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ORIGINAL RESEARCH PAPER

Axisymmetric Magnetohydrodynamic Squeezing flow of Nanofluid in Porous Medium under the influence of Slip Boundary Condition

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ABSTRACT: The various industrial, biological and engineering applications of squeezing flow have been the impetus for the renewed interests in the studies of fluid flow between parallel plates or disks. As a part of the renewed interests, this paper presents the study of axisymmetric magnetohydrodynamic squeezing flow of nanofluid in porous media under the influence of slip boundary condition using differential transformation method. Good agreements are established when the results of the differential transformation method are compared with the results of numerical method Runge-Kutta coupled with shooting method. Also, the analytical solution is used to investigate the effects of porous medium, magnetic field and slip boundary on the steady two-dimensional axisymmetric flow of the nanofluid. It is shown from the results that the velocity of the fluid increases as the magnetic field and porous parameters increase under slip condition while the velocity of the fluid decreases with increase in the magnetic field and porous parameter under no slip condition. By increasing the slip parameter, the velocity of the fluid increases while the velocity of the fluid decreases as the Reynolds number increases. Studies on nanofluidics such as energy conservation, friction reduction and micro mixing biological samples can be enhanced and better understood by the insights given in this present study.

KEYWORDS*: Magnetic field; Nanofluid; Porous media; Slip boundary condition; Squeezing flow*

INTRODUCTION

The increasing industrial, biological and engineering applications of flow of fluid between parallel plates have continued to generate renewed research interests in fluid dynamic analysis. Such types of flows are evident in moving pistons, chocolate fillers, hydraulic lifts, electric motors, flow inside syringes and nasogastric tubes, compression, and injection, power transmission squeezed film and polymers processing. Moreover, following the pioneer work and the basic formulations of the squeezing flows under lubrication assumption by Stefan [1], there have been increasing research interests and many scientific studies on the analysis of squeezing flow. However, the applications of Reynolds equation for the squeezing flow analysis in earlier studies [1-3] led the studies to insufficient and inaccurate analyses as shown by Jackson [4] and Usha and Sridharan [5]. Consequently, further studies have been presented in recent times to give a better insight into the flow phenomena. Among these study, Usha and Sridharan [5] investigated the arbitrary squeezing flow of a viscous fluid between elliptic plates while in an earlier work, Yang [6] considered unsteady laminar boundary layers in an incompressible stagnation flow. In a group of research studies, Kuzma [7], Tichy and Winer [8] and Grimm [9] examined the effects of fluid inertial on squeezing flow.

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Different numerical and approximate analytical methods have been used to analyze the squeezing flow [10-26]. Also, further parametric and methods studies have been presented in literature. Additionally, studies on magnetohydrodynamic flows of fluids through porous medium have also received considerable research interests in the past few years due to its various applications in the study of ground water flow, irrigation problems, crude petroleum recovery, heat-storage beds, thermal and insulating engineering, chromatograph, and chemical catalytic reactors. Siddiqui et al. [27] studied an unsteady squeezing flow of viscous magnetohydrodynamics fluid between parallel plates. With the aids of homotopy perturbation method, Domairry and Aziz [28] presented approximate analysis of magnetohydrodynamic (MHD) squeeze flow between two parallel disk with suction or injection. In a recent work, Acharya et al. [29] analyzed the squeezing flow of Cu-water and Cu-kerosene nanofluid between two parallel plates while Ahmed et al. [30] examined magnetohydrodynamic squeezing flow of a Casson fluid between parallel disks. In another work, Ahmed et al. [31] explored the MHD Flow of a dusty incompressible fluid between dilating and squeezing porous walls. Khan [32] presented a further study on unsteady two-dimensional and axisymmetric squeezing flow between parallel plates. In an earlier work, Khan et al. [33] analyzed MHD squeezing flow between two infinite plates.

