a_{0i}}) \text{ and } 0 \notin C_\alpha(\tilde{D}_{a_{1i}}) \\ \frac{1}{\sigma_{\tilde{a}_{0i}}^2} \times \text{Min}\{(D_{a_{0i}}^L(\alpha))^2, (D_{a_{0i}}^R(\alpha))^2\} + \frac{1}{\sigma_{\tilde{a}_{1i}}^2} \times \text{Min}\{(D_{a_{1i}}^L(\alpha))^2, (D_{a_{1i}}^R(\alpha))^2\} & \text{else} \end{cases}$$

$$\text{Max } FT_i^2(\underline{x}'') = FT_i^{2R}(\alpha) = \frac{1}{\sigma_{\tilde{a}_{0i}}^2} \times \text{Max}\{(D_{a_{0i}}^L(\alpha))^2, (D_{a_{0i}}^R(\alpha))^2\} \quad (54)$$

S.t :

$$\underline{x}'' \in C_\alpha(\underline{x}''^*) + \frac{1}{\sigma_{\tilde{a}_{1i}}^2} \times \text{Max}\{(D_{a_{1i}}^L(\alpha))^2, (D_{a_{1i}}^R(\alpha))^2\}$$

**5.3. Fuzzy Hypothesis Testing to Identify Profile Mode (in-Control/ Out of Control).** There is a close relationship between control charts and hypothesis testing. A control chart is a test of hypothesis that the process is in a state of statistical control. A point within the control limits is equivalent to the lack of sufficient evidence to reject the hypothesis of statistical control and a point outside the control limits is equivalent to rejecting the hypothesis of statistical control. Therefore, a control chart based on  $\widetilde{FEWMA}_I$ ,  $\widetilde{FEWMA}_S$  or  $\widetilde{FT}^2$  statistics tests the following fuzzy hypothesis:

$$\left\{ \begin{array}{l} H_0 : \tilde{A}_0 = \tilde{a}_0 \\ H_1 : \tilde{A}_0 \neq \tilde{a}_0 \end{array} \right. , \quad \left\{ \begin{array}{l} H_0 : \tilde{A}_1 = \tilde{a}_1 \\ H_1 : \tilde{A}_1 \neq \tilde{a}_1 \end{array} \right.$$

In [17], the authors recommended a method for fuzzy hypothesis testing that involved four steps:

1. The area under fuzzy test statistic ( $A_T$ ) is calculated.
2.  $A_R$ , the area of fuzzy test statistic ( $\widetilde{FEWMA}_I$  and  $\widetilde{FEWMA}_S$  or  $\widetilde{FT}^2$ ) greater than  $Q_{1-\beta/2}$  or smaller than  $Q_{\beta/2}$  is calculated, where  $Q_{\beta/2}(Q_{1-\beta/2})$  is the  $\beta/2(1-\beta/2)$ -quantile of the distribution of the crisp test statistic.
3. A value  $\varphi \in (0, 1)$  is selected as the level of credit.

4. If  $\frac{A_R}{A_T} \geq \varphi$ , the fuzzy null hypothesis is rejected ( $RH_0$ ); otherwise, the fuzzy null hypothesis is accepted ( $AH_0$ ).

In this article, only the observed values of the classical random variable ( $Y$ ) are considered as a fuzzy number, while the model for observed values is still as in (12). So the proposed test statistics in sections 5.1 and 5.2 are usual random variables and distributions of which are similar to classical test statistics.

We note that, the quantile  $Q_{\beta/2}$  and  $Q_{1-\beta/2}$  of the distribution of the  $EWMA_I$  and  $EWMA_S$  are determined by means of simulation, so as to give specified in-control ARL. In fuzzy hypothesis testing, there are two types of uncertainty; the first type, controlled via parameter  $\beta$  in testing hypothesis, is related to the randomness of data and the second type is caused by ambiguity and the fuzzy nature of data and the fuzzy hypothesis is controlled by parameter of reliability level  $\varphi$ .

