PROFIT MAXIMIZATION SOLID TRANSPORTATION PROBLEM UNDER BUDGET CONSTRAINT USING FUZZY MEASURES

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Abstract. Fixed charge solid transportation problems are formulated as profit maximization problems under a budget constraint at each destination. Here item is purchased in different depots at different prices. Accordingly the item is transported to different destinations from different depots using different vehicles. Units are sold from different destinations to the customers at different selling prices. Here selling prices, purchasing costs, unit transportation costs, fixed charges, sources at origins, demands at destinations, conveyances capacities are assumed to be crisp or fuzzy. Budget constraints at destinations are imposed. It is also assumed that transported units are integer multiple of packets. So the problem is formulated as constraint optimization integer programming problem in crisp and fuzzy environments. As optimization of fuzzy objective as well as consideration of fuzzy constraint is not well defined, different measures- possibility/necessity/credibility of fuzzy event are used to transform the problem into equivalent crisp problem. The reduced crisp problem is solved following generalized reduced gradient(GRG) method using lingo software. A dominance based genetic algorithm (DBGA) and a particle swarm optimization (PSO) technique using swap sequence are also developed for this purpose and are used to solve the model. The models are illustrated with numerical examples. The results obtained using DBGA and PSO are compared with those obtained from GRG. Moreover, a statistical analysis is presented to compare the algorithms.

1. Introduction

The solid transportation problem (STP) is a generalization of the well-known transportation problem (TP) in which three kinds of constraint sets exist instead of two (source and destination) as in TP [18]. This extra constraint is mainly due to modes of transportation(conveyances). The STP was stated by Shell[39]. Haley[16] showed a comparison of the STP with the classical TP and applied Modi-method to solve the STP. In a TP when fixed charge is considered against transportation of units from a source to a destination, the problem is transformed to fixed charge TP(FCTP). The FCTP was initialized by Hirsch and Dantzig[17]. Gottlieb and Paulman [15] presented a genetic agorithm for the fixed charge transportation problems. Kennington and Unger[23] investigate a new branch and bound

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algorithm for the fixed charge transportation problem. Sun et al.[41] developed a tabu search heuristic procedure for fixed charge transportation problem. In STP, fixed charge is also considered and solved by several researchers in crisp as well as uncertain environments. Yang and Liu[44] developed fuzzy fixed charge STP and developed algorithms to solve the problem using possibility and credibility measures on fuzzy sets. Durai Raj et al.[7] presented some fast heuristic algorithms to solve a single-stage fixed charge transportation problem and compared their performances with the existing best method by making use of benchmark problem instances.

In both investigations, assumed crisp unit transportation cost, Jimnez and Verdegay [20][21] minimized the total transportation cost and evaluated the optimum cost as crisp. Liu[30] considered the fuzzy STP with fuzzy unit transportation costs and sources, demands and conveyance capacities as fuzzy numbers. Using extension principle, the problem was transformed into a pair of problems, upper and lower bounds of the fuzzy total transportation cost were evaluated. Yang and Liu [44] investigated fixed charge STP under fuzzy environment with direct costs, fixed charges, supplies, demands and conveyance capacities as fuzzy variables. The above problem was transformed to three new models expected value model, chance-constrained programming model and dependent chance programming model on the basis of credibility theory. The different equivalent crisp models were obtained and solved by a hybrid intelligent algorithm- tabu search algorithm. They presented the total transportation cost in crisp number. Ojha et al.[35] studied entropy based STP with general fuzzy cost and time. Kundu et al.[24] developed a multi-objective multi-item solid transportation problem in fuzzy environment and solved using global criteria technique. Cui and Sheng [3]assumed uncertain distribution, founded by Liu [30], for the unit cost, sources, demands and conveyance capacities of STP and found the equivalent crisp from using inverse uncertainty distribution. Golnarkar et al. [14] presents solving best path problem on multimodal transportation networks with fuzzy costs. Molla-Alizadeh-Zavardehi et al. [33] investigated a fixed charge STP under a fuzzy environment where both fixed and unit costs are fuzzy. The problem was solved by three metaheuristic methods i.e. variable neighborhood search (VNS), simulated annehealing (SA)and hybrid VNS. Kundu et al.[25], presents fixed charge transportation problem with type-2 fuzzy variables.

In all the above investigations, formulation of a STP as profit maximization problem was not considered. The metaheuristic method-particle swarm optimization (PSO) was not used for solution. Moreover, none obtained the final output i.e. total transportation cost in fuzzy numbers. These above liftouts have been achieved in the proposed analysis of the present investigation.

From above discussions there are some lacunas in the existing STP models, which are summarized below.

- Transportation problems-TP or STP are normally formulated and solved as cost minimization problems, very few might have formulated these as profit maximization problems.
- In the literature, there are several research works on transportation with uncertain sources, demands, conveyances capacities, etc.

  None has investigated FCSTP under destination budget constraints with uncertain purchasing costs, transportation costs, fixed charges, etc. for profit maximization.

- In the case of some items, units are transported as a whole i.e. transported amounts take the integer multiple of packets/bags. In the literature, there are very few integer valued fuzzy STPs.
- Normally objective of a fuzzy transportation problem (FTP) is not optimized directly, rather its crisp equivalent is optimized for marketing decision. This may lead to major loss in present day competitive market.
- A particle swarm optimization (PSO) technique using swap sequence are also developed for this purpose and are used to solve the model. The models are illustrated with numerical examples.
- Moreover, the supremacy of the PSO is illustrated statistically with the help of the statistical analysis.

Overcoming the above mentioned shortcomings, here we have considered profit maximization FCSTP under destination budget constraints with crisp and fuzzy data. Several types of conveyances are used for transportation of units from sources to destinations. In this paper, integer valued crisp and fuzzy FCSTPs are formulated as profit maximization problems under fuzzy resource (budget) constraints. Here the transportation system is formulated with respect to a merchant who purchases the source amounts at different origins and sells the transported amounts at different destinations as per the demands at destinations. Thus purchasing costs and selling prices at different origins and destinations respectively are different and fuzzy in nature. The unit transportation costs, fixed charges of transportation, demands at destinations and conveyance capacities are also fuzzy. The fuzzy objective and constraints are transformed to equivalent crisp forms using possibility, necessity and credibility measures. The above transportation problems are solved by GRG, DBGA and PSO techniques in such a way that the transported amounts are integers only i.e. transported units are integer multiple of packets of the item. The methods are illustrated with numerical examples and the optimum results from three methods are compared.

Rest of the paper is organized as follows. Section 1 gives the literature survey and gist of the paper. In section 2, some preliminaries and deductions required for mathematical formulation of the models are presented. In section 3, assumptions and notations of the proposed FCSTP models are listed and mathematical formulation of the proposed FCSTP models are presented. Imprecise optimization problems arising from the models are transformed into equivalent deterministic optimization problems in section 4. In section 5, an outline of DBGA is presented which is used to solve the single-objective deterministic models. In section 6, an outline of PSO is presented which is also used to solve the model. Numerical examples to illustrate the models are provided in section 7. A brief discussion of the models are presented in sections 8. The statistical analysis is given in section 9. Finally a conclusion is drawn in section 10.

2. **Preliminaries**

Definition 2.1. A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line $\Re$, with membership function $\mu_{\widetilde{A}} : \Re \to [0,1]$, satisfying the following conditions:

- There exists exactly one interval $I \in \Re$ such that $\mu_{\widetilde{A}}(x) = 1, \forall x \in I$.
- The membership function $\mu_{\widetilde{A}}(x)$ is piecewise continuous.

Definition 2.2. A Triangular Fuzzy Number (TFN) $\widetilde{A}$ is specified by three parameters (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\widetilde{A}}(x)$: $X \to [0,1]$ as follows :

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.3. Let $\tilde{A}$ be a fuzzy number with membership function $\mu_{\tilde{A}}(x)$ then its α - cut is denoted by $\tilde{A}(\alpha) = [A_L(\alpha), A_R(\alpha)]$, and is defined as

$$A_L(\alpha) = inf\{x \in R : \mu_{\tilde{A}}(x) \ge \alpha\}$$
$$A_R(\alpha) = sup\{x \in R : \mu_{\tilde{A}}(x) \ge \alpha\}$$

Definition 2.4. (see e.g. [45]) If $\tilde{A}$, $\tilde{B} \in \Re$ and $\tilde{C} = f(\tilde{A}, \tilde{B})$ where $f : \Re \times \Re \longrightarrow \Re$ be a binary operation then membership function $\mu_{\tilde{C}}$ of $\tilde{C}$ is defined as

$$\mu_{\tilde{C}}(z) = sup\{min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)),\ x, y \in \Re\ and\ z = f(x,y), \forall\, z \in \Re\}$$

Definition 2.5. Let R represent the set of real numbers and $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively. Then taking degree of uncertainty as the semantics of fuzzy number, according to (Zadeh [45]; Dubois and Prade [5]; Liu [29]) .

$$Pos(\tilde{A} * \tilde{B}) = sup\{min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \Re, x * y\}$$

where the abbreviation 'Pos' represent possibility and * is any one of the relations $>, <, =, \ge, \le$. Analogously, if $\tilde{B}$ is a crisp number, say b, then

$$Pos(\tilde{A} * b) = sup\{min(\mu_{\tilde{A}}(x), 1), x \in \Re, x * b\}$$

On the other hand, necessity measure of an event $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ is a dual of possibility measure. The grade of necessity of an event is the grade of impossibility of the opposite event and is defined as:

$$Nes(\tilde{A} * \tilde{B}) = 1 - Pos\overline{(\tilde{A} * \tilde{B})}$$

where the abbreviation 'Nes' represents necessity measure and $\overline{\tilde{A} * \tilde{B}}$ represents complement of the event $\tilde{A} * \tilde{B}$.

$$\text{If } \tilde{A}\ and\ \tilde{B} \in \Re\ and\ \tilde{C} = f(\tilde{A}, \tilde{B}),\ \ where\ \ f : \Re \times \Re \to \Re$$

is a binary operation then membership function $\mu_{\tilde{C}}$ of C can be obtained using Fuzzy Extension Principle (Zadeh [45]) as

$$\mu_{\tilde{C}}(z) = sup\{min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \Re \ and \ z = f(x, y), \forall z \in \Re\}$$