Nomenclature

b	induced magnetic fields	T	Cauchy stress tensor
B	total magnetic field	V	velocity
BO	imposed magnetic fields		**Abbreviations**
E	electric field	HAM	Homotopy analysis method
f	body force	DTM	Differential transformation method
h	half distance between two plates	HPM	Homotopy perturbation method
J	electric current density	ADM	Adomian Decomposition method
p	pressure	VIM	Variation iteration method
r	radius	OHAM	Optimal homotopy asymptotic method

Hayat et al. [34] focused their study on MHD squeezing flow of second grade fluid between parallel disks and the MHD effects on unsteady squeezing flow of Casson and viscous fluids passing through porous medium was analyzed by Khan [35] and Ullah [36], respectively. The impacts of magnetic field on the squeezing flow have been presented in some further studies [37-41].

The effects of magnetic field, flow characteristics and fluid properties on the squeezing flow under have been analyzed in the above reviewed studies but they are based on the assumptions of no slip conditions. However, in polymeric liquids, there is slip at the boundary when the weight of molecule is high. Indisputably, the no-slip boundary condition is not applicable in the flow analysis of such liquid. Additionally, in many cases such as thin film problems, nanofluids, rarefied fluid problems, fluids containing concentrated suspensions, and flow on multiple interfaces, the no-slip boundary condition fails to work. Therefore, Navier [42] proposed the general boundary condition which demonstrates the fluid slip at the surface. The slip condition is of great importance especially when fluids with elastic character are under consideration [43]. Under the studies of the slip effects on the flow process, Ebaid [44] investigated the effects of magnetic field and wall slip conditions on the peristaltic transport in an asymmetric channel. Hayat et al. [45] analyzed the influence of slip on the peristaltic motion of third-order fluid in asymmetric channel. In another study, Hayat and Abelman [46] examined the effects of slip condition on the rotating flow of a third grade fluid in a nonporous medium. Abelman et al. [47] presented an extended work of Hayat and Abelman [46] by considering a flow in a porous medium and obtaining the numerical solutions for the steady magnetohydrodynamics flow of a third grade fluid in a rotating frame. Ullah et al. [48] presented approximation of first grade MHD squeezing fluid flow with slip boundary condition. The reviewed studies above are based on viscous fluids. To the best of the authors' knowledge, the study of squeezing flow under the influences of magnetic field and hydrodynamic slip boundary condition has not been extended to the flow nanofluid through a porous medium. Therefore, in this work, axisymmetric magnetohydrodynamic squeezing flow of nanofluid in porous media under the influence of slip boundary condition using differential transformation method. A further investigation is carried out to analyze the applications and limitations of the DTM to the fluid problem when the fluid flow situation.

PROBLEM FORMULATION

Consider axisymmetric squeezing flow of nanofluid in porous media between two large cylindrical plates separated by a small distance 2h approaching each other with a low constant velocity V in the presence of a magnetic field, as shown in Figure 1.

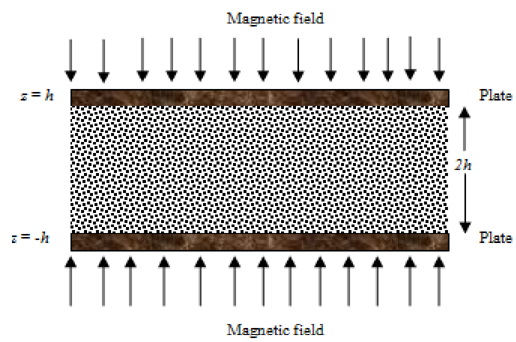

Fig. 1. Model of the squeezing flow of nanofluid under transverse uniform magnetic field

It is assumed that the flow of the nanofluid is laminar, stable, incompressible, isothermal, non-reacting chemically, the nano-particles and base fluid are in thermal equilibrium and the physical properties are constant. The fluid conducts electrical energy as it flows unsteadily under magnetic force field. The fluid structure is everywhere in thermodynamic equilibrium and the plate is maintained at constant temperature. Following the assumptions, the governing equations of motion of the flow are given as:

