FUZZY GRADE OF I.P.S. HYPERGROUPS OF ORDER 7

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Abstract. i.p.s. hypergroups are canonical hypergroups such that

$$[\forall\,(a,x),\ a+x\ni x]\Longrightarrow[a+x=x].$$

i.p.s. hypergroups were investigated in [1], [2], [3], [4] and it was proved that if the order is less than 9, they are strongly canonical (see [13]). In this paper we obtain the sequences of fuzzy sets and of join spaces determined (see [8]) by all i.p.s. hypergroups of order seven. For the meaning of the hypergroups ${}^iH$ and the notations, see [7], [8].

Definition. A W-hypergroupoid H has fuzzy grade $f.g.(H)=m$ if $\forall\, i<m$, ${}^iH$ and ${}^{i+1}H$ are not isomorphic (${}^\circ H=H$) and $\forall\, s:\ \ s>m$, ${}^sH$ is isomorphic to ${}^mH$. We say that H has strong fuzzy grade $s.f.g.(H)=m$ if $f.g.(H)=m$ and $\forall\, s:\ s>m$, ${}^sH={}^mH$.

Theorem. *Let $IPS(7)$ be the class of i.p.s. hypergroups of order 7. Then among the 36 hypergroups of $IPS(7)$,*

- *26 of the H_i are such that any two ${}^1H_i$ are non isomorphic,*
- *28 of the H_i have $f.g.(H)=1$,*
- *26 have $s.f.g.(H)=1$,*
- *7 have $f.g.(H)=2$,*
- *1 has $f.g.(H)=3$.*

1) Set

$H=H_1=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		2	3	0	4	5	6
2			0	1	4	5	6
3				2	4	5	6
4					5	K	6
5						4	6
6							A_6

where
$K=\{0,1,2,3\}$, $A_6=H_1-\{6\}$.
We have ${}^1\mu(0)={}^1\mu(1)=$
$={}^1\mu(2)={}^1\mu(3)=0,666$
${}^1\mu(4)={}^1\mu(5)=0,916$,
${}^1\mu(6)=1$.
It follows that

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${}^1H_1 =$

	0	1	2	3	4	5	6
0	K	K	K	K	A_6	A_6	H
1		K	K	K	A_6	A_6	H
2			K	K	A_6	A_6	H
3				K	A_6	A_6	H
4					4, 5	4, 5	4, 5, 6
5						4, 5	4, 5, 6
6							6

and hence:
${}^2\mu(0) = 0,195$
${}^2\mu(4) = 0,223$
${}^2\mu(6) = 0,267.$
Then ${}^1H_1 = {}^rH_1$, $\forall\, r > 1$.

2) Set

$H_2 =$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	3	2	4	5	6
2			0	1	4	5	6
3				0	4	5	6
4					5	K	6
5						4	6
6							A_6

In this case the membership function is the same as in 1), so ${}^1H$ also, and therefore we have again $\forall\, r,\ {}^rH_2 = {}^1H_2$.

3) Set

$H_3 =$

	0	1	2	3	4	5	6
0					4	5	6
1					4	5	6
2		$\mathbb{K}$			4	5	6
3					4	5	6
4					6	K	5
5						6	4
6							K

where $\mathbb{K} \in \{\mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2\}$.
We have
${}^1\mu(0) = {}^1\mu(1) = {}^1\mu(2) =$
$= {}^1\mu(3) = 0,679.$
${}^1\mu(4) = {}^1\mu(5) = {}^1\mu(6) = 1.$
It follows that

${}^1H_3 =$

	0	1	2	3	4	5	6
0	K	K	K	K	H	H	H
1		K	K	K	H	H	H
2			K	K	H	H	H
3				K	H	H	H
4					4, 5, 6	4, 5, 6	4, 5, 6
5						4, 5, 6	4, 5, 6
6							4, 5, 6

and we have:
${}^2\mu(0) = 0,186$
${}^2\mu(4) = 0,195.$
Then $\forall\, r$,
${}^rH_3 = {}^2H_3 = {}^1H_1.$

4) Set

$H_4 =$

	0	1	2	3	4	5	6
0					4	5	6
1					4	5	6
2		$\mathbb{K}$			4	5	6
3					4	5	6
4					K	6	5
5						K	4
6							K

where $\mathbb{K} \in \{\mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2\}$.
${}^1\mu$ and ${}^2\mu$ have the values computed in 3) and hence:
$\forall\, r > 1,\ {}^1H_3 = {}^1H_4 = {}^rH_4.$

5) Set

$H_5 =$

	0	1	2	3	4	5	6
0					4	5	6
1					4	5	6
2		$\mathbb{K}$			4	5	6
3					4	5	6
4					K	5	6
5						6	$K \cup \{4\}$
6							5

where $\mathbb{K} \in \{\mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2\}$.
We have:
${}^1\mu(0) = {}^1\mu(1) = {}^1\mu(2) =$
$= {}^1\mu(3) = 0,664$
${}^1\mu(4) = 0,865$
${}^1\mu(5) = {}^1\mu(6) = 1.$
It follows that

${}^1H_5 =$

	0	1	2	3	4	5	6
0	K	K	K	K	$K \cup \{4\}$	H	H
1		K	K	K	$K \cup \{4\}$	H	H
2			K	K	$K \cup \{4\}$	H	H
3				K	$K \cup \{4\}$	H	H
4					4	$4,5,6$	$4,5,6$
5						$5,6$	$5,6$
6							$5,6$

and hence:
${}^2\mu(0) = 0,197$
${}^2\mu(4) = 0,214$
${}^2\mu(5) = 0,234.$
It follows:
${}^rH_5 = {}^1H_5,$
$\forall\, r > 1.$