Definition 2.6. Credibility measure was presented by Liu and Liu [28]. For a fuzzy variable $\tilde{A}$ with membership function $\mu_{\tilde{A}}(x)$ and then for any set B $\subset$ R of real numbers, credibility measure of fuzzy event $\{\tilde{A} \in B\}$ is defined as $Cr\{\tilde{A} \in B\} = \frac{1}{2}(Pos\{\tilde{A} \in B\} + Nec\{\tilde{A} \in B\})$, where possibility and necessity measures of $\{\tilde{A} \in B\}$ are respectively defined as

$$Pos\{\tilde{A} \in B\} = \sup_{x \in B} \mu_{\tilde{A}}(x) \ \& \ Nec\{\tilde{A} \in B\} = 1 - \sup_{\tilde{A} \in B^c} \mu_{\tilde{A}}(x)$$

$\tilde{A}$ is a triangular fuzzy variable given by $\tilde{A} = (a_1, a_2, a_3)$, $0 \le a_1 \le a_2 \le a_3$.

$$Cr(\tilde{A} \le r) = \begin{cases} 0 & \text{if } r \le a_1 \\ \dfrac{r - a_1}{2(a_2 - a_1)} & \text{if } a_1 \le r \le a_2 \\ \dfrac{r + a_3 - 2a_2}{2(a_3 - a_2)} & \text{if } a_2 \le r \le a_3 \\ 1 & \text{if } r \ge a_3 \end{cases}$$

Definition 2.7. (see e.g. [29]) Let $\tilde{A}$ be a fuzzy variable and $\beta \in [0, 1]$. Then β-optimistic value of $\tilde{A}$ is denoted by $\tilde{A}_{sup}(\beta)$ and is defined as

$$\tilde{A}_{sup}(\beta) = sup\{r : Cr\{\tilde{A} \ge r\} \ge \beta\}$$

Similarly β pessimistic value of $\tilde{A}$ is denoted by $\tilde{A}_{inf}(\beta)$ and is defined as

$$\tilde{A}_{inf}(\beta) = inf\{r : Cr\{\tilde{A} \le r\} \ge \beta\}$$

Definition 2.8. (see e.g. [29]) Let $\tilde{X}$ be any normalized fuzzy variable. The expected value of the fuzzy variable $\tilde{X}$ is denoted by $E(\tilde{X})$ and defined by

$$E(\tilde{X}) = \int_0^{\infty} Cr(\tilde{X} \ge r)dr - \int_{-\infty}^{0} Cr(\tilde{X} \le r)dr$$

provided that at least one of the two integral is finite. If $\tilde{A} = (a_1, a_2, a_3)$ be a TFN, then according to definition above expected value of $\tilde{A}$ is given by

$$E(\tilde{A}) = \frac{1}{4}[a_1 + 2a_2 + a_3]$$

Definition 2.9. If $\tilde{A}$ be a fuzzy number defined on $\Re$ then the centroid of $\tilde{A}$ is defined by

$$C(\tilde{A}) = \frac{\int_{-\infty}^{\infty} x\mu_{\tilde{A}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\tilde{A}}(x)dx}$$

In particular if $\tilde{A} = (a_1, a_2, a_3)$ be a TFN, then according to definition above centroid method of $\tilde{A}$ is given by $C(\tilde{A}) = (a_1 + a_2 + a_3)/3$.

Definition 2.10. An interval $\bar{A}$ on $\Re$ has two bounds A_L and A_R and is defined as

$$\bar{A} = \{x : A_L \leq x \leq A_R, x \in \Re\}$$

where A_L, A_R are the left and right limit of the interval $\bar{A}$ on the real line $\Re$ respectively. Alternatively, it is represented as $\bar{A} = [m_A, w_A]$ where $m(A) = (A_L + A_R)/2$ and $w(A) = (A_R - A_L)/2$, the mid point and half width of $\bar{A}$ respectively. Clearly an α-cut of a fuzzy number can be treated as an interval.

Definition 2.11. (see e.g. [40]) There are several approaches in comparison of interval numbers. A detailed discussion about the merits and demerits of these methods is made by Sengupta and Pal [40]. According to Sengupta and Pal, fuzzy preference ordering scheme gives a complete interval ranking method and defines different sets of 'pairs of intervals' for which there exits strict and fuzzy preference relation and indifference between the interval-attributes. They used following assumptions for the maximization problem:

1. More profit is better than less profit.
2. More certainty is better than less certainty.
3. If more profit is associated with more uncertainty, a decision maker (DM) undergoes a trade-off between the two.
4. To a pessimistic DM, Assumption 2 is somewhat more important than Assumption 1(obviously to an optimistic DM, Assumption 1 is somewhat more important than Assumption 2).

According to this set of assumptions they ordered any pair of intervals A and B, as (A,B) if $m(A) \leq m(B)$ and classified into two sets S_1 and S_2 as follows:

(i) $(A, B) \in S_1$, if $w(A) \geq w(B)$
(ii) $(A, B) \in S_2$, if $w(A) < w(B)$

Then for maximization problem:

- For $(A, B) \in S_1$ unless A and B are identical, B is always the best choice.
- $(A, B) \in S_2$ fuzzy preference between A and B may be constructed.

In order to develop fuzzy preference between the pair (A,B) in S_2, a fuzzy set B' as rejection of B in S_2 is defined as $B' = \{(X, B) \in S_2/X = [X_L, X_R] = < m(X), W(X) >, m(X) \leq m(B), w(X) < w(B)\}$ with the membership function $\mu_{B'}(X, B)(\mu_{B'}$ being a function $S_2 \longrightarrow$ [0,1]), given by

$$\mu_{B'}(X, B) = \begin{cases} 1 & \text{if } m(X) = m(B) \\ max\{0, \dfrac{(m(X) - (B_L + w(X))}{(m(B) - (B_L + w(X))}\} & \text{if } m(B) \leq m(X) \leq B_L + w(X) \\ 0 & \text{otherwise} \end{cases}$$

According to definition above following conclusions are obvious:

- If $\mu_{B'}(X, B)$=1, then B is definitely rejected compared to X.
- If $\mu_{B'}(X, B)$=0, then B is definitely accepted compared to X.
- If $\mu_{B'}(X, B) \in [0, 1]$, then B is accepted/rejected according to DM's preference.

Lemma 2.12. *Let* $\tilde{a}$=(a_1, a_2, a_3) *be TFN with* $a_1 > 0$ *and* f *be a crisp number, then* $Pos(\tilde{a} \leq f) \geq \alpha$ *iff* $\frac{f - a_1}{a_2 - a_1} \geq \alpha$.

Proof.

$$Pos(\tilde{a} \leq f) = \begin{cases} 1 & \text{for } a_2 \leq f \\ \frac{f-a_1}{a_2-a_1} & \text{for } a_1 \leq f \leq a_2 \\ 0 & \text{otherwise.} \end{cases}$$

Since $0 \leq \alpha \leq 1$, $Pos(\tilde{a} \leq f) \geq \alpha$ iff $\frac{f-a_1}{a_2-a_1} \geq \alpha$. □

Lemma 2.13. *Let* $\tilde{a}$=(a_1, a_2, a_3) *be TFN with* $a_1 > 0$ *and* f *be a crisp number, then* $Pos(\tilde{a} \geq f) \geq \alpha$ *iff* $\frac{a_3-f}{a_3-a_2} \geq \alpha$.

Proof.

$$Pos(\tilde{a} \geq f) = \begin{cases} 1 & \text{for } a_2 \geq f \\ \frac{a_3-f}{a_3-a_2} & \text{for } a_2 \leq f \leq a_3 \\ 0 & \text{otherwise.} \end{cases}$$

Since $0 \leq \alpha \leq 1$, $Pos(\tilde{a} \geq f) \geq \alpha$ iff $\frac{a_3-f}{a_3-a_2} \geq \alpha$. □

Lemma 2.14. *Let* $\tilde{a}$=(a_1, a_2, a_3) *be TFN with* $a_1 > 0$ *and* f_1 *be a crisp number, then* $Nes(\tilde{a} \leq f_1) \geq \alpha$ *iff* $\frac{a_3-f_1}{a_3-a_2} \leq 1-\alpha$.

Proof. We have $Nes(\tilde{a} \leq f_1) \geq \alpha$
$\Rightarrow 1 - Pos(\tilde{a} > f_1) \geq \alpha$
$\Rightarrow Pos(\tilde{a} > f_1) \leq 1 - \alpha$

$$Pos(\tilde{a} > f_1) = \begin{cases} 1 & \text{for } a_2 > f_1 \\ \frac{a_3-f_1}{a_3-a_2} & \text{for } a_2 < f_1 < a_3 \\ 0 & \text{otherwise.} \end{cases}$$

Since $0 \leq \alpha \leq 1$, $Pos(\tilde{a} > f_1) \leq 1 - \alpha$ iff $\frac{a_3-f_1}{a_3-a_2} \leq 1 - \alpha$. Hence the result. □

Lemma 2.15. *Let* $\tilde{a}$=(a_1, a_2, a_3) *be TFN with* $a_1 > 0$ *and* f_1 *be a crisp number, then* $Nes(\tilde{a} \geq f_1) \geq \alpha$ *iff* $\frac{f_1-a_1}{a_2-a_1} \leq 1-\alpha$.

Proof. We have $Nes(\tilde{a} \geq f_1) \geq \alpha$
$\Rightarrow 1 - Pos(\tilde{a} < f_1) \geq \alpha$
$\Rightarrow Pos(\tilde{a} < f_1) \leq 1 - \alpha$

$$Pos(\tilde{a} < f_1) = \begin{cases} 1 & \text{for } a_1 > f_1 \\ \frac{f_1-a_1}{a_2-a_1} & \text{for } a_1 < f_1 < a_2 \\ 0 & \text{otherwise.} \end{cases}$$

Since $0 \leq \alpha \leq 1$, $Nes(\tilde{a} > f_1) \geq 1 - \alpha$ iff $\frac{f_1-a_1}{a_2-a_1} \leq 1 - \alpha$. Hence the result. □

Lemma 2.16. *If* $\tilde{a} = (a_1, a_2, a_3)$ *and* $\tilde{b} = (b_1, b_2, b_3)$ *be TFNs with* $0 < a_1$ *and* $0 < b_1$ *then* $Nes(\tilde{a} > \tilde{b}) > \alpha$ *iff* $\dfrac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2} < 1 - \alpha$.

