A COMPARATIVE PERFORMANCE OF GRAY LEVEL IMAGE THRESHOLDING USING NORMALIZED GRAPH CUT BASED STANDARD S MEMBERSHIP FUNCTION

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Abstract. In this research paper, we use a normalized graph cut measure as a thresholding principle to separate an object from the background based on the standard S membership function. The implementation of the proposed algorithm known as fuzzy normalized graph cut method. This proposed algorithm compared with the fuzzy entropy method [25], Kittler [11], Rosin [21], Sauvola [23] and Wolf [33] method. Moreover, we examine that in most cases, our algorithm gives the lowest absolute error that improves the segmentation process of gray images. Finally, we change different parameter values in fuzzy normalized graph cut and the effect of the substitutes is studied. Also, we analyze the computational complexity of fuzzy weight matrix (fuzzification) results with a weight matrix (classical) results.

1. Introduction

In image processing, the gray levels of pixels belonging to objects are significantly different from those belonging to the background. According to a certain norm, the partition of the set of pixels in the image into a finite set of regions (subsets), that is, the image is dissolved into a worthwhile component for further investigation.

Compare each pixel in an image with an assured threshold value is the base idea behind the thresholding rule. The determination of a relevant threshold value, which splits or partitions a gray level image into objects and background regions, is the nub of a threshold problem. An optimum threshold value is a gray level that splits an image into two subdivisions. i.e., an object segment and background segment. However, automatic determination of the optimum threshold value is recurrently a tough function. While a number of accesses for automatic threshold determination have been suggested over past several decades, executes new ideas and concepts to image thresholding rest an impressive and challenging research area.

Elegant surveys on early thresholding methods can be found in [22, 26, 27]. Glassbey [8] and Lee et al. [12] expressed the studies of the comparative performance of global thresholding techniques for image segmentation. Otsu [17] presented an automatic threshold method for image segmentation. The automatic threshold section is granted in [11], it is based on image statistics which can be

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derived without histogramming. Rosin [21] conferred a technique for image segmentation that is explicitly designed to endure with substantially unimodel distributions rather than the more frequently bimodel or multimodel distribution. The binarization of document images is separation of foreground from background, i.e. the foreground text in black and noisy background in white. Different types of binarization methods such as Niblack, Sauvola, Feng and Wolf have been estimated in [16, 23, 6, 33] for distinct types of document images.

Several methods have been presented for determining the threshold t of an image [18, 19]. In spite of, considering the fuzzy set theory [9, 34] has treated well in the analysis of models that commence ambiguity and highly noisy data, this theory is a desirable alternative for determining the best threshold, in order to attain a good segmentation of the image considered [2, 3, 4]. Within the framework of this theory, the most famous algorithms are those that use the thought of fuzzy entropy [7]. In [25] discussing the optimal thresholding using fuzzy entropy for image enhancement and compared with bi-level and multi-level thresholding.

Fuzzy rules for locating the intensities of the disorganization regions were expressed by M. Seetharaman Prasad et al. in [24]. Tizhoosh [30] reported a new thresholding technique, which actions thresholding as type II fuzzy sets. Fuzzy measure to threshold the image histogram was narrated by Tobias and Seara in [31]. Moallem et al. [14] presented the fuzzy inference system based potato color image segmentation. The intuitionistic fuzzy C means algorithm for brain image segmentation compared with fuzzy C means algorithm in [20]. The intuitionistic fuzzy set approach for gray image segmentation presented in [15].

Image thresholding is a very crucial problem in computer vision. Even if many years of research, the generalized result of image segmentation is still a very difficult work because segmentation is naturally ill-posed. Through the distinct segmentation methods, graph theoretical approaches have several good appearances in practical applications. It formulates the image elements into mathematically well described structures, making the formation of image segmentation problem more fascinating and the computation more valuable. Graph partitioning methods can effectually be used for image thresholding [1].

Felzenszwalb [5] defined the boundary between two regions using a graph based representation of the image. The feature vectors of image classification are color, shape and texture. Jiang and coenen [10] introduced a frame of reference for graph based image segmentation using these structural details. In [13] presented a graph-cut-based interactive segmentation method with a randomized texton-searching algorithm. Wenbing Tao [29] constructed thresholding algorithm to improve image segmentation execution at low computational cost. The presented algorithm uses a normalized graph cuts measures as the thresholding ethic to separate an object from the background. In [32] S. Wang and J. M. Siskind expressed a new cost function, ratio cut, for separating images using graph based methods. Jianbo Shi et al. [28] considered the image segmentation as a graph partitioning problem and suggested a novel global form, the normalized cut, for segmenting the graph.