6) Set

$H_6 =$

	0	1	2	3	4	5	6
0					4	5	6
1					4	5	6
2		$\mathbb{K}$			4	5	6
3					4	5	6
4					K	5	6
5						$K \cup \{4\}$	6
6							A_6

where $\mathbb{K} \in \{\mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2\}$.
We have:
${}^1\mu(0) = {}^1\mu(1) = {}^1\mu(2) =$
$= {}^1\mu(3) = 0,66$
${}^1\mu(4) = 0,862$
${}^1\mu(5) = 0,924$
${}^1\mu(6) = 1.$
So we obtain

${}^1H_6 =$

	0	1	2	3	4	5	6
0	K	K	K	K	$K \cup \{4\}$	A_6	H
1		K	K	K	$K \cup \{4\}$	A_6	H
2			K	K	$K \cup \{4\}$	A_6	H
3				K	$K \cup \{4\}$	A_6	H
4					4	$4,5$	$4,5,6$
5						5	$5,6$
6							6

We now have:
${}^2\mu(0) = 0,202$
${}^2\mu(4) = 0,233$
${}^2\mu(5) = 0,267$
${}^2\mu(6) = 0,293$
It follows that $\forall\, r \geq 2,$
${}^1H_6 = {}^rH_6$

7) Set

$H_7 =$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		2	0	3	4	5	6
2			1	3	4	5	6
3				6	$0,1,2$	4	5
4					5	6	3
5						3	$0,1,2$
6							4

We have:
${}^1\mu(0) = 0,524 = {}^1\mu(1) = {}^1\mu(2)$
${}^1\mu(3) = {}^1\mu(4) = {}^1\mu(5) = {}^1\mu(6) = 1$
Therefore the join space
associated is as follows:

${}^1H_7=$

	0	1	2	3	4	5	6
0	$0,1,2$	$0,1,2$	$0,1,2$	H	H	H	H
1		$0,1,2$	$0,1,2$	H	H	H	H
2			$0,1,2$	H	H	H	H
3				$3,4,5,6$	$3,4,5,6$	$3,4,5,6$	$3,4,5,6$
4					$3,4,5,6$	$3,4,5,6$	$3,4,5,6$
5						$3,4,5,6$	$3,4,5,6$
6							$3,4,5,6$

We clearly have:

$$ {}^2\mu(0) = {}^2\mu(1) = {}^2\mu(2) = 0,195,\ {}^2\mu(3) = {}^2\mu(4) = {}^2\mu(5) = {}^2\mu(6) = 0,186, $$

whence $\forall\, r \geq 2$, ${}^rH_7 = {}^1H_7$.

8) Set

$H_8 =$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		2	0	3	4	5	6
2			1	3	4	5	6
3				4	$0,1,2$	5	6
4					3	5	6
5						6	$0,1,2,3,4$
6							5

We have
${}^1\mu(0) = {}^1\mu(1) = {}^1\mu(2) = 0,581$
${}^1\mu(3) = {}^1\mu(4) = 0,822$
${}^1\mu(5) = {}^1\mu(6) = 1$
It follows that

${}^1H_8=$

	0	1	2	3	4	5	6
0	$0,1,2$	$0,1,2$	$0,1,2$	$0,1,2,3,4$	$0,1,2,3,4$	H	H
1		$0,1,2$	$0,1,2$	$0,1,2,3,4$	$0,1,2,3,4$	H	H
2			$0,1,2$	$0,1,2,3,4$	$0,1,2,3,4$	H	H
3				$3,4$	$3,4$	$3,4,5,6$	$3,4,5,6$
4					$3,4$	$3,4,5,6$	$3,4,5,6$
5						$5,6$	$5,6$
6							$5,6$

Hence: ${}^2\mu(0) = 0,216$, ${}^2\mu(3) = 0,225$, ${}^2\mu(5) = 0,238$. We conclude that $\forall\, r \geq 2$, ${}^rH_8 = {}^1H_8$.

9) Set

$H_9 =$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		2	0	3	4	5	6
2			1	3	4	5	6
3				4	$0,1,2$	5	6
4					3	5	6
5						$0,1,2,3,4$	6
6							A_6

We have
${}^1\mu(0) = {}^1\mu(1) = {}^1\mu(2) =$
$= 0,576$
${}^1\mu(3) = {}^1\mu(4) = 0,819$
${}^1\mu(5) = 0,924$
${}^1\mu(6) = 1$
It follows that
${}^1H_9$ is as follows:

$^1H_9=$

	0	1	2	3	4	5	6
0	$0,1,2$	$0,1,2$	$0,1,2$	$0,1,2,3,4$	$0,1,2,3,4$	A_6	H
1		$0,1,2$	$0,1,2$	$0,1,2,3,4$	$0,1,2,3,4$	A_6	H
2			$0,1,2$	$0,1,2,3,4$	$0,1,2,3,4$	A_6	H
3				$3,4$	$3,4$	$3,4,5$	$3,4,5,6$
4					$3,4$	$3,4,5$	$3,4,5,6$
5						5	$5,6$
6							6

We have: $^2\mu(0) = 0,220$, $^2\mu(3) = 0,239$, $^2\mu(5) = 0,269$, $^2\mu(6) = 0,297$. Therefore $\forall\, r \geq 1$, $^rH_9 = {}^2H_9 = {}^1H_9$.