Proof. We have $\text{Nes}(\tilde{a} > \tilde{b}) > \alpha$

$\Rightarrow \{1 - Pos(\tilde{a} \leq \tilde{b})\} > \alpha$

$\Rightarrow Pos(\tilde{a} \leq \tilde{b}) < 1 - \alpha$

$Pos(\tilde{a} \leq \tilde{b}) = \delta = \dfrac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2}$

and hence the result follows. □

Lemma 2.17. *If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$ then $Pos(\tilde{a} > \tilde{b}) \geq \alpha$ iff $\dfrac{a_3 - b_1}{a_3 - a_2 + b_2 - b_1} \geq \alpha$.*

Proof. follows from Lemma-2.16. □

Lemma 2.18. *Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy variable. Then its β-optimistic value is*

$$\tilde{A}_{sup}(\beta) = \begin{cases} 2\beta a_2 + (1 - 2\beta)a_3 & \text{if } \beta \leq 0.5 \\ (2\beta - 1)a_1 + 2(1 - \beta)a_2 & \text{if } \beta > 0.5 \end{cases}$$

Lemma 2.19. *Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy variable. Then its β-pessimistic value is*

$$\tilde{A}_{inf}(\beta) = \begin{cases} (1 - 2\beta)a_1 + 2\beta a_2 & \text{if } \beta \leq 0.5 \\ 2(1 - \beta)a_2 + (2\beta - 1)a_3 & \text{if } \beta > 0.5 \end{cases}$$

3. Proposed FCSTPs with Budget Constraints

3.1. **Assumptions and Notations.** In order to construct the mathematical model for the unbalanced FCSTP under destination budget constraints, the following notations are introduced:

(i) M : number of origins/sources of the transportation problem.
(ii) N : number of destinations/demands of the transportation problem.
(iii) K : number of conveyances i.e. different modes of transporting units from sources to destinations.
(iv) A_i : amount (number of packets of the item) available at the i-th origin.
(v) B_j : demand at the j-th destination.
(vi) E_k : amount of the product (number of packets) which can be carried by k-th conveyance.
(vii) C_{ijk}: per unit(packet/bag) transportation cost from i-th origin to j-th destination by k-th conveyance.
(viii) f_{ijk} : fixed transportation charge for transporting per packet from i-th origin to j-th destination by k-th conveyance.
(ix) S_j : selling price per packet at the j-th demand point.
(x) P_i : purchasing cost per packet at the i-th origin.
(xi) x_{ijk}: the amount (number packets of the item) transported from i-th origin to j-th destination by k-th conveyance.
(xii) Bud_j: total budget at the j-th destination point.

Symbols ˜ is used with the above notations to represent fuzzy parameters.

If the transportation activity is assigned from source i to destination j by conveyance k, then the fixed charge will be costed. This implies that if $x_{ijk} > 0$ we must add the fixed charge to the total transportation cost. Thus for the convenience of modelling, the following notation is introduced:

$$y_{ijk} = \begin{cases} 1 & \text{for } x_{ijk} > 0 \\ 0 & \text{otherwise} \end{cases}$$

3.2. **Mathematical Model Formulation.** Fixed charge solid transportation problems are formulated as profit maximization problems under a budget constraint at each destination. Here item is purchased at different depots in different prices. Accordingly the item is transported to different destinations from different depots using different vehicles. Units are sold from different destinations to the customers at different selling prices. Here selling prices, purchasing costs, unit transportation costs, fixed charges, sources at origins, demands at destinations, conveyances capacities are assumed to be crisp or fuzzy. Budget constraints at destinations are imposed. It is also assumed that transported amount are integer multiples of packets of units.

Model-I(Crisp Model): In this model, selling prices, purchasing costs, unit transportation costs, fixed charges, sources at origins, demands at destinations, conveyances capacities are crisp. So the crisp model mathematically takes the following form:

$$\left.\begin{array}{ll}
\text{Maximize } Z = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{S_j - P_i - C_{ijk}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} f_{ijk}y_{ijk} & \\
\text{subject to } \quad \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \le A_i & i = 1, 2, ..., M \\
\qquad \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \ge B_j & j = 1, 2, ..., N \\
\qquad \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \le E_k & k = 1, 2, ..., K \\
\qquad \sum_{i=1}^{M}\sum_{k=1}^{K} (x_{ijk}P_i + C_{ijk}x_{ijk} + f_{ijk}y_{ijk}) \le Bud_j \quad j = 1, 2, ..., N & \\
\qquad x_{ijk} = 0 \text{ or +ve integer} \qquad ((i, j, k) = 1, 2, ..., (M, N, K)) &
\end{array}\right\}$$

The constraint arise as transported units are integer multiple of packets. Model-I is solved using GRG, DBGA which is discussed in section-5 and PSO also which is presented in section-6

Model-II(Fuzzy Model): In this model, transportation costs per packet and fixed charges are fuzzy in nature. Selling prices, purchasing costs, Sources, destinations, conveyances capacities of transportation problem are also taken as TFNs and let $\tilde{S}_j = (S_{j1}, S_{j2}, S_{j3})$, $\tilde{P}_i = (P_{i1}, P_{i2}, P_{i3})$, $\tilde{C}_{ijk} = (C_{ijk1}, C_{ijk2}, C_{ijk3})$, $\tilde{f}_{ijk} = (f_{ijk1}, f_{ijk2}, f_{ijk3})$ for i=1,2,...,M, j=1,2,...,N, k=1,2,...,K. So the above crisp model is transformed to the following fuzzy model.

$$\left.\begin{array}{ll}
\text{Maximize } \tilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{\tilde{S}_j - \tilde{P}_i - \tilde{C}_{ijk}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} \tilde{f}_{ijk}y_{ijk} & \\
\qquad = (Z_1, Z_2, Z_3) & \\
\text{subject to } \quad \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \le \tilde{A}_i & i = 1, 2, ..., M \\
\qquad \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \ge \tilde{B}_{jp} & j = 1, 2, ..., N \\
\qquad \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \le \tilde{E}_k & k = 1, 2, ..., K \\
\qquad \tilde{H}_j \le Bud_j & j = 1, 2, ..., N \\
\qquad x_{ijk} = 0 \text{ or +ve integer} \quad ((i, j, k) = 1, 2, ..., (M, N, K)) &
\end{array}\right\}$$

$$\left.\begin{array}{l}
\text{Where } \tilde{H}_j = \sum_{i=1}^{M}\sum_{k=1}^{K}(x_{ijk}\tilde{P}_i + \tilde{C}_{ijk}x_{ijk} + \tilde{f}_{ijk}y_{ijk}) \\
H_{j1} = \sum_{i=1}^{M}\sum_{k=1}^{K}(x_{ijk}P_{i1} + C_{ijk1}x_{ijk} + f_{ijk1}y_{ijk}) \\
H_{j2} = \sum_{i=1}^{M}\sum_{k=1}^{K}(x_{ijk}P_{i2} + C_{ijk2}x_{ijk} + f_{ijk2}y_{ijk}) \\
H_{j3} = \sum_{i=1}^{M}\sum_{k=1}^{K}(x_{ijk}P_{i3} + C_{ijk3}x_{ijk} + f_{ijk3}y_{ijk}) \\
Z_1 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{S_{j1} - P_{i3} - C_{ijk3}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} f_{ijk3}y_{ijk} \\
Z_2 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{S_{j2} - P_{i2} - C_{ijk2}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} f_{ijk2}y_{ijk} \\
Z_3 = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{S_{j3} - P_{i1} - C_{ijk1}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K} f_{ijk1}y_{ijk}
\end{array}\right\}$$

4. Deterministic Equivalent of Imprecise Model

Till date, for the best of our knowledge, optimization of fuzzy objectives with fuzzy constraints are not well defined. As a result model-II can not be solved in the present form. So to deal with such problem an equivalent crisp problem is considered and its solution is taken as approximate solution of the problem.

4.1. **Deterministic Equivalent of Model-II.** Deterministic equivalent of Model-II can be derived in different approaches. Each approach has some merits and demerits. Here five approaches are used, which are discussed below.

Approach-4.1.1. In this approach expected value and centroid of the objective function is maximized and constraints are checked using possibility or necessity measure of fuzzy events with degree of possibility or necessity α according as DM is optimistic or pessimistic. Normally DM considers values of α near to 1 or 0 according as possibility and necessity measure is followed. So using this approach deterministic equivalent of Model-II is as below:

Model-IIA:

$$\left.\begin{array}{lr}
\text{Maximize } EorC[\tilde{Z}] = EorC\Big[\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{\tilde{S}_j - \tilde{P}_i - \tilde{C}_{ijk}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\tilde{f}_{ijk}y_{ijk}\Big] & \\
\text{s.t. pos/nes}\Big(\sum_{j=1}^{N}\sum_{k=1}^{K}x_{ijk} \le \tilde{A}_i\Big) \ge \alpha & i = 1, 2, ..., M \\
\text{pos/nes}\Big(\sum_{i=1}^{M}\sum_{k=1}^{K}x_{ijk} \ge \tilde{B}_j\Big) \ge \alpha & j = 1, 2, ..., N \\
\text{pos/nes}\Big(\sum_{i=1}^{M}\sum_{j=1}^{N}x_{ijk} \le \tilde{E}_k\Big) \ge \alpha & k = 1, 2, ..., K \\
\text{pos/nes}(\tilde{H}_j) \le Bud_j) \ge \alpha & j = 1, 2, ..., N \\
x_{ijk} = 0 \text{ or +ve integer} & ((i, j, k) = 1, 2, ..., (M, N, K))
\end{array}\right\}$$

