

FUZZY LOGISTIC REGRESSION BASED ON LEAST SQUARE APPROACH AND TRAPEZOIDAL MEMBERSHIP FUNCTION

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ABSTRACT. Logistic regression is a non-linear modification of the linear regression. The purpose of the logistic regression analysis is to measure the effects of multiple explanatory variables which can be continuous and response variable is categorical. In real life situations sometimes we deal with the information that is vague in nature and where each and every case has not been explained precisely. In this regard, we have used the concept of possibilistic odds and fuzzy approach. Fuzzy logic deals with linguistic uncertainties and extracts valuable information from linguistic terms. In our study, we have developed fuzzy possibilistic logistic model with trapezoidal membership function and fuzzy possibilistic logistic model is a tool that help us to deal with imprecise observations. Comparison of the fuzzy logistic regression model with classical logistic regression has been done by goodness of fit criteria on real life example.

1. Introduction

The regression analysis is widely used for the purpose of forecasting, predictions and making inferences. Regression analysis is a statistical process that studies the relationship among variables. It measures the strength of the relationship between an explained variable and one or more explanatory variables. More specifically, the regression tells how the particular value of explaining variable changes due to changes in the explanatory variables. There are two main types of regression analysis, i.e. simple regression and multiple regressions [7]. In the simple regression, one independent variable is used to predict the dependent variable while in multiple regressions; two or more independent variables are used to predict the dependent variable. Regression helps to establish a mathematical relationship between variables where both the explained and the explanatory variables are of continuous type. This relationship is usually expressed in terms of a straight line i.e. by simple linear regression. Binary Logistics is used when the nature of explained variable is binary. It has the boundaries between zero and one and estimates the probability of occurring of an event. Here our interest is in the fact that whether the probability is one (the event occurs) or zero (event does not occur). There are different types of logistic regression model and the model is chosen according to the type of variable. When the categories of the dependent variable are ordinals (e.g. important, more important and most important) then we use the ordinal logistic

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regression. The ordinal regression model is based on a proportional odds model which converts the ordinal scale into binary cutoff points. When there are more than two categories of nominal dependent variables, we use the multinomial logistic regression. Maximum likelihood method (MLE) [2] is the method of estimating the unknown parameters which gives the maximum of a known likelihood distribution. Wald test [6] is used to estimate the unknown population parameter on the basis of a sample statistic and it is used in the multivariate case. It helps in testing the significance of particular predictors in a statistical model. The Wald statistic is the square of t-statistic and gives equivalent results for a single parameter. Likelihood Ratio Test is an alternative to Wald test. It is the ratio of likelihoods of data under two hypotheses, null hypothesis and alternative hypothesis. This test compares the fit of two models, where one model (null) is nested within another (alternative) model. The Likelihood ratio statistic is calculated as $D = -2 \ln(\text{likelihood for null model}) - (\text{likelihood for alternative model})$. The D statistic is compared to the critical value table to conclude whether to accept null model or reject, D statistic asymptotically follows the chi square distribution. The Goodness of Fit tells how well a model fits a data, in other words, it tells whether a given data represents a particular distribution or not. In logistic regression models Hosmer Lemeshow (HL) [8] test is used for determining the goodness of fit. This test tells whether the rates of observed events in the model population match with the expected rates or not. The odds ratio is an important concept in the logistic regression, it is a measure of association which compares the odds of an event happens or not. In logistic regression the dependent variable is a log of odds ratios (logit). Usually we deal with precise data but when we deal with imprecise data then we move towards fuzzy regression. Fuzzy logic [8] is an approach that recognizes more than simple true or false values and it deals with the "degree of truthiness" where the true value may lie between completely true or completely false. The truth value of fuzzy logic [13] variables ranges between zero and one and includes different stages of truth in between. For example when two fat women are compared then the result may be "over-weight" instead of fat or smart. Fuzzy theory performs better to deal with these uncertainties and impreciseness. Fuzzy techniques have been used in image understanding applications such as detection of edges, feature extraction, classification and clustering. Fuzzy logic is introduced by Zadeh in 1960's [3] and it is concerned with the possibilistic theory that deals with the vagueness. When we are concerned with the linguistic variables for example slow/fast, hot/cold, tall/short etc. then their degrees of truth is managed by using fuzzy logic [4]. Fuzzy reasoning is based on the theory of fuzzy sets and it includes theories from logic to pure and applied Mathematics like graph theory, topology and optimization. Fuzzy set [9] theory is commonly used in fuzzy models. Linguistic are mostly used in scientific researches as compared to mathematical terminology. The linguistic terms are irreducibly vague. Fuzzy logic addresses the linguistic ambiguities and it is an approach of reasoning. Boolean or binary logic is the special case of fuzzy logic. It is applicable in many fields medicine, engineering and economics as discussed in [2] and [4]. Fuzzy logic studies the possibility approach and it deals with degrees of truth instead of true or false. It is concerned with the uncertainty data. The