In general, image segmentation techniques based on graph cuts require high computational complexity and contribute poor real-time performance. Therefore, in many image segmentation problems may not be worked out practically. In this paper, we use a fuzzy normalized graph cut measure as the thresholding principle to separate objects from the background. Related to the existing techniques, the suggested method formulates a fuzzy weighted graph by treating each pixel as a vertex and joining each pair of pixels by an edge. The fuzzy weight edge has a value zero, it means no link between that pixels. Otherwise a fuzzy relation exists between them. In particular, we represent fuzzy weight edge has a value 0.5, it means either pixel belongs to the background or to the object. For this reason we use fuzzy concepts. Membership function can take various types depending on the application. Here we construct this fuzzy weight edge using standard S membership function.

The purpose of this paper is to advance a simple and effective thresholding approach with extremely reduced computational cost using fuzzy sets. Such cost reduction is obtained by representing a graph using 256×256 symmetrical fuzzy matrix based on gray levels rather than the $I \times I$ symmetrical fuzzy matrix based on pixels, where I is the number of pixels in the image. Because I is much larger than 256, the size of the fuzzy weight matrix based on the gray levels is much smaller than $I \times I$ and thresholding value is obtained by representing a graph using a symmetrical fuzzy matrix based on gray levels and their corresponding pixel's membership values. As a consequence, it becomes workable to quickly attain graph cut values for every accessible threshold t from this fuzzy weight matrix.

The rest of this paper is organized as follows: In section 2, an improved thresholding method based on normalized graph cut using standard S membership function and procedure for finding the thresholding value in a given image X is presented. The performance estimation of the proposed thresholding method using a variety of gray images and comparison with other existing methods is evaluated in section 3. In section 4, we examined the effect of the parameter changes and the computational cost is compared to other graph-cut based image segmentation techniques. Finally, the conclusions of the work are presented in section 5.

2. Thresholding Method Based on Fuzzy Normalized Graph Cut

2.1. **Proposed Approach.** In this section, we prefer an improved approach which is represented by the pursuing steps:

i) An image X of $I \times J$ dimensions and L denote the gray levels of the image X. i.e. $L = \{0, 1, ..., 255\}$. Let V denote the collection of pixels of the image X. i.e. V=$\{(i,j)$: i = 0, 1, ..., X_h-1; j= 0, 1, ..., X_w-1$\}$, where X_h and X_w are the height and width of the image X, respectively. Let (i,j) denote the pixel location and the corresponding pixel's gray level value denoted by $x(i,j)$. Then V and $x(i,j)$ satisfy

$$x(i,j) \in L \quad \forall (i,j) \in V. \tag{1}$$

$$V_k = \{(l,m) : x(l,m) = k, (l,m) \in V\}, \quad k \in L \tag{2}$$

$$\bigcup_{k=0}^{255} V_k = V, \quad V_n \cap V_k = \emptyset, \quad k \neq n, \quad k,n \in L. \tag{3}$$

ii) A fuzzy subset of a set S is a mapping $\mu : S \to [0,1]$. For any pixel (i,j) belonging to S, $\mu_x(x(i,j))$ or $\mu_x(x_{ij})$ is called the degree of membership of (i,j) in S. We construct the standard S membership values of each set of pixel V_k $(k \in L)$ of the given image X, using the equation (4).

$$\mu_x(x_{ij}) = S(x_{ij}, a, b, c) = \begin{cases} 0 & if \;\; x_{ij} \leq a, \\ \frac{1}{2}\left(\frac{x_{ij}-a}{b-a}\right)^2 & if \;\; a < x_{ij} \leq b, \\ 1 - \frac{1}{2}\left(\frac{x_{ij}-c}{c-b}\right)^2 & if \;\; b < x_{ij} \leq c, \\ 1 & if \;\; x_{ij} \geq c. \end{cases} \tag{4}$$

Where 'a' and 'c' are min and max value of the set V_k respectively, and 'b' vary between 0 and L-1.

iii) Construct the fuzzy weighted graph G=(V, E) by taking each pixel as a vertex(V) and joining each pair of pixels by an edge(E). The fuzzy weight of an edge should resonate the prospect that the two pixels belong the same subdivision.

iv) Using the brightness of the pixels, spatial location and their membership value of the pixels, we can define the fuzzy weight of the graph edge joining two vertices u and v as (5), where F(u), X(u) and M(u) are the gray scale, spatial location and membership value of vertex u, respectively, and $\|.\|_2$ denotes the vector norm. d_I, d_X and d_M are positive scaling factors that determine the sensitivity of w(u, v) to the intensity difference, spatial location between two vertices and membership value between two vertices, respectively, and r is a positive integer that indicate the number of adjacent vertices involved the fuzzy weight computations.