Model-IIA-1: When DM is optimistic, i.e., when possibility measure of the constraints are considered, using Lemmas-2.12,-2.13 and definition of expected value of fuzzy numbers (Definition-2.8), the Model-IIA reduces to

$$\left.\begin{array}{lr}
\text{Maximize } E[\tilde{Z}] = (Z_1 + 2Z_2 + Z_3)/4.0 & \\
\text{subject to } \sum_{j=1}^{N}\sum_{k=1}^{K}x_{ijk} \le A_{i3} - \alpha(A_{i3} - A_{i2}) & i = 1, 2, ..., M \\
\sum_{i=1}^{M}\sum_{k=1}^{K}x_{ijk} \ge B_{j1} + \alpha(B_{j2} - B_{j1}) & j = 1, 2, ..., N \\
\sum_{i=1}^{M}\sum_{j=1}^{N}x_{ijk} \le E_{k3} - \alpha(E_{k3} - E_{k2}) & k = 1, 2, ..., K \\
H_{j1} + \alpha(H_{j2} - H_{j1}) \le Bud_j & j = 1, 2, ..., N \\
\text{and } x_{ijk} = 0 \text{ or +ve integer} & ((i, j, k) = 1, 2, ..., (M, N, K))
\end{array}\right\}$$

Model-IIA-2: When DM is pessimistic, i.e., when necessity measure of the constraints are considered, using Lemmas-2.14,-2.15 and definition of centroid method of fuzzy number (Definition-2.9), the Model-IIA reduces to

$$\left.\begin{array}{ll}
\text{Maximize } C[\tilde{Z}] = (Z_1 + Z_2 + Z_3)/3.0 & \\
\text{s. t. } \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \alpha A_{i1} + (1-\alpha)A_{i2} & i = 1,2,...,M \\
\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \alpha B_{j3} + (1-\alpha)B_{j2} & j = 1,2,...,N \\
\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \alpha E_{k1} + (1-\alpha)E_{k2} & k = 1,2,...,K \\
\alpha H_{j3} + (1-\alpha)H_{j2} \leq Bud_j & j = 1,2,...,N \\
\text{and } x_{ijk} = 0 \text{ or +ve integer } ((i,j,k) = 1,2,...,(M,N,K)) &
\end{array}\right\}$$

Approach-4.1.2. In this approach minimum profit level with some degree (α_1) of possibility/necessity is maximized and constraints are checked using possibility or necessity measure of fuzzy events with degree of possibility or necessity α according as DM is optimistic or pessimistic. Normally DM considers value of α_1 and α near to 1 or 0 according as possibility or necessity measure is follows. So using this approach deterministic equivalent of Model-II is as below:

Model-IIB:

$$\left.\begin{array}{l}
\text{Maximize } f \\
\text{pos/nes}\Big((Z_1, Z_2, Z_3) \geq f\Big) \geq \alpha_1 \\
\text{and constraints of Model-IIA .}
\end{array}\right\}$$

Model-IIB-1: When DM is optimistic, i.e., when possibility measure of the objective and constraints are considered, using Lemmas-2.12,-2.13 , the Model-IIB reduces to

$$\left.\begin{array}{l}
\text{Maximize } O_1 = Z_3 - \alpha_1(Z_3 - Z_2) \\
\text{subject to constraints of Model-IIA-1 .}
\end{array}\right\}$$

Here O_1 is the minimum profit level with degree of possibility α_1.

Model-IIB-2: When DM is pessimistic, i.e., when necessity measure of the constraints are considered, using Lemmas-2.14,-2.15, the Model-IIB reduces to

$$\left.\begin{array}{l}
\text{Maximize } O_2 = Z_1 + (1-\alpha_1)(Z_2 - Z_1) \\
\text{subject to constraints of Model-IIA-2.}
\end{array}\right\}$$

Here O_2 is the minimum profit level with degree of necessity β.

Approach-4.1.3. In literature, possibility measure and necessity measure are often used to construct mathematical model for fuzzy programming. In fact, credibility measure is superior to possibility measure and necessity measure in doing this work. For instance, a fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity achieves 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0. In the sense of this fact, credibility measure is more suitable in dealing with fuzzy optimal problem. Let f be the minimum profit with degree of credibility β and constraints are satisfied with some degree of credibility β. Then the Model-II reduces to

Model-IIC

$$\left.\begin{array}{lr} \text{Maximize } f & \\ Cr\Big((Z_1, Z_2, Z_3) \geq f\Big) \geq \beta & \\ Cr\Big(\sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq \tilde{A}_i\Big) \geq \beta & i = 1, 2, ..., M \\ Cr\Big(\sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq \tilde{B}_j\Big) \geq \beta & j = 1, 2, ..., N \\ Cr\Big(\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq \tilde{E}_k\Big) \geq \beta & k = 1, 2, ..., K \\ Cr\Big(\tilde{H}_j \leq Bud_j\Big) \geq \beta & j = 1, 2, ..., N \\ x_{ijk} = 0 \text{ or +ve integer} & ((i, j, k) = 1, 2, ..., (M, N, K)) \end{array}\right\}$$

Model-IIC-1: When DM is optimistic, then he/she assumes value of $\beta \leq 0.5$ i.e. $(0 \leq \beta \leq 0.5)$ and hence using Lemmas-2.18,-2.19, the Model-IIC reduces to

$$\left.\begin{array}{llr} \text{Maximize } \tilde{Z}_{sup}(\beta) = 2\beta Z_2 + (1-2\beta)Z_3 & & \\ \text{s.t.} & \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq 2\beta A_{i2} + (1-2\beta)A_{i3} & i = 1, 2, ..., M \\ & \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq (1-2\beta)B_{j1} + 2\beta B_{j2} & j = 1, 2, ..., N \\ & \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq 2\beta E_{k2} + (1-2\beta)E_{k3} & k = 1, 2, ..., K \\ & (1-2\beta)H_{j1} + 2\beta H_{j2} \leq Bud_j & (j = 1, 2, ..., N) \\ & x_{ijk} = 0 \text{ or +ve integer} & ((i, j, k) = 1, 2, ..., (M, N, K)) \end{array}\right\}$$

Model-IIC-2: When DM is pessimistic, then he/she assumes value of $\beta \geq 0.5$ i.e. $(0.5 \leq \beta \leq 1)$ and hence using Lemmas-2.18,2.19, the Model-IIC reduces to

$$\left.\begin{array}{llr} \text{Maximize } \tilde{Z}_{sup}(\beta) = (2\beta-1)Z_1 + 2(1-\beta)Z_2 & & \\ \text{s.t.} & \sum_{j=1}^{N}\sum_{k=1}^{K} x_{ijk} \leq (2\beta-1)A_{i1} + 2(1-\beta)A_{i2} & i = 1, 2, ..., M \\ & \sum_{i=1}^{M}\sum_{k=1}^{K} x_{ijk} \geq 2(1-\beta)B_{j2} + (2\beta-1)B_{j3} & j = 1, 2, ..., N \\ & \sum_{i=1}^{M}\sum_{j=1}^{N} x_{ijk} \leq (2\beta-1)E_{k1} + 2(1-\beta)E_{k2} & k = 1, 2, ..., K \\ & 2(1-\beta)H_{j2} + (2\beta-1)H_{j3} \leq Bud_j & j = 1, 2, ..., N \\ & x_{ijkp} = 0 \text{ or +ve integer} & ((i, j, k) = 1, 2, ..., (M, N, K)) \end{array}\right\}$$

Approach-4.1.4. In this approach α_1-cut of fuzzy objective is taken as an equivalent objective of the model. Constraints are treated similarly as approach-4.1.1. Accordingly for pessimistic and optimistic DM the Model-II reduces to Model-IID-1 and Model-IID-2 respectively as below:

Model-IID-1: Here possibility measure of the constraints are considered and hence similar as Model-IIA-1 problem reduces to

$$\left.\begin{array}{l} \text{Maximize } [Z_L(\alpha_1), Z_R(\alpha_1)] = [Z_1 + \alpha_1 * (Z_2 - Z_1), Z_3 - \alpha_1 * (Z_3 - Z_2)] \\ \text{subject to constraints of Model-IIA-1 .} \end{array}\right\}$$

Model-IID-2: Here necessity measure of the constraints are considered and hence similar as Model-IIB-2 problem reduces to

$$\left.\begin{array}{l}\text{Maximize } [Z_L(\alpha_1), Z_R(\alpha_1)] = [Z_1 + \alpha_1 * (Z_2 - Z_1), Z_3 - \alpha_1 * (Z_3 - Z_2)] \\ \text{subject to constraints of Model-IIA-2.}\end{array}\right\}$$

Considering α_1-cut of a fuzzy number as interval, Models-IID-1 and -IID-2 are solved using DBGA which is discussed in section5 , where FPOI is used to compare two chromosomes(solutions).

Approach-4.1.5. In this approach fuzzy objective is directly optimized using GA where comparison between two values of the objective is made using possibility/necessity measure of fuzzy event(Subsection5.2 of Evaluation process). In this case also two subcase may arise

Model-IIE-1: Here possibility measure of constraints are considered and comparison of fuzzy objectives are made using possibility measure. So Model-II reduces to

$$\left.\begin{array}{l}\text{Maximize } \tilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{\tilde{S}_j - \tilde{P}_i - \tilde{C}_{ijk}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\tilde{f}_{ijk}y_{ijk} \\ \quad = (Z_1, Z_2, Z_3) \\ \text{subject to constraints of Model-IIA-1 .}\end{array}\right\}$$

Model-IIE-2: Here necessity measure of constraints are considered and comparison of fuzzy objectives are made using necessity measure. So Model-II reduces to

$$\left.\begin{array}{l}\text{Maximize } \tilde{Z} = \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\{\tilde{S}_j - \tilde{P}_i - \tilde{C}_{ijk}\}x_{ijk} - \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{K}\tilde{f}_{ijk}y_{ijk} \\ \quad = (Z_1, Z_2, Z_3) \\ \text{subject to constraints of Model-IIA-2.}\end{array}\right\}$$

Models-I,-IIA-1,IIA-2,-IIB-1,IIB-2,-IIC-1,IIC-2 are solved using both GRG, DBGA and PSO method (PSO is discussed in section-6). Models IID-1,IID-2,IIE-1,IIE-2 are solved using DBGA which is discussed in section-5.

