AN EPQ MODEL FOR AN IMPERFECT PRODUCTION PROCESS WITH FUZZY CYCLE TIME AND QUALITY SCREENING

HARUN ÖZTÜRK, SALİH AYTAR, AND FATİH AHMET SENEL

Abstract. This study has developed a production inventory model where the cycle time is fuzzy, the existence of defective products is assumed in each batch and product screening is performed both in-production and after-production. Triangular fuzzy numbers serve to model uncertainties in the cycle time, and a fuzzified total inventory profit function is created by the defuzzification method known as the signed distance method. The classical approach is used to determine the optimal policy, with the ideal cycle time matched to the total profit. Although assuming asymmetric triangular fuzzy numbers prevents the calculation of a clear analytical solution, the method approaches as closely as possible to an analytical solution. A numerical solution to only one equation is needed to obtain the optimal configuration. Conversely, there is a positive trade-off, with an analytical solution to the optimization problem if there is an assumption of symmetrical triangular fuzzy numbers. The proposed model is illustrated by a numerical example. The paper presents results and sensitivity analyses, in both tables and graphic illustrations. The effects on total profit are discussed in relation to various parameters. From the numerical studies, it is observed that the level of fuzziness influences the cycle time and an approximately linear relationship, in the opposite direction, was found between the total profit and the level of fuzziness, when it was increased.

1. Introduction

Research in logistics and inventory systems has often focused on two basic models: Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ). Although these give the user a general idea of the performance of an inventory system under given input parameters, they are inadequate to deal with more complex inventory situations, such as discounts for quantity or when considering items with deteriorating or imperfect quality. This paper addresses a situation with a shipment or production lot that includes defective items needing rework. There is extensive literature on production models with rework but the main limitation is the assumption that screening for items of imperfect quality occurs continuously, with rework following immediately when an imperfect item is found [53]. This is an impractical assumption, especially for automated processes with high production

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rates. When quality control and assurance departments were introduced in manufacturing setups, inspection was recognised as an important part of the process. Several research studies, including [60, 68] have compared strategies for timing the screening, either after or during production, and for locating the screening stations, either at the end of each stage in the production line, or at the end of the final stage. Like this paper, several published research works have assumed that screening takes place at the end of production [66]. This paper considers onsite, manual inspection where new items may be consumed before the start of the screening process begins. This model applies to systems which require strict quality measures, such as the food industry and other process-based industries. In the United Kingdom (UK), for example, a change in the food safety law means that retailers are no longer covered by suppliers' warranties but are required to ensure the quality of the food they sell [45]. In this situation, the retailer inspects newly received items while shelving them for sale to customers before setting aside time for a thorough inspection of the complete batch. Unsatisfactory food items cannot be reworked, but the retailer can return them to the supplier. Another shortcoming of earlier inventory models is that they ignore the impact of the screening process on inventory policies. Ullah and Kang [49] produced an inventory model for a group-technology-based manufacturing environment. The model of Ullah and Kang was extended by Kang et al. [14], who incorporated human inspection errors to increase the model's realism in addressing industry problems. These inventory models examine the ways that optimum lot size is influenced by imperfect production and inspection; however, it is treated as a problem in lot-sizing for the process inventory, not in the inventory of finished goods. Moussawi-Haidar et al. [43] integrate screening time into a production model with rework and examine the ways that rework and inspection affect the finished goods inventory. They considered the case, common in the real world, where defective items are found in quality checks during purchasing, either by the buyer or the seller. They also look at the case of manual inspection while items are consumed during the production run, resulting in a manual inspection rate that is equal to the demand rate. The screening process for identification of items that need to be reworked takes place immediately after production stops. Two inventory models are analysed; the first model assumes the sale of defective items at a discount on completion of the screening process. The second model incorporates reworking of defective items at a constant rate. They do not allow shortages. They demonstrate that screening has a significant effect on the optimal lot size, both analytically and numerically. The results show that their derived closed-form expression for the optimal lot size depends on the screening rate. Öztürk [24] corrected some mathematical expressions in [43].

The different models for EOQ usually apply to a continuous-review setting, and assume that inventory monitoring is possible at every moment. However, it is essential to recognise the importance of production factors in the management of the supply chain, especially in a process-based industry. Such applications need solutions that allow efficient production but keep the inventory low [36]. This trade-off issue in many supply chains. In the supply chains of process industries in the Nordic countries, there are a few, large producing companies and quite numerous distributors operating independently from the producers. These producers face uncertainties in their production decisions that often are not amenable to capture by probabilistic measures; such cases often require fuzzy measures [37, 11]. One source of uncertainty is that production does not stop precisely at the desired batch size, making the cycle time or production batch size often uncertain. There is also substantial uncertainty in market conditions [36], which may lead management to want to increase batch sizes in order to produce goods for stock. There may also be other reasons for uncertainty in batch sizes, all of which will lead to uncertain cycle times. Therefore, models have been devised to take account of these uncertainties. This paper contributes to the research track of fuzzy EPQ-models. A recent focus of studies has been on fuzzifying inventory models to describe different situations, to make them better resemble reality (see [6]). To the authors' knowledge, however, there is no fuzzified version of the model of Moussawi-Haidar et al. [43], which represents common production situations. In view of the limitations of previous models in the context of quality and production-inventory control, of this paper principally aims to develop a profit maximization model including a screening policy and rework for defective items to obtain the optimal production cycle time. It is assumed that the screening of produced items is performed both during the production and after the production. The fuzziness of the cycle time, which has been explored in previous studies by Björk [33], and Mezei and Björk [29] is consistent with the uncertainty caused by market conditions and higher competition in process industry as reported by Björk and Carlsson [37]. Therefore, this article assumes that the production cycle time can also be described as a triangular fuzzy number. This paper takes a novel approach, since only few papers have focused on modelling realistic inventory assumptions. We use a solution methodology similar to that of Mezei and Björk [29] and Björk [35], where the signed distance method [30] is used to defuzzify the fuzzy model and then derivatives are used to find the solution. A clear analytical solution is impossible with the assumption of asymmetric triangular fuzzy numbers, but solutions as close to an analytical solution as possible will be produced. Only one equation needs to be solved numerically to obtain the optimal solution. Conversely, a positive trade-off provides an analytical solution to the optimization problem. An analytical solution can be found by assuming symmetrical triangular fuzzy numbers to describe the cycle time and defuzzifying the objective function before beginning the optimization process. Our focus is an investigation of the combined effects of production run time, quality screening, and a rework/repair procedure on an imperfect production system. The proposed model can be considered to be an extension of the fuzzy inventory models, investigating the effect of fuzzy production cycle time on the total profit.

This study is organized as follows: The next section provides a review of the literature. Section 3 presents the crisp model. Section 4 presents an introduction to the fuzzy set theory and a defuzzification method. Section 5 presents the fuzzy model which was then defuzzified in a similar manner to that used by Chang [20] and Mezei and Björk [29]. Defuzzification will be performed with the signed distance method so that the analytical solution can be obtained from the first order derivative. Section 6 gives numerical examples and sensitivity analysis in order to illustrate the developed fuzzy model. The last section concludes and summarizes the paper.