$$w(u,v) = \begin{cases} e^{-\left[\frac{\|F(u)-F(v)\|_2^2}{d_I} + \frac{\|X(u)-X(v)\|_2^2}{d_X} + \frac{\|M(u)-M(v)\|_2^2}{d_M}\right]}, & if \; \|X(u)-X(v)\|_2 < r \\ 0, & otherwise \end{cases} \tag{5}$$

v) For any t $(0 \leq t \leq 255)$, we can attain a unique bipartition V=(A, B) of the corresponding graph G=(V, E), where sets A and B can be generated as

$$A = \bigcup_{k=0}^{t} V_k, \quad B = \bigcup_{k=t+1}^{255} V_k, \quad k \in L. \tag{6}$$

Then,

$$\begin{aligned} cut(A,B) &= \sum_{u \in A}\left[\sum_{v \in B} w(u,v)\right] \\ &= \sum_{i=0}^{t}\left[\sum_{j=t+1}^{255}\sum_{v \in V_j} w(u,v)\right] \\ &= \sum_{i=0}^{t}\sum_{t+1}^{255}\left[\sum_{u \in V_i, v \in V_j} w(u,v)\right] \\ &= \sum_{i=0}^{t}\sum_{t+1}^{255} cut(V_i, V_j) \end{aligned} \tag{7}$$

where

$$cut(V_i, V_j) = \sum_{u \in V_i, v \in V_j} w(u, v) \tag{8}$$

is the total collection between all vertices in V_i (whose gray level is i) and all vertices in V_j (whose gray level is j). Similarly

$$asso(A, A) = \sum_{u \in A, v \in A} w(u, v) = \sum_{i=0}^{t} \sum_{j=i}^{t} \left[\sum_{u \in V_i, v \in V_j} w(u, v) \right] = \sum_{i=0}^{t} \sum_{j=i}^{t} cut(V_i, V_j) \tag{9}$$

$$asso(B, B) = \sum_{u \in B, v \in B} w(u, v) = \sum_{i=t+1}^{255} \sum_{j=i}^{255} \left[\sum_{u \in V_i, v \in V_j} w(u, v) \right] = \sum_{i=t+1}^{255} \sum_{j=i}^{255} cut(V_i, V_j) \tag{10}$$

Note that the following relation hold:

$$asso(A, V) = asso(A, A) + cut(A, B) \tag{11}$$

$$asso(B, V) = asso(B, B) + cut(A, B) \tag{12}$$

Therefore,

$$Ncut(A, B) = \frac{cut(A, B)}{asso(A, A) + cut(A, B)} + \frac{cut(A, B)}{asso(B, B) + cut(A, B)} \tag{13}$$

vi) Let N be the 256×256 symmetrical matrix with elements $n_{i,j} = n_{j,i} = cut(V_i, V_j)$. Only the elements in the upper triangle are shown in Figure 1 due to symmetry, $N = [n_{i,j}]_{256 \times 256}$ can be uniquely derived from a given image by measuring all the fuzzy weights on the edges joining each pair of pixels in the image.

Illustration of symmentrical matrix $N = [i, j]_{256 \times 256}$ with $n_{i,j} = n_{j,i}$ is given below:

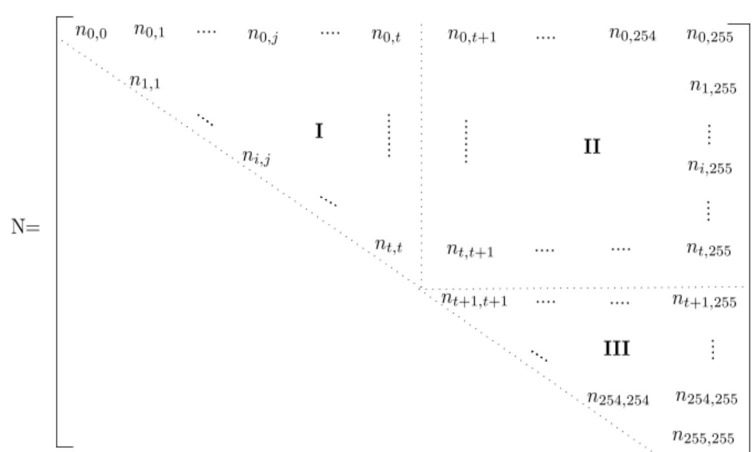


Figure 1. Graphical Representation of Symmetrical Matrix N

From matrix N, we measure $cut(A,B)$, $asso(A,A)$ and $asso(B,B)$ of the corresponding bipartition of the fuzzy weight graph for each threshold t. As presented in Figure 1, the elements of matrix N are segmented into three sections. First, the sum of all the elements in section I is the value of $asso(A,A)$. Second, the sum of all the elements in section II is the values of $cut(A,B)$. Finally, the sum of all the elements in section III gives the values of $asso(B,B)$. In that event, the fuzzy normalized cuts can now be easily measured from N for every possible threshold t.