5. Dominance Based Genetic Algorithm (DBGA)

Use of GA in complex decision making problem is already well established (Holland[19]; Michalewicz[32]). A simple GA starts with a set of potential solutions (called initial population) of the decision making problem under consideration. Individual solutions are called chromosomes. Crossover and mutation operations happen among the potential solutions with some probabilities p_c and p_m respectively to get a new set of solutions and it continues until terminating conditions are encountered. Behavior and performance of a GA is directly affected by the interaction between the parameters, i.e., selection process of chromosomes for mating pool, p_c, p_m, etc. Poor parameter settings usually leads to several problems such as premature convergence (Davis and Principe[4]). Extensive research work has been made to improve the performance of GA for single/multi-objective continuous/discrete optimization problems during last two decades. Michalewicz[32] proposed a genetic algorithm named contractive mapping genetic algorithm (CMGA), where movement from old population to new population takes place only when average fitness of new population is better than the old one and proved the asymptotic convergence of the algorithm by Banach fixed point theorem. Bessaou and Siarry[2] proposed

a GA, where initially more than one population of solutions are generated. Genetic operations are done on every population for a finite number of times to find a promising zone of optimum solution. Finally a population of solutions is generated in this zone and genetic operations are done on this population a finite number of times to get a final solution. Last and Eyal[26] developed a GA with varying population size, where chromosomes are classified into young, middle age and old according to their age and lifetime. Nezmabadi-Pour et al. [34] presents a solution to an economic dispatch problem by fuzzy adaptive genetic algorithm. In recent years different authors use GA to solve different transportation problems (Gen et al.[12]; Ojha et al.[36][37]; Giri et al.[13], Nezamabadi-Pour and others).
Following Last and Eyal [26], a GA with varying population size is proposed. Here chromosomes are classified into young, middle age and old (in fuzzy sense) according to their age and lifetime. Following comparison of fuzzy numbers using possibility theory (Dubois and Prade [6]; Liu and Iwamura [31]), crossover probability is measured as a function of parents' age interval (a fuzzy rule base on parents' age limit is also used for this purpose). In this GA, a subset of better children is included with the parent population for next generation and maximum size of this subset is a percentage of the size of its parent set. To control memory overflow at the runtime of the GA, an upper limit of population size is imposed ($Maxsize$). Chromosomes with age exceeds lifetime are discarded from the population at the beginning of every iteration. Algorithm terminates when the difference between maximum fitness ($Maxfit$) of chromosome and average fitness ($Avgfit$) of the population becomes negligible or when number of iterations exceeds some predefined upper limits($Maxgen$). To deal with single as well as multi-objective optimization problem fitness of a solution is taken as the ratio of the number of solutions dominated by it and current population size. To deal with transportation problem chromosomes are generated in a special way and cyclic crossover and mutation are used.

5.1. General Structure of DBGA is Presented Below.

1. Set iteration counter t, $Maxsize$, $\epsilon = 0.0001$, $Maxgen$ and p_m.
2. Randomly generate initial population $P(t)$ of size *pop_size*.//According to representation in subsection-5.2.
3. Evaluate initial population $P(t)$.//Evalution process according to subsection-5.2.
4. Determine lifetime of all chromosomes of $P(t)$.//Determine lifetime of chromosome according to subsection-5.2.
5. Set $Maxfit$= Maximum fitness in $P(t)$ and $Avgfit$=Average fitness of $P(t)$.
6. While ($Maxfit - Avgfit > \epsilon$ and $t \leq Maxgen$) do
7. $t = t + 1$.
8. Increase age of each chromosome.
9. For each pair of parents do
10. Determine probability of crossover $\tilde{p}_c$ for the selected pair of parents using fuzzy rule base and possibility theory.

11. Perform crossover with probability $\tilde{p}_c$.// Crossover operation according to subsection-5.2.
12. For each offspring perform mutation with probability p_m.// Mutation operation according to subsection-5.2.
13. Store offsprings into offspring set.
14. End do
15. Remove from $P(t)$ all individuals with age grater than their lifetime.
16. Select a percent of better offsprings from the offspring set and insert into $P(t)$, such that maximum size of the population is less than maximize.
17. Evaluate $P(t)$.
18. Determine lifetime of all child chromosomes of $P(t)$.
19. Remove all offsprings from the offspring set.
20. End While
21. Output: Some optimal solutions of $P(t)$.
22. End Algorithm

5.2. **DBGA Procedures for The Proposed Model.** There are different steps of DBGA presented below.

5.2.1. **Representation.** A MNK dimensional integer vector $T = (t_1, t_2, ..., t_{MNK})$ is employed to represent a chromosome corresponding to a solution X, where $t_j \in \{1, 2, ..., MNK\}$ and $t_j \neq t_k$ for any $j \neq k$, $j, k = 1, 2, ..., MNK$. That is the sequence $(t_1, t_2, ..., t_{MNK})$ is a rearrangement of $\{1, 2, ..., MNK\}$. Since the fixed charge will be costed if the transportation activity occurs, in order to minimize the total transportation cost, it is better to decrease the occurrence of the transportation activity if possible. Also, it is easy to see that the more the products are transported, the more the transportation cost will be spent. Thus the optimal solution should be obtained when the demand of each destination is at least satisfied (Yang and Liu[44]). Also as units are normally transported in packets, here integer multiple of packets is considered as allocated units. For this reason, the following method is employed to decode the solution X of crisp model-I from the chromosome T. For problems with fuzzy sources, demands and capacities of conveyances values of $\tilde{A}_i$, $\tilde{B}_j$, $\tilde{E}_k$ are taken as corresponding limits of the transformed crisp constraints :

1. for $i = 1$ to M step 1 do
   $a_i \leftarrow A_i$
   end for
2. for $i = 1$ to K step 1 do
   $e_i \leftarrow E_i$
   end for
3. Let n = 1.
4. Suffixes i, j, k of x_{ijk} corresponding to n are calculated using the formulas

$$j = \begin{cases} N & \text{if } N|t_n \\ (t_n) \bmod N & \text{otherwise} \end{cases}$$

$$i = \begin{cases} M & \text{if } MN|t_n \\ \dfrac{(t_n) \bmod MN - j}{N} + 1 & \text{otherwise} \end{cases}$$

$$k = \frac{t_n - (i-1)N - j}{MN} + 1$$

5. Let val=rand$(0, a_i)$, when rand$(0, a_i)$ represent random integer between 0 and A_i.
6. If $val > e_k$ then $val = e_k$.
7. Set $x_{ijk} = val$, $a_i = a_i - val$, $e_k = e_k - val$
8. Set $n = n + 1$.
9. if $n \leq MNK$ and go to Step 4.
10. for $j = 1$ to N step 1 do

    if $\sum_{i=1}^{M} \sum_{k=1}^{K} x_{ijk} < B_j$

    goto step1. // if constraints are not satisfied then regenerate the solution.

    end if
11. end for.
12. Stop.

5.2.2. **Initialization.** pop_size number of such chromosomes T_i, $i = 1, 2, ...,$ pop_size, are randomly generated by random number generator. The solution X_i is generated from the chromosome T_i following above mentioned steps of subsection-5.2.

5.2.3. **Constraint Checking.** For constrained optimization problems, at the time of generation of each individuals X_i of $P(1)$, constraints are checked using a separate subfunction check_constraint(X_i), which returns 1 if X_i satisfies the constraints, otherwise it returns 0. If check_constraint(X_i)=1, X_i is included in $P(1)$ otherwise X_i is again generated and it continues until constraints are satisfied.

5.2.4. **Evaluation Process.** Fitness of a solution X_i in the population is taken as the ratio of the number of solutions dominated by X_i and pop_size. A solution X_i dominates a solution X_j if one or more objectives of X_i are better than that of X_j and others are equal. Depending upon the nature of the objectives, comparison of objectives of solutions are made using different approaches, which are listed below.

- Crisp objective: For crisp objective, comparison between two solutions is made using comparison of the real objective values.
- Interval objective: For interval objective, comparison between two solutions is made using comparison of the real interval objectives following FPOI(cf. 2.11).
- Fuzzy objective: For fuzzy objective comparison is made using possibility/ necessity measure of fuzzy events. Using this approach a solution X_1 with fuzzy objective $\tilde{F}_1$ is better than another solution X_2 with objective $\tilde{F}_2$ if $pos(\tilde{F}_1 > \tilde{F}_2) > \alpha$ (cf. Lemma-2.17, see section-2) or if $nes(\tilde{F}_1 > \tilde{F}_2) > \alpha$(cf. Lemma-2.16, see section-2.) according as DM is optimistic or pessimistic.

5.2.5. **Determination of Lifetime.** Let fitness of X_i be $Z(X_i)$. At the time of initialization age of each solution is set to zero. Following Michalewicz [32], at the time of birth life-time of X_i is computed by using the following formula:

$$\text{If } Avgfit \geq Z(X_i) \qquad lifetime(X_i) = Minlt + \frac{K(Z(X_i) - Minfit)}{Avgfit - Minfit},$$

$$\text{If } Avgfit < Z(X_i) \quad lifetime(X_i) = \frac{Minlt + Maxlt}{2} + \frac{K(Z(X_i) - Avgfit)}{Maxfit - Avgfit}$$

where $Maxlt$ and $Minlt$ refers to maximum and minimum allowed lifetime of a chromosome, $K = (Maxlt - Minlt)/2$. $Maxfit$, $Avgfit$, $Minfit$ represent the best, average and worst fitness of the current population. To solve proposed STP, it is assumed that $Maxlt = 7$ and $Minlt = 1$, $N = 10$. According to the age, a chromosome can belong to any one of age intervals - young, middle-age or old, whose membership functions are presented in Figure 1.

