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Achieving Optimal Path of Extracting Groundwater Resources Considering Side Effects in Hamadan-Bahar Plain

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ABSTRACT

Stable modelling in water resources management, on one hand, requires recognizing the relations between the different applications of water and their long-term results and on the other hand taking into account the current and future access to water resources and demands for them. In recent years, the basin of Kabudarahang plain has been identified as one of the critical groundwater areas in Hamedan province. In this plain, the groundwater level has been decreased remarkably due to lack of the proper water usage management. The purpose of this study is to provide a model to maximize the net benefit of farmers considering the stability of the groundwater reservoir. Since the effect of the groundwater discharge is accumulated over time, time is taken as an essential variable in solving water optimization problems. Accordingly, applying dynamic models such as the optimal control method is appropriate for this purpose. The optimal path for water extraction from groundwater resources can be determined using the optimal control model. In this model, the additional cost of water extraction due to the further exploitation has been considered as a constraint in the modeling process and the effect of the cost internalization are determined by the optimal path of the extraction and the price. Considering the optimal use of water and maximum welfare of the farmers, the results show that it will take about 38 years to increase the water level from 1716 meters to the optimal level of 1749 meters. During this period, the price of water will decrease from 1820 to 1180 Riyals per cubic meter and the annual harvest from the groundwater resources will reduce to the level of 1.7 million cubic meters. Therefore, the observation of the specified limit not only results in stable groundwater resources, but also leads to a sustainable agricultural development and the increase of farmers' income in this area.

1. Introduction

Water is the most valuable natural resource.Water as one of the main factors of agricultural production, which plays an important role in sustainable development in this sector (Chizari *et al*., 2006). Water scarcity is a major problem in many countries, especially countries with the growing populations. The only solution to this crisis is the optimum use and the increase of water productivity in different sectors, especially agriculture (Keramatzadeh *et al*, 2007). The average rainfall of 800 mm in the world but in Iran the average annual rainfall is 250 mm. The amount of rainfall is less than one-third of global precipitation in Iran, due to semi-arid climate with dry conditions. Distribution and considerable fluctuation in rainfall in the country contributes to the problem is a shortage of water to the agricultural sector. Groundwater resources in Iran are the most important and most valuable water resources, especially in the agricultural sector. Groundwater provides more than 90% of the agricultural sector (Anonymous. 2009). Uncontrolled exploitation of groundwater leads to problems such as land has been sinking. Hamedan Province is located in West of Iran. The agricultural areas of Kabudarahang Plain are located in Hamedan province. Kabudarahang plain groundwater has dropped 16 meters over the past two decades. Studies show that 69 percent of the province's groundwater resources has forbidden status, so that the limit of 17.5 percent, and 13.5 percent for the prohibited status is critical (Mosavi and porhaghighat, 2007). In determination of the optimum utilization of groundwater resources over time and the dynamic methods such as optimal control, few studies have been conducted by researchers abroad. Iran studies on utilization of water resources to consider such an approach is not just limited. The study of Pitafi and Roumasset (2003) to estimate water use efficiency in the region was in Ohio. They compared the welfare of farmers both with and without their optimal control model. Their results show that the water in the optimization mode leads to increase the welfare. Chaitra

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and Chandrakanth (2005) used the principles of maximization Pontryagin[1] and specified the path of sustainable economic exploitation. The results showed that by optimizing water use, the life of water wells increases compared to uncontrolled harvesting. It also increases the present value of the actual net profit at optimum harvesting. Ponghijvorasin *et al* (2008) used the optimal control methods to apply of groundwater resources in an ecosystem seaside. The results of their study showed that the water level rises in the optimum state. Roumasset and Christopher (2010) showed that the path of sustainable groundwater extraction should be determined in order to maximize the welfare of the adopted dynamic optimization model. The objective of their study was to achieve stable performance for water usage. Therefore, in determining the optimal path, they used watershed protection strategy and the recycled sewage.