2. Literature review

First, the imperfect inventory literature relating to inventory models is reviewed. The next section reviews the literature on fuzzy inventory models. Interested readers will find a complete discussion in the following works, which provide a through review of many problems in inventory management [2].

2.1. **Review of inventory models.**

2.1.1. *Models for items with imperfect quality.* Basic production-inventory control models allowing for defective items have been developed by [17, 46]. In the inventory model proposed by Salameh and Jaber [47] for imperfect quality (defective), defective items are screened out and then sold as one batch at a discounted price. Papachristos and Konstantaras [62] revised this process by subtracting the defective items from the inventory on completion of the production cycle. Since the last few decades several EPQ models which consider defective items in light of various assumptions have been developed [10].

2.1.2. *Models for items with imperfect quality and screening constraint.* Inspection is defined as "the process of measuring, examining, testing, or otherwise comparing the unit to applicable requirements" [13]. Its two primary purposes are: (i) to check if a product matches specifications and (ii) to assess whether it is possible to use a nonconforming product in another process. The impact of incorporating screening and quality control on the behavior of inventory systems is gaining interest in the recent literature [56, 21, 23]. There is no existing work relating to the realistic case in which screening occurs onsite and defective items are inspected at the same rate as demand before the production cycle is completed. Konstantaras et al. [26] addressed the manual inspection of items, with an economic order quantity (EOQ) model taking into account the effect of learning on the inspection procedure. Glock and Jaber [12] also considered manual inspection of items, with a model which accounts for both learning and forgetting. A production inventory model discussed by Sett et al. [9] has online inspection continuing during the production process following a time variable, with an additional human-based inspection policy considered offline to identify defective items on completion of the production cycle.

2.1.3. *Models for items with imperfect quality and rework.* Flapper et al. [57] based their systematic description of rework in a process industry on two principal categories of rework situations: in-line versus off -line rework, and single-stage versus multiple-stage production. Jamal et al. [7] took the basic inventory model and extended it with the assumption that reworking is possible for all defective items and rework produces no defective items; thus, reworking defective items produces 100% satisfactory products. Chiu [64] considered an EPQ model for an imperfect production process that is subject to random breakdowns. Taleizadeh et al. [1] and

Chen and Tsao [65] are among many researchers who made contributions in this area in recent years.

2.2. **Review of fuzzy inventory models.**

2.2.1. *Fuzzy set theory.* There are many distinct facets of fuzzy logic, which overlap one another and have imprecise boundaries. Of these facets, four are extremely important. These are: the (i) logical; (ii) set-theoretic; (iii) relational; and (iv) epistemic facets. Most applications of fuzzy logic involve the set-theoretic facet [41]. Inventory management is one area where fuzzy set theory has found application [6]. Alongside such applications of fuzzy sets, fuzzy set theoretical considerations have been explored by theorists such as [28, 4, 27]. Building on these studies, Zadeh [42] introduced the concept of type-2 fuzzy sets, in addition to the ordinary fuzzy set, i.e., type-1 fuzzy sets. Mizumoto and Tanaka [48] studied the set theoretic operations with type-2 fuzzy sets and analysed the membership grades of such sets. A further generalization of the ordinary fuzzy set is the interval-valued fuzzy set. This concept was first introduced by Zadeh [42], after which some authors conducted further investigations and reached meaningful conclusions. For example, Gorzałczany [44] and Cui and Zeng [8] investigated interval-valued fuzzy sets in approximate reasoning, Turksen [25] in normal forms; Zeng and Li [67] examined the relationship between similarity measures and entropy in interval-valued fuzzy sets and their representation theorems, Lin [19] in job-shop scheduling problems; and Mondal [61] in solving first-order differential equations with interval-valued fuzzy sets as initial values. Moreover, some authors noted a strong connection between intuitionistic fuzzy sets and interval valued fuzzy sets. For more details, we refer readers to [39, 63].

2.2.2. *Decision making under uncertainty.* Real-life decision-making is often uncertain. Since Zadeh [40] initiated the concept of fuzzy sets to deal with ill-defined objects, several researchers have engaged in studies of fuzzy sets and ways they can be applied to automata, languages, pattern recognition, decision making and logic. With an insignificant level of uncertainty, a classical EPQ- or EOQ-formula may be usable. However, today's uncertainties are often significant, so the models must accommodate them. Stochastic modelling is sometimes possible [55, 32], but probabilistic methods frequently cannot captured them, leaving the opinions of experts in inventory management as the only option. This is typically true of new products, or products with sizeable variation for seasonal and other reasons. For such extensive uncertainties, fuzzy numbers can be used as an alternative to probabilistic techniques [18]. A number of researchers, for example [16, 52], have studied *one*-dimension or n-dimension fuzzy numbers. As further theories and applications of fuzzy numbers are developed, the concept becomes increasingly important.

2.2.3. *Analytically solved fuzzy inventory models.* Several authors have contributed to this field. Among them are Chang [20], who studied fuzzy modifications to the model developed by Salameh and Jaber [47]. As another example, Park [38] explored fuzzy inventory costs having employed the Extension Principle and relevant arithmetic. Other related studies including [3, 58, 50] and their references. Björk

[33], in addition to the studies listed above, explored the limited production rate and fuzzy cycle time under the classic EPQ model. After defining the cycle time as a triangular fuzzy number, Björk obtained the equation for optimal cycle time analytically, having solved the total cost function with defuzzification by signed distance. Following this study, Björk has published a series of studies offering analytical solutions to different inventory control models. For example, Björk [34] has expressed the lead time and demand rate in terms of triangular fuzzy numbers in the classical EOQ model which allows for shortage and calculated the optimal order and shortage quantity analytically. Björk [35] developed the equation for optimal cycle time, and thus the equation for optimal production rate, using the signed distance defuzzification method in a multi-product economic production quantity model. Moreover, Mezei and Björk [29] also explored an EPQ model with respect to fuzzy cycle times and backorders.

2.2.4. *Numerically solved fuzzy inventory models.* Lee and Yao [22] developed an EPQ model for fuzzy demand and fuzzy production quantity. They used a numerical optimization method, namely Nelder-Mead, to solve the optimization problem. Islam and Roy [59] analysed an EPQ model with a constraint of limited storage space and solved by a geometric programming technique. In respect of deteriorating items, Chakraborty et al. [15] created a multi-product production-inventory model involving fuzzy random circumstances, with limits on space and budget. Sadeghi et al. [31] devised a multi-item EPQ model with fuzzy demand, where shortages are fully backordered and there is limited warehouse space. The optimal production quantities and maximum backorder levels are found numerically by the Newton–Raphson method.