10) Set

$H_{10} =$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		2	0	3	4	5	6
2			1	3	4	5	6
3				$0,1,2$	4	5	6
4					5	$0,1,2,3$	6
5						4	6
6							A_6

We have:
$^1\mu(0) = {}^1\mu(1) =$
$= {}^1\mu(2) = 0,571$,
$^1\mu(3) = 0,741$,
$^1\mu(4) = {}^1\mu(5) = 0,917$,
$^1\mu(6) = 1$.
Hence:

$^1H_{10}=$

	0	1	2	3	4	5	6
0	$0,1,2$	$0,1,2$	$0,1,2$	$0,1,2,3$	A_6	A_6	H
1		$0,1,2$	$0,1,2$	$0,1,2,3$	A_6	A_6	H
2			$0,1,2$	$0,1,2,3$	A_6	A_6	H
3				3	$3,4,5$	$3,4,5$	$3,4,5,6$
4					$4,5$	$4,5$	$4,5,6$
5						$4,5$	$4,5,6$
6							6

It follows that $^2\mu(0) = {}^2\mu(1) = {}^2\mu(2) = 0,223$, $^2\mu(3) = 0,232$,
$^2\mu(4) = {}^2\mu(5) = 0,251$, $^2\mu(6) = 0,284$. Therefore $\forall\, r > 1$, $^rH_{10} = {}^1H_{10}$.

11) Set

$H_{11} =$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		2	0	3	4	5	6
2			1	3	4	5	6
3				$0,1,2$	4	5	6
4					$0,1,2,3$	5	6
5						$0,1,2,3,4$	5
6							A_6

We have:
$^1\mu(0) = {}^1\mu(1) =$
$= {}^1\mu(2) = 0,564$,
$^1\mu(3) = 0,735$,
$^1\mu(4) = 0,837$,
$^1\mu(5) = 0,936$,
$^1\mu(6) = 1$.
Hence:

$^1H_{11}=$

	0	1	2	3	4	5	6
0	$0,1,2$	$0,1,2$	$0,1,2$	$0,1,2,3$	$0,1,2,3,4$	A_6	H
1		$0,1,2$	$0,1,2$	$0,1,2,3$	$0,1,2,3,4$	A_6	H
2			$0,1,2$	$0,1,2,3$	$0,1,2,3,4$	A_6	H
3				3	$3,4$	$3,4,5$	$3,4,5,6$
4					4	$4,5$	$4,5,6$
5						5	$5,6$
6							6

It follows that $^2\mu(0) = {}^2\mu(1) = {}^2\mu(2) = 0,229$, $^2\mu(3) = 0,249$, $^2\mu(4) = 0,272$, $^2\mu(5) = 0,291$, $^2\mu(6) = 0,310$, whence we have: $\forall\, r \geq 1$, $^rH_{11} = {}^1H_{11}$.

12) Set

$H_{12} =$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		2	0	3	4	5	6
2			1	3	4	5	6
3				$0,1,2$	4	5	6
4					$0,1,2,3$	6	5
5						$0,1,2,3$	4
6							$0,1,2,3$

We have: $^1\mu(0) = {}^1\mu(1) = {}^1\mu(2) = 0,583$, $^1\mu(3) = 0,75$, $^1\mu(4) = {}^1\mu(5) = {}^1\mu(6) = 1$. Therefore

$^1H_{12}=$

	0	1	2	3	4	5	6
0	$0,1,2$	$0,1,2$	$0,1,2$	$0,1,2,3$	H	H	H
1		$0,1,2$	$0,1,2$	$0,1,2,3$	H	H	H
2			$0,1,2$	$0,1,2,3$	H	H	H
3				3	$3,4,5,6$	$3,4,5,6$	$3,4,5,6$
4					$4,5,6$	$4,5,6$	$4,5,6$
5						$4,5,6$	$4,5,6$
6							$4,5,6$

It follows that $^2\mu(0) = {}^2\mu(1) = {}^2\mu(2) = {}^2\mu(4) = {}^2\mu(5) = {}^2\mu(6) = 0,214$, $^2\mu(3) = 0,212$. Therefore

$^2H_{12} =$

	0	1	2	3	4	5	6
0	A_3	A_3	A_3	H	A_3	A_3	A_3
1		A_3	A_3	H	A_3	A_3	A_3
2			A_3	H	A_3	A_3	A_3
3				3	H	H	H
4					A_3	A_3	A_3
5						A_3	A_3
6							A_3

We conclude that $^3\mu(0) = {}^3\mu(1) = {} = {}^3\mu(2) = {}^3\mu(4) = {} = {}^3\mu(5) = {}^3\mu(6) \neq {}^3\mu(3)$. Hence $\forall\, r > 2$, $^rH_{12} = {}^2H_{12} \neq {}^1H_{12}$.

13) Set

$H_{13}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			$0,1$	3	4	5	6
3				$0,1,2$	4	5	6
4					$0,1,2,3$	6	5
5						$0,1,2,3$	4
6							$0,1,2,3$

We have:
${}^1\mu(0)={}^1\mu(1)=0,512$,
${}^1\mu(2)=0,635$,
${}^1\mu(3)=0,75$,
${}^1\mu(4)={}^1\mu(5)=$
$={}^1\mu(6)=1$.
The corresponding join space is

${}^1H_{13}=$

	0	1	2	3	4	5	6
0	$0,1$	$0,1$	$0,1,2$	$0,1,2,3$	H	H	H
1		$0,1$	$0,1,2$	$0,1,2,3$	H	H	H
2			2	$2,3$	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$
3				3	$3,4,5,6$	$3,4,5,6$	$3,4,5,6$
4					$4,5,6$	$4,5,6$	$4,5,6$
5						$4,5,6$	$4,5,6$
6							$4,5,6$

We obtain that ${}^2\mu(0)={}^2\mu(1)=0,252$, ${}^2\mu(2)=0,250$, ${}^2\mu(3)=0,239$, ${}^2\mu(4)=$ ${}^2\mu(5)={}^2\mu(6)=0,225$. So it's clear that $\forall\, r>1$, ${}^rH_{13}={}^1H_{13}$.