## 6. Case Study

One of the main processes in ceramic and tile industry is the process of grinding for producing slurry. The main purpose of the grinding process is to achieve a desired particle size distribution without adding any metal or other impurities in the final product along with decreasing production costs and increasing operational power of the grinding circuit. The grinding process is difficult to control for various reasons, e.g. unknown and nonlinear structure of the process, imprecision in mathematical models, variables with mutual interaction, operational conditions and inability to use accurate and reliable sensors. To produce the slurry required in ceramic and tile industry, a combination of materials are mixed together according to a predetermined formula and transferred to a ball mill device. To control performance of the device and ensure controlling particle size of the slurry, the operator must gather samples from the content of the device at regular intervals and measure the roughness of the product (mass percent remaining on the sieve) using a standard sieve. The sieve size is determined by the type of application. For example, for producing the slurry required for glaze, a sieve with standard size of 325 is used. The slurry roughness is a linguistic variable shown in Figure 3. The mass distribution of particles with different sizes during the grinding process is assumed to be exponential with regard to the grinding time [10]. In other words, the process profile in under control mode is as follows:

$$\tilde{y}(t) = \tilde{a}e^{-\tilde{b}t}. \quad (55)$$

Where,  $\tilde{y}(t)$  is a dependent variable, which is linguistic and related to the operator's evaluation of slurry roughness,  $\tilde{a}$ ,  $\tilde{b}$  are profile parameters and  $t$  is an independent variable of the profile, representing time. Putting  $\tilde{y}' = \ln(\tilde{y})$ , we have:

$$\tilde{y}'(t) = \ln(\tilde{y}) = \ln(\tilde{a}) - \tilde{b}t = \tilde{c} - \tilde{b}t \quad (56)$$

If the membership function of a triangular fuzzy number  $\tilde{A} = (a, \lambda, \beta)$  is as in (1), the membership function of  $\tilde{B} = \ln(\tilde{a})$  is as follows:

$$\mu_{\tilde{B}(y)} = \begin{cases} \frac{e^y - (a - \lambda)}{\lambda} & \ln(a - \lambda) < y \leq \ln(a) \\ \frac{(a + \beta) - e^y}{\beta} & \ln(a) \leq y \leq \ln(a + \beta) \\ 0 & \text{otherwise} \end{cases} \quad (57)$$