The aim of this study is to provide a model to optimize the utilization of groundwater resources in the plain of Kabudarahang. To this end, it provides a model for determining the optimal path of extraction and price of water over time from this source, which is, in fact, the innovation of this research.

2. Materials and methods

Kabudarahang plain is located between east longitude 48°,33´ to 48°,17´ and 35°,02´ to 34°,49´ north latitude. This plain with an area of 880 square kilometers is forbidden (Anonymous, 2008). In most areas of this plain, the river is not permanent. In this area, because of low average rainfall and lack of timeliness, surface water in the agricultural sector has a limited role. Use of groundwater in the plain, supply more than 80 percent of the water needed for agriculture and 50 percent of municipal drinking water. Figure (1) shows the area's aquifer.

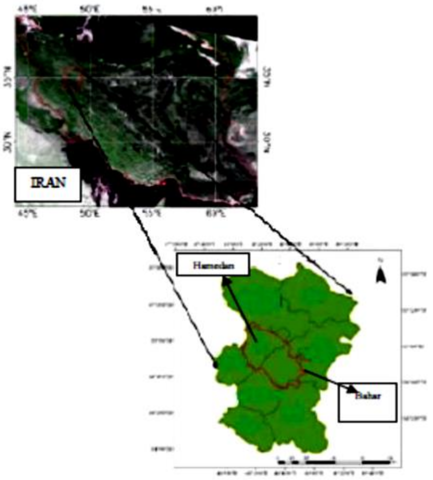

Figure 1. Hamedan- Bahar Plain

In this area, more than 57 percent of the acreage devoted to potato crop. Tables (1) shows the crops and their share of irrigated land in Kabudarahang plain. Since the potato crop has the highest water consumption per unit area and assigned to the high share of agricultural lands plains. The data required for this study was based on data collected from the potato crop in farmers' fields

Table 1. Percentage of major agricultural crops in the plains of Hamedan- Bahar

Products	Percent of cultivated area
Potato	57.4
Wheat and barley	20.5
Alfalfa	16.1
Vegetable	3.6
Sugar beet	1
beans	0.3
CORN	0.3
Other products	0.8
Total	100

Source: Agriculture Organization of Hamedan province, 2014

In this study, we used two sets of data. The first group data collected through field study and questionnaires in 2015. The second group used data from that organization. The study population consisted of farmers in four villages in the plain of Kabudarahang. Sampling was carried out using equation (1), the ratio stratified random sampling method.

$$n = \frac{N \times t^2 \times s^2}{N \times d^2 + t^2 \times s^2} \qquad (1)$$

In equation (1):
n: number of samples required
N: number of community members (farmers who use wells)
t: t-student statistic
s: sample variance
d: error

For this purpose, 30 questionnaires were used and calculated the variance in the depth of the well (water level) is equal to 0.29. Therefore, the choice of 121 wells in operation. (Equation 2).

$$\mathrm{n} = \frac{2143 \times (1.96)^2 \times (0.29)^2}{2143 \times (0.05)^2 + (1.96)^2 \times (0.29)^2} = 115 \qquad (2)$$

To determine the optimal path, it is to estimate the demand for water extraction. The method is used to calculate the demand for water. There's one way to minimize the costs and the second method is profit maximization. First method is used if the production function characterized by increasing returns to scale (IRS)[2] and if they have

[1] Principal Pontryagin'sMaximum

[2] Increasing Return to Scale

diminishing returns to scale (DRS)[3] the second method is used (Hossinzadeh, 2011). In the agricultural sector, it is generally assumed that the production function, DRS. For this reason, in this study, there is used the second method. For this purpose, the form of the production function is as equation (3):

$$Y = f(X_1, X_2, X_3, X_4) \quad (3)$$

In equation (3):

Y: production (kg)

