MULTI-ATTRIBUTE DECISION MAKING METHOD BASED ON BONFERRONI MEAN OPERATOR AND POSSIBILITY DEGREE OF INTERVAL TYPE-2 TRAPEZOIDAL FUZZY SETS

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Abstract. This paper proposes a new approach based on Bonferroni mean operator and possibility degree to solve fuzzy multi-attribute decision making (FMADM) problems in which the attribute value takes the form of interval type-2 fuzzy numbers. We introduce the concepts of interval possibility mean value and present a new method for calculating the possibility degree of two interval trapezoidal type-2 fuzzy sets (IT2 TrFSs). Then, we develop two aggregation techniques, which are called the interval type-2 trapezoidal fuzzy Bonferroni mean (IT2TFBM) operator and the interval type-2 trapezoidal fuzzy weighted Bonferroni mean (IT2TFWBM) operator. We study their properties and discuss their special cases. Based on the IT2TFWBM operator and the possibility degree, a new method of multi-attribute decision making with interval type-2 trapezoidal fuzzy information is proposed. Finally, an illustrative example is given to verify the developed approaches and to demonstrate their practicality and effectiveness.

1. Introduction

The purpose of multi-attribute group decision making (MAGDM) is to find a desirable solution from finite alternatives by a group of experts assessing on multiple attributes with different types of decision information, such as crisp numbers [13],interval values [3], linguistic scales [28], and fuzzy numbers [17]. Fuzzy Multi-attribute decision making (FMADM) problem is to find the most desirable alternative from a set of feasible alternatives, where the information provided by a group of decision makers is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human thinking. In recent years, some methods have been presented to deal with FMADM problems based on traditional type-1 fuzzy sets (T1 FSs). Chen [4] presented an extension of the TOPSIS method for FMADM problem. Chen [5] presented a method to evaluate the rate of aggregative risk in software development using fuzzy sets under the fuzzy group decision making environment. Li [18] presented a method for FMADM based on the particular measure of closeness to ideal solution which is developed from the fuzzy weighted Minkowski distance used as an aggregating function in a compromise programming method. Xu [37] established

Received: March 2014; Revised: December 2015; Accepted: June 2016

Key words and phrases: Multi-attributes group decision making, Interval type-2 fuzzy sets, Bonferroni mean operator, IT2TFWBM operator.

a practical interactive procedure for solving the FMADM problems, in which the information about attribute weights is partly known. Fan and Liu [14] presented a method for group decision making based on the multi-granularity uncertain linguistic information. Lin and Wu [19] presented a causal analytical method for group decision making in the fuzzy environment. Tsai and Wang [27] presented a method for computing coordination based fuzzy group decision making for web service. Wang and Lin [29] presented a method for FMAGDM to select configuration items for software development. Wu and Chen [30] presented a method for maximizing deviation for group multi-attributes decision-making in a linguistic environment. Li [20] presented some different distances measure and develop a method for solving FMAGDM problems with non-homogeneous information. However, the above FMAGDM methods are based on traditional T1 FSs.

The concept of type-2 fuzzy sets (T2 FSs), initially introduced by Zadeh [42], can be regarded as an extension of the concept of T1 FSs. The main difference between the two kinds of fuzzy sets is that the memberships of T1 FSs are crisp numbers whereas the memberships of T2 FSs are T1 FSs [31]; hence, T2 FSs involve more uncertainties than T1 FSs. Since its introduction, type-2 fuzzy sets are receiving more and more attention. Because the computational complexity of using general T2 FSs is very high, to date, interval type-2 fuzzy sets (IT2 FSs) [23] are the most widely used type-2 fuzzy sets and have been successfully applied to many practical fields [24, 25, 26, 32, 33, 34]. In particular, some authors have applied IT2 FSs theory to the field of FMADM. Wu and Mendel [26, 34] presented a method using the linguistic weighted average and IT2 FSs for handling fuzzy multiple criteria hierarchical group decision-making problems. Chen and Lee [6] presented a method for FMADM based on ranking values and the arithmetic operations of IT2 TrFSs. Chen and Lee [7] presented an interval type-2 fuzzy TOPSIS method to handle FMADM problems based on IT2 TrFSs. Wang and Liu [35] investigated the FMADM problems under IT2 fuzzy environment, and developed an approach to handling the situations where the attribute values are characterized by IT2 TrFSs, and the information about attribute weights is partially known. Chen and Yang [8] proposed a method for FMADM based on the ranking method of IT2 TrFSs. Zhang [43] proposed a novel approach to FMADM by using interval trapezoidal type-2 fuzzy soft sets. Chen and Chang [9] developed an extended QUALIFLEX method for handling MADM problems in the context of IT2 TrFSs and applications to medical decision making.

As two extensions of the arithmetic average (AA) and the geometric mean (GM), the Bonferroni mean (BM) and the geometric Bonferroni mean (GBM) are two very useful aggregation operators, which consider the interrelationships among arguments. The BM originally introduced by Bonferroni [1] and then generalized by Yager [41]. Due to its capability to capture the interrelationship between input arguments, BM is very useful in various application fields and has attracted a lot of attentions from researchers. Liu and Jin [21], Wei et al. [36] and Li et al. [22] proposed some fuzzy linguistic BM operators and applied to linguistic multi-attribute decision making. Xu and Yager [38] investigated the BM operators under intuitionistic fuzzy environment. Xia et al. [39] developed a GBM and applied it to multi-criteria decision making problems. Zhu et al. [44] and Xia et al. [40] proposed some intuitionistic and hesitant GBM operators and applied to fuzzy multi-attribute decision making. Gong et al. proposed [15] some interval type-2 fuzzy GBM operators and applied to interval type-2 fuzzy multi-attribute decision making.

Up to now, few studies have focused on interval type-2 fuzzy decision making problems consider the interrelationship between input interval type-2 fuzzy arguments. Therefore, it is necessary to pay attention this issue. The aim of this paper is to develop some approaches to interval type-2 fuzzy decision making problems consider the interrelationship between input interval type-2 fuzzy arguments. In order to do so, we further extend the BM operator to interval type-2 fuzzy environment. We first develop two aggregation techniques called the interval type-2 trapezoidal fuzzy Bonferroni mean (IT2TFBM) operator and the interval type-2 trapezoidal fuzzy weighted Bonferroni mean (IT2TFWBM) operator. We study its properties and discuss its special cases. Then, we present a new method to deal with FMADM problems based on the IT2TFWBM operator and the possibility degree of IT2 FSs. The remainder of this paper is organized as follows. In section 2, we give a review of basic concepts and operations related to IT2 TrFSs. In Section 3, we introduce the concepts of lower and upper possibility mean value of IT2 TrFSs. Then, we present a new method for calculating the possibility degree of two IT2 TrFSs based on the interval-valued possibility mean values. In section 4, the interval type-2 trapezoidal fuzzy Bonferroni mean (IT2TFBM) operator and the interval type-2 trapezoidal fuzzy weighted Bonferroni mean (IT2TFWBM) operator are developed, some desirable properties of these operators are studied and some special cases are discussed. Section 5 introduces a procedure for FMADM problem based on IT2TFWBM operator and the possibility degree of two IT2 TrFSs. Section 6 we use global supplier selection problem to illustrate the proposed method. The conclusions are discussed in Section 7.

2. The Basic Concepts and Arithmetic Operations of IT2 FSs

In this section, the basic concepts and arithmetic operations of IT2 FSs are introduced below to facilitate future discussions. Type-2 fuzzy sets, characterized by primary and secondary membership, are the extension of type-1 fuzzy sets.