3. Model description

Consider a manufacturing process has a constant production rate α, which is larger than the demand rate, β. In this manufacturing process, both perfect and imperfect quality items are produced. The production cost per unit is C_p, and h is the unit holding cost per unit time. The good items are sold at a regular price S per unit. The manufacturing process generates P percent of defective items at a rate, d. All items produced are screened and an inspcetion process is performed during production and at the end of production. Different costs apply to inspection during production, d_1, and after production, d_2, where $d_1 > d_2$. During production, demand is met from good items only. Thus, the inspection rate is the same as the demand rate during production. After production, the remaining items from the produced lot are screened at a rate, x, where $x > \beta$. All defective items produced are reworked/repaired to meet quality control requirements at a rate of α_1, where $\alpha_1 < \beta$, and the reworking of defective items starts when the inspection process ends in each cycle, with unit reworking cost C_r. The unit holding cost for reworked items h_1 is greater than the unit holding cost of the good items h, i.e. $h_1 > h$. The rework process is assumed to be perfect. Let d denote the production rate of defective items during the manufacturing process, then d can be expressed as the product of production rate α, times the proportion of defective items, P.

Thus, $d = \alpha P$. For reading ease, we adopt the same notation available in the mathematical modeling and formulation of Moussawi-Haidar et al. [43] as follows. α the production rate, β the demand rate, α_1 the rework rate of defective items per unit time, y the total number of items produced during a production cycle, T the production cycle, t_1 the production run time, $t_1 = y/\alpha$, t_2 the inspection time after the production, t_3 the time needed to rework the defective items, t_4 the time to consume all on-hand inventories after the rework, z_1 the maximum level of on-hand inventory of good items when the production is completed, z_2 the level of on-hand inventory of good items when the inspection is completed, z_3 the maximum level of on-hand inventory of good items when the rework is completed, K the production setup cost, P the proportion of defective items, x the inspection rate, d the production rate of defective items, d_1 unit inspection cost during production, d_2 unit inspection cost after production, h the holding cost per unit per unit time, h_1 the holding cost of defective items to be reworked per unit per unit time, S the selling price of good items per unit, C_p the production cost per unit, C_r the rework cost per unit. The basic assumption of the classic EPQ model is that the production rate α must always be greater than or equal to the sum of demand rate β and the production rate of defective items d, as

$$\alpha - (d + \beta) \geqslant 0 \text{ or } P \leq 1 - \beta/\alpha. \tag{1}$$

the inspection time t_2 should be finished before the end of the cycle. That is

$$t_2 < t_3 + t_4 \text{ or } x > 2\beta \left(1 - \frac{\beta}{\alpha\,(1-P)}\right) / \left(1 - P - \frac{\beta}{\alpha}\right). \tag{2}$$

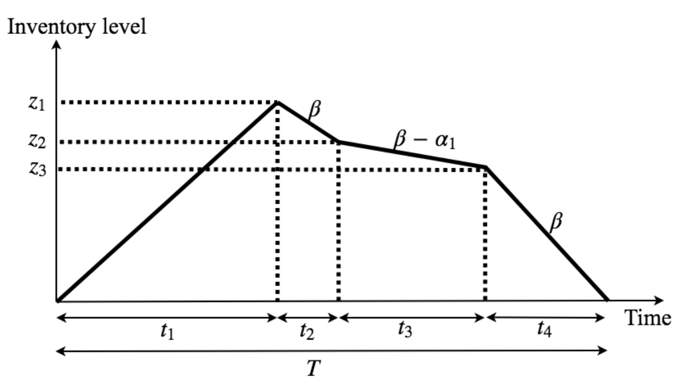

Figure 1. Inventory for Nondefective Items

Figure 1 represents the behavior of the on-hand inventory of good items. The production cycle length T is the sum of the production run time t_1, the inspection time t_2, the rework time t_3 and the production downtime t_4, that is

$$T = t_1 + t_2 + t_3 + t_4.$$

From Figure 2, the defective items produced during production, t_1, can be computed as in (3).

$$dt_1 = d\,(y/\alpha) = Py. \tag{3}$$

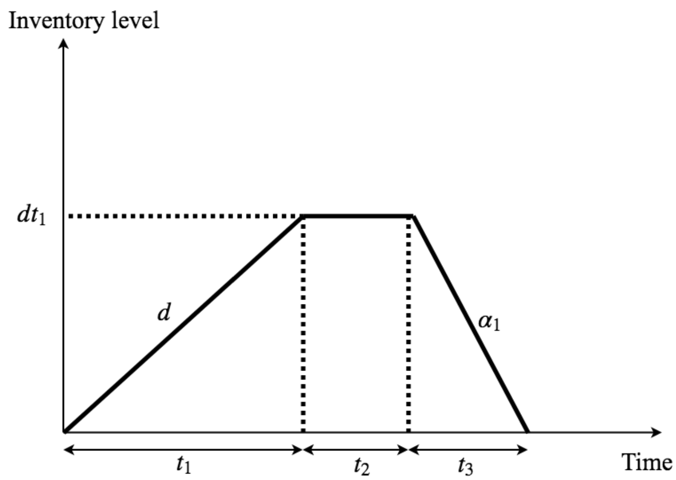

Figure 2. Inventory for Defective Items

Following is a solution procedure is like the one published by Moussawi-Haidar et al. [43]. The expressions of production time, t_1, screening time after production, t_2, reworking time t_3, production downtime, t_4, maximum levels of on-hand inventory z_1, z_2 and z_3 are as follows:

$$t_1 = y/\alpha = z_1/\left(\alpha - d - \beta\right),$$

$$t_2 = \left[y\left(1 - \frac{\beta}{\alpha}\right) - y\left(\frac{P\beta}{\alpha\,(1-P)}\right)\right]/x,$$

$$t_3 = dt_1/\alpha_1 = dy/\left(\alpha\alpha_1\right),$$

$$t_4 = z_3/\beta,$$

$$z_1 = y\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right),$$

$$z_2 = z_1 - \beta t_2 = y\left[\left(1 - \frac{\beta}{\alpha} - \frac{d}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha} - \frac{P\beta}{\alpha\,(1-P)}\right)\right],$$

$$z_3 = z_2 - \left(\beta - \alpha_1\right)t_3 = y\left[\begin{array}{c} \left(1 - P - \frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha(1-P)}\right) \\ -\frac{(\beta-\alpha_1)P}{\alpha_1} \end{array}\right].$$

thus, the cycle length will be

$$T = \frac{y}{\beta}. \tag{4}$$

The total cost function $TC(y)$ is given by

$$\begin{aligned} TC(y) &= K + C_p y + C_r P y + d_1 y\left(\frac{\beta}{\alpha(1-P)}\right) + d_2 y\left(1 - \frac{\beta}{\alpha(1-P)}\right) \\ &+h\left(\frac{z_1 t_1}{2} + \frac{(z_1+z_2)(t_2)}{2} + \frac{(z_2+z_3)(t_3)}{2} + \frac{z_3(t_4)}{2}\right) \\ &+h\left(\frac{dt_1(t_1)}{2} + dt_1(t_2)\right). \end{aligned}$$