Continuity equation

$$\nabla . \mathrm{v} = 0 \tag{1}$$

Momentum equation

$$\rho_{nf}\left[\frac{\partial \bar{v}}{\partial t} + \left(\nabla . \bar{v}\right)\bar{v}\right] = \rho_{nf}\, f - \nabla . p + \mu_{nf}\nabla^2 \bar{v} - \frac{\mu_{nf}\bar{v}}{k} - \sigma \mathrm{B}_0^2 \bar{v} \tag{2}$$

where

$$\rho_{nf} = \rho_f \left(1-\phi\right) + \rho_s \phi$$

$$\mu_{nf} = \frac{\mu_f}{\left(1-\phi\right)^{2.5}}$$

For axial symmetry, ν is represented by ν=(ν_r,0,ν_z), neglecting the body force, the Navier-Stokes equation in cylindrical coordinates are given by [1, 6, and 10]:

Continuity equation

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \qquad (3)$$

Momentum equation

$$-\rho_{nf}\left(\bar{v}\times\bar{w}\right) + \bar{\nabla}\left(\frac{\rho_{nf}}{2} + p\right) = \mu_{nf}\bar{\nabla}\times\bar{w} - \frac{\mu_{nf}}{k}\bar{v} - \sigma B_0^2 \bar{v} \qquad (4)$$

where

$$\Omega\left(r,z\right) = -\frac{1}{r}E^2\psi \qquad (5)$$

If we introduce the stream function Ψ(r,z), we have

$$\bar{v}_r = \frac{1}{r}\frac{\partial\psi}{\partial z},\ \bar{v}_z = -\frac{1}{r}\frac{\partial\psi}{\partial r}. \qquad (6)$$

On eliminating the pressure term from Equations 3 and 4, we arrived at

$$\rho_{nf}\left[\frac{\partial\left(\psi, E^2\psi / r^2\right)}{\partial\left(r,z\right)}\right] = -\frac{\mu_{nf}}{r}E^4\psi + \frac{1}{r}\left(\frac{\mu_{nf}}{k} + \sigma B_0^2\right)\frac{\partial^2\psi}{\partial z^2} \qquad (7)$$

where

$$E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \qquad (8)$$

If we define the stream function as $P = \frac{\rho nf}{2}(\bar{v}_r^2 + \bar{v}_z^2)$, the compatibility Equation 7 reduces to Equation 9

$$f^{(iv)}\left(z\right) - \left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu_{nf}}\right) f''\left(z\right) + \frac{2\rho_{nf}}{\mu_{nf}} f\left(z\right) f'''\left(z\right) = 0 \qquad (9)$$

And the slip boundary conditions become

$$f\left(0\right) = 0,\ f''\left(0\right) = 0,$$
$$f\left(h\right) = \frac{v}{2},\ f'\left(h\right) = \beta f''\left(h\right) \qquad (10)$$

Applying the following dimensionless parameters in Equation 11

$$F^* = \frac{f}{v/2},\ z^* = \frac{z}{h},\ R = \frac{\rho_f H v}{\mu_f},$$
$$G = h\sqrt{\left(\frac{1}{k} + \frac{\sigma B_0^2}{\mu_{nf}}\right)} = \sqrt{\left(Da + m^2\right)} \qquad (11)$$

we can then write Equation 9 and Equation 10 as

$$F^{(iv)}\left(z\right) + R\left(\left(1-\phi\right) + \phi\frac{\rho_s}{\rho_f}\right)\left(1-\phi\right)^{2.5}$$
$$F\left(z\right)F'''\left(z\right) - G^2 F''\left(z\right) = 0 \qquad (12)$$

and the boundary conditions as

$$z = H,\quad \nu_r = 0\ \ and\ \nu_z = -V$$
$$z = 0, \nu_z = 0\ \ and\ \frac{\partial \nu_r}{\partial z} = 0 \qquad (13)$$

It should be noted that the asterisk (*) is omitted from Equation 12 and 13 for the sake of conveniences.

Application of the differential transform method to the present problem

The above nonlinear Equation 12 can be solved using differential transformation method as introduced by Zhou [49].