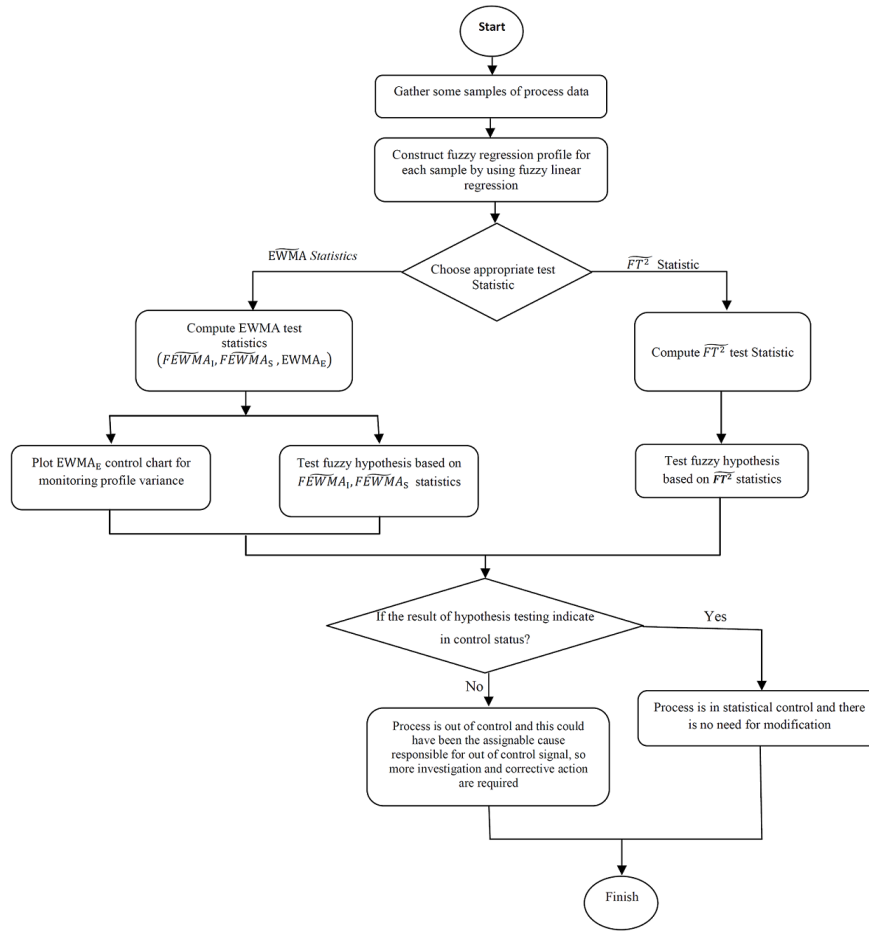


FIGURE 2. Flowchart of Proposed Method for Monitoring Fuzzy Profiles in Phase II

This change of variable, changes the fuzzy exponential profile of the equation (55) to a fuzzy linear profile. The process fuzzy profile function, based on the under control data set using the method presented for determining parameters of the process fuzzy profile, was found to be  $\tilde{y} = (11.42, 2.16, 2.12)e^{(-0.18, 0.05, 0.03)t}$ . The data set in Table 1 is the result of sampling from each time period of the operation of a ball mill device after 3, 6 and 9 h of operation. The resulting values for  $\widetilde{FEWMA}$  and  $\widetilde{FT}^2$  are presented in Table 2. Using simulation with 10,000 runs, we set the control method parameters based on  $\widetilde{FEWMA}$  and  $\widetilde{FT}^2$  statistics to obtain an in-control ARL of approximately 200. The value of the aforementioned parameters are  $Q_{\beta/2} = -1.0052$ ,  $Q_{1-\beta/2} = 1.0052$ ,  $\varphi = 0.51$ ,  $\theta = 0.2$  and  $\beta = 0.005$ ,  $\varphi = 0.61$ , respectively.

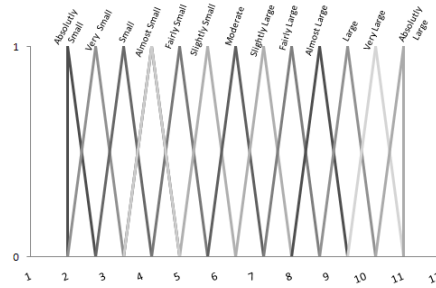


FIGURE 3. Linguistic Variable "Slurry Roughness"

The values of  $\widetilde{FEWMA}_{I(i)}$  and statistics, which are triangular fuzzy numbers, and  $\widetilde{FT}^2$  statistic, which is nonlinear, are given in Table 2. The membership functions of the  $\widetilde{FT}^2$  statistic related to profiles 1 to 8 is shown in Figure 4. The value of the  $\widetilde{EWMA}_{E(i)}$  statistic for  $i = 0, 1, \dots, 30$  is equal to zero. The area of the fuzzy test statistic of  $\widetilde{FEWMA}_S$  larger than the quantile  $Q_{1-\beta/2}$  or smaller than  $Q_{\beta/2}$  of the  $\widetilde{EWMA}$  distribution is zero for all the samples. Thus, the results of fuzzy hypothesis testing for  $\widetilde{FEWMA}_{S(i)}$  statistic do not show any out of control signal. The value of the  $\frac{\widetilde{AS}_R}{\widetilde{AS}_T}$ , as a basis of fuzzy hypothesis testing for the intercept of the profile, are also presented in Table 2. Despite the fact that control charts based on  $\widetilde{FEWMA}_{S(i)}$  and  $\widetilde{EWMA}_{E(i)}$  statistics indicate that the process is under statistical control, results of the analysis show the departure of grinding process from in-control mode, because from sample number 22 onward, the  $\widetilde{FEWMA}_I$  chart, which is related to the intercept of profile, displays an out of control signal.