The suggested threshold technique searches the optimal threshold value, minimizing the corresponding fuzzy normalized graph cuts of the image. The performance algorithm is presented in the subdivision 2.2, where T is the optimum threshold, $0 \leq t \leq 255$ is a variable threshold, and $ncut_{min}$ is the minimum value of the fuzzy normalized graph cuts.

2.2. **Performance of the Proposed Algorithm.** The theoretical discussions planned above section (2.1) enable us to write following general algorithm for calculating the threshold of an image.

Step I: Starts t=0, fuzzy $Ncut_{min} = 2$, Define the aspect explanation matrix for a given image and a fuzzy weighting function.

Step II: Build a fuzzy weighted graph G=(V, E) and measure the edge weights, i.e., the elements of matrix N.

Step III: From matrix N, figure out fuzzy Ncut of the bipartition $V = \{A, B\}$ corresponding threshold t.

Step IV: Determine if the resulting fuzzy Ncut is less than fuzzy $Ncut_{min}$. If $Ncut(A,B) < Ncut_{min}$. let $Ncut_{min} = Ncut(A,B)$ and T=t.

Step V: Go to next step if step IV is satisfied. Untill repeat the process step III for $t <= 255$.

Step VI: Threshold the image using the optimal threshold value T.

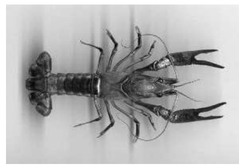
3. Performance Estimation and Comparison among Fuzzy Entropy, Kittler, Rosin, Sauvola, Wolf and Proposed Algorithm

A set of images are used to measure the implementation of the suggested algorithm along with some of the frequently used algorithms given in the literature. Performance estimation and comparisons are executed by using real images, where the object can be exactly separated from the background using some appropriate threshold method. Used in all illustrations in this section, the parameters used in equation (5) are set to $d_I = 625$, $d_X = 4$, $d_M = 1$, and r=2.

The examples are shown in Figures 2-6. Each figure shows the original gray-level image, the optimally threshold image, the histogram of the image, the values of fuzzy Ncut as a function of threshold t, and the threshold images using the preferred along with other methods used in the comparison. The gray scale images Crayfish (223 × 300), Joshua tree (300 × 268), Ship (300 × 291), Airplane (399 × 217), Brontosaurus (224 × 300) are used for comparison with other methods. The optimum threshold value for each image is selected as the one interrelated to the smallest value of fuzzy Ncut.

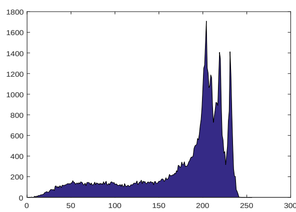
The major comparison norm is the absolute error ratio. The absolute error is defined as the absolute difference in the number of object pixels between the optimum thresholded image and the thresholded image attained by each method. The absolute error ratio is resolved as the ratio between the absolute error n_{diff} and the total number of pixels M of an image, i.e.,

$$r_{err} = \frac{n_{diff}}{M} \times 100\% \tag{14}$$

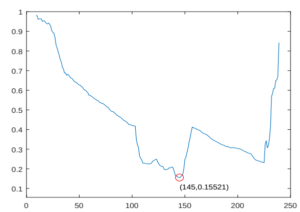


(A) Original Image (Crayfish-223×300)

(B) Thresholding Result by the Proposed Method (T=145)



(C) Histogram of the Original Image



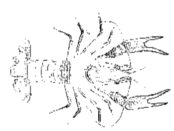
(D) Plot of Fuzzy Ncut Versus Threshold t

(E) Fuzzy Entropy (T=176)

(F) Manually Thresholding (T=157)

(G) Kittler (T=163)

(H) Rosin (T=212)



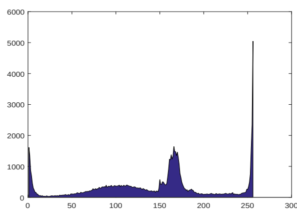
(I) Sauvola (T=35)

(J) Wolf (T=28)

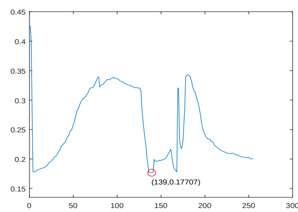
Figure 2. Performance Comparison for Crayfish Image

(A) Original Image (Joshua tree- 300×268)

(B) Thresholding Result by the Proposed Method (T=139)



(C) Histogram of the Original Image



(D) Plot of Fuzzy Ncut Versus Threshold t

(E) Fuzzy Entropy (T=153)