The basic definitions and the operational properties of the method are as follows

If u(t) is analytic in the domain T, then the function u(t) will be differentiated continuously with respect to time *t*.

$$\frac{d^p u(t)}{dt^p} = \varphi(t,p)\ \text{for all}\ t \in T \qquad (14)$$

for t=t_i, then φ(t,p)=φ(t_i,p), where *p* belongs to the set of non-negative integers, denoted as the *p*-domain.
We can therefore write Equation 14 as

$$U(p) = \varphi(t_i, p) = \left[\frac{d^p u(t)}{dt^p}\right]_{t=t_i} \qquad (15)$$

Where U_p is called the spectrum of u(t) at $t=t_i$

Expressing u(t) in Taylor's series as

$$u(t)=\sum_{p}^{\infty}\left[\frac{(t-t_i)^p}{p!}\right]U(p) \qquad (16)$$

where Equation16 is the inverse of U(K) us symbol 'D' denoting the differential transformation process and combining 15 and 16, we have

$$u(t)=\sum_{p=0}^{\infty}\left[\frac{(t-t_i)^p}{p!}\right]U(p)=D^{-1}U(p) \qquad (17)$$

Table 1

Operational properties of differential transformation method.

Coefficient	magnitude	coefficient
1	$u(t)\pm v(t)$	$U(p)\pm V(p)$
2	$\alpha u(t)$	$\alpha U(p)$
3	$\frac{du(t)}{dt}$	$(p+1)\,U(p+1)$
4	$u(t)v(t)$	$\sum_{r=0}^{p}V(r)U(p-r)$
5	$u^m(t)$	$\sum_{r=0}^{p}U^{m-1}(r)U(p-r)$
6	$\frac{d^n u(t)}{dx^n}$	$(p+1)(p+2)\cdots(p+n)U(p+n)$
7	$\sin(\omega t+\alpha)$	$\frac{\omega^p}{\text{p!}}\sin\left(\frac{\pi p}{2!}+\alpha\right)$
8	$\cos(\omega t+\alpha)$	$Z(p)=\frac{\omega^p}{\text{p!}}\cos\left(\frac{\pi p}{2!}+\alpha\right)$

The differential transforms of Equation 12 is given as

$$\begin{aligned}&(k+1)(k+2)(k+3)(k+4)F[k+4]\\ &R\left[(1-\phi)+\phi\frac{\rho_s}{\rho_f}\right](1-\phi)^{2.5}\\ &\left(\sum_{l=0}^{k}(k-l+3)\right.\\ &\left.(k-l+2)(k-l+1)F[l]F[k-l+3]\right)\\ &-Ha^2\left((k+1)(k+2)F[k+2]\right)+=0\end{aligned} \qquad (18)$$

And the differential transforms of the boundary conditions in Equation 13 as

$$\begin{aligned}&\tilde{F}[0]=0,\ \tilde{F}[1]=a,\ \tilde{F}[2]=0,\ \tilde{F}[3]=b,\\ &\sum(k+1)F[k+1]=\gamma\sum(k+1)(k+2)F[k+2]\end{aligned} \qquad (19)$$

where a and b are unknowns to be determined later using the boundary conditions of Equation 13b.

Using Equations 18 and 19, the value of $\tilde{F}(i), i=1,2,3,4,5,...19,20.$ are

$$\begin{aligned}&\tilde{F}[4]=\tilde{F}[6]=\tilde{F}[8]=\tilde{F}[10]=\tilde{F}[12]\\ &=\tilde{F}[14]=\tilde{F}[16]=\tilde{F}[18]=\tilde{F}[20]=0\end{aligned}$$

$$\tilde{F}[5]=\frac{1}{20}\left\{bG^2-abR\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\right\}$$

$$\tilde{F}[7]=\frac{1}{840}\left(\begin{aligned}&bG^4-6b^2R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ &-4abG^2R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ &+3a^2bR^2\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\end{aligned}\right)$$

$$\tilde{F}[9]=\frac{1}{60480}\left(\begin{aligned}&bG^6-72b^2G^2R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ &-9abG^4R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ &+96ab^2R^2\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ &+23a^2bG^2R^2\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ &-15a^3bR^3\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\end{aligned}\right)$$