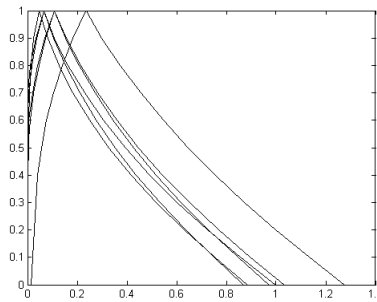


FIGURE 4.  $\widetilde{FT}^2$  Membership Function of Profiles Number 1 to 8

The area of the fuzzy test statistic  $\widetilde{FT}^2$  larger than the quantile  $Q_{1-0.005}$  of the  $\chi^2$  distribution with two degrees of freedom is zero for all the samples; therefore, the  $\widetilde{FT}^2$  statistic does not show any out of control signal. The conducted studies

Row <i>i</i>	Data Set			Regression Coefficients		
	<i>t</i> = 3	<i>t</i> = 6	<i>t</i> = 9	intercept	Slope	<i>MSE<sub>i</sub></i>
1	Slightly Large	Small	Very Small	(1.496, 0.241, 0.194)	(-0.162, 0.016, 0.009)	0.1348
2	Fairly Large	Almost Small	Very Small	(1.546, 0.194, 0.163)	(-0.178, 0.028, 0.016)	0.036
3	Slightly Large	Small	Very Small	(1.496, 0.241, 0.194)	(-0.162, 0.016, 0.009)	0.1348
4	Slightly Large	Fairly Small	Very Small	(1.496, 0.163, 0.140)	(-0.162, 0.037, 0.022)	0.0396
5	Moderate	Small	Very Small	(1.442, 0.241, 0.194)	(-0.143, 0.016, 0.009)	0.0853
6	Moderate	Fairly Small	Small	(1.562, 0.163, 0.140)	(-0.103, 0.018, 0.012)	0.0092
7	Moderate	Fairly Small	Small	(1.562, 0.163, 0.140)	(-0.103, 0.018, 0.012)	0.0092
8	Fairly Large	Fairly	Very Small	(1.546, 0.163, 0.140)	(-0.178, 0.037, 0.022)	0.0239
9	Fairly Large	Very Small	Absolutely Small	(1.386, 0.319, 0.241)	(-0.231, 0.032, 0.014)	0.3277
10	Slightly Large	Small	Very Small	(1.496, 0.241, 0.194)	(-0.162, 0.016, 0.009)	0.1348
11	Moderate	Slightly Small	Absolutely Small	(1.283, 0.140, 0.123)	(-0.196, 0.081, 0.045)	0.4693
12	Slightly Large	Almost Small	Absolutely Small	(1.337, 0.194, 0.163)	(-0.215, 0.065, 0.033)	0.0625
13	Fairly Small	Almost Small	Absolutely Small	(1.151, 0.194, 0.163)	(-0.153, 0.067, 0.035)	0.201
14	Fairly Large	Small	Very Small	(1.546, 0.241, 0.194)	(-0.178, 0.0160, 0.009)	0.1896
15	Slightly Small	Almost Small	Absolutely Small	(1.221, 0.194, 0.163)	(-0.176, 0.066, 0.034)	0.133
16	Large	Slightly Large	Moderate	(2.062, 0.109, 0.098)	(-0.063, 0.003, 0.002)	0.0137
17	Almost Large	Fairly Large	Moderate Small	(2.020, 0.098, 0.090)	(-0.050, 0.006, 0.004)	0.0074
18	Very Large	Fairly Large	Slightly Small	(2.038, 0.098, 0.090)	(-0.096, 0.010, 0.008)	0.0049
19	Very Large	Fairly Large	Slightly Small	(2.038, 0.098, 0.090)	(-0.096, 0.010, 0.008)	0.0049
20	Large	Fairly Large	Moderate	(2.062, 0.098, 0.090)	(-0.063, 0.006, 0.004)	0.0013
21	Absolutely Large	Slightly Large	Moderate	(2.135, 0.109, 0.098)	(-0.088, 0.003, 0.002)	0.0486
22	Absolutely Large	Very Large	Large	(2.325, 0.076, 0.071)	(-0.024, 0.001, 0.001)	0.0001
23	Large	Fairly Large	Moderate	(2.289, 0.098, 0.090)	(0.013, 0.006, 0.004)	0.0242
24	Absolutely Large	Very Large	Large	(2.325, 0.076, 0.071)	(-0.024, 0.001, 0.001)	0.0001
25	Large	Moderate	Slightly Small	(2.284, 0.123, 0.109)	(-0.038, 0.003, 0.003)	0.0053
26	Absolutely Large	Large	Fairly Large	(2.239, 0.082, 0.076)	(-0.053, 0.004, 0.003)	0.001
27	Almost Large	Fairly Large	Slightly Small	(2.325, 0.098, 0.090)	(-0.024, 0.010, 0.008)	0.001
28	Large Large	Fairly Large	Fairly Small	(2.325, 0.098, 0.090)	(-0.024, 0.016, 0.012)	0.002
29	Absolutely Large	Very Large	Fairly Large	(2.239, 0.076, 0.071)	(-0.053, 0.005, 0.004)	0.016
30	Absolutely Large	Very Large	Fairly Large	(2.239, 0.076, 0.0716)	(-0.053, 0.005, 0.004)	0.016