Then, the total profit per cycle, $TP(y)$, is obtained as follows [43]

$$\begin{aligned} TP(y) = Sy &- \left[K + C_p y + C_r P y + d_1 y\left(\frac{\beta}{\alpha(1-P)}\right)\right] \\ &- d_2 y\left(1 - \frac{\beta}{\alpha(1-P)}\right) \\ &- hy^2 \begin{bmatrix} \frac{1}{2\alpha}\left(1-\frac{\beta}{\alpha}\right) - \frac{1}{2x}\left(1-P-\frac{\beta}{\alpha}\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\ +\frac{1}{2x}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\ +\frac{P}{\beta}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right) \\ +\frac{P}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right) + \frac{1}{2\beta}\begin{pmatrix} P^2 - 2P\left(1-\frac{\beta}{\alpha}\right) \\ -\frac{2\beta^3}{x^2\alpha(1-P)} + \frac{\beta^4}{x^2\alpha^2(1-P)^2} \end{pmatrix} \\ -\frac{(\beta-\alpha_1)P^2}{2\beta\alpha_1} + \frac{1}{2\beta}\left(\left(1-\frac{\beta}{\alpha}\right)^2 + \frac{\beta^2}{x^2}\right) \end{bmatrix} \\ &- \frac{h_1 P^2 y^2}{2\alpha_1}. \end{aligned}$$

Hence, the total profit per unit time is obtained as follows

$$\begin{aligned} TPU(y) = S\beta - \frac{KD}{y} &- \left[C_p + C_r P + \frac{d_1\beta}{\alpha(1-P)} + d_2\left(1-\frac{\beta}{\alpha(1-P)}\right)\right] D \\ -hDy &\begin{bmatrix} \frac{1}{2\alpha}\left(1-\frac{\beta}{\alpha}\right) - \frac{1}{2x}\left(1-P-\frac{\beta}{\alpha}\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\ +\frac{1}{2x}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\ +\frac{P}{\beta}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right) + \frac{P}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right) \\ +\frac{1}{2\beta}\left(P^2 - 2P\left(1-\frac{\beta}{\alpha}\right) - \frac{2\beta^3}{x^2\alpha(1-P)} + \frac{\beta^4}{x^2\alpha^2(1-P)^2}\right) \\ -\frac{(\beta-\alpha_1)P^2}{2\beta\alpha_1} + \frac{1}{2\beta}\left(\left(1-\frac{\beta}{\alpha}\right)^2 + \frac{\beta^2}{x^2}\right) \\ +\frac{h_1\beta P^2 y}{2\alpha_1} \end{bmatrix}. \end{aligned}$$

The decision variable, y, can be exchanged with the cycle time, T, according the formula in (4). This will yield the following results

$$
\begin{aligned}
TPU(T) &= S\beta - \tfrac{K}{T} - \left[C_p + C_r P + \tfrac{d_1\beta}{\alpha(1-P)} + d_2\left(1-\tfrac{\beta}{\alpha(1-P)}\right)\right] D \\
&-hD^2T \left[\begin{array}{c}
\frac{1}{2\alpha}\left(1-\frac{\beta}{\alpha}\right) - \frac{1}{2x}\left(1-P-\frac{\beta}{\alpha}\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\
+\frac{1}{2x}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\
+\frac{P}{\beta}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right) \\
+\frac{P}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right) \\
+\frac{1}{2\beta}\left(P^2 - 2P\left(1-\frac{\beta}{\alpha}\right) - \frac{2\beta^3}{x^2\alpha(1-P)} + \frac{\beta^4}{x^2\alpha^2(1-P)^2}\right) \\
-\frac{(\beta-\alpha_1)P^2}{2\beta\alpha_1} + \frac{1}{2\beta}\left(\left(1-\frac{\beta}{\alpha}\right)^2 + \frac{\beta^2}{x^2}\right)
\end{array}\right] \\
&+\frac{h_1\beta^2P^2T}{2\alpha_1}.
\end{aligned} \tag{5}
$$

It is assumed that all variables and parameters used are greater than zero always. A version of the classical (crisp) EPQ model is given in (5). Maximization of the total profit per unit time means minimizing the total cost per unit of time as the revenue is independent of production cycle length. Derivatives can be used to solve the classical EPQ model as all the terms in (5)are convex. The optimal solution will be obtained using

$$
T^* = \frac{1}{\beta}\sqrt{\frac{K}{\left(\begin{array}{l}
h\left[\begin{array}{c}
\frac{1}{2\alpha}\left(1-\frac{\beta}{\alpha}\right) - \frac{1}{2x}\left(1-P-\frac{\beta}{\alpha}\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\
+\frac{1}{2x}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right)\left(1-\frac{\beta}{\alpha(1-P)}\right) \\
+\frac{P}{\beta}\left(\left(1-P-\frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right)\right) \\
+\frac{P}{x}\left(1-\frac{\beta}{\alpha(1-P)}\right) \\
+\frac{1}{2\beta}\left(P^2 - 2P\left(1-\frac{\beta}{\alpha}\right) - \frac{2\beta^3}{x^2\alpha(1-P)} + \frac{\beta^4}{x^2\alpha^2(1-P)^2}\right) \\
-\frac{(\beta-\alpha_1)P^2}{2\beta\alpha_1} + \frac{1}{2\beta}\left(\left(1-\frac{\beta}{\alpha}\right)^2 + \frac{\beta^2}{x^2}\right)
\end{array}\right] \\
+\frac{h_1P^2}{2\alpha_1}
\end{array}\right)}}. \tag{6}
$$

From (6), the optimal production quantity is obtained as $Q^* = \beta T^*$.

4. Background for Fuzzy Set Theory

This section consist of some basic concepts related to theory of fuzzy numbers. A fuzzy number is a function X from the set $\mathbb{R}$ of real numbers to unit closed interval, satisfying the conditions: normality, fuzzy convexity, upper semi-continuity and support boundedness (see [54, 5] for more details). A special case of fuzzy numbers is that of triangular fuzzy numbers, which is very important for applications and is defined as follows.

Definition 4.1. A fuzzy number $\tilde{A} = (a, b, c)$ on the set $\mathbb{R}$ of real numbers, where $a < b < c$ is called a triangular fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, x \in [a, b] \\ \frac{c-x}{c-b}, x \in [b, c] \\ 0, \text{otherwise} \end{cases}.$$

Definition 4.2. A fuzzy interval $[a_\alpha, b_\alpha]$ on $\mathbb{R}$ is called α—level fuzzy interval if its membership function is

$$\mu_{[a_\alpha, b_\alpha]}(x) = \begin{cases} \alpha, x \in [a, b] \\ 0, \text{otherwise} \end{cases}.$$

The α—cut of a fuzzy number $\tilde{A}$ is defined as $\tilde{A}(x) = \{x \mid \mu_{\tilde{A}}(x) \geqslant \alpha\} = [E_L(\alpha), E_U(\alpha)]$. The membership function of the fuzzy number $\tilde{A}$ is $\mu_{\tilde{A}}(x) = \mu_{\bigcup_{\alpha \in [0,1]}[E_L(\alpha), E_U(\alpha)]}(x)$.