And X_1 until X_4, respectively, the amount of water per hectare (cubic meters), the amount of fertilizer per hectare (kg), the amount of seed per hectare (kg) and the number of labor force per hectare (people). In order to estimate the production function, the most appropriate functional form of the model is determined. For this purpose, the flexible[4] and inflexible[5] functions were estimated Generelized Quadratic function [6] and Cobb -Dauglass[7]. According to statistics, correlation coefficients, F، LR and distribution of residuals normality test, was selected Generelized Quadratic function. The general form of this function is as equation (4):

$$Y = A + \sum_{i=1}^{4} \beta_i x_i + \frac{1}{2}\sum_{i=1}^{4} \beta_{ii}(x_i)^2 + \sum_{i=1}^{4}\sum_{j=1}^{4} \beta_{ij}(x_i)(x_j) \qquad i \neq j \quad (4)$$

In equation (4):

A: coefficient of Technology

X_i : represents inputs that are introduced in the equation (3).

After estimating the model, t test was used to test the significant coefficients and the Durbin-Watson test for testing the lack of correlation between the error terms. The equation (5) is used in the calculation of water consumption.

$$W = 3.6L.H.D \quad (5)$$

In equation (5):

W: amount of pumping from wells, in cubic meters

L: instantaneous discharge, (lps), H: number of hours pumping from wells

D: The number of days pumping from wells per year

In this study, in order to determine the optimum water harvesting, there has been used the dynamic optimization and optimal control approach. It is a dynamic system, the control variables, the status variables, goals and a first time and an end time. Control variables are influenced on instrument system, which is shown by $u(t)$ .The decision maker will show at any point in time in the best way to determine the value of $u(t)$ and it has a favorable effect on his system. The situation variables, which are characterized by $X(t)$, show system status at any time. Over time, during the route and it is route state. Obviously, any value assigned to the control variables, it will be determined according to the situation. Relationship status and control variable can be written as a differential equation which describes the movement of the system. Therefore, it is said the equation of motion (Ahrabi and Shakeri, 1999). The aim of this research is to achieve the optimum water extraction with the side effects of withdrawal of ground water resource. Because of shared groundwater resources, it can be seen side effects in the use of this resource among farmers. High harvest of this resource leads to increased costs in terms of loss of water extracted. Therefore, to achieve optimal extraction path, and the optimum value of water, in this case, side effects should be calculated. Given the discussion above, the objective function is defined by equation (6). This relationship means to maximize the present value of welfare surplus producer (Sori, 2009).

$$NB = \int_0^T e^{-rt}\left(\int_0^q D_t^{-1}(q)dq - c(h_t)q_t\right)dt \quad (6)$$

In equation (6):

NB: The present value of benefits in water consumption during the period T (welfare surplus producer)

q: Water demand.

D_t^{-1} : Inverse demand equation

$\int_0^q D_t^{-1}(q)dq$: The area under the demand curve, which shows surplus by subtracting the cost of water extraction,

$c(h_t)$: The cost of water extraction that is a function of the height of the water surface.

q_t : Amount of water extracted.

r : Discount rate

Recharge groundwater resources, the sum of net rainfall infiltration and surface water returned from irrigation and water penetration are shown. The output from underground sources, the total crop farmers and water leakage from canals and fountains are shown. Equations of motion, the height of water level over time, shown by the equation (7):

$$\gamma(\dot{h}_t) = R - l(h_t) - q_t \quad (7)$$

γ : Conversion factor of cubic meters of water to the water level in meters. It is expressed in $l(h_t)=kh^2$ the relationship between the level of water extraction from aquifers and aqueducts and fountains (13). k is the coefficient for the aquifer. Thus, model (8) is introduced in order to withdraw

[3] Decreasing Return to Scale

[4] Flexible functional form

[5] Inflexible functional form

[6] Generalized Quadratic functional form

[7] Cobb-Douglass functional form

from the aquifer to maximize the present value of the welfare of farmers.

$$Max\int_0^T e^{-rt}(\int_0^q D_t^{-1}(q)dq - c(h_t)q_t)dt \quad (8)$$
$$s.t:$$
$$\gamma(\dot{h}_t) = R - l(h_t) - q_t$$
$$h(t) \geq h_s$$