information which is compiled from real world problems has different forms of ambiguity. Fuzzy logic [4] is a helping instrument to consider these ambiguities. In our research, we have developed fuzzy logistic regression diagnostic model and estimated the values of parameters. Here the input variables are independent variables and output variables are dependent variables. Before this, researchers have been explored an application of fuzzy Logistic regression model in different fields, see [13], [10], and [11].

2. Preliminaries

In this study, we have developed fuzzy logistic regression model using least square trapezoidal membership function. Before this, the vagueness of logistic model has been studied by triangular and least square method in [13]. The aim of this study is to develop a possibilistic fuzzy logistic regression and to present the logistic regression diagnostic model with trapezoidal fuzzy membership technique. A comparison has been done between the results obtained by classical and fuzzy logistic regression techniques and definite methodology has been formulated for fuzzy logistic regression model. The concept of possibilistic odd has been used and it is defined as in [13]

Possibilistics Odds Let $\mu_i, i = 1, 2, \dots, n$ represents the possibility of achievement. Then the ratio $\frac{\mu_i}{1-\mu_i}$ is reflected as the possibilistic odds of the i th situation that notices opportunity of achievement as compared to the possibilistic odd of not achievement.

2.1. Fuzzy Logistic Regression Model. Let the independent and dependent variables consists of the number of observations $(x_{i0}, x_{i1}, \dots, x_{in}, Y_i)$ and $1 < i < m$ and Y_i is a fuzzy observation detecting the status of each case relative to binary response categories i.e. it takes two labels: approximately 1 or approximately 0 instead of 1 or 0. In our study, we consider the fuzzy logistic model with input variables are fuzzy and output variable is categorical. The proposed fuzzy logistic regression model is given below

$$W = \ln\left(\frac{\mu_i}{1-\mu_i}\right) = b_0 + b_1x_{1i} + \dots + b_5x_{5i} + \epsilon \tag{1}$$

where in (1), W is fuzzy estimated response variable, b_0, b_1, \dots, b_5 exposed fuzzy association. We suppose $b_5 = (al, au, am, ar)$ in equation (1) then the predictable outputs become trapezoidal fuzzy numbers and al, au, am, ar represents left, right middle, left middle and right points.

Fuzzy Least Square Technique

Fuzzy least square approach (which is proposed by Diamond [5] and Celmins [3]) is an addition of ordinary least square technique. This technique encompasses of goodness of fit and entails a distance between the fuzzy values estimated by the model and ambiguous data is really observed. Mathematically it is expressed as

$$d(A_1, A_2) = \left[\int_0^1 f(\alpha) d^2(A_1, A_2) \alpha d\alpha \right]^{\frac{1}{2}}, \tag{2}$$

where in (2), A_1, A_2 are arbitrary fuzzy numbers. d^2 (trapezoidal distance) is represented as [1]

$$d^2(A_1, A_2) = (a_{1l} - a_{2l})^2 + 0.5(a_{1u} - a_{2u})^2 + 0.5 * (a_{1m} - a_{2m})^2 + (a_{1r} - a_{2r})^2$$

$$A_1 = [a_{1l}, a_{1u}, a_{1m}, a_{1r}],$$

$$A_2 = [a_{2l}, a_{2u}, a_{2m}, a_{2r}],$$

where A_1 and A_2 are α -cuts, a_{il} denotes left values and shown a_{ir} right values, whereas a_{im} and a_{iu} are the left and right middle points, $i = 1, 2$. The function $f(\alpha)$ represents as weighting function.