TABLE 1. Fuzzy Gathered Data Set and Quality Profiles in Grinding Process

showed that the balls used as grinding media became deformed and eroded from grinding over time. When new balls were used, the process profile returned to the control mode.

j	$\widetilde{FEWMA}_{S(i)}$		$\widetilde{FEWMA}_{I(i)}$		$\widetilde{FT}_i^2$			
	Statistic value	$\frac{AS_R}{AS_T}$	Statistic value	$\frac{AS_R}{AS_T}$	Center	left Spread	Right Spread	$\frac{AS_R}{AS_T}$
1	(0.02, 0.31, 0.32)	0	(0.05, 0.67, 0.67)	0	0.066	0.066	0.93	0
2	(0.01, 0.30, 0.31)	0	(0.11, 0.66, 0.66)	0	0.11	0.11	0.93	0
3	(0.03, 0.28, 0.30)	0	(0.13, 0.67, 0.67)	0	0.07	0.07	0.93	0
4	(0.04, 0.28, 0.30)	0	(0.16, 0.65, 0.66)	0	0.07	0.07	0.81	0
5	(0.06, 0.26, 0.29)	0	(0.15, 0.66, 0.67)	0	0.05	0.05	0.84	0
6	(0.11, 0.25, 0.29)	0	(0.19, 0.65, 0.65)	0	0.23	0.22	1.04	0
7	(0.16, 0.24, 0.28)	0	(0.23, 0.63, 0.64)	0	0.23	0.22	1.04	0
8	(0.13, 0.25, 0.29)	0	(0.25, 0.62, 0.64)	0	0.11	0.11	0.87	0
9	(0.06, 0.25, 0.28)	0	(0.21, 0.67, 0.67)	0	0.05	0.05	0.88	0
10	(0.06, 0.24, 0.28)	0	(0.22, 0.68, 0.67)	0	0.07	0.07	0.93	0
11	(0.04, 0.29, 0.30)	0	(0.15, 0.65, 0.65)	0	0.02	0.02	0.71	0
12	(0, 0.31, 0.31)	0	(0.11, 0.64, 0.65)	0	0.02	0.02	0.72	0
13	(0.02, 0.33, 0.32)	0	(0.02, 0.64, 0.65)	0	0.14	0.14	1.06	0
14	(0.02, 0.30, 0.31)	0	(0.09, 0.66, 0.66)	0	0.11	0.11	1.02	0
15	(0.02, 0.32, 0.32)	0	(0.02, 0.65, 0.66)	0	0.05	0.05	0.84	0
16	(0.11, 0.29, 0.30)	0	(0.26, 0.62, 0.63)	0	1.74	1.07	1.86	0
17	(0.20, 0.26, 0.28)	0	(0.44, 0.59, 0.61)	0.003	1.60	1	1.77	0
18	(0.23, 0.24, 0.28)	0	(0.59, 0.56, 0.59)	0.05	1.53	0.97	1.73	0
19	(0.26, 0.23, 0.27)	0	(0.71, 0.54, 0.58)	0.13	1.53	0.97	1.73	0
20	(0.30, 0.21, 0.26)	0	(0.81, 0.53, 0.57)	0.24	1.74	1.05	1.82	0
21	(0.32, 0.20, 0.25)	0	(0.92, 0.52, 0.56)	0.40	1.98	1.16	1.95	0
22	(0.39, 0.18, 0.25)	0	(1.07, 0.50, 0.54)	0.66	3.25	1.41	2.20	0
23	(0.48, 0.18, 0.24)	0	(1.18, 0.49, 0.54)	0.81	3.29	1.51	2.38	0
24	(0.51, 0.17, 0.24)	0	(1.28, 0.48, 0.53)	0.91	3.25	1.41	2.20	0
25	(0.53, 0.16, 0.23)	0	(1.35, 0.49, 0.53)	0.96	2.95	1.52	2.40	0
26	(0.53, 0.16, 0.23)	0	(1.38, 0.48, 0.53)	0.98	2.63	1.28	2.05	0
27	(0.56, 0.16, 0.24)	0	(1.44, 0.47, 0.52)	0.99	3.25	1.54	2.39	0
28	(0.58, 0.17, 0.24)	0	(1.49, 0.47, 0.52)	0.99	3.25	1.56	2.42	0
29	(0.57, 0.16, 0.24)	0	(1.50, 0.46, 0.52)	1	2.63	1.26	2.03	0
30	(0.56, 0.16, 0.24)	0	(1.50, 0.46, 0.51)	1	2.63	1.26	2.03	0