Definition 2.1. [23] Let $\widetilde{A}$ be a type-2 fuzzy set, i.e.

$$\widetilde{A} = \{((x,u), \mu_{\widetilde{A}}(x,u)) | \forall x \in X, \forall u \in J_X \subseteq [0,1], 0 \leq \mu_{\widetilde{A}}(x,u) \leq 1\} \tag{1}$$

where X denotes the domain of $\widetilde{A}$ and $\mu_{\widetilde{A}}$ denotes the membership function of $\widetilde{A}$.

Definition 2.2. [23] For a type-2 fuzzy set $\widetilde{A}$, if all $\mu_{\widetilde{A}}(x,u) = 1$, then $\widetilde{A}$ is called an interval type-2 fuzzy set, i.e.

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_X} 1/(x,u) \tag{2}$$

where $J_X \subseteq [0,1]$.

In this paper, we present a method to use interval type-2 trapezoidal fuzzy number (IT2 TrFN) for handling FMADM problems, where the reference points and the heights of the upper and the lower membership functions of IT2 FSs are used to characterize IT2 FSs.

Definition 2.3. [6, 7, 8] Suppose that $\widetilde{A}_1$ and $\widetilde{A}_2$ are two non-negative IT2 TrFNs, where $\widetilde{A}_1 = (\widetilde{A}_1^U, \widetilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; h_1^U), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; h_1^L))$ and $\widetilde{A}_2 = (\widetilde{A}_2^U, \widetilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; h_2^U), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; h_2^L))$. The arithmetic operations between $\widetilde{A}_1$ and $\widetilde{A}_2$ are defined as follows:

(1)Addition operation

$$\widetilde{A}_1 \oplus \widetilde{A}_2 = ((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; min(h_1^U, h_2^U)), (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; min(h_1^L, h_2^L))) \tag{3}$$

(2) Multiplication operation

$$\widetilde{A}_1 \otimes \widetilde{A}_2 = ((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; min(h_1^U, h_2^U)), (a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L; min(h_1^L, h_2^L))) \tag{4}$$

(3) Multiplication by real number operation

$$k\widetilde{A}_1 = ((ka_{11}^U, ka_{12}^U, ka_{13}^U, ka_{14}^U; h_1^U), (ka_{11}^L, ka_{12}^L, ka_{13}^L, ka_{14}^L; h_1^L)) \tag{5}$$

(4) Power operation

$$(\widetilde{A}_1)^k = (((a_{11}^U)^k, (a_{12}^U)^k, (a_{13}^U)^k, (a_{14}^U)^k; h_1^U), ((a_{11}^L)^k, (a_{12}^L)^k, (a_{13}^L)^k, (a_{14}^L)^k; h_1^L)) \tag{6}$$

3. The Interval-valued Possibility Mean Value and Possibility Degree of IT2 TrFNs

In this section, we extended the concept of Carlsson and Fullér [11] about the possibilistic mean value of type-1 fuzzy numbers. We first introduce the lower and upper possibility mean value of IT2 TrFSs. If an IT2 TrFN $\tilde{A} = (\tilde{A}^U, \tilde{A}^L)$, the upper membership function$\tilde{A}^U$ and the lower membership function $\tilde{A}^L$ have pseudo level sets with $\tilde{A}^U_\alpha = [a_1^U(\alpha), a_2^U(\alpha)], \alpha \in [0, h_U]$ and $\tilde{A}^U_\beta = [a_1^U(\beta), a_2^U(\beta)], \beta \in [0, h_L]$, where h_L, h_U are the maximum membership function value of $\tilde{A}^U$ and $\tilde{A}^L$, then we present the following concepts:

Definition 3.1. The lower possibility mean value of an IT2 TrFN $\widetilde{A} = (\widetilde{A}^U, \widetilde{A}^L)$is defined as

$$M_*(\widetilde{A}) = \int_0^{h_U} \alpha a_1^U(\alpha) d\alpha + \int_0^{h_L} \beta a_1^L(\beta) d\beta \tag{7}$$

Obviously,$M_*(\widetilde{A})$is nothing else but the level-weight average of the arithmetic means of all pseudo level sets, that is , the weight of the arithmetic mean of $a_1^U(\alpha)$ and $a_1^U(\alpha)$. In a similar manner, we introduce the upper possibility mean value $M^*(\widetilde{A})$ as follows

Definition 3.2. The upper possibility mean value of an IT2 TrFN $\widetilde{A} = (\widetilde{A}^U, \widetilde{A}^L)$ is defined as

$$M^*(\widetilde{A}) = \int_0^{h_U} \alpha a_2^U(\alpha)d\alpha + \int_0^{h_L} \beta a_2^L(\beta)d\beta \tag{8}$$

Let us introduce the notation

$$M(\tilde{A}) = [M_*(\widetilde{A}), M^*(\widetilde{A})] \tag{9}$$

That is, $M(\tilde{A})$ is a closed interval bounded by the lower and upper possibility mean values of IT2 FSs $\tilde{A}$.

Definition 3.3. Let $M(\tilde{A}_1) = [M_*(\tilde{A}_1), M^*(\tilde{A}_1)]$ and $M(\tilde{A}_2) = [M_*(\tilde{A}_2), M^*(\tilde{A}_2)]$ be interval-valued possibility mean values of IT2 TrFNs $\tilde{A}_1$and$\tilde{A}_2$,respectively, then we define the possibility degree formula of IT2 TrFNs as follows:

$$p(\tilde{A}_1 \succ \tilde{A}_2) = min\{max(\frac{M^*(\tilde{A}_1) - M_*(\tilde{A}_2)}{M^*(\tilde{A}_1) - M_*(\tilde{A}_1) + M^*(\tilde{A}_2) - M_*(\tilde{A}_2)}, 0), 1\} \tag{10}$$

Theorem 3.4. *The possibility degree $p(\tilde{A}_1 \succ \tilde{A}_2)$ of IT2 TrFNs $\tilde{A}_1$ and $\tilde{A}_2$ has the following properties* [15]*:*

(1) $0 \leq p(\tilde{A}_1 \succ \tilde{A}_2) \leq 1$, $0 \leq p(\tilde{A}_2 \succ \tilde{A}_1) \leq 1$

(2) If $M^*(\tilde{A}_1) \leq M_*(\tilde{A}_2)$, *then* $p(\tilde{A}_1 \succ \tilde{A}_2) = 0$

(3) If $M^*(\tilde{A}_1) \geq M_*(\tilde{A}_2)$, *then* $p(\tilde{A}_1 \succ \tilde{A}_2) = 1$

(4) $p(\tilde{A}_1 \succ \tilde{A}_2) + p(\tilde{A}_2 \succ \tilde{A}_1) = 1$, *specially* $p(\tilde{A}_1 \succ \tilde{A}_1) = 0.5$

(5)For IT2 FSs $\tilde{A}_1, \tilde{A}_2$ *and* $\tilde{A}_3$, *if* $p(\tilde{A}_1 \succ \tilde{A}_2) \geq 0.5$ *and* $p(\tilde{A}_2 \succ \tilde{A}_3) \geq 0.5$ *then* $p(\tilde{A}_1 \succ \tilde{A}_2) + p(\tilde{A}_2 \succ \tilde{A}_3) \geq p(\tilde{A}_1 \succ \tilde{A}_3)$

Example 3.5. Two IT2 TrFNs $\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((0.1, 0.3, 0.6, 0.7; 1), (0.2, 0.3, 0.5, 0.6; 0.9))$ and $\tilde{A}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = ((0.2, 0.4, 0.6, 0.8; 1), (0.2, 0.3, 0.5, 0.8; 0.8))$. Then, the low and upper possibility mean value is computed respectively as follows:

$$\tilde{M}_*(\tilde{A}_1) = 0.237, \tilde{M}^*(\tilde{A}_1) = 0.557, \tilde{M}_*(\tilde{A}_2) = 0.273, \tilde{M}^*(\tilde{A}_2) = 0.573$$