According to the model (8), Hamilton's function for equation (9)

$$H=(\int_0^q D_t^{-1}(q)dq - c(h_t)q_t) + \lambda\gamma(R - l(h_t) - q_t) \quad (9)$$

Shows the shadow price of each unit of ground water. The first order condition to maximize the Hamilton function is written as equation (10), (11) and (12):

$$\dot{h}_t = \frac{\partial H}{\partial \lambda_t} = \gamma\left(R - l(h_t) - q_t\right) \quad (10)$$

$$\dot{\lambda} = r\lambda_t - \frac{\partial H}{\partial h_t} = r\lambda_t + C^{'}(h_t)q_t + \lambda_t\gamma L^{'}(h_t) \quad (11)$$

$$\frac{\partial H}{\partial q_t} = D_t^{-1}(q_t) - c(h_t) - \lambda_t \leq 0 \qquad if \prec then \quad \Rightarrow q_t = 0 \quad (12)$$

In order to achieve an optimum height aquifer, the optimal route of the aquifer altitude, the optimal route of the water price and the optimal route of the groundwater extraction, obtained from the above equations. The optimal size of the aquifer height.

3. Results and discussion

According to ratio random sampling method, 115 farmers were studied in Kabudarahang Plain. Farm size is very different, a minimum of 0.8 and maximum of 125 hectares, with the average of 8/5 hectares. To estimate the water demand needed, the optimal path for water extraction should be determined. In this study, the water demand of profit maximization is determined. The same study is performed by Pitafi and Roumasset (2003), Roumasset and Christopher (2010). They determined the most appropriate functional form model to estimate the production function. Therefore, the production function of inflexible functions, Generelized Quadratic function and Cobb-Douglass are estimated to express the relationship between factors of production and potato production. It shows the estimated coefficients and the coefficient of determination, which is the proper function of two forms. F statistic is significant in both models and it is a sign, and the regression is significant. Therefore, based on a lower number of parameters, easily interpreted, generalized quadratic function was selected.

This form is function (13).

$$Y = A + \sum_{i=1}^{4}\beta_i x_i + \frac{1}{2}\sum_{i=1}^{4}\beta_{ii}(x_i)^2 + \sum_{i=1}^{4}\sum_{j=1}^{4}\beta_{ij}(x_i)(x_j) \qquad i \neq j \quad (13)$$

In equation (13):

Y :produce,

A :Coefficient technology.

Xi: Inputs, Including water, fertilizer, seed and labor.

(8)
(9)

Table 2. Results of potato production estimate

Variable	Coefficient	Standard error	T-statistic	Significant level.
C	725	254	2.85	0.00
β_W	11.7	2.54	4.6	0.00
β_N	23.4	4.25	5.5	0.00
β_{WW}	-0.16	0.02	-8.6	0.00
β_{NN}	1.18	0.31	3.85	0.00
β_{WN}	-0.88	0.16	-5.46	0.00
$R^2 = 0.75$		$F = 487.7^{**}$		$n = 115$
$\bar{R}^2 = 0.74$		Jarque-Bera =5/7^{ns}	Ramsey RESET = 0/8^{ns}	Glejser = 1/5^{ns}

Source: Research Findings ** Shows a significant level of 5%

The results of production function shown in the table (2). W and N are in the table representing the amount of water and fertilizer. Remove seeds and labor variables are not significant parameters. According to the culture of agriculture and agricultural operations in the region, almost all farmers used to be a size per unit area of labor and seed. Adjusted coefficient of determination is equal to 0.75. The high coefficient refers to a good model. Significant statistic F, rejected the null hypothesis of variables.

The results of estimating the generalized quadratic production function are shown in Table (3). In this table, W and N represent the amount of water and fertilizer, respectively.