2.2. Estimation of Model Parameters. In order to obtain an optimal solution of equation (1) by the least square method, the sum of squared errors between w and W should be minimized. By using distance equation in equation (2), the sum of squared errors is,

$$SSE = \sum_{i=1}^m [d(w, W)]^2. \quad (3)$$

In equation(3)

$$d(w, W) = \left[\int_0^1 f(\alpha) d^2(w, W) d\alpha \right]^{\frac{1}{2}},$$

w represents the possibilistic odds which is reflected as observed outputs and W represents the estimated output value. Now using the (3) on w and W we have

$$W = [f(a_l, a_u, a_m, a_r)],$$

where

$$f(a_l) = al_0 + \dots + al_n x_{in}, \quad (4)$$

$$f(a_u) = au_0 + \dots + au_n x_{in}, \quad (5)$$

$$f(a_m) = am_0 + \dots + am_n x_{in}, \quad (6)$$

$$f(a_r) = ar_0 + \dots + ar_n x_{in}, \quad (7)$$

The above equations (4), (5), (6) and (7) expresses the function contains left values, left middle values, right middle values and right values. Now we expressed W in terms of Trapezoidal fuzzy number.

$$(W) = [f(au) - f(al)\alpha + f(al), f(ar) - \alpha f(ar) - f(am)],$$

Similarly the observed value (w) is based on

$$w = [k_1, k_2],$$

where

$$k_1 = [f(au) - f(al)\alpha + f(al)],$$

$$k_2 = [f(ar) - \alpha f(ar) - f(am)],$$

which gives lower and upper bound of W .

$$(w) = \left[\ln\left(\frac{k_1}{1-k_1}\right), \ln\left(\frac{k_2}{1-k_2}\right) \right],$$

where (w) gives observed values. Using the distance formula we have.

$$d^2[w, W] = \left[\ln\left(\frac{k_1}{1-k_1}\right) - \{f(au) - f(al)\alpha + f(al)\} \right]^2 + \left[\ln\left(\frac{k_2}{1-k_2}\right) - f(ar) + \alpha\{f(ar) - f(am)\} \right]^2 \quad (8)$$

Then by putting equation (8) in equation (3), we get

$$SSE = \sum_{i=1}^m \int_0^1 f(\alpha) [\ln(\frac{k_1}{1-k_1}) - \{f(au) - f(al)\}\alpha + f(al)]^2 + [\ln(\frac{k_2}{1-k_2}) - f(ar) + \alpha\{f(ar) - f(am)\}]^2 \quad (9)$$

The minimization process includes setting the partial derivatives of SSE with respect to al, au, am, ar we get these equations.

$$\sum_{i=1}^m (\int_0^1 2\alpha(\alpha-1)x_{ij} [\ln(\frac{k_1}{1-k_1}) - \{f(au) - f(al)\}\alpha + f(al)] d\alpha) \quad (10)$$

$$\sum_{i=1}^m (\int_0^1 2\alpha(\alpha-1)x_{ij} [\ln(\frac{k_1}{1-k_1}) - \{f(au) - f(al)\}\alpha + f(al)] d\alpha) \quad (11)$$

$$\sum_{i=1}^m (\int_0^1 2\alpha x_{ij} [-\ln(\frac{k_2}{1-k_2}) + f(ar) - \alpha\{f(ar) - f(am)\}] d\alpha) \quad (12)$$

$$\sum_{i=1}^m (\int_0^1 2\alpha(\alpha-1)x_{ij} [\ln(\frac{k_2}{1-k_2}) - f(ar) + \alpha\{f(ar) - f(am)\}] d\alpha) \quad (13)$$