Denote F as set of triangular fuzzy numbers. If $\tilde{A} \in F$, then we have $\tilde{A}(\alpha) = [a + \alpha\,(b - a)\,, c - \alpha\,(c - b)]$

Definition 4.3. For any $a \in \mathbb{R}$, the signed distance from a to 0 is defined as $d_0(a, 0) = a$, where

$$d_0(a, 0) = \begin{cases} a, a > 0 \\ -a, a < 0 \end{cases}.$$

For the fuzzy number $\tilde{A} \in F$, the signed distance from $\tilde{A}$ to 0_1 is

$$\begin{aligned} d(\tilde{A}, 0_1) &= \frac{1}{2}\int_0^1 d\left([E_L(\alpha), E_U(\alpha)], 0_1\right) d\alpha \\ &= \frac{1}{2}\int_0^1 \left(E_L(\alpha) + E_U(\alpha)\right) d\alpha. \end{aligned} \tag{7}$$

where 0_1 is the characteristic function of zero, i.e. $0_1(x) = \begin{cases} 1, x = 0 \\ 0, x \neq 0 \end{cases}$.

In this paper we use the signed distance defuzzification method. If $\tilde{A} \in F$, then we get

$$d(\tilde{A}, 0_1) = \frac{a + b + c}{3}.$$

5. A Fuzzy EPQ Model with Fuzzy Cycle Times

Business managers commonly face the need to make decisions under uncertain conditions which are inherently fuzzy. This is why a set of EPQ models is needed to accurately capture the fuzzy uncertainties. A fuzzy EPQ model and its numerical and analytical solutions are presented in this section. The assumption here is that the cycle time is uncertain and can be expressed with a triangular fuzzy number (not necessarily symmetric). The uncertainty in case of asymmetrical triangular fuzzy numbers is represented by the parameter Δ_1, and Δ_2, i.e. the width of the support on the left and right-side of the centre of the fuzzy number. Then the cycle time T will be

$$\check{T} = (T - \Delta_1, T, T + \Delta_2)\,. \tag{8}$$

The total profit per unit time in the fuzzy sense is given by

$$\begin{aligned} TPU(\check{T}) &= (S\beta)_1 - TCU(\check{T}) \\ &= (S\beta)_1 - \left\{ \begin{array}{c} \left[\left(\begin{array}{c} C_p + C_r P + \frac{d_1\beta}{\alpha(1-P)} \\ +d_2\left(1 - \frac{\beta}{\alpha(1-P)}\right) \end{array}\right)\beta\right]_1 \\ +K\frac{1}{\check{T}} + h\beta^2\varphi\check{T} + \frac{h_1\beta^2P^2}{2\alpha_1}\check{T} \end{array} \right\} \end{aligned} \tag{9}$$

where

$$\begin{aligned} \varphi \;=\; & \frac{1}{2\alpha}\left(1 - \frac{\beta}{\alpha}\right) - \frac{1}{2x}\left(1 - P - \frac{\beta}{\alpha}\right)\left(1 - \frac{\beta}{\alpha\,(1-P)}\right) \\ & + \frac{1}{2x}\left(\left(1 - P - \frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha\,(1-P)}\right)\right)\left(1 - \frac{\beta}{\alpha\,(1-P)}\right) \\ & + \frac{P}{\beta}\left(\left(1 - P - \frac{\beta}{\alpha}\right) - \frac{\beta}{x}\left(1 - \frac{\beta}{\alpha\,(1-P)}\right)\right) + \frac{P}{x}\left(1 - \frac{\beta}{\alpha\,(1-P)}\right) \\ & + \frac{1}{2\beta}\left(P^2 - 2P\left(1 - \frac{\beta}{\alpha}\right) - \frac{2\beta^3}{x^2\alpha\,(1-P)} + \frac{\beta^4}{x^2\alpha^2\,(1-P)^2}\right) \\ & - \frac{(\beta - \alpha_1)\,P^2}{2\beta\alpha_1} + \frac{1}{2\beta}\left(\left(1 - \frac{\beta}{\alpha}\right)^2 + \frac{\beta^2}{x^2}\right). \end{aligned}$$

It is necessary to define signed distances to defuzzify the total profit function. The signed distance of $TPU(\check{T})$ and 0_1 is given, as follows;

$$\begin{aligned} d\left(TPU(\check{T}), 0_1\right) &= d(S\beta, 0_1) - d\left(TCU(\check{T}), 0_1\right) \\ &= d(S\beta, 0_1) - \left\{ \begin{array}{c} d\left(\left(\begin{array}{c} C_p + C_r P + \frac{d_1\beta}{\alpha(1-P)} \\ +d_2\left(1 - \frac{\beta}{\alpha(1-P)}\right) \end{array}\right)\beta, 0_1\right) \\ +d\left(K\frac{1}{\check{T}}, 0_1\right) + d\left(h\beta^2\varphi\check{T}, 0_1\right) \\ +d\left(\frac{h_1\beta^2P^2}{2\alpha_1}\check{T}, 0_1\right) \end{array} \right\} \end{aligned} \tag{10}$$

Simplifying (10) we get

$$d\left(TPU(\check{T}), 0_1\right) = S\beta - \left\{ \begin{array}{c} \left[C_p + C_r P + \frac{d_1\beta}{\alpha(1-P)}\right]\beta \\ +d_2\beta\left(1 - \frac{\beta}{\alpha(1-P)}\right) + d\left(K\frac{1}{\check{T}}, 0_1\right) \\ +Kd\left(\frac{1}{\check{T}}, 0_1\right) + h\beta^2\varphi d\left(\check{T}, 0_1\right) \\ +\frac{h_1\beta^2P^2}{2\alpha_1}d\left(\check{T}, 0_1\right) \end{array} \right\}. \tag{11}$$

Here, $d\left(\check{T},0_1\right)$ and $d\left(\frac{1}{\check{T}},0_1\right)$ are calculated. The left and right endpoints of the α—cut of $\check{T}$ are written as

$$T_L(\alpha)=T-\Delta_1+\left(T-\left(T-\Delta_1\right)\right)\alpha=T-\Delta_1+\Delta_1\alpha,$$

and

$$T_U(\alpha)=T+\Delta_2+\left(T+\Delta_2-T\right)\alpha=T+\Delta_2-\Delta_2\alpha.$$

From (7), the following can be obtained;

$$d\left(\check{T},0_1\right)=\frac{1}{4}\left(\left(T-\Delta_1\right)+2T+\left(T+\Delta_2\right)\right)=T+\frac{\Delta_2-\Delta_1}{4}. \tag{12}$$

Similarly, the left and right endpoints of the α—cut of $\frac{1}{\check{T}}$ are written as

$$\left(\frac{1}{T}\right)_L(\alpha)=\frac{1}{T_U(\alpha)}=\frac{1}{T+\Delta_2-\Delta_2\alpha},$$

and

$$\left(\frac{1}{T}\right)_U(\alpha)=\frac{1}{T_L(\alpha)}=\frac{1}{T-\Delta_1+\Delta_1\alpha}.$$

From (7), the following can be obtained;

$$\begin{aligned} d\left(\frac{1}{\check{T}},0_1\right) &= \frac{1}{2}\int_0^1\left(\left(\frac{1}{T}\right)_L(\alpha)+\left(\frac{1}{T}\right)_U(\alpha)\right)d\alpha \\ &= \frac{1}{2}\left(\frac{1}{\Delta_1}\ln\left(\frac{T}{T-\Delta_1}\right)-\frac{1}{\Delta_2}\ln\left(\frac{T}{T+\Delta_2}\right)\right). \end{aligned} \tag{13}$$

which is positive since $\Delta_1>0$, $\Delta_2>0$, $\ln\left(\frac{T}{T-\Delta_1}\right)>0$ and $\ln\left(\frac{T}{T+\Delta_2}\right)<0$.