Profit function is to estimate the demand equation (14):

$$\pi = P_y \times (A + \sum_{i=1}^{2}\beta_i x_i + \frac{1}{2}\sum_{i=1}^{2}\beta_{ii}(x_i)^2 + \sum_{i=1}^{2}\sum_{j=1}^{2}\beta_{ij}(x_i)(x_j)) - (C_f - \sum_{i=1}^{2} r_i X_i) \tag{14}$$

In equation (14):

π : Profit

P_y : Market price

A: Technology coefficient,

X: Inputs

C_f: Fixed costs

r_i : Price per unit of input

According to the first condition maximization (FOC) and based on the Hotelling[8] case, demand and the optimum amount of input can be calculated. The equations (15) and (16) are shown below:

$$\frac{\partial \pi}{\partial W} = P_y\left[11.7 - 0.32W - 0.88N\right] - r_W = 0 \tag{15}$$

$$\frac{\partial \pi}{\partial N} = P_y\left[23.4 - 2.32W + 0.88N\right] - r_N = 0 \tag{16}$$

By solving simultaneous equations (15) and (16), written demand for water, as eq. (17):

$$W = 54.3 - \frac{1}{P_y}(2.23r_w - 2.4r_N) \tag{17}$$

This function specifies that water consumption was negatively correlated with its price. This means that an increase in water prices leads to the reduction of water consumption, and water demand has positive relationship with the price of fertilizer and product price. The demand for water by substituting the average price of water and fertilizer, and crop, calculated optimum amount of water in the form of equation (18):

$$W = 954.3 - (2.23r_W - 2.4r_N)\frac{1}{P_y} = 6570 \quad (m^3/ha) \tag{18}$$

According to equation (18) optimum amount of water is equal to 6870 cubic meters per hectare. The amount of water leads to maximum benefit farmers. Recommendation of this research is 6672 cubic meters (Seyedan and Ghadami, 2009). Therefore, the recommendation of the present study is roughly equal to optimum water consumption. Table (3) shows the state of agriculture in the region regarding the usage of water. Therefore, the consumption of 25.7 percent of farmers are less than optimum levels and 74.3 percent higher than this amount.

Calculate the price elasticity of water demand equation (19):

$$E_W = \frac{\partial W}{\partial r_W} \cdot \frac{r_W}{W} = -1.4 \tag{19}$$

Equation (19) shows that an increase of one percent water prices, demand decreases 1.4 percent. This finding is consistent with results from research Ponghijvorasin *et al* (2008), Pyndyck (1978). Despite the differences in quantity but stated that the elasticity of water demand. Of course Shapoorabadi (2002), Assadi (1997) in their research findings show that water demand is inelastic.

Therefore, according to table (3), the profit function is governed by (13).

Table 3. Optimal water economy in the potato fields

Unit: m^3/ha

Range	Frequency	Percent
< 6870	30	25.7
> 6870	85	74.3
Total	115	100

Source: Research Findings

Table (4) shows the result of the cost function of water. It is significant, at a percentage rate of well depth factor. This coefficient shows that the depth of the well has a positive effect on the cost of water extraction. The amount R^2 shows that 75 percent of the cost changes of water is explained by the size of the depth of the well. Water cost function, written in the equation (20).

$$c(h_t) = 723 + 28.2h_t \tag{20}$$

Some researchers have used this model to estimate the cost of water. The results in this study is very close to the results Taghizadeh and Soltani (2013), Fathi and Zibai (2011), and Tahamipour *et al* (2005), but it is different with the result obtained coefficient of Khalilian and mehrjardi (2005). Basically, the situation in the region and years of research is effective on results. Equations of motion, as restrictions on the model (21):

$$\dot{h} = 0.0000000517 \times (260300000 - 5.8h_t^2 - q_t) \tag{21}$$

In plain of Kabudarahang is the effective porosity of 5 percent (Anonymous, 2009). So γ amount equal to 0.0000000517. This implies that each cubic meter of water taken from underground sources reduces the water level to 0.0000000517 meters. The average go to the source of groundwater is 260.3 million cubic meters. Water leaks