TABLE 2.  $\widetilde{FT}^2$  and  $\widetilde{FEWMA}$  Statistics in Grinding Process

### 7. Performance Comparison of Proposed Methods

In this section, the performance of control charts  $\widetilde{FEWMA}$  and  $\widetilde{FT}^2$  are evaluated using the ARL criterion and simulation. The profile model used for the simulation is as follows:

$$\tilde{Y} = (13, 0.2) + (2, 0.2)X$$

The set of simulated data was produced after applying a shift to the parameters of the regression line. The  $X_i$  values were -3, -1, 1, and 3. To produce an under control ARL of almost equal to 200, the control method parameters based on  $\widetilde{FEWMA}$  statistic were set to  $Q_{\beta/2} = -1.0052$ ,  $Q_{1-\beta/2} = 1.0052$ ,  $\varphi = 0.51$ ,  $\theta = 0.2$  and those of  $\widetilde{FT}^2$  statistic were set to  $\beta = 0.005$ ,  $\varphi = 0.61$ . A simulation study was performed with 10,000 replications for each out of control ARL value and triple of profile parameters. The values for ARL are shown in Tables 3, 4 and 5 and Figures 5, 6 and 7. The plots in each figure are based on ARL values for 100 equally spaced shifts within each range of shifts considered.

Results of the analysis indicate that, for small changes in slope, the performance of the method based on  $\widetilde{FEWMA}$  statistic is better than that of  $\widetilde{FT}^2$ .

The results show that  $\widetilde{FEWMA}$  is more efficient for small and medium changes than  $\widetilde{FT}^2$ .