(F) Manually Thresholding (T=145)

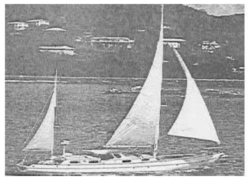
(G) Kittler (T=251)

(H) Rosin (T=231)

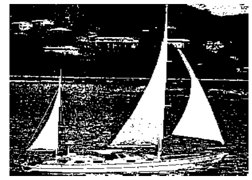
(I) Sauvola (T=66)

(J) Wolf (T=48)

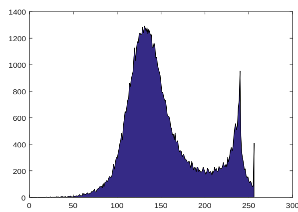
Figure 3. Performance Comparison for Joshua Tree Image



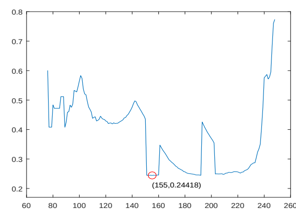
(A) Original Image (Ship- 300 × 291)



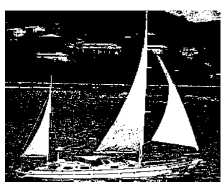
(B) Thresholding Result by the Proposed Method (T=155)



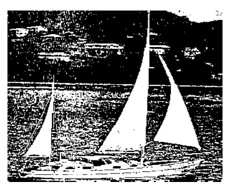
(C) Histogram of the Original Image



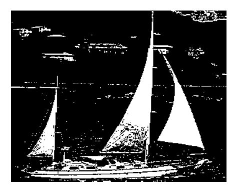
(D) Plot of Fuzzy Ncut Versus Threshold t



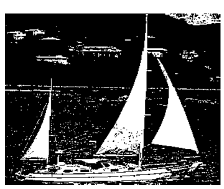
(E) Fuzzy Entropy (T=178)



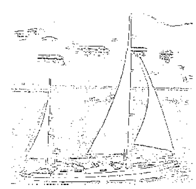
(F) Manually Thresholding (T=153)



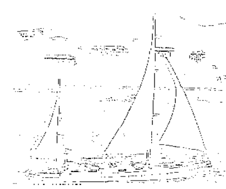
(G) Kittler (T=200)



(H) Rosin (T=184)



(I) Sauvola (T=97)

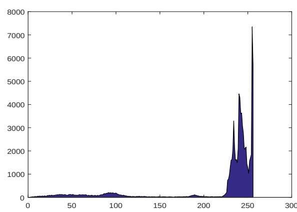


(J) Wolf (T=94)

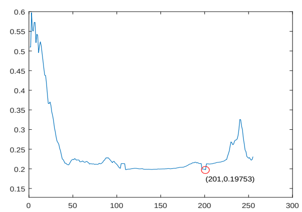
FIGURE 4. Performance Comparison for Ship Image

(A) Original Image (Airplane- 399 × 217)

(B) Thresholding Result by the Proposed Method (T=201)



(C) Histogram of the Original Image



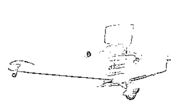
(D) Plot of Fuzzy Ncut Versus Threshold t

(E) Fuzzy Entropy (T=222)

(F) Manually Thresholding (T=217)

(G) Kittler (T=212)

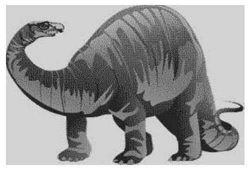
(H) Rosin (T=255)



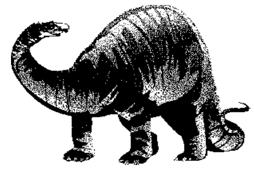
(I) Sauvola (T=45)

(J) Wolf (T=40)

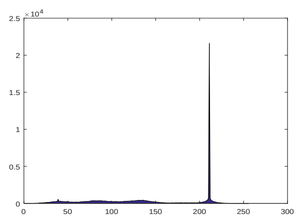
Figure 5. Performance Comparison for Airplane Image



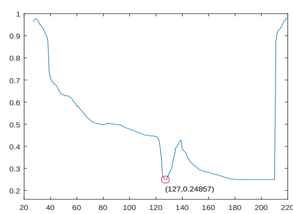
(A) Original Image (Brontosaurus-224 × 300)



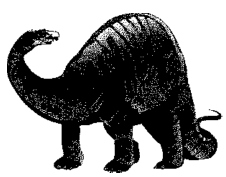
(B) Thresholding Result by the Proposed Method (T=127)