14) Clearly we obtain the same join space ${}^1H_{13}$ starting from the i.p.s. hypergroup

$H_{14}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			$0,1$	3	4	5	6
3				$0,1,2$	4	5	6
4					$0,1,2,3$	6	5
5						4	$0,1,2,3$
6							4

Indeed, H_{13} and H_{14} determine the same membership function ${}^1\mu$,
so ${}^1H_{13}={}^1H_{14}$.

15) Set

$H_{15}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			$0,1$	3	4	5	6
3				$0,1,2$	4	5	6
4					5	$0,1,2,3$	6
5						4	6
6							A_6

Then
${}^1\mu(0)={}^1\mu(1)=0,5$,
${}^1\mu(2)=0,625$,
${}^1\mu(3)=0,741$,
${}^1\mu(4)={}^1\mu(5)=0,917$,
${}^1\mu(6)=1$.

Hence, we have:

$^1H_{15}=$

	0	1	2	3	4	5	6
0	0,1	0,1	0,1,2	0,1,2,3	A_6	A_6	H
1		0,1	0,1,2	0,1,2,3	A_6	A_6	H
2			2	2,3	2,3,4,5	2,3,4,5	2,3,4,5,6
3				3	3,4,5	3,4,5	3,4,5,6
4					4,5	4,5	4,5,6
5						4,5	4,5,6
6							6

Therefore $^2\mu(0) = {}^2\mu(1) = 0,260$, $^2\mu(2) = 0,263$, $^2\mu(3) = 0,262$, $^2\mu(4) = {}^2\mu(5) = 0,265$, $^2\mu(6) = 0,293$.

We obtain a structure $^2H_{15}$ which is isomorphic to $^1H_{15}$. Hence:

$\forall\, r > 1$, $^rH_{15} \sim {}^1H_{15}$, $^{2k}H_{15} = {}^2H_{15}$, $^1H_{15} = {}^{2k+1}H_{15}$.

16) Set

$H_{16}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			0,1	3	4	5	6
3				0,1,2	4	5	6
4					0,1,2,3	5	6
5						6	0,1,2,3,4
6							5

We have: $^1\mu(0) = {}^1\mu(1) = 0,498$, $^1\mu(2) = 0,623$, $^1\mu(3) = 0,739$, $^1\mu(4) = 0,84$, $^1\mu(5) = {}^1\mu(6) = 1$. Hence the associated join space is

$^1H_{16}=$

	0	1	2	3	4	5	6
0	0,1	0,1	0,1,2	0,1,2,3	0,1,2,3,4	H	H
1		0,1	0,1,2	0,1,2,3	0,1,2,3,4	H	H
2			2	2,3	2,3,4	2,3,4,5,6	2,3,4,5,6
3				3	3,4	3,4,5,6	3,4,5,6
4					4	4,5,6	4,5,6
5						5,6	5,6
6							5,6

and it follows that $^2\mu(0) = {}^2\mu(1) = {}^2\mu(5) = {}^2\mu(6) = 0,262$, $^2\mu(2) = {}^2\mu(4) = 0,267$, $^2\mu(3) = 0,271$. So the join space associated with $^1H_{16}$ is

$^2H_{16}=$

	0	1	2	3	4	5	6
0	0,1,5,6	0,1,5,6	A_3	H	A_3	0,1,5,6	0,1,5,6
1		0,1,5,6	A_3	H	A_3	0,1,5,6	0,1,5,6
2			2,4	2,3,4	2,4	A_3	A_3
3				3	2,3,4	H	H
4					2,4	A_3	A_3
5						0,1,5,6	0,1,5,6
6							0,1,5,6

We now have: ${}^3\mu(0) = {}^3\mu(1) = {}^3\mu(5) = {}^2\mu(6) = 0,195$, ${}^3\mu(2) = {}^3\mu(4) = 0,223$, ${}^3\mu(3) = 0,267$. Therefore $\forall r > 2$, ${}^rH_{16} = {}^2H_{16} \neq {}^1H_{16}$.

17) Set

$H_{17}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			0,1	3	4	5	6
3				0,1,2	4	5	6
4					0,1,2,3	5	6
5						0,1,2,3,4	6
6							A_6

We have:
${}^1\mu(0) = {}^1\mu(1) = 0,493$,
${}^1\mu(2) = 0,619$,
${}^1\mu(3) = 0,735$,
${}^1\mu(4) = 0,837$,
${}^1\mu(5) = 0,924$,
${}^1\mu(6)=1$.

The join space associated with the former ${}^1\mu$ is the following:

${}^1H_{17}=$

	0	1	2	3	4	5	6
0	0,1	0,1	0,1,2	0,1,2,3	0,1,2,3,4	A_6	H
1		0,1	0,1,2	0,1,2,3	0,1,2,3,4	A_6	H
2			2	2,3	2,3,4	2,3,4,5	2,3,4,5,6
3				3	3,4	3,4,5	3,4,5,6
4					4	4,5	4,5,6
5						5	5,6
6							6

The corresponding membership function is: ${}^2\mu(0) = {}^2\mu(1) = 0,265$, ${}^2\mu(2) = 0,274$, ${}^2\mu(3) = 0,283$, ${}^2\mu(4) = 0,291$, ${}^2\mu(5) = 0,303$, ${}^2\mu(6) = 0,318$. Hence: $\forall r > 1$, ${}^rH_{17} = {}^1H_{17}$.