Using (12) and (13) in (11) ensures the defuzzified total profit function

$$d\left(TPU(\check{T}),0_1\right)=S\beta-\left\{\begin{array}{c}\left[C_p+C_rP+\frac{d_1\beta}{\alpha(1-P)}+d_2\left(1-\frac{\beta}{\alpha(1-P)}\right)\right]\beta \\ +K\left[\frac{1}{2}\left(\frac{1}{\Delta_1}\ln\left(\frac{T}{T-\Delta_1}\right)-\frac{1}{\Delta_2}\ln\left(\frac{T}{T+\Delta_2}\right)\right)\right] \\ +h\beta^2\varphi\left[T+\frac{\Delta_2-\Delta_1}{4}\right]+\frac{h_1\beta^2P^2}{2\alpha_1}\left[T+\frac{\Delta_2-\Delta_1}{4}\right]\end{array}\right\}. \tag{14}$$

$d\left(TPU(\check{T}),0_1\right)$ which is the total profit per unit of time estimated in fuzzy terms. It is necessary to see whether or not the defuzzified objective function is concave to solve it using derivatives. Maximization of the total profit per unit time means minimizing the total cost per unit of time as the revenue is independent of production cycle time T.

$$\begin{aligned} W &= d\left(TCU(\check{T}), 0_1\right) \\ &= \left[C_p + C_r P + \frac{d_1 \beta}{\alpha(1-P)} + d_2\left(1 - \frac{\beta}{\alpha(1-P)}\right)\right]\beta \\ &\quad + K\left[\frac{1}{2}\left(\frac{1}{\Delta_1}\ln\left(\frac{T}{T-\Delta_1}\right) - \frac{1}{\Delta_2}\ln\left(\frac{T}{T+\Delta_2}\right)\right)\right] \\ &\quad + h\beta^2\varphi\left[T + \frac{\Delta_2 - \Delta_1}{4}\right] + \frac{h_1 \beta^2 P^2}{2\alpha_1}\left[T + \frac{\Delta_2 - \Delta_1}{4}\right]. \end{aligned} \tag{15}$$

As it is known that the latter two terms in (15) are linear, the fact that linear functions can be both convex and concave eliminates the need for a proof of convexity. Additionally, the fact that the first term is a positive constant will not influence the convexity of the function and only the second term will need to be analysed. It is necessary to calculate derivatives (first and second grade) to assess the convexity. The following function will be examined:

$$f(T) = \frac{1}{\Delta_1}\ln\left(\frac{T}{T-\Delta_1}\right) - \frac{1}{\Delta_2}\ln\left(\frac{T}{T+\Delta_2}\right).$$

Then, the first and second derivatives of the function $f(T)$ are

$$f^{'}(T) = \frac{1}{\Delta_1}\left(\frac{1}{T} - \frac{1}{T-\Delta_1}\right) - \frac{1}{\Delta_2}\left(\frac{1}{T} - \frac{1}{T+\Delta_2}\right),$$

$$f''(T) = \frac{1}{\Delta_1}\left(-\frac{1}{T^2} + \frac{1}{(T-\Delta_1)^2}\right) - \frac{1}{\Delta_2}\left(-\frac{1}{T^2} + \frac{1}{(T+\Delta_2)^2}\right), \tag{16}$$

(16) shows that the absolute value of the first term is positive while that of the second term is negative. This is an indication of the convexity of (15). The conclusion here can be that $W = d\left(TCU(\check{T}), 0_1\right)$ (and $d\left(TPU(\check{T}), 0_1\right)$) is convex (concave) for all values of $T > 0$. Having assigned zero to the derivative of (15) and solving the equality will give the minimum value of (15);

$$\frac{dW}{dT} = \frac{K}{2}\left[\frac{1}{\Delta_1}\left(\frac{1}{T} - \frac{1}{T-\Delta_1}\right) - \frac{1}{\Delta_2}\left(\frac{1}{T} - \frac{1}{T+\Delta_2}\right)\right] + h\beta^2\varphi + \frac{h_1\beta^2 P^2}{2\alpha_1}.$$

This can be solved by simplification according to

$$\frac{K}{2\Delta_1}\left(\frac{1}{T} - \frac{1}{T-\Delta_1}\right) - \frac{K}{2\Delta_2}\left(\frac{1}{T} - \frac{1}{T+\Delta_2}\right) + h\beta^2\varphi + \frac{h_1\beta^2 P^2}{2\alpha_1} = 0.$$

We can find the optimal value of T, if we solve (17):

$$-2KT + 2\left(h\beta^2\varphi + \frac{h_1\beta^2 P^2}{2\alpha_1}\right)T\left(T-\Delta_1\right)\left(T+\Delta_2\right) = 0. \tag{17}$$

Hence, the numerical solution of (17) will give us the optimal value of T.

Assume that the cycle time is defined using symmetric triangular fuzzy numbers as it was the case in [33, 34]. Hence, it is possible to describe T from (8) with the assumption of $\Delta_1 = \Delta_2 = \Delta$, as follows:

$$T^* = \sqrt{\frac{K}{h\beta^2\varphi + \frac{h_1\beta^2P^2}{2\alpha_1}} + \Delta^2}. \tag{18}$$

(18) indicates that the problem under investigation can be solved analytically, and an analytical solution can be obtained to the case where the cycle time is expressed as symmetric triangular fuzzy numbers. In (18), the first term in the square root gives the optimal solution for the classic EPQ problem as expressed in (6). When the fuzzy solution and the optimal solution of the classic EPQ problem are compared, a difference as much as Δ^2 is found. It can be said that the optimal cycle time in the fuzzy environment increases depending on the uncertainty in cycle time. Using (18), if no uncertainty is assumed in the cycle time, i.e. $\Delta = 0$, then the fuzzy optimal cycle time will be reduced to the optimal result of the classic problem as shown in (6). These results agree with the results obtained by [33, 35].