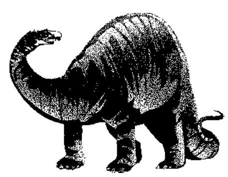
(C) Histogram of the Original Image



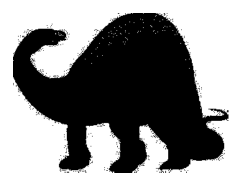
(D) Plot of Fuzzy Ncut Versus Threshold t



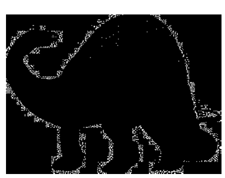
(E) Fuzzy Entropy (T=118)



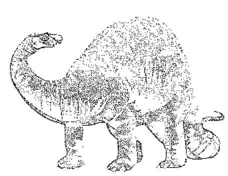
(F) Manually Thresholding (T=131)



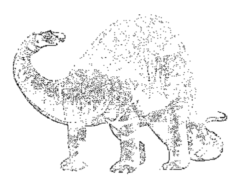
(G) Kittler (T=200)



(H) Rosin (T=212)



(I) Sauvola (T=67)



(J) Wolf (T=54)

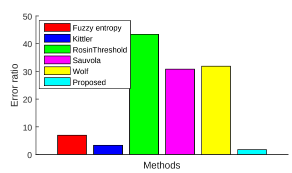
Figure 6. Performance Comparison for Brontosaurus Image

Image	Ref. value	Thresholding Algorithms											
		Fuzzy		Kittler		Rosin		Sauvola		Wolf		Proposed	
	T0	T	Error	T	Error	T	Error	T	Error	T	Error	T	Error
Crayfish	157	176	6.9625	163	3.3333	212	43.35	35	30.807	28	31.881	145	**1.768**
Joshua-tree	145	153	1.7813	251	44.861	231	40.801	66	36.307	48	37.985	139	**1.434**
Ship	153	178	12.879	200	18.446	184	14.644	97	59.19	94	60.86	155	**1.621**
Airplane	217	222	**0.136**	212	0.154	255	83.328	45	15.397	40	15.87	201	0.476
Brontosaurus	131	118	6.9747	200	14.685	212	52.707	67	33.409	54	89.89	127	**1.866**

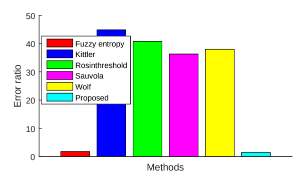
Table 1. Error Comparison for Fuzzy Entropy, Kittler, Rosin, Sauvola, Wolf and Proposed Algorithm

The variations from the optimal threshold value are shown in Table 1 for different methods. In Table 1, the error indicates the numerical values corresponding, in each case, to the similarity among the manual thresholding(T0) determined from the associated fuzzy entropy based image thresholding [25], Kittler [11], Rosinthreshold [21] and some adaptive binarization techniques like Sauvola [23] and Wolf [33] method. Then we choose from a set of thresholds of an image the best one. We have noticed in bold type the lowest error value. Please note that in all of the cases, leaving out for the fourth case, the lowest error is that of the numerical value given by the fuzzy entropy method. It is noticeable from Table 1 and Figure 7 reveals that the proposed algorithm gives improved segmentation performance than the other methods.

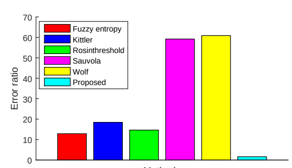
We would specify that the best threshold collection achieved by proposing a method does not regularly sync with the image that visibly separates background from the object best. At the time that, the finding of the best threshold was very nonobjective and depends in each case on the application we are operating on. For this reason, in future, we must develop our algorithms with suitable techniques that will implement us to fit the algorithms to the interrelated application.