$$\tilde{F}[11]=\frac{1}{6652800}\left(\begin{array}{l} bG^{8}-414b^{2}G^{4}R\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}-16abG^{6}R\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ +1296b^{3}R^{2}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}+1716ab^{2}Ha^{2}R^{2}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ +86a^{2}bG^{4}R^{2}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}-1446a^{2}b^{2}R^{3}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ -176a^{3}bG^{2}R^{3}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}+105a^{4}bR^{4}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \end{array}\right)$$

$$\tilde{F}[13]=\frac{1}{1037836800}\left(\begin{array}{l} (bG^{10}-1896b^{2}G^{6}R\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}-25abG^{8}R\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ +43848b^{3}G^{2}R^{2}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}+14892ab^{2}G^{4}R^{2}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ +230a^{2}bG^{6}R^{2}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}-66456ab^{3}R^{3}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ -35424a^{2}b^{2}G^{2}R^{3}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}-950a^{3}G^{4}R^{3}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ +23580a^{3}b^{2}R^{4}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5}+1689a^{4}bG^{2}R^{4}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \\ -945a^{5}bR^{5}\left(\left(1-\phi\right)+\phi\frac{\rho_{s}}{\rho_{f}}\right)\left(1-\phi\right)^{2.5} \end{array}\right)$$

$$\tilde{F}[15]=\frac{1}{217945728000}\left(\begin{array}{l} bG^{12}-7974b^2G^8R\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-36abG^{10}R\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +703296b^3G^4R^2\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+98544ab^2G^6R^2\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +505a^2bG^8R^2\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-1362096b^4R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ -2874096ab^3G^2R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-427716a^2b^2G^4R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ -3480a^3bG^6R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+2540304a^2b^3R^4\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +753936a^3b^2G^2R^4\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+12139a^4bG^4R^4\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ -428310a^4b^2R^5\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-19524a^5bG^2R^5\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +10395a^6bR^6\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5} \end{array}\right)$$

$$\tilde{F}[17]=\frac{1}{59281238016000}\left(\begin{array}{l} bG^{14}-32472b^2G^{10}R\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-49abG^{12}R\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +8496576b^3G^6R^2\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+574104ab^2G^8R^2\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +973a^2bG^{10}R^2\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-93783744b^4G^2R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ -61494336ab^3G^4R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-3844560a^2b^2G^6R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ -10045a^3bG^8R^3\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+151160256ab^4R^4\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +137564928a^2b^3G^2R^4\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+12050448a^3b^2G^4R^4\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +57379a^4bG^6R^4\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-90984960a^3b^3R^5\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ -17320920a^4b^2G^2R^5\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-177331a^5bG^4Re^5\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ +8711640a^5b^2R^6\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+264207a^6bG^2R^6\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\ -135135a^7bR^7\left((1-\phi)+\phi\dfrac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5} \end{array}\right)$$

$$\tilde{F}[19]=\frac{1}{20274183401472000}\left(\begin{array}{l}bG^{16}-130686b^2G^{12}R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-64abG^{14}R\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\+89650368b^3G^8R^2\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+3121068ab^2G^{10}R^2\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\+1708a^2bG^{12}R^2\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-3168258624b^4G^4R^3\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\-978609024ab^3G^6R^3\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-29567250a^2b^2G^8R^3\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\-24640a^3bG^{10}R^3\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+4090611456b^5R^4\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\+12617990784ab^4G^2R^4\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+3791037312a^2b^3G^4R^4\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\+141160488a^3b^2G^6R^4\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+208054a^4bG^8R^4\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\-11321698752a^2b^4R^5\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-6076050048a^3b^3G^2R^5\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\-354431586a^4b^2G^4R^5\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-1038016a^5bG^6R^5\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\+3295339200a^4b^3R^6\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+436149036a^5b^2G^2R^6\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\+2924172a^6bG^4R^6\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}-198236430a^6b^2R^7\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\\-4098240a^7bG^2R^7\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}+2027025a^8bR^8\left((1-\phi)+\phi\frac{\rho_s}{\rho_f}\right)(1-\phi)^{2.5}\end{array}\right)$$