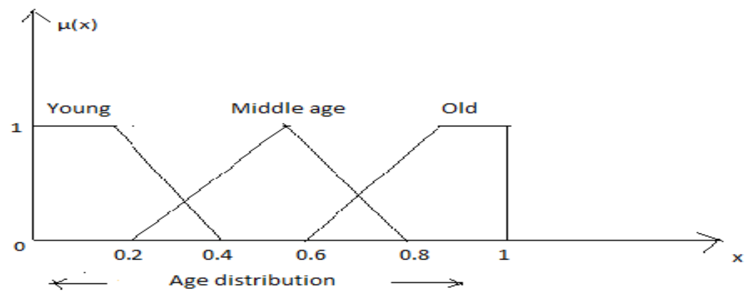

FIGURE 1. Membership Function of Age Intervals

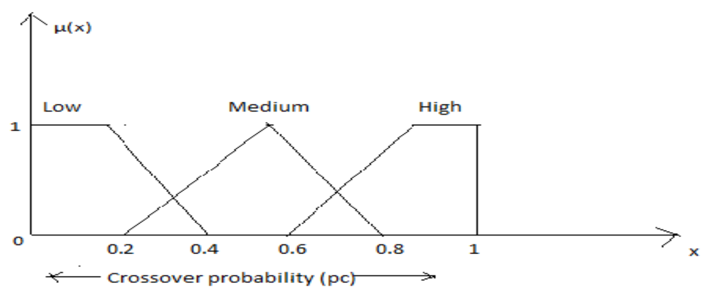

FIGURE 2. Membership Functions of Crossover Probabilities

5.2.6. **Crossover.**

5.2.6.1. **Determination of Probability of Crossover ($\tilde{p}_c$).** Probability of crossover $\tilde{p}_c$, for a pair of parents (X_i, X_j) is determined as below:

5.2.6.1.1. At first, age intervals (young, middle-age, old) of X_i and X_j are determined by making possibility measure of fuzzy numbers- young, middle-age, old in respect of their age using Lemma-2.13 (see section-2).

5.2.6.1.2. After determination of age intervals of the parents, their crossover probability ($\tilde{p}_c$) is determined as a linguistic variable (low, medium or high) using a fuzzy rule base as presented in Table-1. Membership function of these linguistic variables are presented in FIGURE 2.

5.2.6.2. **Crossover Process.** Cyclic crossover process (Oliver et al.[38]) is used for this purpose. The cycle crossover focuses on subsets of genes that occupy the same subset of positions in both parents. Then, these genes are copied from the first parent to the offspring (at the same positions), and the remaining positions are filled with the genes of the second parent. In this way, the position of each gene is inherited from one of the two parents. However, many edges can be broken in the process, because the initial subset of genes is not necessarily located at consecutive positions in the parent. For each pair of parent solutions X_i and X_j, a random number c is generated from the range [0, 1] and if $Nes(c < \tilde{p}_c) > \beta$ (cf. Lemma-2.15, see section-2), the crossover operation is made on X_i and X_j, where $\beta\,(0 < \beta < 1)$ is a predefined necessity level. For the proposed model it is assumed that $\beta = 0.5$. To make crossover operation on each pair of coupled solutions X_i and X_j, cyclic crossover is made on every column of T_i with corresponding column of T_j. To illustrate the process let each column of a chromosome consisting of 9 genes and consider k-th column of two parents T_i, T_j as below:

$$T_{ik} : 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9$$

$$T_{jk} : 3\ \ 4\ \ 5\ \ 1\ \ 2\ \ 9\ \ 8\ \ 7\ \ 6$$

Let CT_1, CT_2 be two children born after crossover. The mechanism of construction of k-th column of CT_1, CT_2 using cyclic crossover is explained with the help of following steps:

- Randomly generate an integer in the range [1 ... 9]. Let it be 3.
- As $T_{ik}[3] = 3$, 3-rd element of CT_1 is 3, i.e., $CT_1[3] = 3$.
- T_{jk}, is then searched to check the presence of element 3 and it has been found in the 1-st position. Then 1-st element of CH_1 is selected from the 1-st element of T_{ik}, i.e., $CT_1[1] = T_{ik}[1] = 1$.
- T_{jk}, is again searched for the presence of element 1 and it has occurred at the 4-th position. Thus 4-th element of T_{ik} has been copied as the 4-st element of CT_1, i.e., $CT_1[4] = T_{ik}[4] = 4$. Similarly, following are obtained:

$$CT_1[2] = T_{ik}[2] = 2, \quad CT_1[5] = T_{ik}[5] = 5$$

This completes one cycle because element 5 is seen to be present at the 3-rd position of T_{jk} and the corresponding 3-rd position element of T_{ik} is element 3, which has already been selected as the starting element of the cycle.

- The remaining elements of CT_1 are selected directly from T_{jk} as follows:

$$CT_1[6] = T_{jk}[6] = 9, \quad CT_1[7] = T_{ik}[7] = 8$$

$$CT_1[8] = T_{jk}[8] = 7, \quad CT_1[9] = T_{ik}[9] = 6$$

- Final forms of k-th columns of CT_1 and CT_2 are as below:

$$CT_{1k} : 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 9\ \ 8\ \ 7\ \ 6$$

$$CT_{2k} : 3\ \ 4\ \ 5\ \ 1\ \ 2\ \ 6\ \ 7\ \ 8\ \ 9$$

For constrained optimization problems, if a child solution satisfies the constraints of the problem, it is included in the offspring set, otherwise it is excluded.

Parent 2	Parent 1		
	Young	Middle-age	Old
Young	Low	Medium	Low
Middle-age	Medium	High	Medium
Old	Low	Medium	Low

Table 1. Fuzzy Rule Base for Crossover Probability

5.2.7.**Mutation.**

5.2.7.1. **Selection for Mutation.** For each offspring generate a random number 'r' from the range $[0, 1]$. If $r < p_m$, the solution is taken for mutation, where p_m is the probability of mutation of the current population.

5.2.7.2. **Mutation Process.** To mutate a chromosome T_m, one of its column is randomly selected. Let it is k. Then, two random integers i, j are selected in the range $[1, MNK]$. Then interchange $T_{mk}[i]$, $T_{mk}[j]$ to get child chromosome CT_m. This process is named two point mutation. If child solution does not satisfy the constraint, the parent solution will not be replaced by child solution. Constraint checking of a child solution CT_m is made by using check_constraint (CT_m) function.

5.2.8. **Selection of Offsprings.** Maximum population growth in a generation is assumed as forty percent. So, not all offsprings belong to the parent set for next generation. At first, offspring set is arranged in descending order in fitness. Then better solutions are selected and entered into parent set such that the population size does not exceeds $Maxsize$. Here it is assumed as $Maxsize = 200$.

5.2.9. **Termination Condition.** Algorithm terminates when difference between maximum fitness $(Maxfit)$ of chromosome, (i.e., fitness of the best solution of the population) and average fitness $(Avgfit)$ of the population becomes negligible.

5.2.10. **Implementation.** With the above function and values the algorithm is implemented using C-programming language in a personal computer consist of Intel(R) Core(TM) i5-2450M (2.50 GHz) CPU with Windows 8.1 operating system and 4 GB RAM.

6. Swap Sequence Based Particle Swarm Optimization (SSPSO) for STP

PSOs are exhaustive search algorithms based on the emergent motion of a flock of birds searching for food (Eberhart and Kennedy [8],Kennedy and Eberhart [22]) and has been extensively used/ modified to solve complex decision making problems in different field of science and technology (Engelbrecht [9]; Esmin et al. [10]; Feng [11]; Liang et al. [27]). A PSO normally starts with a set of potential solutions (called swarm) of the decision making problem under consideration. Individual solutions are called particles and food is analogous to optimal solution. In simple terms the particles are flown through a multi-dimensional search space, where the position of each particle is adjusted according to its own experience and that of its neighbors. Each particle i has a position vector $(X_i(t))$, a velocity vector $(V_i(t))$, the position at which the best fitness ($pbesti$) encountered by the particle so far, and the best position of all particles ($gbest$) in current generation t. In generation $(t+1)$, the position and velocity of the particle are changed to $X_i(t+1)$ and $V_i(t+1)$ using following rules:

$$V_i(t+1) = wV_i(t) + c_1r_1(X_{pbesti}(t) - X_i(t)) + c_2r_2(X_{gbest}(t) - X_i(t)) \quad (1)$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (2)$$

The parameters c_1 and c_2 are set to constant value, which are normally taken as 2, r_1 and r_2 are two random values, uniformly distributed in [0, 1], $w(0 < w < 1)$ is inertia weight which controls the influence of previous velocity on the new velocity. It is mainly used to solve continuous optimization problems. It is also used to solve travelling salesman problems where swap sequence and swap operations are used to find velocity of a particle and its updating (Wang et al.[42]; Yan et al.[43]; Akhand et al.[1]). In a travelling salesman problems a potential solution is represented by a sequence of nodes. Swap operations on different nodes is used to update a solution. A Swap sequence represents a sequence of swap operations used to transform a solution to another solution. A PSO that uses swap sequence and swap operation is called SSPSO. If potential solution of a STP is represented by a sequence of cells (as represented in DBGA subsection-5.2) then SSPSO can be used to find marketing decision of the STP. In this research paper SSPSO is used to find solution of STP where potential solutions are represented by a sequence of cost cells as in DBGA.

6.1. **Basic Operations of SSPSO.** Different steps of SSPSO are briefly presented below.

6.1.1. **Representation of a Solution.** A solution (particle) of the STP represented in the same way as a solution (chromosome) is represented in DBGA (see subsection-5.2)

6.1.2. **Swap Operator.** Consider a normal solution sequence of STP $T = (t_1, t_2, ..., t_{MNK})$ corresponding to a solution X, where $t_j \in \{1, 2, ..., MNK\}$ and $t_j \neq t_k$ for any $j \neq k$, $j, k = 1, 2, ..., MNK$. Here we define swap operator, $SO(t_i, t_j)$ as exchanging cell t_i and cell t_j in sequence T. Then we define $T' = T + SO(t_i, t_j)$ as a new sequence on operating operator $SO(t_i, t_j)$ on T. So the plus sign $'+'$, above has its new meaning. It can be given a concrete example: Suppose there is a STP problem with eight nodes, and $T = (1, 3, 5, 2, 4, 6, 8, 7)$ be a sequence. The swap operator is $SO(2, 5)$, then $T' = T + SO(2, 5) = (1, 3, 5, 2, 4, 6, 8, 7) + SO(2, 5) = (1, 4, 5, 2, 3, 6, 8, 7)$.

6.1.3 **Swap Sequence.** A swap sequence SS is made up of one or more swap operators. $SS = (SO_1, SO_2, ..., SO_n)$, where $SO_1, SO_2, ..., SO_n$ are swap operators. Here the order of the swap operator in SS is important.
swap sequence acting on a solution means all the swap operators of the swap sequence act on the solution in order. This can be described by the following formula:

$$T' = T + SS = T + (SO_1, SO_2, ..., SO_n) = (((T + SO_1) + SO_2)... + SO_n)$$

Different swap sequences acting on the same solution may produce the same new solution. All these swap sequences are named the equivalent set of swap sequences.