From formula (10), the possibility degree of IT2 TrFNs $\tilde{A}_1$ and $\tilde{A}_2$ is

$$p(\tilde{A}_1 \succ \tilde{A}_2) = min\{max(\frac{0.557 - 0.273}{0.557 - 0.237 + 0.573 - 0.273}, 0), 1\} = 0.458$$

Let $\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L)$ be an IT2 TrFN, the fuzzy preference matrix P can be obtained, shown as follows:

$$P = \begin{bmatrix} p(\tilde{A}_1 \succ \tilde{A}_1) & p(\tilde{A}_1 \succ \tilde{A}_2) & \dots & p(\tilde{A}_1 \succ \tilde{A}_n) \\ p(\tilde{A}_2 \succ \tilde{A}_1) & p(\tilde{A}_2 \succ \tilde{A}_2) & \dots & p(\tilde{A}_2 \succ \tilde{A}_n) \\ \vdots & \vdots & \vdots & \vdots \\ p(\tilde{A}_n \succ \tilde{A}_1) & p(\tilde{A}_n \succ \tilde{A}_2) & \dots & p(\tilde{A}_n \succ \tilde{A}_n) \end{bmatrix} \tag{11}$$

Then, the ranking value of IT2 TrFNs $Rank(\tilde{A}_i)$ is calculated as follows [36]:

$$Rank(\tilde{A}_i) = \frac{1}{n(n-1)}(\sum_{k=1}^{n} p(\tilde{A}_i \succ \tilde{A}_k) + \frac{n}{2} - 1) \tag{12}$$

Where $1 \leq i \leq n$ and $\sum_{i=1}^{n} Rank(\tilde{A}_i) = 1$ .The larger ranking value $Rank(\tilde{A}_i)$, the greater the IT2 FSs $\tilde{A}_i$.

Using the thirteen sets of interval type-2 trapezoidal fuzzy numbers provided by Bortolan and Degani [2], we computed the possibility degrees to compare those fuzzy numbers based on different definitions proposed by Chen et al. [6], Hu et al. [16], and equtaion (10) developed by us respectively. The calculated results are shown in Table1.

Sets of fuzzy number		Chen et al. method	Hu et al. method	The proposed method
Set1	A_1	0.520	0.423	0.250
	A_2	0.480	0.576	0.750
Set2	A_1	0.400	0.250	0.380
	A_2	0.600	0.750	0.620
Set3	A_1	0.360	0.375	0.250
	A_2	0.640	0.625	0.750
Set4	A_1	0.390	0.431	0.475
	A_2	0.330	0.292	0.305
	A_3	0.280	0.277	0.220
Set5	A_1	0.400	0.487	0.390
	A_2	0.320	0.333	0.340
	A_3	0.280	0.180	0.270
Set6	A_1	0.390	0.487	0.400
	A_2	0.340	0.333	0.320
	A_3	0.270	0.180	0.280
Set7	A_1	0.500	0.500	0.500
	A_2	0.500	0.500	0.500
Set8	A_1	0.280	0.294	0.290
	A_2	0.350	0.337	0.320
	A_3	0.370	0.369	0.390
Set9	A_1	0.280	0	0.250
	A_2	0.720	1	0.750
Set10	A_1	0.490	0.590	0.750
	A_2	0.510	0.410	0.250
Set11	A_1	0.250	0	0.250
	A_2	0.750	1	0.750
Set12	A_1	0.630	0.750	0.750
	A_2	0.370	0.250	0.250
Set13	A_1	0.630	0.820	0.750
	A_2	0.370	0.180	0.250

Table 1. A Comparison of the Ranking Results for Different Methods

From Table1, we can see the drawbacks of the existing ranking methods, described as below:

(1) From Set 1 of Table1, we can see that Hu et al.'s method and the proposed method get the same ranking order: "$\tilde{A}_1 \prec \tilde{A}_2$", while Chen and Lee's method failed to produce a correct order. The reason is that Chen and Lee's method, overall, has enhanced the influence of the value of right branch.

(2) From Set 10, the result from Chen and Lee's method is inconsistent with other approaches, because the membership effect of the interval type-2 fuzzy number has been enlarged.

(3) From Set 9 and Set 11, we can see that Hu et al.'s method and other methods get the same ranking order: "$\tilde{A}_1 \prec \tilde{A}_2$". However, Hu et al.'s method is too simple discrimination for fuzzy uncrossed numbers.

4. The Interval Type-2 Trapezoidal Fuzzy Bonferroni Mean Operators

Bonferroni [1] originally introduced a mean type aggregation operator, called Bonferroni mean, which can provide for aggregation lying between the max, min operators and the logical "or" and "and" operators, which was defined as follows:

Definition 4.1. [1] Let $p, q \geq 0$, and $a_i > 0 (i = 1, 2, \ldots, n)$ be a collection of nonnegative numbers. The aggregation functions:

$$BM^{p,q}(a_1, a_2, \ldots, a_n) = (\frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^{n} a_i^p a_j^q)^{1/p+q} \tag{13}$$

is called the Bonferroni mean (BM) operator.

The BM operator, however, has usually been used in situations where the input arguments are the non-negative real numbers. We shall extend the BM operators to accommodate the situations where the input arguments are interval type-2 fuzzy variables. In this section, we shall investigate the BM operator under interval type-2 trapezoidal fuzzy environments. Based on Definition 4.1, we give the definition of the interval type-2 trapezoidal fuzzy Bonferroni mean (IT2TFBM) operator as follows:

Definition 4.2. Let $\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; h_i^U), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; h_i^L))$ $(i = 1, 2, \ldots n)$ be a collection of the IT2 trapezoidal fuzzy variables, and $p, q \geq 0$, then, we call:

$$IT2TFBM^{p,q}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = (\frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^{n} \tilde{A}_i^p \otimes \tilde{A}_j^q)^{1/p+q} \tag{14}$$

an IT2 Trapezoidal Fuzzy Bonferroni Mean (IT2TFBM) operator.

According to the operations of the IT2 trapezoidal fuzzy variables, we can get the following result.

Theorem 4.3. *Let $\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; h_i^U), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; h_i^L))$ $(i = 1, 2, \ldots n)$ be a collection of the IT2 trapezoidal fuzzy variables, and $p, q \geq 0$, then, the aggregated result by Equtaion (14) is also an IT2 trapezoidal fuzzy variable, and*

$$IT2TFBM^{p,q}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \tilde{A} = (\tilde{A}^U, \tilde{A}^L) \tag{15}$$

where

$$\tilde{A}^U = ((\frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^{n} (a_{i1}^U)^p (a_{j1}^U)^q)^{1/p+q}, (\frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^{n} (a_{i2}^U)^p (a_{j2}^U)^q)^{1/p+q},$$

$$(\frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^{n} (a_{i3}^U)^p (a_{j3}^U)^q)^{1/p+q}, (\frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^{n} (a_{i4}^U)^p (a_{j4}^U)^q)^{1/p+q};$$

$$\min_{1\leq i\leq n}\{h_i^U\}) \tag{16}$$

and

$$\tilde{A}^L = ((\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(a_{i1}^L)^p(a_{j1}^L)^q)^{1/p+q}, (\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(a_{i2}^L)^p(a_{j2}^L)^q)^{1/p+q},$$
$$(\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(a_{i3}^L)^p(a_{j3}^L)^q)^{1/p+q}, (\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(a_{i4}^L)^p(a_{j4}^L)^q)^{1/p+q};$$
$$\min_{1\leq i\leq n}\{h_i^L\})) \tag{17}$$

We use mathematical induction to prove this theorem as follows.