According to the definition of DTM, the solution is

$$\begin{aligned}&F(z)=\tilde{F}[0]+z\tilde{F}[1]+z^2\tilde{F}[2]+z^3\tilde{F}[3]\\&+z^4\tilde{F}[4]+z^5\tilde{F}[5]+z^6\tilde{F}[6]+z^7\tilde{F}[7]\\&+z^8\tilde{F}[8]+z^9\tilde{F}[9]+z^{10}\tilde{F}[10]+z^{11}\tilde{F}[11]\\&++z^{12}\tilde{F}[12]+z^{13}\tilde{F}[13]+z^{14}\tilde{F}[14]+\\&z^{15}\tilde{F}[15]+z^{16}\tilde{F}[16]+z^{17}\tilde{F}[17]+z^{18}\tilde{F}[18]\\&+z^{19}\tilde{F}[19]+z^{20}\tilde{F}[20]\end{aligned} \quad (20)$$

RESULTS AND DISCUSSION

The above analysis shows the application of approximate analytical methods of differential transformation method for the analysis of a steady two-dimensional axisymmetric flow of an incompressible nanofluid in a porous medium under the influence of a uniform transverse magnetic field with slip boundary condition. Using DTM, a series solution was obtained which provides excellent approximations to the solution of the non-linear equation with high accuracy as shown in Table 2.

Table 2
Comparison of results.

z	NM	DTM
0.00	0.000000	0.000000
0.10	0.075739	0.075739
0.20	0.152935	0.152935
0.30	0.233046	0.233046
0.40	0.317540	0.317540
0.50	0.407893	0.407893
0.60	0.505591	0.505591
0.70	0.612134	0.612134
0.80	0.729034	0.729034
0.90	0.857813	0.857813
1.00	1.000000	1.000000

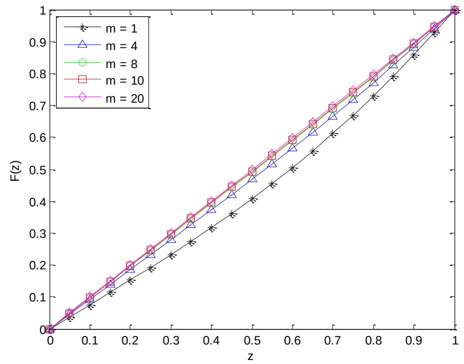

Fig. 2. Effects of magnetic parameter on the flow behavior of the fluid under the influence of slip condition

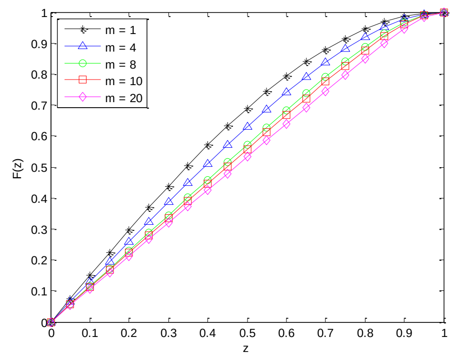

Fig. 3. Effects of magnetic field parameter on the flow behavior of the fluid for no-slip condition

In order to get an insight into the problem, the effects of pertinent flow, magnetic field and slip parameters on the velocity profile of the fluid are investigated. Figure 2 shows the effects of magnetic field and porous parameter (porous-magnetic parameter), m on the velocity of the fluid under the influence of slip condition, while Figure 3 depicts the combined influence of the porosity and magnetic field on the velocity of the fluid under no-slip condition. It could be inferred from the figures that the velocity of the fluid increases with increase in the porous-magnetic parameter under slip condition while an opposite trend was recorded during no-slip condition as the velocity of the fluid decreases with increase in the parameter that represent the combined influence of porosity and magnetic field under the no slip condition.

The magnetic field response is due to the boundary layer thickness caused by the Lorentz or magnetic force field. Figure 4 shows the influence of the slip parameter γ on the fluid velocity. By increasing γ, it is observed that the velocity of the fluid increases. The behavior of the fluid under the slip condition can be physically explained due to increase in slip components that leads to a corresponding decrease in shear stress. Figure 5 presents the effects of Reynold's number on the velocity of the fluid. It is observed from the figure that by increasing the value R, the velocity of the fluid decreases.