6. Numerical Examples and Sensitivity Analysis

To illustrate the results of the proposed model, we consider an inventory system with an imperfect production process. Most of the following data in the inventory system is taken from the Moussawi-Haidar et al. [43]. The problem is solved for one product, where the demand rate is $\beta = 1200$units/year and the production rate $\alpha = 1600$units/year. The machine setup cost is $K = \$1500$ for each production and the production cost is $C_p = \$104$/unit. The holding cost per unit is $h = \$20$/unit. The production process produces both perfect and imperfect quality items. The selling price for good items is $S = \$200$/unit. To identify the defective items produced, a screening process is conducted during production and at the end of production. The inspection cost per item during production is $d_1 = \$0.6$, and the inspection cost per item after production is $d_2 = \$0.5$. The defective items are reworked at the end of the screening process at a rate of $\alpha_1 = 100$units/year. The repair cost per defective item is $C_r = \$8$. The holding cost per unit of the items being reworked is $h = \$22$/year. However, instead of assuming that the defective rate P is uniformly distributed with mean 0.05, we consider that the defective rate is around $P = 0.05$ (i.e., the good items rate is around 0.95). When we use triangular fuzzy numbers $\check{T} = (T - \Delta_1, T, T + \Delta_2)$ with $0 < \Delta_1 < T$ and $0 < \Delta_2$ to fuzzify the crisp total profit function, $TPU(T)$, in (5), we get fuzzy total profit function $TPU(\check{T})$ in (9). Using signed distance to defuzzify, we obtain the defuzzified total profit function $d\big(TPU(\check{T}), 0_1\big)$ in (14). Note that in practical situations, Δ_1 and Δ_2 are determined by the decision-makers due to the uncertainty of the problem. The Δ_1 and Δ_2 parameters in the fuzzy case are assumed to be 0.005 and 0.01, respectively. Given these parameters, the following results are obtained.

The optimal production cycle times for the EPQ model with defective items is then found using (6) as $T^* = 0.7029$ years and the optimal production lot size is calculated as $y^* = 843.48$ units. The corresponding total profit per unit of time is calculated as $TPU(T^*) = \$109757.243$ In addition, the assumptions in (1) and (2) are provided. Concavity of the total profit function $TPU(T^*)$ is also presented in Figure 3. For the fuzzy case, $T^* = 0.7017$ years and the corresponding optimal production lot size and total profit per unit of time in the fuzzy sense are calculated as 842.04 units, \$109757.160, respectively. The results for this example are given in Table 1. The results in Table 1 indicate that the production cycle time would decrease by 0.17% if the uncertainties were accounted for in a suitable manner. This would also require a decrease in the total profit.

Case	T^*	y^*	$TPU(T^*)$
Crisp	0.7029	843.48	109757.243
Fuzzy	0.7017	842.04	109757.160

TABLE 1. Optimal Results

In Table 2, the sensitivity of Δ_1 and Δ_2 parameters is analyzed (the "*"-sign denotes the base case scenario value). We solve the optimal production cycle time (T) and the maximum total profit $TPU(T)$ in the fuzzy sense for various given sets of (Δ_1, Δ_2). The variations in the values are arranged arbitrarily and their defuzzified values are determined using the signed distance method and are shown in the fifth and sixth columns of the tables. With the increase in the estimate of production cycle time in the fuzzy sense $d\left(\check{T}, 0_1\right)$, total profit in the fuzzy sense will decrease. Moreover, when the value of $d\left(\frac{1}{\check{T}}, 0_1\right)$ decreases, the total profit in the fuzzy sense will also decrease. Further, as the absolute value $|\Delta_1 - \Delta_2|$ increases, the difference in the total profit decreases; hence, the smaller the difference between Δ_1 and Δ_2 (i.e., the less the uncertainty of production cycle time), the smaller the variation of the solutions between fuzzy case and crisp case.

Δ_1	Δ_2	T	y	$d\left(\check{T}, 0_1\right)$	$d\left(\frac{1}{\check{T}}, 0_1\right)$	$TPU(T)$	$\|\Delta_1 - \Delta_2\|$
0.0050*	0.0100	0.7017	842.04	0.7030	1.4226	109757.2	0.0050
0.0970	0.2900	0.6792	815.04	0.7275	1.4074	109705.7	0.1930
0.4838	1.7088	0.8239	988.68	1.1302	1.2430	108729.6	1.2250
0.2446	3.0126	0.6608	792.96	1.3528	1.2297	108073.6	2.7680
0.4032	4.0055	0.7547	905.64	1.6553	1.1775	107233.7	3.6023
0.3897	4.6431	0.7437	892.44	1.8071	1.1657	106790.5	4.2534
0.0911	5.7574	0.5628	675.36	1.9794	1.1792	106247.1	5.6663
0.3705	6.1133	0.7277	873.24	2.1634	1.1436	105741.8	5.7428
0.3488	6.5251	0.7128	855.36	2.2569	1.1410	105461.9	6.1763
0.1772	8.5196	0.6048	725.76	2.6904	1.1376	104150.9	8.3424

TABLE 2. The Effect of Δ_1 and Δ_2 on The Optimal Solution

Figure 4a and Figure 4b show the percentage change in the level of fuzziness and the percentage change in total profit, respectively, in accordance with Table 2.

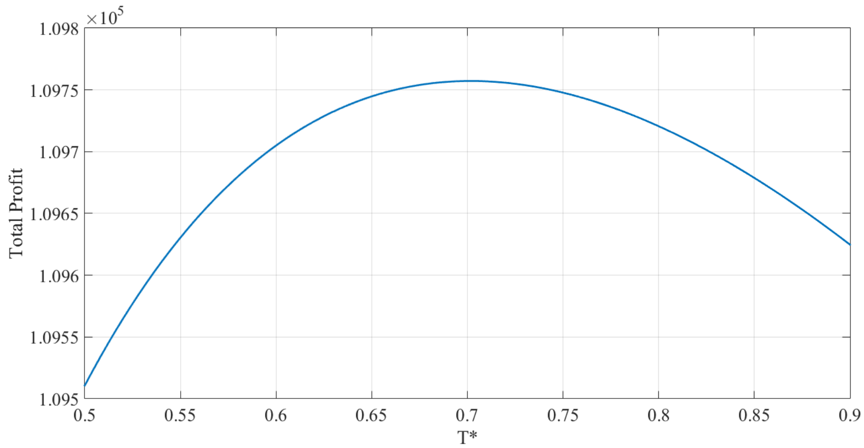

FIGURE 3. The Graph of The Function $d\left(TPU(\check{T}), 0_1\right)$

A closer look into Figure 4 shows an approximately linear relationship on opposite direction between the level of fuzziness and the total profit.

Similar to Kazemi et al.[51], this study also involved a simple regression analysis of the percentage change in the level of fuzziness and the change in total profit and the relationship is obtained as expressed in (19).

$$\Delta TPU(\%) = -0.01393 \times \Delta d\left(\check{T}, 0_1\right)(\%) - 0.3443 \tag{19}$$

where $R^2 = 0.3934$.

We investigate the effects of changes in the values of the setup cost, the demand rate, the holding cost per unit and the holding cost for reworked items on the optimal solutions. We change one parameter at a time while the other parameters being equal. The results are given in Table 3.