Proof. (1) Firstly, we need to prove that:

$$\sum_{i,j=1,i\neq j}^{n}\tilde{A}_i^p\otimes\tilde{A}_j^q = ((\sum_{i,j=1,i\neq j}^{n}(a_{i1}^U)^p(a_{j1}^U)^q, \sum_{i,j=1,i\neq j}^{n}(a_{i2}^U)^p(a_{j2}^U)^q, \sum_{i,j=1,i\neq j}^{n}(a_{i3}^U)^p(a_{j3}^U)^q$$
$$\sum_{i,j=1,i\neq j}^{n}(a_{i4}^U)^p(a_{j4}^U)^q; \min_{1\leq i\leq n}\{h_i^U\})), (\sum_{i,j=1,i\neq j}^{n}(a_{i1}^L)^p(a_{j1}^L)^q, \sum_{i,j=1,i\neq j}^{n}(a_{i2}^L)^p(a_{j2}^L)^q,$$
$$\sum_{i,j=1,i\neq j}^{n}(a_{i3}^L)^p(a_{j3}^L)^q, \sum_{i,j=1,i\neq j}^{n}(a_{i4}^L)^p(a_{j4}^L)^q; \min_{1\leq i\leq n}\{h_i^L\}))) \tag{18}$$

By the operations equtaions (3)-(6) of the IT2 trapezoidal fuzzy variables, we have

$$\tilde{A}_i^p\otimes\tilde{A}_j^q = (((a_{i1}^U)^p(a_{j1}^U)^q, (a_{i2}^U)^p(a_{j2}^U)^q, (a_{i3}^U)^p(a_{j3}^U)^q, (a_{i4}^U)^p(a_{j4}^U)^q; min(h_i^U, h_j^U)),$$
$$((a_{i1}^L)^p(a_{j1}^L)^q, (a_{i2}^L)^p(a_{j2}^L)^q, (a_{i3}^L)^p(a_{j3}^L)^q, (a_{i4}^L)^p(a_{j4}^L)^q; min(h_i^L, h_j^L))) \tag{19}$$

(a) When $n = 2$, we can get

$$\sum_{i,j=1,i\neq j}^{2}\tilde{A}_i^p\otimes\tilde{A}_j^q = ((\sum_{i,j=1,i\neq j}^{2}(a_{i1}^U)^p(a_{j1}^U)^q, \sum_{i,j=1,i\neq j}^{2}(a_{i2}^U)^p(a_{j2}^U)^q, \sum_{i,j=1,i\neq j}^{2}(a_{i3}^U)^p(a_{j3}^U)^q$$
$$\sum_{i,j=1,i\neq j}^{2}(a_{i4}^U)^p(a_{j4}^U)^q; min(h_1^U, h_2^U)), (\sum_{i,j=1,i\neq j}^{2}(a_{i1}^L)^p(a_{j1}^L)^q, \sum_{i,j=1,i\neq j}^{2}(a_{i2}^L)^p(a_{j2}^L)^q,$$
$$\sum_{i,j=1,i\neq j}^{2}(a_{i3}^L)^p(a_{j3}^L)^q, \sum_{i,j=1,i\neq j}^{2}(a_{i4}^L)^p(a_{j4}^L)^q; min(h_1^L, h_2^L)))$$

So, when $n = 2$, equation (18), is right.

(b) Suppose when $n = k$, equation (18) is right, i.e.

$$\sum_{i,j=1,i\neq j}^{k}\tilde{A}_i^p\otimes\tilde{A}_j^q = ((\sum_{i,j=1,i\neq j}^{k}(a_{i1}^U)^p(a_{j1}^U)^q, \sum_{i,j=1,i\neq j}^{k}(a_{i2}^U)^p(a_{j2}^U)^q, \sum_{i,j=1,i\neq j}^{k}(a_{i3}^U)^p(a_{j3}^U)^q$$
$$\sum_{i,j=1,i\neq j}^{k}(a_{i4}^U)^p(a_{j4}^U)^q; \min_{1\leq i\leq k}\{h_i^U\})), (\sum_{i,j=1,i\neq j}^{k}(a_{i1}^L)^p(a_{j1}^L)^q, \sum_{i,j=1,i\neq j}^{k}(a_{i2}^L)^p(a_{j2}^L)^q,$$

$$\sum_{i,j=1,i\neq j}^{k}(a_{i3}^{L})^{p}(a_{j3}^{L})^{q},\ \sum_{i,j=1,i\neq j}^{k}(a_{i4}^{L})^{p}(a_{j4}^{L})^{q};\ \min_{1\leq i\leq k}\{h_{i}^{L}\}))) \tag{20}$$

then, when $n = k + 1$, we have

$$\sum_{i,j=1,i\neq j}^{k+1}\tilde{A}_{i}^{p}\otimes\tilde{A}_{j}^{q}=\sum_{i,j=1,i\neq j}^{k}\tilde{A}_{i}^{p}\otimes\tilde{A}_{i}^{q}+\sum_{i,j=1,i\neq j}^{k}\tilde{A}_{i}^{p}\otimes\tilde{A}_{k+1}^{q}+\sum_{i,j=1,i\neq j}^{k}\tilde{A}_{k+1}^{p}\otimes\tilde{A}_{i}^{q} \tag{21}$$

By the operations equtaions (3)-(6) of the IT2 trapezoidal fuzzy variables, we can get

$$\sum_{i=1}^{k}\tilde{A}_{i}^{p}\otimes\tilde{A}_{k+1}^{q}=((\sum_{i=1}^{k}(a_{i1}^{U})^{p}(a_{k+1,1}^{U})^{q},\sum_{i=1}^{k}(a_{i2}^{U})^{p}(a_{k+1,2}^{U})^{q},\sum_{i=1}^{k}(a_{i3}^{U})^{p}(a_{k+1,3}^{U})^{q}$$
$$\sum_{i=1}^{k}(a_{i4}^{U})^{p}(a_{k+1,4}^{U})^{q};\min_{1\leq i\leq k}\{h_{i}^{U}\},h_{k+1}^{U})),(\sum_{i=1}^{k}(a_{i1}^{L})^{p}(a_{k+1,1}^{L})^{q},\sum_{i=1}^{k}(a_{i2}^{L})^{p}(a_{k+1,2}^{L})^{q},$$
$$\sum_{i=1}^{k}(a_{i3}^{L})^{p}(a_{k+1,3}^{L})^{q},\sum_{i=1}^{k}(a_{i4}^{L})^{p}(a_{k+1,4}^{L})^{q};\min_{1\leq i\leq k}\{h_{i}^{L}\},h_{k+1}^{L}))) \tag{22}$$

and

$$\sum_{j=1}^{k}\tilde{A}_{k+1}^{p}\otimes\tilde{A}_{j}^{q}=((\sum_{j=1}^{k}(a_{j1}^{U})^{q}(a_{k+1,1}^{U})^{p},\sum_{j=1}^{k}(a_{j2}^{U})^{q}(a_{k+1,2}^{U})^{p},\sum_{j=1}^{k}(a_{j3}^{U})^{q}(a_{k+1,3}^{U})^{p}$$
$$\sum_{j=1}^{k}(a_{j4}^{U})^{q}(a_{k+1,4}^{U})^{p};\min_{1\leq j\leq k}\{h_{j}^{U}\},h_{k+1}^{U})),(\sum_{j=1}^{k}(a_{j1}^{L})^{q}(a_{k+1,1}^{L})^{p},\sum_{j=1}^{k}(a_{j2}^{L})^{q}(a_{k+1,2}^{L})^{p},$$
$$\sum_{j=1}^{k}(a_{j3}^{L})^{q}(a_{k+1,3}^{L})^{p},\sum_{j=1}^{k}(a_{j4}^{L})^{q}(a_{k+1,4}^{L})^{p};\min_{1\leq j\leq k}\{h_{j}^{L}\},h_{k+1}^{L}))) \tag{23}$$