18) Set

$H_{18}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			0,1	3	4	5	6
3				0,1	4	5	6
4					0,1	5	6
5						0,1,2,3,4	6
6							A_6

The corresponding ${}^1\mu$ is:
${}^1\mu(0) = {}^1\mu(1) = 0,552$,
${}^1\mu(2) = 0,728$,
${}^1\mu(3) = 0,796$, ${}^1\mu(4) = 0,837$,
${}^1\mu(5) = 0,924$,
${}^1\mu(6) = 1$. We therefore have:
${}^1H_{18} = {}^1H_{17}$.

19) Set

$H_{19}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			0,1	3	4	5	6
3				4	0,1,2	5	6
4					3	5	6
5						6	0,1,2,3,4
6							5

We have:
${}^1\mu(0) = {}^1\mu(1) = 0,510$,
${}^1\mu(2) = 0,633$,
${}^1\mu(3) = {}^1\mu(4) = 0,822$,
${}^1\mu(5) = {}^1\mu(6) = 1$.
Then the associated
join space is the following:

$^1H_{19}=$

	0	1	2	3	4	5	6
0	0,1	0,1	0,1,2	0,1,2,3,4	0,1,2,3,4	H	H
1		0,1	0,1,2	0,1,2,3,4	0,1,2,3,4	H	H
2			2	2,3,4	2,3,4	2,3,4,5,6	2,3,4,5,6
3				3,4	3,4	3,4,5,6	3,4,5,6
4					3,4	3,4,5,6	3,4,5,6
5						5,6	5,6
6							5,6

From $^1H_{19}$ we obtain the following membership function: $^2\mu(0)=^2\mu(1)=0,233$, $^2\mu(2)=0,249$, $^2\mu(3)=^2\mu(4)=0,247$, $^2\mu(5)=^2\mu(6)=0,238$. Therefore $^2H_{19} = {}^1H_{19}$ and hence: $\forall\, r > 1$, $^rH_{19} = {}^1H_{19}$.

20) Set

$H_{20}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			0,1	3	4	5	6
3				4	0,1,2	5	6
4					3	5	6
5						0,1,2,3,4	6
6							A_6

The corresponding $^1\mu$ is as follows: $^1\mu(0) = {}^1\mu(1) = 0,505$, $^1\mu(2) = 0,629$, $^1\mu(3) = {}^1\mu(4) = 0,819$, $^1\mu(5) = 0,924$, $^1\mu(6) = 1$. The associated join space is

$^1H_{20}=$

	0	1	2	3	4	5	6
0	0,1	0,1	0,1,2	0,1,2,3,4	0,1,2,3,4	A_6	H
1		0,1	0,1,2	0,1,2,3,4	0,1,2,3,4	A_6	H
2			2	2,3,4	2,3,4	2,3,4,5	2,3,4,5,6
3				3,4	3,4	3,4,5	3,4,5,6
4					3,4	3,4,5	3,4,5,6
5						5	5,6
6							6

This determines the following membership function: $^2\mu(0) = {}^2\mu(1) = 0,257$, $^2\mu(2) = 0,255$, $^2\mu(3) = {}^2\mu(4) = 0,261$, $^2\mu(5) = 0,281$, $^2\mu(6) = 0,305$ from which we obtain:

$^2H_{20}=$

	0	1	2	3	4	5	6
0	0,1	0,1	0,1,2	0,1,3,4	0,1,3,4	0,1,3,4,5	A_2
1		0,1	0,1,2	0,1,3,4	0,1,3,4	0,1,3,4,5	A_2
2			2	0,1,2,3,4	0,1,2,3,4	A_6	H
3				3,4	3,4	3,4,5	3,4,5,6
4					3,4	3,4,5	3,4,5,6
5						5	5,6
6							6

The corresponding membership function is: $^3\mu(0) = {}^3\mu(1) = 0,257$, $^3\mu(2) = 0,289$, $^3\mu(3) = {}^3\mu(4) = 0,256$, $^3\mu(5) = 0,279$, $^3\mu(6) = 0,304$, and the associated join space $^3H_{20}$ is:

$^3H_{20}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2, 5	0, 1, 3, 4	0, 1, 3, 4	0, 1, 5	0, 1, 2, 5, 6
1		0, 1	0, 1, 2, 5	0, 1, 3, 4	0, 1, 3, 4	0, 1, 5	0, 1, 2, 5, 6
2			2	A_6	A_6	2, 5	2, 6
3				3, 4	3, 4	0, 1, 3, 4, 5	H
4					3, 4	0, 1, 3, 4, 5	H
5						5	2, 5, 6
6							6

Hence we obtain: $^4\mu(0) = {}^4\mu(1) = 0,255$, $^4\mu(2) = 0,292$, $^4\mu(3) = {}^4\mu(4) = 0,252$, $^4\mu(5) = 0,270$, $^4\mu(6) = 0,311$.
Therefore $^4H_{20} = {}^3H_{20}$, whence $\forall\, r > 3$, $^rH_{20} = {}^3H_{20} \neq {}^2H_{20} \neq {}^1H_{20}$.

21) Set

$H_{21}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			0, 1	3	4	5	6
3				5	0, 1, 2	4	6
4					5	3	6
5						0, 1, 2	6
6							A_6

The associated membership function is:
$^1\mu(0) = {}^1\mu(1) = 0,524$,
$^1\mu(2) = 0,646$,
$^1\mu(3) = {}^1\mu(4) =$
$= {}^1\mu(5) = 0,907$,
$^1\mu(6) = 1$.