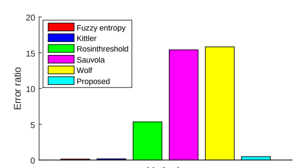
(A) Crayfish



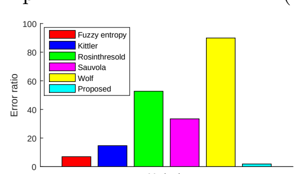
(B) Joshua Tree



(C) Ship



(D) Airplane

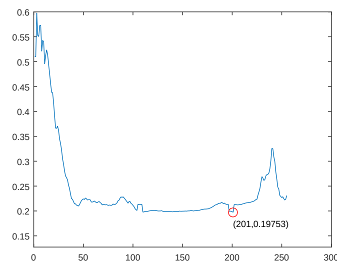


(E) Brontosaurus

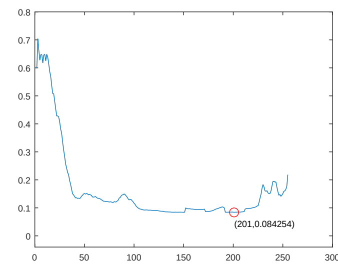
Figure 7. Error Ratio Comparison

4. **Results and Discussion**

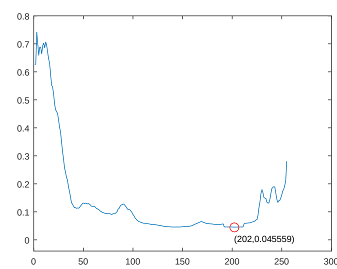
4.1. **Effect of the Parameters.** The fuzzy weight matrix of a graph edge joining two vertices is concerned by a number of parameters that must be relevantly determined. As presented in equation (5), these parameters include d_I, d_X, d_M, and r, where d_I controls the outcome of grayscale difference between two vertices to the fuzzy weight, and d_X controls the outcome of spatial position difference between two vertices to the fuzzy weight and d_M controls the outcome of membership value difference between the two vertices to the fuzzy weight. An appropriate pair of parameters d_I, d_X and d_M can be chosen to combine the gray, spatial features and membership values of the pixels to effectual threshold an image. The parameter r resolves the number of neighboring vertices involved the computations. A large value of r more perfectly exposes the relationship between the vertices in a graph at the consumption of higher computational cost in the measure of fuzzy weight matrix N. Therefore, we must take the value of r as small as possible to present suitable segmentation results. But a larger value of r is needed when d_X is large.



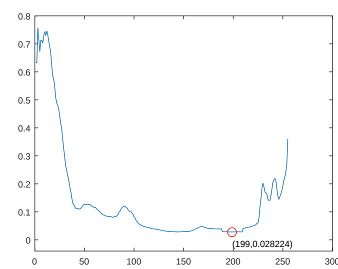
(A) r=2, $d_I = 625$, $d_X = 4$, $d_M = 1$



(B) r=4, $d_I = 625$, $d_X = 8$, $d_M = 1$



(C) r=8, $d_I = 625$, $d_X = 16$, $d_M = 1$

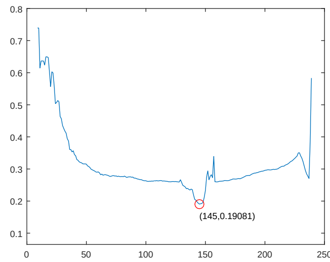


(D) r=16, $d_I = 625$, $d_X = 32$, $d_M = 1$

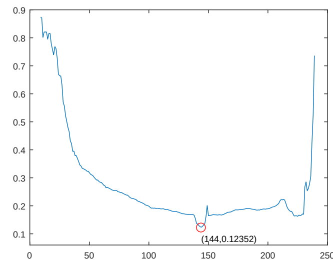
FIGURE 8. Plots of Fuzzy Ncut of the Airplane Image as a Function of Threshold t Using Different Parameter Settings

Figure 8 indicates the fuzzy ncut for the Airplane image corresponding to different values of d_X and r. The plot shows uniform, and minimum valley point becomes more distinguished as d_X, r increased and d_I, d_M are fixed.

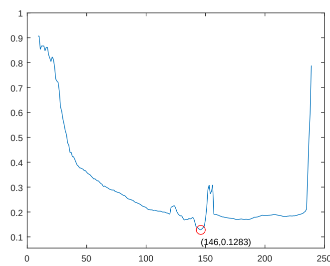
Figure 9 reveals the values of fuzzy ncut with respect to the threshold t for the Crayfish image. We establish that the threshold result attained by the proposed method is incurious to change in r, d_I and d_X. We also noticed that the parameter value of d_I increases as the values of d_X and r increase. However, the parameter value in some application, we manage that the proposed method is appropriately hard to parameters d_I, d_X, d_M, and r because the parameter value of d_I is usually much higher examined with the value of d_X.



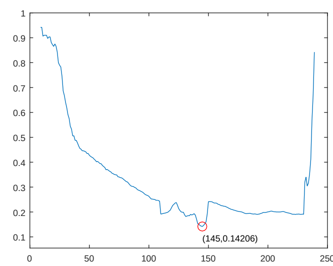
(A) r=2, $d_I = 1000$, $d_X = 4$, $d_M = 1$



(B) r=4, $d_I = 2000$, $d_X = 8$, $d_M = 1$



(C) r=6, $d_I = 3000$, $d_X = 10$, $d_M = 1$



(D) r=8, $d_I = 5000$, $d_X = 16$, $d_M = 1$

FIGURE 9. Plots of Fuzzy Ncut of the Crayfish Image as a Function of Threshold t Using Different Parameter Settings