[8] Hetling

through aqueducts and fountains and its relationship with water surface height shown in equation (22).

$$L(h_t)=5.8h_t^2 \tag{22}$$

Table 4. Results of regression, the cost of water extraction

Variables	Coefficient	Standard error	T-statistic..
Constant	723	821.6	0.88
Well depth (m)	28.2	6.6	4.26
$R^2 = 0.86$	$F = 456.7^{**}$	$n = 115$	
$\overline{R}^2 = 0.84$	$S.E = 4859$	$DW = 2.08$	
Jarque-Bera = 3/96^{ns}	Ramsey RESET = 2/16^{ns}	Glejser = 0/87^{ns}	

Source: Research Findings *,** Shows a significant level of 1% and 5% respectively

Thus, in order to maximize the present value of the welfare of farmers, extracted from the aquifer shown in Equation (23):

$$\text{Max}\int_0^T e^{-rt}\left(\int_0^q (-2.5r_N - P_y(63.07+0.89q_t)dq - (723+28.2h_t)q_t\right)dt$$
$$\text{s.t}:$$
$$\gamma\left(\dot{h}_t\right) = 0.0000000517(260300000 - 5.8h_t^2 - q_t)$$
$$h_t \geq h_s \tag{23}$$

In equation (23) the first part of the objective function is the inverse function of water demand obtained from the equation (17). According to equation (23) is written Hamilton function in equation (24).

$$H = \left(\int_0^q (-2.5r_N - P_y(63.07+0.98q_t)dq - (723+28.2h_t)q_t\right.$$
$$\left.+\lambda_t\left[0.0000000517\times(260300000-5.8h_t^2-q_t)\right)\right] \tag{24}$$

In equation (24) represents the shadow price of each unit of groundwater. The first order condition of the maximization of the Hamilton function is obtained, equations (25), (26) and (27):

$$\dot{h}_t = \frac{\partial H}{\partial \lambda_t} = 0.0000000517\times(260300000-5.8h_t^2-q_t) \tag{25}$$

$$\dot{\lambda} = r\lambda_t - \frac{\partial H}{\partial h_t} = r\lambda_t + C'(h_t)q_t + \lambda_t\gamma L'(h_t) =$$
$$r\lambda_t - 28.2q_t + \lambda_t\left[0.0000000517\times 11.6\right] \tag{26}$$

$$\frac{\partial H}{\partial q_t} =$$
$$(-2.5r_N - P_y(63.07+0.98q_t) - 723 + 28.2h_t - \lambda_t = 0 \tag{27}$$

Equation (25), (26) and (27) is consistent with the results of Chaitra and Chandrakanth (2005), Hossein Zadeh *et al* (2011). The value r (increasing the price of water) is equal to 9 percent. According to equation (27) obtained equation (28):

$$\lambda_t = (-2.5r_N - P_y(63.07+0.98q_t) - 723 + 28.2h_t \tag{28}$$

Equation (29) could be derived from equation (28) and combining it with equations (25) and (26).

$$(-2.5r_N - P_y\times(63.07+0.98q_t) - \left(723+28.2h_t\right) = \frac{0.0000000517\times(260300000-5.8h_t^2)\times 28.2}{r+0.0000000517\times 11.6h_t} \tag{29}$$

If the water diverted is equal to net input ($R_t = q_t$). In equation (29) can be calculated optimum height. Therefore, the optimum height is 1733.2. By equation (30) and demand for water is obtained, water optimum price. Therefore, the optimal price, is $\left(P^* = 2200\right)$ Reals.

$$\dot{h}_t = 0.0000000517\times(260300000-5.8h_t^2-q_t) \tag{30}$$

If the primary level is lower than the optimal equilibrium $h^* \prec h_t^*$. The higher price is the optimum price $P^* \succ P_t$. In this mode through water demand management policies reduce water consumption. In this way it is possible to obtain optimal route extraction of ground water, and the optimal route height, and the optimal route price of water from the equations (31), (32) and (33). Equation (31) indicates the maximum amount of water that can be extracted from groundwater.