So, by the equtaions (19)-(23), we get

$$\sum_{i,j=1,i\neq j}^{k+1}\tilde{A}_{i}^{p}\otimes\tilde{A}_{j}^{q}=((\sum_{i,j=1,i\neq j}^{k+1}(a_{i1}^{U})^{p}(a_{j1}^{U})^{q},\sum_{i,j=1,i\neq j}^{k+1}(a_{i2}^{U})^{p}(a_{j2}^{U})^{q},\sum_{i,j=1,i\neq j}^{k}(a_{i3}^{U})^{p}(a_{j3}^{U})^{q}$$
$$\sum_{i,j=1,i\neq j}^{k+1}(a_{i4}^{U})^{p}(a_{j4}^{U})^{q};\min_{1\leq i\leq k+1}\{h_{i}^{U}\})),(\sum_{i,j=1,i\neq j}^{k+1}(a_{i1}^{L})^{p}(a_{j1}^{L})^{q},\sum_{i,j=1,i\neq j}^{k+1}(a_{i2}^{L})^{p}(a_{j2}^{L})^{q},$$
$$\sum_{i,j=1,i\neq j}^{k+1}(a_{i3}^{L})^{p}(a_{j3}^{L})^{q},\sum_{i,j=1,i\neq j}^{k+1}(a_{i4}^{L})^{p}(a_{j4}^{L})^{q};\min_{1\leq i\leq k+1}\{h_{i}^{L}\})))$$

Thus, when $n = k + 1$, equtaion (18) is right. So, equtaion (18) is right for all n.

(2) Then, we can prove equtaion (15) is right. By the operations of the IT2 trapezoidal fuzzy variables defined in equtaions (3)-(6) and equtaion (18) , we can get

$$IT2TFBM^{p,q}(\tilde{A}_{1},\tilde{A}_{2},\ldots,\tilde{A}_{n})=(\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}\tilde{A}_{i}^{p}\otimes\tilde{A}_{j}^{q})^{1/p+q}=\tilde{A}=(\tilde{A}^{U},\tilde{A}^{L})$$

is right for all n. □

It can be easily proved that the IT2TFBM operator has the following properties.

Corollary 4.4. *(Idempotency)Let* $\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=((a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;h_i^U),(a_{i1}^L,a_{i2}^L,a_{i3}^L,a_{i4}^L;h_i^L))(i=1,2,\dots n)$ *be a set of the IT2 trapezoidal fuzzy variables. If all* $\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)$ *are equal, i.e.*$\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=\tilde{A}_0=(\tilde{A}_0^U,\tilde{A}_0^L)$ *for all i, then*

$$IT2TFBM^{p,q}(\tilde{A}_1,\tilde{A}_2,\dots,\tilde{A}_n)=\tilde{A}_0=(\tilde{A}_0^U,\tilde{A}_0^L) \tag{24}$$

Corollary 4.5. *(Boundedness)Let* $\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=((a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;h_i^U),(a_{i1}^L,a_{i2}^L,a_{i3}^L,a_{i4}^L;h_i^L))(i=1,2,\dots n)$ *be a set of the IT2 trapezoidal fuzzy variables, and let*

$$\tilde{A}_-=(\tilde{A}_-^U,\tilde{A}_-^L)=((\min_i\{a_{i1}^U\},\min_i\{a_{i2}^U\},\min_i\{a_{i3}^U\},\min_i\{a_{i4}^U\};\min_i\{h_i^U\}),$$
$$(\min_i\{a_{i1}^L\},\min_i\{a_{i2}^L\},\min_i\{a_{i3}^L\},\min_i\{a_{i4}^L\};\min_i\{h_i^L\}))$$
$$\tilde{A}_+=(\tilde{A}_+^U,\tilde{A}_+^L)=((\max_i\{a_{i1}^U\},\max_i\{a_{i2}^U\},\max_i\{a_{i3}^U\},\max_i\{a_{i4}^U\};\max_i\{h_i^U\}),$$
$$(\max_i\{a_{i1}^L\},\max_i\{a_{i2}^L\},\max_i\{a_{i3}^L\},\max_i\{a_{i4}^L\};\max_i\{h_i^L\}))$$

Then

$$\tilde{A}_-\le IT2TFBM^{p,q}(\tilde{A}_1,\tilde{A}_2,\dots,\tilde{A}_n)\le\tilde{A}_+ \tag{25}$$

Corollary 4.6. *(Monotonicity)Let* $\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=((a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;h_i^U),(a_{i1}^L,a_{i2}^L,a_{i3}^L,a_{i4}^L;h_i^L))$ *and* $\tilde{B}_i=(\tilde{B}_i^U,\tilde{B}_i^L)=((b_{i1}^U,b_{i2}^U,b_{i3}^U,b_{i4}^U;h_i^U),(b_{i1}^L,b_{i2}^L,b_{i3}^L,b_{i4}^L;h_i^L))(i=1,2,\dots n)$ *be two set of the IT2 trapezoidal fuzzy variables, if* $a_{ik}^U\le b_{ik}^U$ *and* $a_{ik}^{LU}\le b_{ik}^L(k=1,2,3,4)$ *for all i, then*

$$IT2TFBM^{p,q}(\tilde{A}_1,\tilde{A}_2,\dots,\tilde{A}_n)\le IT2TFBM^{p,q}(\tilde{B}_1,\tilde{B}_2,\dots,\tilde{B}_n) \tag{26}$$

Corollary 4.7. *(Commutativity)Let* $\tilde{A}_i'=(\tilde{A}_i'^U,\tilde{A}_i^L)=((a_{i1}'^U,a_{i2}'^U,a_{i3}'^U,a_{i4}'^U;h_i'^U),(a_{i1}'^L,a_{i2}'^L,a_{i3}'^L,a_{i4}'^L;h_i'^L))$ *is any permutation of the IT2 trapezoidal fuzzy variables* $\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=((a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;h_i^U),(a_{i1}^L,a_{i2}^L,a_{i3}^L,a_{i4}^L;h_i^L))(i=1,2,\dots,n)$ *then*

$$IT2TFBM^{p,q}(\tilde{A}_1,\tilde{A}_2,\dots,\tilde{A}_n)=IT2TFBM^{p,q}(\tilde{A}_1',\tilde{A}_2',\dots,\tilde{A}_n') \tag{27}$$

By assigning different values to the parameters p and q, some special cases of the IT2TFBM can be obtained as follows:

Case1: If $q=0$, then the equtaion (15) reduces to an IT2 trapezoidal fuzzy generalized mean operator. It follows that:

$$IT2TFBM^{p,q}(\tilde{A}_1,\tilde{A}_2,\dots,\tilde{A}_n)=(((\frac{1}{n}\sum_{i=1}^n(a_{i1}^U)^p)^{1/p},(\frac{1}{n}\sum_{i=1}^n(a_{i2}^U)^p)^{1/p},(\frac{1}{n}\sum_{i=1}^n(a_{i3}^U)^p)^{1/p},$$
$$(\frac{1}{n}\sum_{i=1}^n(a_{i4}^U)^p)^{1/p};\min_{1\le i\le n}\{h_i^U\}),((\frac{1}{n}\sum_{i=1}^n(a_{i1}^L)^p)^{1/p},((\frac{1}{n}\sum_{i=1}^n(a_{i2}^L)^p)^{1/p},(\frac{1}{n}\sum_{i=1}^n(a_{i3}^L)^p)^{1/p},$$
$$(\frac{1}{n}\sum_{i=1}^n(a_{i4}^L)^p)^{1/p};\min_{1\le i\le n}\{h_i^L\}))) \tag{28}$$