Parameter	Change	T	$TPU(T)$	Parameter	Change	T	$TPU(T)$
K	1000	0.5727	110468.48	h	15	0.7933	110249.49
	1500	0.7017	109757.16		20	0.7017	109757.16
	2000	0.8104	109045.84		25	0.6359	109316.01
	2500	0.9062	108334.53		30	0.5856	108912.78
	3000	0.9928	107623.21		35	0.5457	108539.10
β	800	0.6099	71129.00	h_1	18	0.7102	109808.08
	1000	0.6292	90274.89		20	0.7059	109782.54
	1200	0.7017	109757.16		22	0.7017	109757.16
	1400	0.9119	129725.59		24	0.6976	109731.93
	1600	infeasible	infeasible		26	0.6935	109706.84

TABLE 3. Effects of Parameter Changes on The Optimal Solution

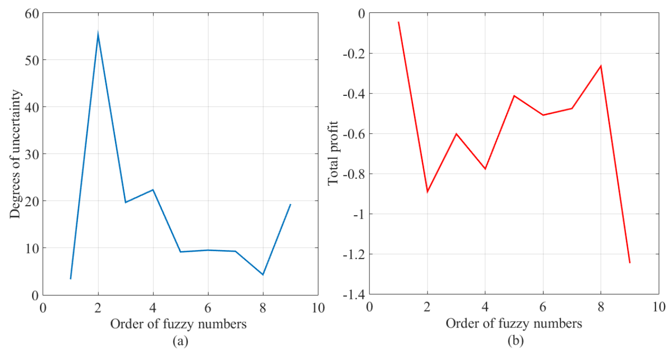

FIGURE 4. The effect of changes in the fuzzy numbers as a percentage on the level of fuzziness and the total profit

We can see that, with a higher fixed setup cost, we will have a smaller effect regarding the increase in both total profits and the cycle time because of the fuzziness introduced. However, if the manufacturer can reduce the setup cost, it may slightly increase the expected annual profit.

One of the ways to increase total profit is to use factors which will make it possible to increase the demand; marketing and promotion efforts are among the means of making this possible. Nevertheless, we may conclude the following result from Table 3. When the annual demand is increased from 1400 to 1600, the problem becomes infeasible as the assumptions in (1) and (2) are not satisfied, and it will not be possible to obtain the optimal solution.

When the holding costs increase, the optimal cycle length decreases significantly, while the expected total annual profit decreases gradually. Considering the other parameters of the model are kept constant, and the fact that the decrease in cycle time will translate into reduced optimal production, this result will reduce the annual average inventory level. As a result, it will reduce the annual inventory holding costs which will clearly increase the total profit.

As distinct from other production models, it is assumed that the inspection time continues after the end of production. Therefore, two separate inspection costs, one for in-production and the other for after-production, d_1 and d_2, were used. Doing this made it inevitable to analyse the effect of relevant costs on the optimal solution. Figure 5 shows the effect of d_1 and d_2 on the total profit. For example, new total profits were obtained having increased each value for $d_1 = 0.7$ and $d_2 = 0.5$ reference points by 20%. As a result, it was found that the effect on the total profit of the increase in d_1 by 20% is greater than the effect of the increase in d_2 by 20%. This result supports the assumption that the increase in d_1 reduces the total profit more rapidly; in other words, in-production screening is costlier than after-production screening.

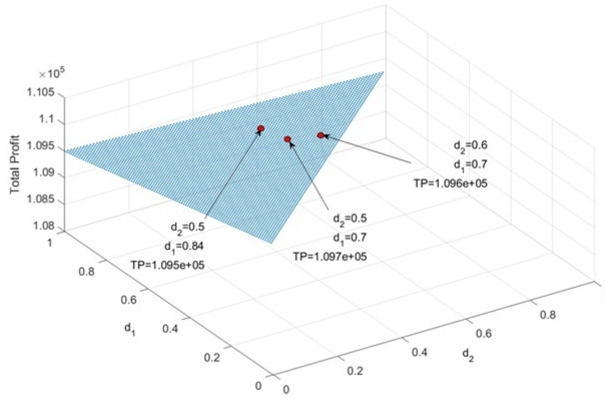

FIGURE 5. Variation in The Fuzzy Total Profit According to d_1 and d_2

7. Conclusion

For many years, emphasis has been placed on the importance of efficient inventory management. Although the first version of Harris's EOQ-model was produced as early as 1913, this type of model is still very relevant. However, management today is deeply concerned with increasing uncertainty. Probabilistic measures are incapable of capturing all of these uncertainties, so 'possibilistic' measures as judged by experts are required, making further development of fuzzy EPQ- and EOQ theories important. In this study, a production inventory model is analysed where the cycle time is fuzzy, and consequently production batches are uncertain. To the authors' knowledge, there has not been an analytical solution of this problem in the published literature. The first significant contribution of the model illustrated in the paper is the integration of inspection time into an economic production model. No previous model has considered the realistic case of screening onsite, resulting in the inspection of defective items at the same rate as demand before the end of the production cycle. Our model offers three practical features: (i) Screening starts immediately at the end of the production run, in response to a fast production process that makes screening during production impractical; (ii) only non-defective items are used to meet demand during production; and (iii) defective items, once identified, are kept in the inventory and reworked at a constant rate. As our second contribution, this formula is used to extend the traditional and fuzzy EPQ model, incorporating uncertain cycle time. The signal distance method was used to defuzzify the fuzzy model and, by use of derivatives, reduced to a simple equation. With asymmetric triangular fuzzy numbers for the cycle time, a numerical solution to this equation is needed. However, in contrast, this paper shows that an analytical solution can be obtained using the signed distance defuzzification method, following the assumption of symmetric triangular fuzzy numbers. This model builds on the previous studies by Björk [33, 34, 35] and, Mezei and Björk [29] which obtained the solution of the fuzzy inventory model both analytically and numerically. However, as distinct from these studies, this study used asymmetric triangular fuzzy numbers when expressing the fuzziness of the cycle time. Nevertheless, in the classical inventory model, the basis of this study, the assumptions are made of defective products occurring in each batch and product screening both in-production and after production. Our model is suitable for the solution of optimization problems in the context of a process industry. This revised fuzzy model is illustrated with a theoretical, but realistic, example. The result of this example showed that the level of fuzziness influences the cycle time, and thus, the optimal production quantity. It was found that the optimal production quantity changes approximately by 0.2% for the randomly selected fuzzy number when compared to the classical model. In contrast, when moving from the optimum as found by the crisp case to the optimum found by the fuzzy case, the reduction in total profits is not very great. The results allow us to conclude that, if the uncertainty in the cycle time does not greatly influence the total profit, management is able give the production planner an additional degree of freedom to depart from the crisp optimum. Such flexibility can be very useful in the dynamic and unpredictable environment in which many process industries operate. The changes in the total profit were analysed in terms of percentages for the changes in the level of fuzziness as calculated for a set of random fuzzy numbers as part of the numerical example. The results of this application revealed an approximately linear relationship with opposite directions between the increased level of fuzziness and the total profit. Therefore, managers are able to use these relationships to make a quick sensitivity analysis of the effects of changes in their inventory policies.

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Harun Öztürk*, Süleyman Demİrel University, Department of Business Administration, 32260, Isparta, TURKEY
Email address: harunozturk@sdu.edu.tr

Salİh Aytar, Süleyman Demİrel University, Department of Mathematics, 32260, Isparta, TURKEY
Email address: salihaytar@sdu.edu.tr

Fatİh Ahmet Senel, Süleyman Demİrel University, Department of Computer Engineering, 32260, Isparta, TURKEY
Email address: fatihsenel@sdu.edu.tr

∗ Corresponding author