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Short Note

RS-BL-ALGEBRAS ARE MV-ALGEBRAS

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ABSTRACT. We prove that RS-BL-algebras are MV-algebras.

1. Introduction

Since Hájek published his book [1], fuzzy logic originally introduced by Zadeh in [5] has received a strong impetus into the direction of mathematical logic; to date mathematical fuzzy logic has been analyzed in several hundreds of scientific works. One of the fundamental discoveries of Hájek are Basic fuzzy logic and BL-algebras; they are in the same relation with each other than classical logic and Boolean algebras are. A particular subclass of BL-algebras are MV-algebras, the algebraic counterpart of Lukasiewicz infinite valued logic. Other subclasses of BL-algebras are Product algebras and G-algebras corresponding to Product fuzzy logic and Gödel logic, respectively. We assume the reader is familiar with BL-algebras, MV-algebras and related topics, if not suitable references are [1, 2, 4].

The authors of [3] introduced the notion of right stabilizer in BL-algebras and defined a class of BL-algebras, called RS-BL-algebras. Moreover, in [3] the concept of semi RS-BL-algebras is introduced and it is proved that semisimple BL-algebras and semi RL-BL-algebras are RS-BL-algebra. In this short note we prove, however, that RS-BL-algebras are not really a new class of BL-algebras but coincide with MV-algebras.

2. RS-BL-algebras are MV-algebras

In their recent paper [3] entitled *A new class of BL-algebras*, the authors define an algebraic structure called *RS-BL-algebra*, a particular BL-algebra i.e. the algebraic counterpart of Hájek's Basic fuzzy logic. The authors define (Definition 4.1 and Theorem 4.3 (6) in [3]) a BL-algebra A to be an RS-BL-algebra if, for all elements $a \in A$ holds

$$R_a = \{x \in A : a \to x = x\} = \{x \in A : x \to a = a\} = L_a. \tag{1}$$

We aim to show that RS-BL-algebras are MV-algebras and vise versa.

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Since MV-algebras are such BL-algebras that $(a \to x) \to x = (x \to a) \to a$ holds for all $x, a \in A$, it is an easy task to show that MV-algebras are RS-BL-algebras. Indeed, if $x \in R_a$ then $(a \to x) \to x = 1$, hence $(x \to a) \to a = 1$ whence $(x \to a) \leq a$. In turn, $a \leq x \to a$ holds in all BL-algebras, therefore $x \to a = a$ and so $x \in L_a$. By a similar argument we see that $L_a \subseteq R_a$. To show that MV-algebras are the *only* BL-algebras that are RS-BL-algebras, observe first that the following holds in all RS-BL-algebras

$$\text{if } x^* = 0, \text{ then } x = 1, \tag{2}$$

where $x^* = x \to 0$. Now look at page 6 in [2] about ordinal sums of BL-chains. If the ordinal sum contains at least two non-trivial BL-chains, then the complement x^* of all elements x above the bottom chain is 0. Therefore condition (2) cannot hold. Therefore the ordinal sum is composed of only one BL-chain. For (non-trivial) Gödel and Product chains (2) is not valid. For any MV-chain condition (2) holds. Conclusion: for linear BL-chains, condition (2) holds iff the chain is an MV-chain. Since any BL-algebra is a subdirect product of BL-chains, condition (2) holds only for MV-algebras. Thus,

Theorem 2.1. *RS-BL-algebras and MV-algebras coincide.*

In [3], Definition 4.8 the authors define what they call semi RS-BL-algebras and prove in Theorem 4.10 that such algebras are RS-BL-algebras, thus we have

Corollary 2.2. *Semi RS-BL-algebras are MV–algebras.*

3. **Conclusion**

In this note we have proved that, unlike claimed in the title of [3], RS-BL-algebras are not a new class of BL-algebras but coincide with well known MV-algebras, a subclass of BL-algebras.

Compliance with ethical standards

Conflict of interest The author declares that there are no conflict of interest.

References

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