4.2. **Computational Cost.** The proposed algorithm consists of two sections for image thresholding. The first section T_1 is to construct the weight matrix N using fuzzy sets, and the second section T_2 is to calculate the value of fuzzy Ncut for every possible threshold t $(0 \leq t \leq 255)$ from a fuzzy weight matrix. The main advantage is to reduce the dimension of the fuzzy matrix from $I \times I$ to $L \times L$, where L is the gray level of an image and is much smaller than the number of pixels in image X. In addition, we derive the computation complexity between the fuzzy weight matrix in equation (5) and weight matrix from [29].

r	**Fuzzy weight matrix** (Fuzzification)		**Weight matrix** (Classical graph cut)	
	t/s	Sum value	t/s	Sum value
r = 2	23.0299	6.1949e+04	871.7534	6.1957e+04
r = 4	23.0013	1.1288e+05	890.6068	1.1290e+05
r = 6	22.6406	1.1566e+05	881.8728	1.1567e+05
r = 8	22.9243	1.1567e+05	885.4553	1.1569e+05
r = 10	22.7396	1.1568e+05	873.6602	1.1570e+05

TABLE 2. Comparison Result of Airplane Image for Different Values of r

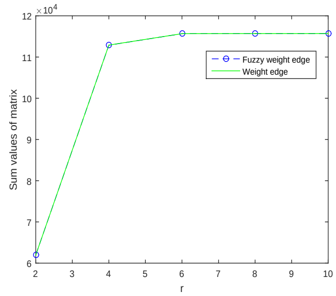


FIGURE 10. Plot Results for Different Values of r Between Fuzzy Weight (fuzzification) Edge Values and Weight (classical) Edge Values of Airplane Image

S. No	Parameters	Elasped time	Sum values
1	r=2, dI=625, dX=4, dM=1	23.0299	6.1949e+04
2	r=4, dI=625, dX=8, dM=1	23.1345	1.9078e+05
3	r=8, dI=625, dX=16, dM=1	23.4210	4.2677e+05
4	r=16, dI=625, dX=32, dM=1	23.5138	8.2906e+05
5	r=32, dI=625, dX=64, dM=1	26.0803	1.5584e+06

TABLE 3. Fuzzy Weight (fuzzification) Matrix Results for Different Values of r and d_X of Airplane Image

S. No	Parameters	Elasped time	Sum values
1	r=2, dI=625, dX=4	871.7534	6.1957e+04
2	r=4, dI=625, dX=8	874.9828	1.9081e+05
3	r=8, dI=625, dX=16	872.0198	4.2689e+05
4	r=16, dI=625, dX=32	871.2699	8.2934e+05
5	r=32, dI=625, dX=64	893.5681	1.5591e+06

TABLE 4. Weight (classical) Matrix Results for Different Values of r and d_X of Airplane Image

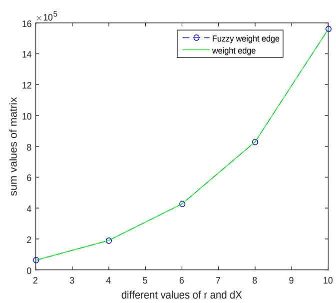


FIGURE 11. Plot Results for Different Values of r and d_X Between Fuzzy Weight (fuzzification) Edge Values and Weight (classical) Edge Values of Airplane Image

The Tables (2-4) exhibit the corresponding execution times versus the fuzzy weight matrix and weight matrix of the Airplane image, respectively. An intel core i3-2330m personal computer with a 2.2 GHz CPU is used. Here, we calculated fuzzy weight matrix and weight matrix for different values of r. i.e (r=2, 4, 6, 8, 10) and different values of d_X. i.e (d_X=4, 8, 16, 32, 64). From this comparison, the Figure 10 and Figure 11 shows that the fuzzy weight edge sums value and weight edge sums value are almost coincided, but the minimum time duration getting only in our fuzzy weight matrix in all the executions compared to weight matrix. Similarly, we will get the same results to all other images. Therefore, the advantage of the proposed algorithm is detaily analyzed.

5. Conclusion

In this research article, we have proposed a thresholding method using the normalized graph cut measure based on the standard S membership function. As a result of the compact and stable size of fuzzy weight matrix, we easily attained the fuzzy normalized graph cut values for all feasible thresholding values ($0 \leq t \leq 255$) and measured the optimum threshold(T) result. Further, we separated an object from the background by the use of the fuzzy normalized graph cut measure as the thresholding principle. Also, in most cases, the proposed algorithm for the gray images showed the better experimental results with respect to those compared with other methods. Numerical results revealed that the proposed algorithm requires significantly less computational cost, such cost reduction is achieved by constructing a new fuzzy weight matrix based on a fuzzy concept and gray levels instead of pixels.

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