Case2: If $p=1$ and $q=0$, then the equtaion (15) reduces to an IT2 trapezoidal fuzzy average operator. It follows that:

$$IT2TFBM^{1,0}(\tilde{A}_1,\tilde{A}_2,\ldots,\tilde{A}_n)=((\frac{1}{n}\sum_{i=1}^{n}a_{i1}^U,\frac{1}{n}\sum_{i=1}^{n}a_{i2}^U,\frac{1}{n}\sum_{i=1}^{n}a_{i3}^U,\frac{1}{n}\sum_{i=1}^{n}a_{i4}^U;$$

$$\min_{1\le i\le n}\{h_i^U\}),(\frac{1}{n}\sum_{i=1}^{n}a_{i1}^L,\frac{1}{n}\sum_{i=1}^{n}a_{i2}^L,\frac{1}{n}\sum_{i=1}^{n}a_{i3}^L,\frac{1}{n}\sum_{i=1}^{n}a_{i4}^L;\min_{1\le i\le n}\{h_i^L\})) \quad (29)$$

Case3: If $p=1$ and $q=1$, then the equtaion (15) reduces to an IT2 trapezoidal fuzzy square mean operator. It follows that:

$$IT2TFBM^{1,1}(\tilde{A}_1,\tilde{A}_2,\ldots,\tilde{A}_n)=(((\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}a_{i1}^Ua_{j1}^U)^{1/2},(\frac{1}{n(n-1)}$$

$$\sum_{i,j=1,i\ne j}^{n}a_{i2}^Ua_{j2}^U)^{1/2},(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}a_{i3}^Ua_{j3}^U)^{1/2},(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}a_{i4}^Ua_{j4}^U)^{1/2};$$

$$\min_{1\le i\le n}\{h_i^U\}),((\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}a_{i1}^La_{j1}^L)^{1/2},(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}a_{i2}^Ua_{j2}^U)^{1/2},$$

$$(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}a_{i3}^La_{j3}^L)^{1/2},(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}a_{i4}^La_{j4}^L)^{1/2};\min_{1\le i\le n}\{h_i^L\})) \quad (30)$$

Considering that the input arguments may have different importance, here we define the IT2 Trapezoidal Fuzzy Weighted Bonferroni Mean (IT2TFWBM) operator.

Definition 4.8. Let $\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=((a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;h_i^U),(a_{i1}^L,a_{i2}^L,a_{i3}^L,a_{i4}^L;h_i^L))$ $(i=1,2,\ldots n)$ be a collection of the IT2 trapezoidal fuzzy variables, and $p,q\ge 0, w=(w_1,w_2,\ldots,w_n)$ is the weight vector of $\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)$, where w_i satisfying $w_i\ge 0$ and $\sum_{i=1}^n w_i=1$ then, we call:

$$IT2TFWBM^{p,q}(\tilde{A}_1,\tilde{A}_2,\ldots,\tilde{A}_n)=(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}(w_i\tilde{A}_i)^p\otimes(w_j\tilde{A}_j)^q) \quad (31)$$

an IT2 Trapezoidal Fuzzy Weighted Bonferroni Mean (IT2TFWBM) operator.

Similar to Theorem4.3, we can get

Theorem 4.9. *Let* $p,q\ge 0,\tilde{A}_i=(\tilde{A}_i^U,\tilde{A}_i^L)=((a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;h_i^U),(a_{i1}^L,a_{i2}^L,a_{i3}^L,$ $a_{i4}^L;h_i^L))(i=1,2,\ldots n)$ *be a collection of the IT2 trapezoidal fuzzy variables,, whose weight vector is* $w=(w_1,w_2,\ldots,w_n)$*,which satisfies* $w_i\ge 0$ *and* $\sum_{i=1}^n w_i=1$. *Then,the aggregated result by Eq. (31) is also an IT2 trapezoidal fuzzy variable, and*

$$IT2TFWBM^{p,q}(\tilde{A}_1,\tilde{A}_2,\ldots,\tilde{A}_n)=\tilde{A}_w=(\tilde{A}_w^U,\tilde{A}_w^L) \quad (32)$$

where

$$\tilde{A}_w^U=((\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}(w_ia_{i1}^U)^p(w_ja_{j1}^U)^q)^{1/p+q},(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}(w_ia_{i2}^U)^p$$

$$(w_ja_{j2}^U)^q)^{1/p+q},(\frac{1}{n(n-1)}\sum_{i,j=1,i\ne j}^{n}(w_ia_{i3}^U)^p(w_ja_{j3}^U)^q)^{1/p+q},$$

$$(\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(w_i a_{i4}^U)^p(w_j a_{j4}^U)^q)^{1/p+q};\min_{1\leq i\leq n}\{h_i^U\}) \tag{33}$$

and

$$\tilde{A}_w^L=((\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(w_i a_{i1}^L)^p(w_j a_{j1}^L)^q)^{1/p+q},(\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(w_i a_{i2}^L)^p (w_j a_{j2}^L)^q)^{1/p+q},(\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(w_i a_{i3}^L)^p(w_j a_{j3}^L)^q)^{1/p+q}, (\frac{1}{n(n-1)}\sum_{i,j=1,i\neq j}^{n}(w_i a_{i4}^L)^p(w_j a_{j4}^L)^q)^{1/p+q};\min_{1\leq i\leq n}\{h_i^L\})) \tag{34}$$

Similarly to IT2TFBM operator, the IT2TFWBM operator owns idempotency, boundedness, monotonicity and commutativity.

5. A New Method for Fuzzy Multi-attributes Group Decision Making Under Interval Type-2 Fuzzy Environment

In this section, we present a new method for handling FMADM problems under interval type-2 fuzzy environment. Assume that there is a set X of alternatives and a set F of attributes, where $X=(x_1,x_2,\ldots,x_n)$ and $F=(f_1,f_2,,f_m)$. Assume that there are l decision-makers $D_1,D_2,\ldots,D_l$. Let $\tilde{R}^{(k)}=(\tilde{A}_{ij}^{(k)})_{n\times m}$ be an IT2 fuzzy decision matrix, where $\tilde{A}_{ij}^{(k)}$ is an IT2 FSs, provided by the DM D_k for the alternative x_i with respect to the attribute f_j.

In general, attributes can be classified into two types: benefit attributes and cost attributes. In other words, the set F of attributes can be divided into two sets F_1 and F_2, where F_1 denotes the set of benefit attributes, F_2 denotes the set of cost attributes, $F_1\cap F_2=\varnothing$, and $F_1\cup F_2=F$. The decision matrices $\tilde{R}^{(k)}$ needs to be normalized unless all the attributes $f_j(j=1,2,\ldots,m)$ are of the same type. In this paper, we choose the following normalized formula to update the decision matrices $\tilde{R}^{(k)}$:

$$\tilde{A}_{ij}^{(k)}=\begin{cases}\tilde{A}_{ij}^{(k)}, & j\in F_1\\ (\tilde{A}_{ij}^{(k)})^c, & j\in F_1\end{cases} \tag{35}$$

where$\tilde{A}_{ij}^{(k)})^c$ is the complement of $\tilde{A}_{ij}^{(k)}$. Hence, we obtain the normalized decision matrices $\tilde{R}^k=(\tilde{A}_{ij}^{(k)})_{n\times m}$

In the process of group decision making, we need to fuse all the individual decision opinion into a group opinion so as to make a final decision. We utilize the equtaion (6) to aggregate all individual normalized decision matrices $\tilde{R}^k=(\tilde{A}_{ij}^{(k)})_{n\times m}$ into the collective normalized decision matrix $\tilde{R}=(\tilde{A}_{ij})_{n\times m}$, where $\tilde{A}_{ij}=\sum_{k=1}^{l}\lambda_k\tilde{A}_{ij}^k$.