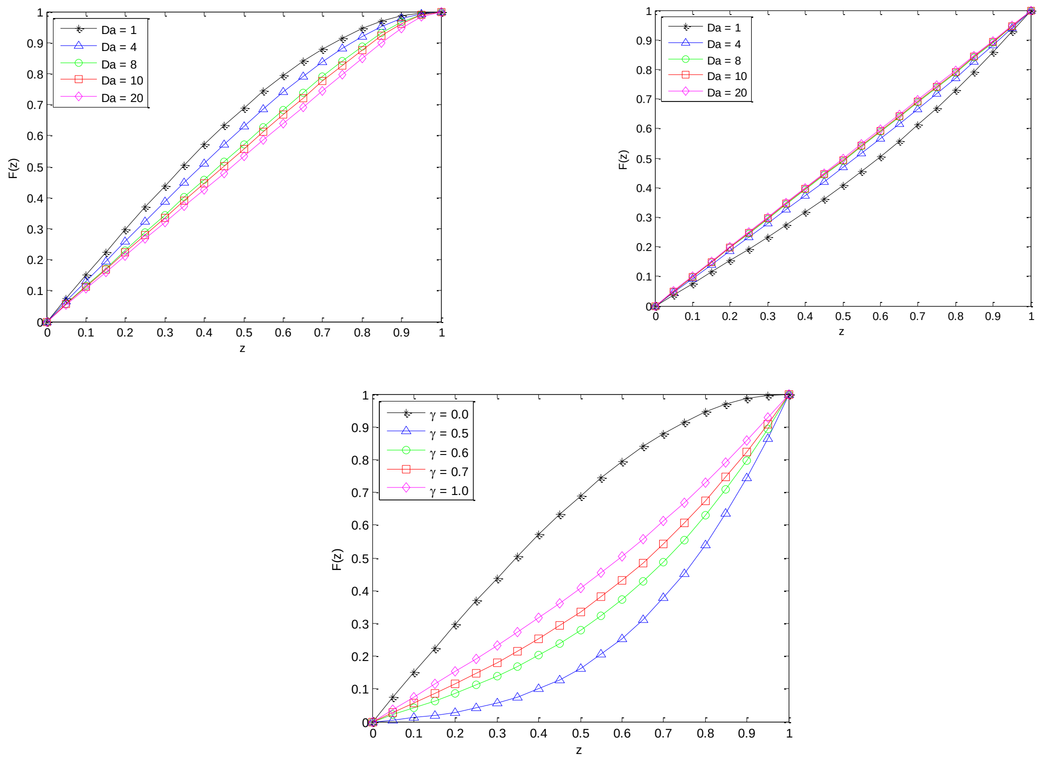

Fig. 4. Effects of slip parameter on the flow behavior of the fluid

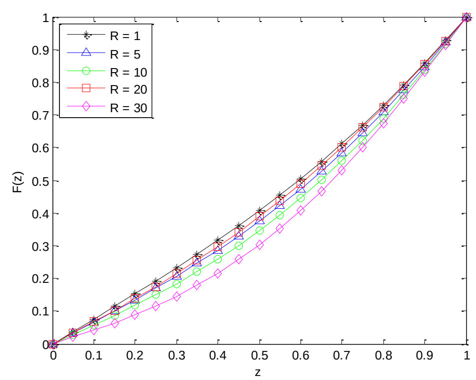

Fig. 5. Effects of Reynolds number on the flow behavior of the fluid under the influence of slip condition

CONCLUSION

In this work, differential transformation method has been employed to develop analytical solution for axisymmetric magnetohydrodynamic squeezing flow of nanofluid in porous media under the influence of slip boundary condition. The developed analytical solution was used to investigate the effects of porous medium, magnetic field and slip boundary on the steady two-dimensional axisymmetric flow of the nanofluid. The results show that the velocity of the fluid increases with increase in the magnetic field and porous parameters under slip condition while the velocity of the fluid decreases with increase in the magnetic field and porous parameter under no slip condition. By increasing the slip parameter, the velocity of the fluid increases and it decrease as the Reynolds number increases.

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