The corresponding join space is as follows:

$^1H_{21}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2	A_6	A_6	A_6	H
1		0, 1	0, 1, 2	A_6	A_6	A_6	H
2			2	2, 3, 4, 5	2, 3, 4, 5	2, 3, 4, 5	2, 3, 4, 5, 6
3				3, 4, 5	3, 4, 5	3, 4, 5	3, 4, 5, 6
4					3, 4, 5	3, 4, 5	3, 4, 5, 6
5						3, 4, 5	3, 4, 5, 6
6							6

We obtain: $^2\mu(0) = {}^2\mu(1) = 0,246$, $^2\mu(2) = 0,235$, $^2\mu(3) = {}^2\mu(4) = {}^2\mu(5) = 0,230$, $^2\mu(6) = 0,267$, and hence:

$^2H_{21}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2	A_6	A_6	A_6	0, 1, 6
1		0, 1	0, 1, 2	A_6	A_6	A_6	0, 1, 6
2			2	2, 3, 4, 5	2, 3, 4, 5	2, 3, 4, 5	0, 1, 2, 6
3				3, 4, 5	3, 4, 5	3, 4, 5	H
4					3, 4, 5	3, 4, 5	H
5						3, 4, 5	H
6							6

The corresponding membership function is as follows: ${}^3\mu(0) = {}^3\mu(1) = 0,251$, ${}^3\mu(2) = 0,232$, ${}^3\mu(3) = {}^3\mu(4) = {}^3\mu(5) = 0,223$, ${}^3\mu(6) = 0,284$. It follows that $\forall\, r > 2$, ${}^rH_{21} = {}^2H_{21} \neq {}^1H_{21}$.

22) Set

$H_{22}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			$0,1$	3	4	5	6
3				$0,1,2$	5	4	6
4					$0,1,2$	3	6
5						$0,1,2$	6
6							A_6

We obtain the same membership functions as for H_{21} and by consequence the same associated join spaces. Therefore we have: $\forall\, r > 2$, ${}^rH_{22}={}^2H_{22}={}^2H_{21}$, ${}^2H_{22} \neq {}^1H_{22} = {}^1H_{21}$.

23) Set

$H_{23}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			$0,1$	3	4	5	6
3				5	$0,1,2$	6	4
4					6	3	5
5						4	$0,1,2$
6							6

We have:
${}^1\mu(0) = {}^1\mu(1) = 0,548$,
${}^1\mu(2) = 0,667$,
${}^1\mu(3) = {}^1\mu(4) =$
$= {}^1\mu(5) = {}^1\mu(6) = 1$.
So the associated join space is the following:

${}^1H_{23}=$

	0	1	2	3	4	5	6
0	$0,1$	$0,1$	$0,1,2$	H	H	H	H
1		$0,1$	$0,1,2$	H	H	H	H
2			2	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$
3				$3,4,5,6$	$3,4,5,6$	$3,4,5,6$	$3,4,5,6$
4					$3,4,5,6$	$3,4,5,6$	$3,4,5,6$
5						$3,4,5,6$	$3,4,5,6$
6							$3,4,5,6$

It follows that ${}^2\mu(0)={}^2\mu(1)=0,234$, ${}^2\mu(2)=0,214$, ${}^2\mu(3)={}^2\mu(4)={}^2\mu(5)={}^2\mu(6)=0,197$. Therefore we have $\forall\, r > 1$, ${}^rH_{23} = {}^1H_{23}$.

24) Set

$H_{24}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			3	$0,1$	4	5	6
3				2	4	5	6
4					$0,1,2,3$	6	5
5						$0,1,2,3$	4
6							$0,1,2,3$

We have:
${}^1\mu(0) = {}^1\mu(1) = 0,536$,
${}^1\mu(2) = {}^1\mu(3) = 0,719$,
${}^1\mu(4)={}^1\mu(5)={}^1\mu(6)=1$.
So the associated join space is as follows:

$^1H_{24}=$

	0	1	2	3	4	5	6
0	$0,1$	$0,1$	$0,1,2,3$	$0,1,2,3$	H	H	H
1		$0,1$	$0,1,2,3$	$0,1,2,3$	H	H	H
2			$2,3$	$2,3$	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$
3				$2,3$	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$
4					$4,5,6$	$4,5,6$	$4,5,6$
5						$4,5,6$	$4,5,6$
6							$4,5,6$

We have: $\mu(0) = \mu(1) = 0,238$, $\mu(2) = \mu(3) = 0,225$, $\mu(4) = \mu(5) = \mu(6) = 0,216$. It clearly follows that $\forall\, r > 1$, $^rH_{24} = {}^1H_{24}$.

25) Set

$H_{25}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			3	$0,1$	4	5	6
3				2	4	5	6
4					5	6	$0,1,2,3$
5						$0,1,2,3$	4
6							5

We have:
$\mu(0) = \mu(1) = 0,536$,
$\mu(2) = \mu(3) = 0,719$,
$\mu(4) = \mu(5) = \mu(6) = 1$.
Hence: $^1H_{25} = {}^1H_{24}$.

26) Set

$H_{26}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			3	$0,1$	4	5	6
3				2	4	5	6
4					$0,1,2,3$	5	6
5						$0,1,2,3,4$	6
6							A_6

The corresponding membership function is as follows:
$^1\mu(0) = {}^1\mu(1) = 0,517$,
$^1\mu(2) = {}^1\mu(3) = 0,702$,
$^1\mu(4) = 0,837$,
$^1\mu(5) = 0,924$, $^1\mu(6) = 1$.
$^1\mu$ determines the join space $^1H_{26}$ where

$^1H_{26}=$

	0	1	2	3	4	5	6
0	$0,1$	$0,1$	$0,1,2,3$	$0,1,2,3$	$0,1,2,3,4$	A_6	H
1		$0,1$	$0,1,2,3$	$0,1,2,3$	$0,1,2,3,4$	A_6	H
2			$2,3$	$2,3$	$2,3,4$	$2,3,4,5$	$2,3,4,5,6$
3				$2,3$	$2,3,4$	$2,3,4,5$	$2,3,4,5,6$
4					4	$4,5$	$4,5,6$
5						5	$5,6$
6							6

From this one obtains: $^2\mu(0) = {}^2\mu(1) = 0,252$, $^2\mu(2) = {}^2\mu(3) = 0,255$, $^2\mu(4) = 0,270$, $^2\mu(5) = 0,292$, $^2\mu(6) = 0,311$. It follows that $^2H_{26} = {}^1H_{26}$ and hence: $\forall\, r > 1$, $^rH_{26} = {}^1H_{26}$.