$$q_t = 356.6\text{-}2056.8t\text{-}56.4t^2 \tag{31}$$

Equation (32) shows the optimal path groundwater elevation.

$$h_t = 0.52t + 1716 \tag{32}$$

The optimal price rout of water is obtained using equations (25) and (26) and demand for water. Equation (33) shows the optimal rout of water prices.

$$p_t = 1820 - 0.22t - 0.0084t^2 \tag{33}$$

Using the equations (32), the optimal path *of* water extraction is shown in Figure (1). According to this figure, water extraction must be reduced, so that over time this source will be restored to optimum levels. According to equation (33) and Figure (2) high water level rises from 1716 meters in 2014. So during 45 years, the optimum height increases and reaches to 1749 meters. This result is somewhat consistent with the results of Roumasset and

Christopher (2010), Hosseinzadeh *et al* (2011). In a study conducted by Hosseinzadeh *et al* (2011), it was estimated to be 36 years, in order to compensate for the depletion of groundwater, and the research conducted by Roumasset and Christopher (2010), 26 years. However, this time depends on the condition of the region. Water prices will change according to equation (33). Figure (3) shows the trend of water price. According to this calculation, the increase in prices in the first year occurs, until reduced water demand. With increasing water height, reduced the cost of extracting and accordingly reduced the price per unit of water. In the final year forecasts (1439) reduced the price per unit of water to 1180 rials. This finding is consistent with the findings, followed by Pitafi and Roumasset (2003), Roumasset and Christopher (2010), Hosseinzadeh *et al* (2011) that the downward trend to reach the optimum level of water. The opposite results, Ballali *et al* (2010), have calculated in 1500 Rials and the price per cubic meter of water in order to eliminate the negative balance of groundwater. Due to the results of this study, the downward trend was occurred in the price of water during the time. By increasing in groundwater levels and reducing the cost of water extraction, water price reaches the end of 1180 rials. Therefore, in such circumstances, Soon Hamedan-Bahar plain has serious problems of water scarcity and loss of groundwater resources. Assuming current trends continue, according to research Rahmani and sdehi (2009) is facing the plains in 2021, with 517 meters decline in groundwater levels, and the average water level will reach 1708 meters. In these circumstances the optimal use of groundwater resources can be changed for the better and sustainable conditions. In contrast to others (Hosseinzadeh and et al, 2011) and (Roumasset and Christopher, 2010) in plain of Kabudarahang due to lack of surface water and lack of treated water from sewage, there is need to repair groundwater resource. In these circumstances, changes in water demand management are the most important actions. In this connection, it is necessary for government legislation to prevent excessive water and lack of licensing, in plain negative balance is essential at least in the medium term to be replaced with agricultural crops with low water requirements of potato. There are recommendations to use the modern irrigation methods in order to reduce the pressure on water leading to water storage. Also, the awareness of farmers should be increased to excessive water consumption and the lack of sustainability of groundwater resources.

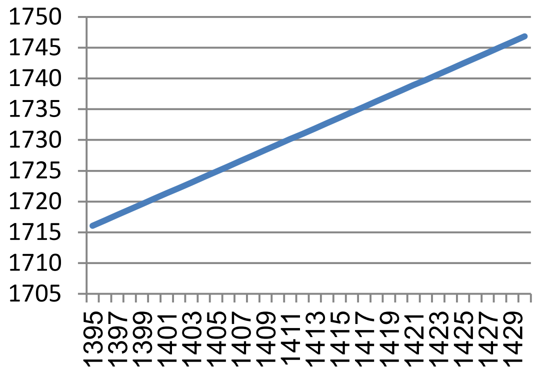

Figure 1. Optimal route of water extraction in the Hamedan - Bahar plain.