Suppose the information about attribute weights is completely known, that is, the weight vector $w=(w_1,w_2,\ldots,w_m)$ of the attributes $f_j(j=1,2,\ldots,m)$ can be completely determined in advance. Then, we utilize the IT2 trapezoidal fuzzy weighted Bonferroni mean (IT2TFWBM) operator to develop an approach to FMADM problems with interval IT2 fuzzy information, which can be described as following:

Step1: Utilize the normalized decision matrix $\tilde{R} = (\tilde{A}_{ij})_{n\times m}$ and the weight vector $w = (w_1, w_2, \dots, w_m)$, the IT2TFWBM operator are shown as follows:

$$\tilde{d}_k = IT2TFWBM_w^{p,q}(\tilde{A}_{k,1}, \tilde{A}_{k,2}, \dots, \tilde{A}_{k,m}) = (\frac{1}{m(m-1)} \sum_{i,j=1, i\neq j} (w_i\tilde{A}_{ki})^p \otimes (w_j\tilde{A}_{kj})^q)^{1/p+q} \quad (36)$$

where $k = (1, 2, , n)$, in general, we can take $p = q = 1$.

Step2: Utilize fuzzy possibility degree equation (10) to calculate the fuzzy preference matrix $P = (p_{ij})_{n\times n}$.

Step3: Utilize the ranking formula (12) to calculate the ranking value $Rank(\tilde{d}_k)$ of the IT2 FSs $\tilde{d}_k$,where $1 \leq k \leq n$. The large the value of $Rank(\tilde{d}_k)$, the more the preference of the alternative $x_k, 1 \leq k \leq n$.

6. **Numerical Example**

In this section, we use an example to illustrate the FMADM process of the proposed method. Table2 shows the linguistic terms “Very Low” (VL), “Low”(L), “Medium Low” (ML), “Medium”(M), “Medium High”(MH), “High” (H), “Very High”(VH) and their corresponding interval type-2 fuzzy sets, respectively [6, 8].

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	((0,0,0,0.1;1),(0,0,0,0.05;0.9))
Low (L)	((0,0.1,0.1,0.3;1),(0.05,0.1,0.1,0.2;0.9))
Medium Low (ML)	((0.1,0.3,0.3,0.5;1),(0.2,0.3,0.3,0.4;0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1),(0.4,0.5,0.5,0.6;0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1),(0.6,0.7,0.7,0.8;0.9))
High (H)	((0.7,0.9,0.9,1;1),(0.8,0.9,0.9,0.95;0.9))
Very High (VH)	((0.9,1,1,1;1),(0.95,1,1,1;0.9))

Table 2. Linguistic Terms and Their Corresponding Interval Type-2 Fuzzy Sets

Assume that the problem discussed here is concerned with a manufacturing company, searching the best global supplier for one of its most critical parts used in assembling process [35]. There are three potential global suppliers x_1, x_2 and x_3 to be evaluated with four attributes (f_1: quality of the product, f_2: risk factor, f_3: service performance of supplier, f_4: supplier's profile (whose weight vector $w = (0.30, 0.15, 0.20, 0.35)$). An expert group is formed which consists of three experts D_1, D_2 and D_3 (whose weight vector is $\lambda = (0.30, 0.45, 0.25)$ from each strategic decision area. The experts D_1, D_2 and D_3 use the linguistic terms shown in Table 1 to represent the characteristics of the potential global suppliers x_1, x_2 and x_3 with respect to different attributes $f_i (i = 1, 2, 3, 4)$, respectively, listed in Table3:

Attribute	Alternatives	Decision makers D_1	D_2	D_3
Quality of the product (f_1)	x_1	MH	H	MH
	x_2	H	MH	H
	x_3	VH	H	MH
Risk facto (f_2)	x_1	H	VH	H
	x_2	MH	H	VH
	x_3	VH	VH	H
Service performance of supplier (f_3)	x_1	VH	H	H
	x_2	H	VH	VH
	x_3	M	MH	MH
Supplier's profile (f_4)	x_1	VH	H	H
	x_2	H	VH	H
	x_3	H	VH	VH

Table 3. Evaluating Values of Alternatives of the Decision-makers with Respect to Different Attributes

Considering that the attributes are the benefit attributes except to the attribute f_2 (risk factor), then based on the formula (35) and Table 3, the decision matrices $\tilde{R}^k = (\tilde{A}^k_{ij})_{3\times4}(k = 1, 2, 3)$ can be updated to the following normalized matrices respectively, listed in Table 4:

Attribute	Alternatives	Decision makers D_1	D_2	D_3
Quality of the product (f_1)	x_1	MH	H	MH
	x_2	H	MH	H
	x_3	VH	H	MH
Risk facto (f_2)	x_1	L	VL	L
	x_2	ML	L	VL
	x_3	VL	VL	L
Service performance of supplier (f_3)	x_1	VH	H	H
	x_2	H	VH	VH
	x_3	M	MH	MH
Supplier's profile (f_4)	x_1	VH	H	H
	x_2	H	VH	H
	x_3	H	VH	VH

Table 4. Evaluating Values of Alternatives of the Decision-makers with Respect to Different Attributes

Based on Table 2, we utilize equtaions (3) and (5) to aggregate all individual normalized IT2 fuzzy decision matrices $\tilde{R}^k = (\tilde{A}^k_{ij})_{3\times4}(k = 1, 2, 3)$ into a collective normalized IT2 fuzzy decision matrix $\tilde{R} = (\tilde{A}_{ij})_{3\times4}$ shown as follows:

$$\tilde{R} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \tilde{A}_{13} & \tilde{A}_{14} \\ \tilde{A}_{21} & \tilde{A}_{22} & \tilde{A}_{23} & \tilde{A}_{24} \\ \tilde{A}_{31} & \tilde{A}_{32} & \tilde{A}_{33} & \tilde{A}_{34} \end{bmatrix}$$

where

$\tilde{A}_{11} = ((0.590, 0.790, 0.790, 0.945; 1), (0.690, 0.790, 0.790, 0.868; 0.9))$
$\tilde{A}_{12} = ((0, 0.055, 0.055, 0.210; 1), (0.028, 0.055, 0.055, 0.133; 0.9))$
$\tilde{A}_{13} = ((0.760, 0.930, 0.930, 1; 1), (0.845, 0.930, 0.930, 0.965; 0.9))$
$\tilde{A}_{14} = ((0.760, 0.930, 0.930, 1; 1), (0.845, 0.930, 0.930, 0.965; 0.9))$
$\tilde{A}_{21} = ((0.610, 0.810, 0.810, 0.955; 1), (0.710, 0.810, 0.810, 0.883; 0.9))$
$\tilde{A}_{22} = ((0.030, 0.135, 0.135, 0.31; 1), (0.083, 0.135, 0.135, 0.223; 0.9))$
$\tilde{A}_{23} = ((0.840, 0.970, 0.970, 1; 1), (0.905, 0.970, 0.970, 0.985; 0.9))$
$\tilde{A}_{24} = ((0.790, 0.945, 0.945, 1; 1), (0.868, 0.945, 0.945, 0.973; 0.9))$
$\tilde{A}_{31} = ((0.710, 0.880, 0.880, 0.975; 1), (0.795, 0.880, 0.880, 0.928; 0.9))$
$\tilde{A}_{32} = ((0, 0.025, 0.025, 0.150; 1), (0.013, 0.025, 0.025, 0.088; 0.9))$
$\tilde{A}_{33} = ((0.440, 0.640, 0.640, 0.840; 1), (0.540, 0.640, 0.640, 0.740; 0.9))$
$\tilde{A}_{34} = ((0.840, 0.970, 0.970, 1; 1), (0.905, 0.970, 0.970, 0.985; 0.9))$

Step1: Suppose $p = q = 1$. By equtaions (33) and (34), we can get the overall performance value,