The simplified form of equations (10),(11), (12) and (13) are written in matrix form

$$AL = Z, L = [al_0 \dots al_n]^T, Z = [\sum_{i=1}^m z_i x_{i0} \dots \sum_{i=1}^m z_i x_{in}]^T \quad (14)$$

$$AU = K, U = [au_0 \dots au_n]^T, K = [\sum_{i=1}^m k_i x_{i0} \dots \sum_{i=1}^m k_i x_{in}]^T \quad (15)$$

$$AM = P, M = [am_0 \dots am_n]^T, P = [\sum_{i=1}^m p_i x_{i0} \dots \sum_{i=1}^m p_i x_{in}]^T \quad (16)$$

$$AR = Q, R = [ar_0 \dots ar_n]^T, Q = [\sum_{i=1}^m q_i x_{i0} \dots \sum_{i=1}^m r_i x_{in}]^T \quad (17)$$

Here $A = X^T X$ where $X = \begin{pmatrix} 1 & \dots & x_{1n} \\ \dots & & \dots \\ 1 & \dots & x_{nn} \end{pmatrix}$

If $\text{rank}(X) = n+1$, then matrix A is positive definite and inverse of A is computable. Then maximization problem has a unique solution. is positive definite and inverse of A is computable. Then maximization problem has a unique solution.

$$L = A^{-1}Z \quad (18)$$

$$U = A^{-1}K \quad (19)$$

$$M = A^{-1}P \quad (20)$$

$$R = A^{-1}Q \quad (21)$$

L, U, M and R represents left, left middle, right middle and right values, Z, K, P and Q provides the results of integral computation, gives us unique solution where

$$\begin{aligned}L &= A^{-1}Z \geq 0 \\U &= A^{-1}K \geq 0 \\M &= A^{-1}P \geq 0 \\R &= A^{-1}Q \geq 0\end{aligned}$$

3. Numerical Computation

The efficiency of the model has been checked by considering the teeth decaying disease and data has been collected from the Raazi Hospital of teeth decaying disease. The primary data was collected by using questionnaire. . The model has been formulated in which dependent variable is teeth decaying, explanatory variables age, family history, gender, sweetness effect and chewable stuff effect. The standard points of the given variables are shown in table 1 after discussion of the doctor. In this research we have collected data from fifty patients, we examine this model. The optimal model is given as below

$$W = \ln\left(\frac{\mu}{1-\mu}\right) = \beta_0 + \beta_1age + \beta_2familyhistory + \beta_3gender + \beta_4sweetnesseffect + \beta_5chewableeffect + \varepsilon,$$

where ε represents error term. We have to estimate β_0, \dots, β_5 by using equations (19) to (21).

$$A = X^T X = \begin{pmatrix} 50 & 28 & 50 & 27 & 38 & 29 \\ 28 & 28 & 25 & 14 & 24 & 18 \\ 50 & 25 & 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 & 7 & 7 \\ 6 & 7 & 6 & 7 & 7 & 7 \\ 9 & 9 & 4 & 4 & 3 & 3 \end{pmatrix}$$

$$L = A^{-1}Z = (1.3250, 0.0003, 0.0004, 0.0001, 0.0002, 0.0001)$$

$$U = A^{-1}K = (0.0884, 0.1800, 1.0566, 1.0261, 1.5880, 0.5476)$$

$$M = A^{-1}P = (1.7902, 0.7246, 1.0638, 0.0684, 2.7819, 0.4685)$$

$$R = A^{-1}Q = (4.3232, 0.3039, 0.0306, 0.2396, 3.8747, 0.5561)$$

The optimal model based on our observations is

$$\begin{aligned}W = \ln\left(\frac{\mu_i}{1-\mu_i}\right) &= (1.3250, 0.0884, 1.7902, 4.3232) + (0.0003, 0.1800, 0.7246, 0.3039)age + \\ &(0.0004, 1.05661, 1.0638, 0.0306)familyhistory + (0.0001, 1.0261, 0.0684, 0.2396)gender + \\ &(0.0002, 1.5880, 2.7819, 3.8747)sweetnesseffect + (0.0001, 0.5476, 0.4685, 0.5561)chewablestuff + \varepsilon\end{aligned}$$