In the equivalent set, the sequence which has the least swap operator is called basic swap sequence of the set or basic swap sequence (BSS) in short.
Several swap sequences can be merged into a new swap sequence, we define the operator $\oplus$ as merging two swap sequences into a new swap sequence. Suppose there is two swap sequences, SSI and $SS2$, SSI and $SS2$ act on one solution T in order, namely SSI first, $SS2$ second, we get a new solution T', and there is another swap sequence SS' acting on the same solution T, then get the same solution T', described as follows:

$$SS' = SS1 \oplus SS2$$

SS' and $SSl \oplus SS2$ are in the same equivalent set.

6.1.4. **The Construction of Basic Swap Sequence.** Suppose there is two solutions, A and B, and our task is to construct a basic swap sequence SS which can act on B to get solution A, we define $SS = A - B$ (Here the sign - also has its new meaning). We can swap the nodes in B according to A from left to right to get SS. So there must be an equation $A = B + SS$. For example, consider two solutions:

$$A = (1, 2, 3, 4, 5), B = (2, 3, 1, 5, 4)$$

Here $A(1) = B(3) = 1$, so the first swap operator is $SO(1, 3)$, $B1 = B + SO(1, 3)$ then we get the following result:

$$B1 : (1, 3, 2, 5, 4)$$

Again $A(2) = B1(3) = 2$, so the second operator is $SO(2, 3)$ and $B2 = (1, 2, 3, 5, 4)$. The third operator is $SO(4, 5)$, then B3=A. Finally we get the basic swap sequence $SS = A - B = (SO(1, 3), SO(2, 3), SO(4, 5))$.

6.1.5. **Transformation of the Formula.** Formula (1), (2) are not suitable for the STP problem. We update it as follows:

$$\begin{aligned} V_i(t+1) &= V_i(t) \oplus r_1 \odot (T_{pbesti}(t) - T_i(t)) \oplus r_2 \odot (T_{gbest}(t) - T_i(t)) \\ T_i(t+1) &= T_i(t) + V_i(t+1) \end{aligned}$$

Here r_1, r_2 are random numbers between 0 and 1. Velocity $V_i(t)$ represent a swap sequence, $T_i(t)$ is sequence of cells of i-th solution $X_i(t)$ in iteration t as discussed earlier. Meaning of $T_{pbesti}(t)$, $T_{gbest}(t)$ are similar. $r_1 \odot (T_{pbesti}(t) - T_i(t))$ means all swap operators in basic swap sequence $(T_{pbesti}(t) - T_i(t))$ should be maintained with the probability of r_1, it is the same as $r_2 \odot (T_{gbest}(t) - T_i(t))$. From here we can see that the bigger the value of r_1 the greater the influence of $X_{pbesti}(t)$ is, for more swap operators in $(T_{gbest}(t) - T_i(t))$ will be maintained, it is also the same as $r_2 \odot (T_{gbest}(t) - T_i(t))$.

6.2. **SSPSO Algorithm for STP.**

1. Initialize $maxgen$, t=0.
2. Randomly generate initial swarm of size s_size, i.e., s_size number of sequence of cells T_i, $i = 1, 2, ..., s\_size$. Then generate corresponding solutions X_i, $i = 1, 2, ..., s\_size$ following subsection-5.2, such that each solution satisfies constraints of the problem.

Models		k	1		2	
		i/j	1	2	1	2
I	C_{ijk}	1	3	6	2	5
		2	5	10	4	9
	f_{ijk}	1	10	9	8	7
		2	11	12	9	10
II	$\tilde{C}_{ijk}$	1	(2,3,4)	(5,6,7)	(4,5,7)	(8,10,12)
		2	(1,2,3)	(4,5,6)	(3,4,5)	(7,9,10)
	$\tilde{f}_{ijk}$	1	(8,10,11)	(7,9,10)	(10,11,12)	(11,12,13)
		2	(7,8,9)	(6,7,9)	(8,9,10)	(9,10,11)

Table 2. Unit Transportation Cost (C_{ijk}) and Fixed Charge (f_{ijk}) of Models-I and -II

3. Find profit for each solution and find X_{gbest} and set T_{gbest} as its corresponding sequence of cells.
4. Initialize velocity $V_i(t)$, $i = 1, 2, ..., s\_size$, where each $V_i(t)$ consists of a randomly generated swap sequence of unit size from the set $\{1, 2, ..., MNK\}$.
5. Set $X_{pbesti} = X_i(t)$, $i = 1, 2, ..., s\_size$. So $T_{pbesti} = T_i(t)$, $i = 1, 2, ..., s\_size$.
6. For $i = 1$ to s_size do
7. $\quad V_i(t+1) = V_i(t) \oplus r_1 \odot (T_{pbesti} - T_i(t)) \oplus r_2 \odot (T_{gbest} - T_i(t))$
8. $\quad T_i(t+1) = T_i(t) + V_i(t+1)$
9. Determine $X_i(t+1)$ from $T_i(t+1)$ such that it satisfies constraints of the problem and find its profit.
10. End for
11. t=t+1
12. For $i = 1$ to s_size do
13. If profit of $X_i(t) >$ profit of X_{pbesti} then set $X_{pbesti} = X_i(t)$, $T_{pbesti} = T_i(t)$
14. If profit of $X_i(t) >$ profit of X_{gbest} then set $X_{gbest} = X_i(t)$, $T_{gbest} = T_i(t)$
15. if $t < maxgen$ goto step-6.
16. output X_{gbest}
17. End Algorithm

In the algorithm comparison of profits (steps 13 and 14) are made in different ways for crisp, fuzzy and interval objectives and are mentioned in subsection-5.2.

7. Numerical Experiment

For illustration of the models, following two transportation problems are considered.

Input Data. For both models, no of origins=2 (i.e. M=2), no of destination=2 (i.e. N=2), no of conveyance=2 (i.e. K=2) are considered. Crisp and fuzzy unit selling prices, unit purchasing costs, unit transportation costs and fixed charges for models-I and -II are given in Table-2. The parametric values of the models-I and -II are given in Table-3. For possibility (necessity) measure of constraints and objective, values of α and α_1 are taken as 0.9 and 0.9 (0.1 and 0.1) respectively. For credibility measure of constraints and objective, values of β are taken as 0.4 and 0.6 respectively.

With the above input data, the transportation Models-I, IIA-1, IIA-2, IIB-1,IIB-2, IIC-1, IIC-2 are solved by GRG(LINGO 11.0 software), DBGA and PSO.

Models	Source (A_1, A_2)	Demand (B_1, B_2)	Capacities of conv. (E_1, E_2)	Unit Selling price (S_1, S_2)	Unit Purchasing Cost (P_1, P_2)	Budjet Budjet Bud_1, Bud_2
-I	(25,24)	(14,21)	(25,22)	(25,22)	(7,5)	(230,288)
-II	{(24,25,26), ,(23,24,25)}	{(12,14,16), (19,21,23)}	{(23,25,27), (20,22,24)}	{(22,25,26), (20,22,23)}	{(6,7,8), ,(4,5,7)}	

TABLE 3. Parametric Values for Models

	Budget constraints	k	1		2		Maximum profit (Z)	Iteration	Executed time
		i/j	1	2	1	2			
GRG	With	1 2	0 1	20 0	4 17	1 0	497	-	-
	With out	1 2	0 5	20 0	2 19	1 0	557		
DBGA	With	1 2	0 0	21 0	0 22	0 0	523	16	14.798
	With out	1 2	4 0	20 1	0 22	0 0	559	19	4.487
PSO	With	1 2	1 0	21 0	0 22	0 0	528	15	13.117
	With out	1 2	2 23	0 0	0 0	22 0	567	18	4.406

TABLE 4. Optimum Results of Model-I

	Models	α/β	k	1		2		$\tilde{Z}$ (Z_1, Z_2, Z_3)	Maximum profit $(E(\tilde{Z})/C(\tilde{Z}))$ $/(O_1/O_2)/\tilde{Z}_{sup}$	Utilized budget
			i/j	1	2	1	2			
GRG	IIA-1	α=.9	1 2	1 0	21 0	3 19	0 0	298,520,658	492	(230.,277.)
	IIA-2	α=.1	1 2	2 15	3 0	0 2	19 0	237,458,588	427.	(216.7,286.7)
	IIB-1	α=.9	1 2	0 5	16 0	4 13	5 0	274,497,632	510.5	(222.3,283.5)
	IIB-2	α=.1	1 2	2 4	15 0	0 14	7 0	257,473,606	451.4	(225.3,287.7)
	IIC-1	β=.4	1 2	2 0	21 0	0 22	0 0	314,543,683	571.	(226.8,273.2)
	IIC-2	β=.6	1 2	2 15	1 0	0 0	21 0	229,437,560	353.8	(225.3,287.7)

TABLE 5. Marketing Decisions of Model-II (With Budget Constraints(Continue))

	Models	α/β	k	1		2		$\tilde{Z}$ (Z_1, Z_2, Z_3)	Maximum profit $(E(\tilde{Z})/C(\tilde{Z}))$ $/(O_1/O_2)/\tilde{Z}_{sup}$	Utilized budget
			i/j	1	2	1	2			
DBGA	IIA-1	α=.9	1 2	1 0	21 0	0 22	0 0	304,528,665	499.	(229.2278.5)
	IIA-2	α=.1	1 2	0 21	1 0	0 0	21 0	268,507,640	471.67	(215.6,287.7)
	IIB-1	α=.9	1 2	0 22	0 0	0 0	22 0	287,532,666	545.4.	(222.1,277.6)
	IIB-2	α=.1	1 2	1 0	22 0	0 21	0 0	299,521,658	498.	(208.2,285.7)
	IIC-1	β=.4	1 2	2 0	21 0	0 22	0 0	314,543,683	571.	(221.4,277.)
	IIC-2	β=.6	1 2	0 20	1 0	0 0	21 0	260,492,622	445.6	(229.5,287.7)
PSO	IIA-1	α=.9	1 2	1 0	21 0	0 22	0 0	304,528,665	506.2	(228.2,279.6)
	IIA-2	α=.1	1 2	1 0	21 0	0 21	0 0	294,512,646	484.	(214.6,287.9)
	IIB-1	α=.9	1 2	1 0	22 0	0 22	0 0	309,537,677	551	(221.1,278.6)
	IIB-2	α=.1	1 2	0 0	24 0	0 21	0 0	301,534,672	511.6	(208.2,287.7)
	IIC-1	β=.4	1 2	2 0	23 0	0 22	0 0	324,561,707	590.	(220.4,279.)
	IIC-2	β=.6	1 2	0 20	1 0	0 0	21 0	260,512,642	461.6	(227.5,287.8)