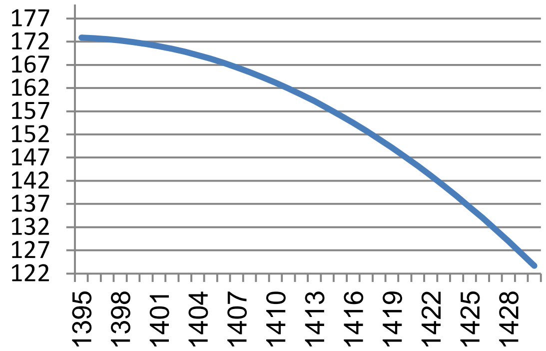

Figure 2. Optimal route of groundwater level in the Hamedan - Bahar plain

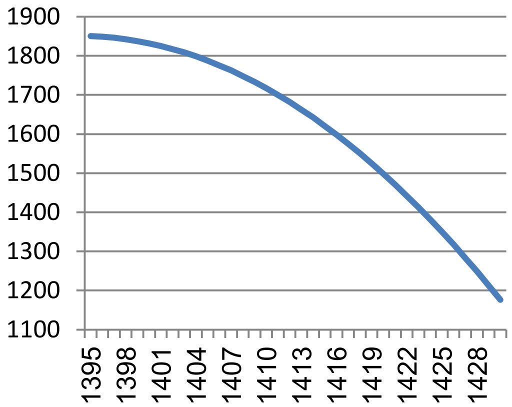

Figure 3. Optimal route of water price in the Hamedan - Bahar plain.

4. Conclusion

In this research, an optimization method and optimal control approach have been used to determine ad determine the optimal economic path for water extraction. The purpose of this study is to achieve optimal water extraction path considering the side effects of groundwater resources removal. The results of this study showed that with this calculation, it is necessary 38 years to increase the height of the water surface to an optimal value or 1749 meters. This result is consistent with the results of Roumasset and Christopher (2014) and Hosseinzadeh *et al*. In Hosseinzadeh's study, the time required to compensate for the deficit of the reservoir was 36 years, and in the researches Roumasset and Christopher 26 years is estimated. This period, of course, depends on the condition of the area in terms of water loss and the rate of withdrawal from groundwater basin. Due to the elasticity of water demand (-1.4), the realization of water prices can be effective in reducing its consumption. This finding is also consistent with the findings of Pitafi and Roumasset (2003), Christopher (23), and Hosseinzadeh *et al*. (2009), which are based on the downward trend of prices to achieve optimal water basin levels. Contrary to the results of Balali *et al*., (2009) that for the removal of the negative balance of groundwater in the Kaboudaradh plain, the price of each cubic meter of water was estimated to be 1500, in this study, water prices have been decreasing over time, groundwater basin and reducing the cost of water extraction The price per cubic meter of water decreases and at the end of the period reaches 1180 Rials. This value shows a significant difference compared to the highest exchange rate of local water in Kabudarahang plain (550 Rials). On this basis, it is suggested that agricultural water tariffs be gradually adjusted to provide a basis for saving and storing this important input. In the absence of optimal harvesting, the Kaboudaragh plain will soon be in serious trouble with water shortages and the loss of groundwater resources. According to Rahmani and Sdehi (2009), with the continuation of the current trend in 1400, the average height of the plain water will reach 1708 m. In this situation, with the optimal understanding of groundwater resources, the situation can be changed to a better and more stable situation. Unlike other studies, in the Kabudarahang plain, because there is no significant surface water and no other source such as treated water from sewage sources is available, the time for the rehabilitation of groundwater basin is longer and about 38 years. In addition to water pricing, other measures should be taken to improve the water demand management approach. In this regard, it is necessary for the government to prevent farmers from harvesting excess water in the formulation of laws and the non-issuance of new licenses. In the condition of the negative balance of the plain, it is necessary to replace potatoes at least during the period of cropping the products with a low water requirement. Advising on the use of advanced irrigation techniques reduces the pressure on the source and allows for the repair of the storage. Also, the ground for raising farmers' awareness of excess water consumption and problems of groundwater instability can be effective in this regard.

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