Consider second patient equation

$$\begin{aligned}W = \ln\left(\frac{\mu_i}{1-\mu_i}\right) &= (1.3250, 0.0884, 1.7902, 4.3232) + (0.0003, 0.1800, 0.7246, 0.3039)1 \\ &(0.0004, 1.05661, 1.0638, 0.0306)0 + (0.0001, 1.0261, 0.0684, 0.2396)1 + \\ &(0.0002, 1.5880, 2.7819, 3.8747)1 + (0.0001, 0.5476, 0.4685, 0.5561)0 \\ W &= (1.3256, 2.8825, 5.3651, 6.2084)\end{aligned}$$

The observed possibilistic odds of this case is as follows. By using extension principle we get,

$$\left(\frac{\mu_2}{1 - \mu_2}\right)(x) = \exp(W_2(x)) = \begin{cases} 1 - \frac{1.3256 - \ln x}{2.8825} & 0.25 \leq \ln x \leq 1.3256 \\ 1 & 1.3256 \leq \ln x \leq 5 \\ 1 - \frac{\ln x - 5.3651}{6.2084} & 5 \leq \ln x \leq 11.57 \end{cases}$$

The standard points of above model are defined as follows.

Standard Points	Age (Binary Logistic)	Family History	Sugar Intake (teaspoon)	Chewable Stuff	Fuzzy Interval
High	17-28(1)	1-above	$1\frac{1}{2} - 2$	≤ 5	0.70
Very High	28 or above(0)	0-0.9	3 and above	≥ 8	0.75

TABLE 1. Standard Points

The graphical representation of observed possibilistic odds as follows.

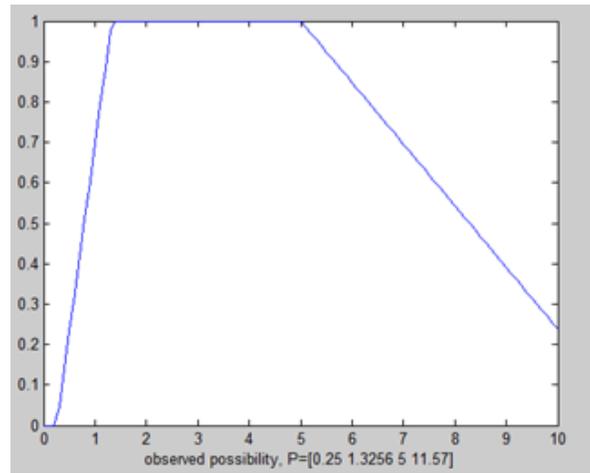


FIGURE 1. Observed Possibilistic Odds

Interpretation of Observed Possibilistic Odds

This figure depicts the observed possibilities of the relationship between explained variable and explanatory variables [0.25, 1.325, 5.0, 11.57] represents left, left middle, right middle and right values of trapezoidal numbers and also denotes age, family history, sweetness effect and chewable stuff effect. In comparison from standard points which are given in table (1.0), the patient has very high chances of teeth decaying disease due to sweetness effect and chewable stuff effect and high chances of family history. Estimated possibility odd of teeth decaying disease for this case is given below,

$$\mu_2(x) = W_2\left(\ln \frac{x}{1-x}\right) = \begin{cases} 1 - \frac{1.3256 - \ln \frac{x}{1-x}}{3.3722} & 0.20 \leq x \leq 0.57 \\ 1 & 0.57 \leq x \leq 0.83 \\ 1 - \frac{\ln \frac{x}{1-x} - 5.3651}{6.2084} & 0.83 \leq x \leq 0.92 \end{cases}$$

The Graphical representation of estimated possibilistic odds as follows.