$\tilde{d}_1 = IT2TFWBM_w^{1,1}(\tilde{A}_{1,1}, \tilde{A}_{1,2}, \tilde{A}_{1,3}, \tilde{A}_{1,4})$
$= ((0.138, 0.177, 0.177, 0.205; 1), (0.158, 0.177, 0.177, 0.191; 0.9))$
$\tilde{d}_2 = IT2TFWBM_w^{1,1}(\tilde{A}_{2,1}, \tilde{A}_{2,2}, \tilde{A}_{2,3}, \tilde{A}_{2,4})$
$= ((0.148, 0.186, 0.186, 0.211; 1), (0.167, 0.186, 0.186, 0.198; 0.9))$
$\tilde{d}_3 = IT2TFWBM_w^{1,1}(\tilde{A}_{3,1}, \tilde{A}_{3,2}, \tilde{A}_{3,3}, \tilde{A}_{3,4})$
$= ((0.134, 0.168, 0.168, 0.195; 1), (0.151, 0.168, 0.168, 0.182; 0.9))$

Step2: Based on equtaions(7) and (8), calculate interval-valued possibility mean values of the weighted decision matrix $D = (\tilde{d}_1, \tilde{d}_2, \tilde{d}_3)$, shown as follows:

$$M(\tilde{d}_1) = [0.151, 0.167], \quad M(\tilde{d}_2) = [0.159, 0.174], \quad M(\tilde{d}_3) = [0.144, 0.159]$$

Based on equtaion (11), we can construct the fuzzy possibility degree preference matrix P, shown as follows:

$$P = \begin{bmatrix} 0.500 & 0.248 & 0.753 \\ 0.752 & 0.500 & 1.000 \\ 0.247 & 0.000 & 0.500 \end{bmatrix}$$

Step3: Based on equtaion (12), the ranking values $Rank(\tilde{d}_j)$ of the IT2 FSs $\tilde{d}_j$ can be calculated, shown as follows:

$$Rank(\tilde{d}_1) = 0.334, \quad Rank(\tilde{d}_2) = 0.459, \quad Rank(\tilde{d}_3) = 0.208$$

Because $Rank(\tilde{d}_2) > Rank(\tilde{d}_1) > Rank(\tilde{d}_3)$, the preference orders of the alternative x_1, x_2 and x_3 is: $x_2 > x_1 > x_3$. That is, the best desirable global supplier among x_1, x_2 and x_3 is x_2. The proposed method does not require complicated computations in the implementation procedure for evaluate global supplier. It provides us with a useful way for dealing with FMADM problems based on IT2 FSs.

A comparative study was conducted to validate the results of the proposed method with those from another approach. In the following, we use the above example to compare the ranking results of the proposed method with the existing methods. Using Gong et al.'s trapezoidal interval type-2 geometric Bonferroni mean

(TIT2FWGBM) operators method [15], suppose $p = q = 1$, the overall performance value can be calculated shown as follows:

$\tilde{d}_1 = IT2TFWGBM_w^{1,1}(\tilde{A}_{1,1}, \tilde{A}_{1,2}, \tilde{A}_{1,3}, \tilde{A}_{1,4})$
$= ((0.638, 0.881, 0.881, 0.942; 1), (0.842, 0.881, 0.881, 0.9171; 0.9))$
$\tilde{d}_2 = IT2TFWGBM_w^{1,1}(\tilde{A}_{2,1}, \tilde{A}_{2,2}, \tilde{A}_{2,3}, \tilde{A}_{2,4})$
$= ((0.831, 0.912, 0.912, 0.956; 1), (0.878, 0.912, 0.912, 0.936; 0.9))$
$\tilde{d}_3 = IT2TFWGBM_w^{1,1}(\tilde{A}_{3,1}, \tilde{A}_{3,2}, \tilde{A}_{3,3}, \tilde{A}_{3,4})$
$= ((0.634, 0.855, 0.855, 0.926; 1), (0.819, 0.855, 0.855, 0.899; 0.9))$

It is noted that operator aggregation values $\tilde{d}_i$ obtained by BM method are smaller than 0 and the ones obtained by Gong et al.'s GBM method are bigger than 0, which indicates that our method can obtain more unfavorable (or pessimistic) expectations, while the one given by Gong et al. has more favorable (or optimistic) expectations. Xia et al. [40] has been a detailed comparison of the two operators. Using Chen and Lee's fuzzy ranking method [6], the upper and lower fuzzy preference matrix P^U, P^L shown as follows:

$$P^U = \begin{bmatrix} 0.500 & 0.194 & 0.592 \\ 0.806 & 0.500 & 0.654 \\ 0.408 & 0.346 & 0.500 \end{bmatrix}$$

$$P^L = \begin{bmatrix} 0.500 & 0.264 & 0.677 \\ 0.736 & 0.500 & 0.834 \\ 0.323 & 0.166 & 0.500 \end{bmatrix}$$

The ranking values $Rank(\tilde{d}_j)$ of the IT2 FSs $\tilde{d}_j$ can be calculated shown as follows:

$$Rank(\tilde{d}_1) = 0.621, \quad Rank(\tilde{d}_2) = 0.838, \quad Rank(\tilde{d}_3) = 0.540$$

Because $Rank(\tilde{d}_2) > Rank(\tilde{d}_1) > Rank(\tilde{d}_3)$, the preference orders of the alternative x_1, x_2 and x_3 is: $x_2 > x_1 > x_3$.

Using Hu et al.'s [16] possibility method, the possibility degree matrix P shown as follows:

$$P = \begin{bmatrix} 0.500 & 0.347 & 0.602 \\ 0.653 & 0.500 & 0.769 \\ 0.398 & 0.231 & 0.500 \end{bmatrix}$$

The ranking values $Rank(\tilde{d}_j)$ of the IT2 FSs $\tilde{d}_j$ can be calculated shown as follows:

$$Rank(\tilde{d}_1) = 0.325, \quad Rank(\tilde{d}_2) = 0.404, \quad Rank(\tilde{d}_3) = 0.272$$

Because $Rank(\tilde{d}_2) > Rank(\tilde{d}_1) > Rank(\tilde{d}_3)$, the preference orders of the alternative x_1, x_2 and x_3 is: $x_2 > x_1 > x_3$.

The ranking order is consistent with the one by ours. But compared with Chen's method, the main advantage of our method is that the values in UMF and LMF are considered simultaneously, and the possibility degree is calculated only once instead of twice in Chen's method, resulting in reduced computing time. Moreover, it is much easier to obtain the wrong order by Chen's method when the trapezoidal interval type-2 fuzzy numbers are closer. The advantages of our method when compared with Hu's method are shown as follows. First, the computation in our possibility degree formula is simpler than the possibility degree formula of Hu's method. Second, the IT2TFWBM operator can be very good to aggregate IT2 FSs information than TIT2-WAA operator.

7. Conclusions

In this paper, the BM operator and possibility degree have been extended to the interval type-2 fuzzy environment to organize and model the uncertainties better within multi- attribute decision analysis. We have presented a new method for FMADM based on the IT2TFWBM operator and the possibility degree of IT2 FSs. Compared with type-1 fuzzy numbers, interval trapezoidal type-2 fuzzy number better represent the uncertainties of decision-maker. We also use one example to illustrate the FMADM process of the proposed method. The result shows that the proposed method provides us with a useful way to deal with FMADM problems based on IT2 FSs.

In the future, we will apply the developed procedures to some other decision-making problems where the information about attribute weights is incomplete, such as making investment choices, hierarchical decision-making and hierarchical and distributed decision making [10], etc. and will consider the situations where the decision makers cannot provide their preference information about attribute weights in the process of decision making [12].

Acknowledgements. The author is very grateful to the editor and the anonymous reviewers for their constructive comments and suggestions that have led to an improved version of this paper. The work was supported in part by the Natural Science Foundation of Jiangsu Province of China (No. BK20130242), and the Fundamental Research Funds for the Central Universities (Nos. 2015B28014, 2015B23914).

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