Method	$k = 0.1$	$k = 0.2$	$k = 0.3$	$k = 0.4$	$k = 0.5$
$\widetilde{FT}^2$	127.23	44.53	17.40	7.85	4.09
$\widetilde{FEWMA}$	51.59	13.67	6.81	4.46	3.38
Method	$k = 0.6$	$k = 0.7$	$k = 0.8$	$k = 0.9$	$k = 1$
$\widetilde{FT}^2$	2.48	1.74	1.36	1.17	1.07
$\widetilde{FEWMA}$	2.73	2.34	2.07	1.90	1.73

TABLE 3. Comparisons of ARL Under Slope Shifts From  $A_1$  To  $A_1 - k(MSE_0)$  (In-control ARL = 200)

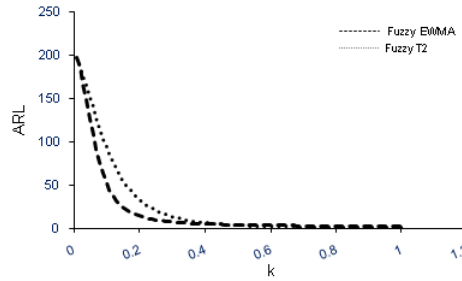


FIGURE 5. Comparisons of ARL Under Slope Shifts  $k$

Method	$k = 0.2$	$k = 0.4$	$k = 0.6$	$k = 0.8$	$k = 1$
$\widetilde{FT}^2$	166.24	109.50	61.79	32.47	16.78
$\widetilde{FEWMA}$	63.67	17.34	8.31	5.27	3.89
Method	$k = 1.2$	$k = 1.4$	$k = 1.6$	$k = 1.8$	$k = 2$
$\widetilde{FT}^2$	9.09	5.27	3.35	2.29	1.7
$\widetilde{FEWMA}$	3.12	2.61	2.29	2.06	1.90

TABLE 4. Comparisons of ARL Under Intercept Shifts From  $A_0$  To  $A_0 + k(MSE_0)$  (In-control ARL = 200)

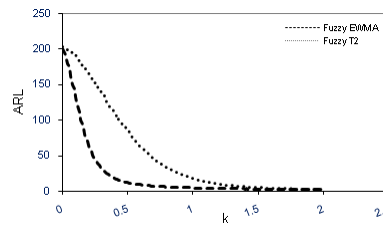


FIGURE 6. Comparisons of ARL Under Intercept Shifts  $k$

### 8. Conclusions

In this paper, monitoring of simple fuzzy linear profiles in phase II is studied for the first time. Two new methods based on the statistics of  $\widetilde{FT}^2$  and  $\widetilde{FEWMA}$  are proposed for monitoring phase II of simple fuzzy linear profiles. The performance of the proposed methods is evaluated and compared using the out of control ARL

Method	$k = 1.2$	$k = 1.4$	$k = 1.6$	$k = 1.8$	$k = 2$
$\widetilde{FT}^2$	44.54	17.20	9.20	5.83	4.22
$\widetilde{FEWMA}$	31.34	12.60	7.62	5.57	4.52
Method	$k = 2.2$	$k = 2.4$	$k = 2.6$	$k = 2.8$	$k = 3$
$\widetilde{FT}^2$	3.31	2.76	2.37	2.11	1.92
$\widetilde{FEWMA}$	3.86	3.38	3.07	2.79	2.59

TABLE 5. Comparisons of ARL under Standard Deviation Shifts From  $MSE_0$  To  $k$  ( $MSE_0$ )(In-control ARL = 200)

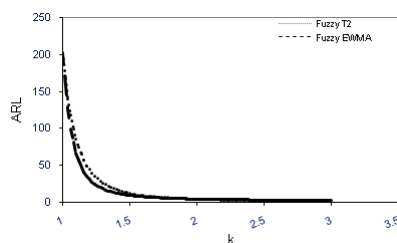


FIGURE 7. Comparisons of ARL under Standard Deviation Shifts  $k$

criterion. The results using simulated data set show that the performance of the method based on  $\widetilde{FEWMA}$  statistic is better than  $\widetilde{FT}^2$  for detecting small and medium changes while the latter performs better in detecting large changes. For future studies, fuzzy profiles with different functional structures such as non-linear, multiple, binary, etc. can be investigated.

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