TABLE 6. Marketing Decisions of Model-II (With Budget Constraints)

	Models	α/β	k i/j	1 1	 2	2 1	 2	$\tilde{Z}$ (Z_1, Z_2, Z_3)	Maximum profit $(E(\tilde{Z})/C(\tilde{Z}))$ $/(O_1/O_2)/\tilde{Z}_{sup})$
GRG	IIA-1	α=.9	1 2	4 0	21 0	0 22	0 0	334,573,719	542.
	IIA-2	α=.1	1 2	2 0	22 0	0 21	0 0	268,507,640	507.
	IIB-1	α=.9	1 2	0 5	20 0	2 19	1 0	308,557,704	571.7
	IIB-2	α=.1	1 2	2 0	22 0	0 21	0 0	309,536,676	513.3
	IIC-1	β=.4	1 2	0 5	20 0	2 19	1 0	308,557,704	586.4
	IIC-2	β=.6	1 2	2 0	22 0	0 21	0 0	309,576,636	490.6
DBGA	IIA-1	α=.9	1 2	4 0	21 0	0 22	0 0	334,573,719	542.
	IIA-2	α=.1	1 2	2 0	22 0	0 21	0 0	309,536,676	507.
	IIB-1	α=.9	1 2	4 0	21 0	0 22	0 0	334,573,719	587.6
	IIB-2	α=.1	1 2	2 0	22 0	0 21	0 0	309,536,676	513.3
	IIC-1	β=.4	1 2	4 0	21 0	0 22	0 0	334,573,719	602.2
	IIC-2	β=.6	1 2	2 0	22 0	0 21	0 0	309,576,636	490.6
PSO	IIA-1	α=.9	1 2	4 0	21 0	0 22	0 0	334,573,719	549.7
	IIA-2	α=.1	1 2	3 0	21 0	0 21	0 0	314,542,682	512.6
	IIB-1	α=.9	1 2	0 1	0 21	24 0	0 0	306,538,680	552.0
	IIB-2	α=.1	1 2	3 0	21 0	0 21	0 0	314,542,682	519.2
	IIC-1	β=.4	1 2	4 0	21 0	0 22	0 0	334,599,745	628.2
	IIC-2	β=.6	1 2	2 0	22 0	0 21	0 0	309,559,699	509

Table 7. Marketing Decisions of Model-II (Without Budget Constraints)

Models	α_1	k i/j	1 1	 2	2 1	 2	$\tilde{Z}$ (Z_1, Z_2, Z_3)	Maximum expected profit
IID-1	0.1	1 2	0 7	17 1	0 14	3 0	246,471,604	440.33
	0.3	1 2	1 20	1 0	0 0	22 0	265,507,645	472.33
	0.5	1 2	0 22	0 0	0 0	22 0	287,532,666	495.
	0.7	1 2	3 0	20 0	0 20	2 0	302,530,671	501.
	0.9	1 2	3 0	22 0	0 20	0 0	309,535,675	506.
IID-2	0.1	1 2	0 2	22 0	0 20	0 0	294,519,655	489.33
	0.3	1 2	0 21	3 0	0 0	19 0	266,505,638	469.67
	0.5	1 2	0 21	0 1	0 0	21 0	261,502,635	466.
	0.7	1 2	0 2	22 0	0 18	0 0	274,487,617	459.
	0.9	1 2	0 19	2 0	0 0	21 0	257,486,616	453.

Table 8. Marketing Decisions of Model-IID Using α_1-cut of Fuzzy-objective Optimization and DBGA

Models	α	k i/j	1 1	 2	2 1	 2	$\tilde{Z}$ (Z_1, Z_2, Z_3)	Maximum expected profit
IIE-1	0.1	1 2	3 0	18 0	1 18	1 2	263.,484.,623.	456.67
	0.3	1 2	4 0	20 1	0 19	0 0	287,511,650	482.67
	0.5	1 2	0 22	0 0	0 0	22 0	287,532,666	495.
	0.7	1 2	0 1	22 0	0 22	0 0	306,536,675	505.67
	0.9	1 2	1 0	22 0	0 22	0 0	309,537,677	507.67
IIE-2	0.1	1 2	1 0	22 0	0 21	0 0	299,521,658	492.67
	0.3	1 2	0 21	2 0	0 0	21 0	273,516,652	480.33
	0.5	1 2	0 21	1 0	0 0	21 0	268,507,640	471.67
	0.7	1 2	0 2	22 0	1 18	0 0	276,495,629	466.67
	0.9	1 2	2 0	22 0	0 18	0 0	279,488,619	462.

Table 9. Marketing Decisions of Model-IIE Using Direct Fuzzy-objective Optimization and DBGA

Algo- rithm.	x	frequency (f)	fx	$\overline{x} = \frac{\sum fx}{\sum f}$	$f(x - \overline{x})^2$	$Var = \frac{\sum f(x-\overline{x})^2}{\sum f}$	$S.D. = \sqrt{\frac{\sum f(x-\overline{x})^2}{\sum f}}$	$S.E = \frac{S.D}{\sqrt{\sum f}}$
PSO	528 527 523	21 4 5	11088 2108 2615	527.03	19.7589 .0036 81.2045	3.36	1.83	.33
		$\sum f = 30$ $\sum f = 30$	$\sum fx$ $= 15811$		$\sum f(x - \overline{x})^2$ $= 100.96$			
DBGA	523 527 517 520	20 2 5 3	10460 1054 2585 1560	521.96	21.6319 50.803 123.08 11.52	6.90	2.62	.48
		$\sum f = 30$ $\sum f = 30$	$\sum fx$ $= 15659$		$\sum f(x - \overline{x})^2$ $= 207.035$			

Table 10. Calculation of Average Value, Standard Deviation, Variance, Sampling Error

The Models IID-1,IID-2,IIE-1 and IIE-2 are solved by DBGA. The optimum profits along with corresponding transported amounts are presented in Tables-4,-5,-6,-7,-8,-9.

8. Discussion

This paper presents fuzzy profit maximization FCSTPs under budget constraints at destinations with different fuzzy measures. Several examples are used for the illustration. Table-4, Tables-5,-6 and -7 furnish the results of crisp(Model-I) and fuzzy (Model-II) profit maximization FCSTP respectively following different approaches. These results are without and with budget constraints. Tables-8 and -9 give the optimum results of models-IID and IIE using different defuzzification techniques.

In all cases, it is observed from Tables-4,-5, -6 and -7 that profits for the models without budget constraint are more than those with budget constraints. To mention one case, in Table-4 (with DBGA), say the profit without budget constraint is 567 units which is more than the profit (523 units) with budget constraint. This is as per the usual expectation.

Moreover, three methods-DBGA, PSO and GRG techniques have been used to solve the maximization problems and in all cases, PSO furnishes better results than the corresponding ones obtained by GRG and DBGA. To cite a case, in Tables-5,-6, profit for the Model IIC-2 using PSO is 461.6 units which is more than the corresponding profit (445.6 units) using DBGA and (353.8 units) using GRG.

In Tables-5,-6 and -7, amongst all the models, IIC-1 and -2 models, give the highest and lowest profits respectively. This is because both fuzzy objective function and transportation constraints have been changed by the credibility measures (average of possibility and necessity measures) in the ranges (0.0-0.5) and (0.5-1.0)for IIC-1 and IIC-2 respectively.

Second highest and lowest profit values are from the Models IIB-1 and IIA-2 respectively. In IIB-1, fuzzy objective and transportation constraints are defuzzified in optimistic sense using possibility measure with α=0.9 and in IIA-2, expected value is used for objective and the fuzzy transportation constraints are replaced by necessity measure with α=0.1 The budget constraints does not have any effect on this nature as both Tables-5, -6, and -7, present the same trend/order. It can be noted that the same trend on the nature of profits is reflected in the case of the methods- PSO, DBGA and GRG.

From the Tables-5, -6,-7, -8 and -9, it is concluded that for the same objective, use of possibility measure for constraints gives more profit than the corresponding one using necessity measure. Again, the Model-IIE using direct fuzzy objective optimization gives slightly more profits than the corresponding results of Model-IID in all cases. This is because in direct optimization no approximation is made.

9. **Statistical Analysis for DBGA and PSO**

Average values(AVs), standard deviations (SDs), variances (Var) and corresponding sampling errors (SEs) have been calculated and presented in Table-10 for crisp problem which is discussed in section-7, using three methods (GRG, DBGA, PSO). In all cases the average results given by GRG, DBGA are less than the corresponding average result PSO. SDs and variance (Var) of PSO is less than DBGA. Its indicate that the method PSO is more stable than DBGA. Its also indicate that results in different run do not differ much from the mean. The sampling error (SEs) of PSO is less than the DBGA. Then the solutions of PSO is nearer to the best optimum solution than DBGA.

10. **Conclusion**

This paper investigates the profit maximization FCSTP under destination budget constraints using different fuzzy measures like possibility, necessity and credibility measures, in which the supplies, demands, capacities of conveyances, unit transportation costs, purchasing costs, selling prices and fixed charges are assumed to be fuzzy in nature. Till now, none has solved the general fuzzy profit maximization TP (two dimensional) or fuzzy profit maximization FCSTP with or without destination budget constraint. This paper presents solutions for the above problems. Here, possibility, necessity and credibility measures have been applied to the objective function and the inequality constraints.

This solution technique can be applied to solve the fuzzy profit maximization models in other areas such as inventory control, management, etc. under destination budget constraints. This problem can be extended to include the additional constraints such as restrictions on the demand for substitutable items and different types of discount like AUD, IQD or IQD in AUD on purchasing cost etc. The technique of formulating a TP/STP problem as a maximization problem can be extended to other types of transportation models

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