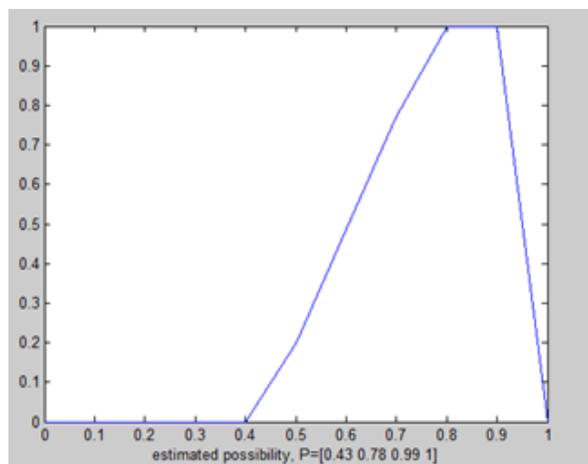


FIGURE 2. Estimated Possibilistic Odds

Interpretation of Estimated Possibilistic Odds

This picture shows the association of the estimated possibilities between dependent variable and independent variables. $[0.20, 0.57, 0.83, 0.92]$ denotes left, left middle, right middle and right values of trapezoidal numbers and also represents age, family history, sweetness effect, chewable stuff effect. In comparison from standard points it has very high chances of teeth decaying due to family history, sweetness effect and chewable effect. It defines the range of the possibilistic odds of having teeth decaying disease is 0.57 and 0.83. On the above pattern, we have studied 50 patient cases and find left, left middle, right middle and right values of trapezoidal numbers. It shows the association of the estimated possibilities between dependent variable (teeth decaying) and independent variables (age, family history, gender, sweetness effect and chewable stuff effect). They provide us precise information reduces vagueness in the responses and values falls between fuzzy limits.

3.1. Discussion on Fuzzy Logistic Regression and Classical Regression.

In the current paper the detail of fuzzy logistic regression model and a numerical example of its application in teeth decaying problem has been discussed. The proposed model is applied when number of observations are vague and the values are reported in the interval $[0,1]$, represents the possibility of having the disease. The fuzzy coefficients in our proposed model has been estimated by using possibilistic approach As concerned to Logistic regression it is suitable when the expected result is dualistic (happening/not happening, success/failure, healthy/unhealthy) and it has only two outcomes that is 0 and 1. In our proposed model, the degree of possibility levels are measured between 0 and 1 and this estimated the possibility levels of each coefficient of having disease in the interval $[0,1]$. The proposed model can also be used in other research area with similar situations.

Observed	t	d	Percentage Correct
Step1 t-d 0	0	1	72.0
1	18	7	72.0
Overall Percentage	7	18	72.0

TABLE 2. Classification Table

Mean Capability Index

To evaluate model goodness of fit, we use mean capability index, see [13] and [10].

$$MCI = \frac{1}{n} \sum_{i=1}^n I_{ui}(w, W) = \frac{1}{50} (36.5)$$

$$MCI = \frac{1}{n} \sum_{i=1}^n I_{ui}(w, W) = 0.73$$

MCI has lowest value is zero and largest value is one, so a value close to one directs good model fitting. n represents number of observations, $\frac{1}{n} \sum_{i=1}^n I_{ui}(w, W)$ this represents the summation of observed and estimated values.

Interpretation of Classification Table 2 It shows us the goodness of fit of classical logistic regression. Classification table 2 from output result summarizes the observed group and the predicted group classification. The overall correctly specified group percentage is 72

4. Conclusion

We have proposed a fuzzy logistic regression model with trapezoidal fuzzy numbers based on a breakdown of the association among a fuzzy explained variable and explanatory crisp variables into four components: two for middle and left middle and two for right and left right of variable. As comparison to logistic regression, it has been observed that ideal assumptions of logistic regression like other statistical methods may not hold in practice and does not cover the vagueness of observation. The proposed fuzzy logistic regression model is based on fuzzy least square method and establishes the relation between crisp inputs and fuzzy outputs. Also the estimated coefficients are fuzzy in nature and it determines the possibility levels of each factor of having teeth decaying problem. Mean capability index has 0.73 value of goodness of

fit for fuzzy logistic regression whereas correctly specified validation for classical logistic regression is 0.72. The results of our proposed fuzzy logistic regression model has shown improvement in precision. In the current paper, the details of a fuzzy logistic regression, its advantages and application are explored by giving a real life example. The proposed model is applied when the observations of the binary response variable are vague (i.e. instead of 0 or 1, they are reported as a value in [0,1] representing the possibility of having the person teeth decaying disease) but the observations of the explanatory variables are precise. We have compared the proposed model with the classical logistic model in which it is difficult to guess on the basis of vague information that a person having a disease or not. This imprecise

information can easily be handled using our proposed model. The proposed model can also be used in other research areas with similar situations.

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