27) Set

$H_{27}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			3	0, 1	4	5	6
3				2	4	5	6
4					0, 1, 2, 3	5	6
5						6	0, 1, 2, 3, 4
6							5

The membership function associated with H_{27} is the following:
${}^1\mu(0) = {}^1\mu(1) = 0,521$,
${}^1\mu(2) = {}^1\mu(3) = 0,706$,
${}^1\mu(4) = 0,84$,
${}^1\mu(5) = {}^1\mu(6) = 1$.

We obtain the join space ${}^1H_{27}$

${}^1H_{27}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2, 3	0, 1, 2, 3	0, 1, 2, 3, 4	H	H
1		0, 1	0, 1, 2, 3	0, 1, 2, 3	0, 1, 2, 3, 4	H	H
2			2, 3	2, 3	2, 3, 4	2, 3, 4, 5, 6	2, 3, 4, 5, 6
3				2, 3	2, 3, 4	2, 3, 4, 5, 6	2, 3, 4, 5, 6
4					4	4, 5, 6	4, 5, 6
5						5, 6	5, 6
6							5, 6

So we have: ${}^2\mu(0) = {}^2\mu(1) = 0,248$, ${}^2\mu(2) = {}^2\mu(3) = 0,247$, ${}^2\mu(4) = 0,249$, ${}^2\mu(5) = {}^2\mu(6) = 0,253$. From this, we obtain:

${}^2H_{27}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2, 3	0, 1, 2, 3	0, 1, 4	0, 1, 4, 5, 6	0, 1, 4, 5, 6
1		0, 1	0, 1, 2, 3	0, 1, 2, 3	0, 1, 4	0, 1, 4, 5, 6	0, 1, 4, 5, 6
2			2, 3	2, 3	0, 1, 2, 3, 4	H	H
3				2, 3	0, 1, 2, 3, 4	H	H
4					4	4, 5, 6	4, 5, 6
5						5, 6	5, 6
6							5, 6

We clearly have: ${}^2H_{27} \simeq {}^1H_{27}$; moreover, ${}^3\mu(0) = {}^3\mu(1) = 0,247$, ${}^3\mu(2) = {}^3\mu(3) = 0,248$, ${}^3\mu(4) = 0,249$, ${}^3\mu(5) = {}^3\mu(6) = 0,253$. That is, ${}^3\mu = {}^1\mu$, and hence: $\forall\, k \geq 1$, ${}^2\mu = {}^4\mu = {}^{2k}\mu$ and ${}^1\mu = {}^{2k+1}\mu$. Finally, $\forall\, k \geq 1$, ${}^{2k}H_{27} = {}^2H_{27} \sim {}^1H_{27} = {}^{2k+1}H_{27}$.

28) Set

$H_{28}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			0, 1	4	3	5	6
3				0, 1	2	5	6
4					0, 1	5	6
5						6	0, 1, 2, 3, 4
6							5

We have:
${}^1\mu(0) = {}^1\mu(1) = 0,557$,
${}^1\mu(2) = {}^1\mu(3) = {}^1\mu(4) = 0,8$,
${}^1\mu(5) = {}^1\mu(6) = 1$.
Hence we obtain:

$^1H_{28}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 2, 3, 4	H	H
1		0, 1	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 2, 3, 4	H	H
2			2, 3, 4	2, 3, 4	2, 3, 4	2, 3, 4, 5, 6	2, 3, 4, 5, 6
3				2, 3, 4	2, 3, 4	2, 3, 4, 5, 6	2, 3, 4, 5, 6
4					2, 3, 4	2, 3, 4, 5, 6	2, 3, 4, 5, 6
5						5, 6	5, 6
6							5, 6

The corresponding membership function is as follows: $^2\mu(0) = {}^2\mu(1) = {}^2\mu(5) = {}^2\mu(6) = 0,231$, $^2\mu(2) = {}^2\mu(3) = {}^2\mu(4) = 0,218$. It follows that the associated join space is

$^2H_{28}=$

	0	1	2	3	4	5	6
0	0, 1, 5, 6	0, 1, 5, 6	H	H	H	0, 1, 5, 6	0, 1, 5, 6
1		0, 1, 5, 6	H	H	H	0, 1, 5, 6	0, 1, 5, 6
2			2, 3, 4	2, 3, 4	2, 3, 4	H	H
3				2, 3, 4	2, 3, 4	H	H
4					2, 3, 4	H	H
5						0, 1, 5, 6	0, 1, 5, 6
6							0, 1, 5, 6

From this we find a function $^3\mu$ which gives the same join space. Indeed: $^3\mu(0) = {}^3\mu(1) = {}^3\mu(5) = {}^3\mu(6) = 0,186$, $^3\mu(2) = {}^3\mu(3) = {}^3\mu(4) = 0,195$. Therefore we have $\forall\, r > 2$, $^rH_{28} = {}^2H_{28} \neq {}^1H_{28}$.

29) Set

$H_{29}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			4	0, 1	3	5	6
3				4	2	5	6
4					0, 1	5	6
5						6	0, 1, 2, 3, 4
6							5

We obtain the same membership function $^1\mu$ associated with 28), and therefore what does "equal sequence" means? $^1H_{29}$ etc. are not sequences!

30) Set

$H_{30}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			4	0, 1	5	3	6
3				5	2	4	6
4					3	0, 1	6
5						2	6
6							A_6

We have:
$^1\mu(0) = {}^1\mu(1) = 0,595$,
$^1\mu(2) = {}^1\mu(3) = {}^1\mu(4) =$
$= {}^1\mu(5) = 0,896$,
$^1\mu(6) = 1$.
Hence the associated join space $^1H_{30}$ is as follows:

$^1H_{30}=$

	0	1	2	3	4	5	6
0	$0,1$	$0,1$	A_6	A_6	A_6	A_6	H
1		$0,1$	A_6	A_6	A_6	A_6	H
2			$2,3,4,5$	$2,3,4,5$	$2,3,4,5$	$2,3,4,5$	$2,3,4,5,6$
3				$2,3,4,5$	$2,3,4,5$	$2,3,4,5$	$2,3,4,5,6$
4					$2,3,4,5$	$2,3,4,5$	$2,3,4,5,6$
5						$2,3,4,5$	$2,3,4,5,6$
6							6

The membership function associated with $^1H_{30}$ is $^2\mu(0) = {}^2\mu(1) = 0,218$, $^2\mu(2) = {}^2\mu(3) = {}^2\mu(4) = {}^2\mu(5) = 0,201$, $^2\mu(6) = 0,244$. Hence the associated join space is as follows:

$^2H_{30}=$

	0	1	2	3	4	5	6
0	$0,1$	$0,1$	A_6	A_6	A_6	A_6	$0,1,6$
1		$0,1$	A_6	A_6	A_6	A_6	$0,1,6$
2			$2,3,4,5$	$2,3,4,5$	$2,3,4,5$	$2,3,4,5$	H
3				$2,3,4,5$	$2,3,4,5$	$2,3,4,5$	H
4					$2,3,4,5$	$2,3,4,5$	H
5						$2,3,4,5$	H
6							6

From this we obtain: $^3\mu(0) = {}^3\mu(1) = 0,223$, $^3\mu(2) = {}^3\mu(3) = {}^3\mu(4) = {}^3\mu(5) = {}^3\mu(6) = 0,267$. Therefore $\forall\, r > 2$, $^rH_{30} = {}^2H_{30} \neq {}^1H_{30}$.

31) Set

$H_{31}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			2	5	$0,1$	6	3
3				$0,1$	6	2	4
4					2	3	5
5						4	$0,1$
6							2

The corresponding membership function is: $^1\mu(0) = {}^1\mu(1) = 0,643$, $^1\mu(2) = {}^1\mu(3) = {}^1\mu(4) = {} = {}^1\mu(5) = {}^1\mu(6) = 1$ and the associated join space is

$^1H_{31}=$

	0	1	2	3	4	5	6
0	$0,1$	$0,1$	H	H	H	H	H
1		$0,1$	H	H	H	H	H
2			$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$
3				$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$
4					$2,3,4,5,6$	$2,3,4,5,6$	$2,3,4,5,6$
5						$2,3,4,5,6$	$2,3,4,5,6$
6							$2,3,4,5,6$

We obtain: ${}^2\mu(0) = {}^2\mu(1) = 0,202$, ${}^2\mu(2)={}^2\mu(3)={}^2\mu(4)={}^2\mu(5)={}^2\mu(6)=0,175$. Hence $\forall\, r > 1$, ${}^rH_{31} = {}^1H_{31}$.

32) Set

$H_{32}=$

	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1		0	2	3	4	5	6
2			4	0, 1	3	5	6
3				4	2	5	6
4					0, 1	5	6
5						0, 1, 2, 3, 4	6
6							A_6

We have:
${}^1\mu(0) = {}^1\mu(1) = 0,552$,
${}^1\mu(2)={}^1\mu(3)={}^1\mu(4)=0,796$,
${}^1\mu(5) = 0,924$,
${}^1\mu(6) = 1$.
From ${}^1\mu$ we obtain
the join space ${}^1H_{32}$ as follows:

${}^1H_{32}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 2, 3, 4	A_6	H
1		0, 1	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 2, 3, 4	A_6	H
2			2, 3, 4	2, 3, 4	2, 3, 4	2, 3, 4, 5	2, 3, 4, 5, 6
3				2, 3, 4	2, 3, 4	2, 3, 4, 5	2, 3, 4, 5, 6
4					2, 3, 4	2, 3, 4, 5	2, 3, 4, 5, 6
5						5	5, 6
6							6

The membership function determined by ${}^1H_{32}$ is: ${}^2\mu(0) = {}^2\mu(1) = 0,235$, ${}^2\mu(2) = {}^2\mu(3) = {}^2\mu(4) = 0,228$, ${}^2\mu(5) = 0,258$, ${}^2\mu(6) = 0,290$. Hence we obtain the join space:

${}^2H_{32}=$

	0	1	2	3	4	5	6
0	0, 1	0, 1	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 5	0, 1, 5, 6
1		0, 1	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 2, 3, 4	0, 1, 5	0, 1, 5, 6
2			2, 3, 4	2, 3, 4	2, 3, 4	A_6	H
3				2, 3, 4	2, 3, 4	A_6	H
4					2, 3, 4	A_6	H
5						5	5, 6
6							6

It follows that ${}^3\mu(0) = {}^3\mu(1) = 0,239$, ${}^3\mu(2) = {}^3\mu(3) = {}^3\mu(4) = 0,220$, ${}^3\mu(5) = 0,269$, ${}^3\mu(6) = 0,297$. Therefore $\forall\, r > 2$, ${}^rH_{32} = {}^2H_{32} = {}^1H_{32}$.

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