



Recognising various Types of Akhtari Karbandi (Discrete, Continuous, Tarkin) Without the Need for Drawing, Based on Mathematical Relationships

Azita Belali Oskoyi^{*,1} and Sepideh Sepehriara¹

**Corresponding Author, ¹Department of Architecture and Urbanism, Tabriz Islamic Art University, Tabriz, Iran.
E-mail: a.oskoyi@tabriziau.ac.ir*

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Abstract

There are various classifications of Karbandis, with Akhtari Karbandi as one of the most widely used classes in traditional architecture. The present research aims to establish a method to identify various types of Akhtari Karbandi without the need for drawing based on mathematical relationships. Moreover, it seeks to answer the question whether there is a mathematical relationship between the number of base circle divisions and the connection distance in the Akhtari Karbandi. Using logical thinking and inductive reasoning, this research investigates the mathematical relationship between the number of base circle divisions and the connection distance in the 8th to 28th bases of Karbandi (including 102 Karbandis) that can be drawn in these fields. The results reveal that the mathematical relationship between the number of base circle divisions and connection distance in Akhtari Karbandi dictates its type: continuous if they share no common divisor, star-discrete if they share one, and discrete regular polygon grid if the number of base circle divisions is divisible by the connection distance.

Keywords: Discrete Akhtari Karbandi, Continuous Akhtari Karbandi, Tarkin Akhtari Karbandi, Mathematical Relationships.

Article Type: Research Article

Introduction

Karbandi, a specific type of arched roofing structure in Iranian architecture, is generally categorised into out-of-plumb (sar-seft in Persian) and plumb (shagouli in Persian) classes (Sherbaf 2006). One can find many various examples of both classes in different buildings, including mosques, bazaars, houses, and so forth. Karbandis are discussed in two faces; theoretical geometry (geometrical aspect) and practical face (executive aspect) (Tehrani *et al.* 2019). While existing literature has addressed Karbandi, the text notes that some ambiguities remain, which this research aims to address. The research will also include a literature review and clarify how it differs from existing studies. The ultimate goal is to improve understanding of Karbandi principles, which will be useful for in designing, repairing, structurally analysing, and identifying different types of Karbandi. Experts

in this field, including Dr.Pirnia, have categorised Karbandi structures from various perspectives, with Akhtari Karbandi being one of the most important (Pirnia 2010). The present research investigates the differentiation of Akhtari Karbandi types. Specifically, it aims to develop a rapid recognition method for these types and to determine if a mathematical relationship exists between the number of base circle divisions and the connection distance between them.

Literature Review on Akhtari Karbandi

Among the various roofing systems in Iranian architecture, Karbandi represents a distinctive technique that integrates geometric precision with both structural and ornamental purposes. The system involves both theoretical geometric principles and practical construction methods, resulting in intricate patterns that continue to attract scholarly attention. Akhtari (stellar) Karbandi, a subtype within this



category, is particularly noteworthy for its unique geometric system based on radial connections and precise divisions of a base circle. In traditional architectural treatises and academic literature, Akhtari Karbandi has often been viewed as an advanced manifestation of geometric logic applied to both ornamentation and structural systems. The most systematic classification of this type is attributed to Mohammad Karim Pirnia, whose typology remains a foundational reference in the study of Persian architectural forms. As noted in *Geometry in Architecture* by Bozorgmehri (2010), Pirnia categorised Akhtari Karbandi into two primary subtypes: continuous (*peyvasteh*) and discrete (*ghesikhté*). The continuous form features uninterrupted polygonal paths that reconnect with their starting point after multiple rotations around the base circle, while the discrete form generates isolated star-like or polygonal figures that do not form a continuous closed loop. Mohammadi *et al.* (2020) expanded on Pirnia's framework by identifying a third subtype, termed Tarkin. This subtype is characterised by the generation of highly symmetrical, visually balanced configurations through the rotation of a straight line around its midpoint. Despite the significance of these categorisations, most studies addressing Akhtari Karbandi have primarily relied on traditional drawing-based methods and the intuitive expertise of master builders. As described by Sherbaf (2006), these patterns were historically developed through manual chord connections within a divided circle, following empirical trial-and-error techniques. While these methods possess considerable cultural and historical values, they often lack formal mathematical analysis, thus limiting opportunities for computational modelling, systematic classification, and algorithmic interpretation. In contrast, in recent decades, several scholars have adopted computational and mathematical perspectives to study Karbandis — and traditional Iranian vaulting systems more broadly, unlocking new possibilities for understanding and utilising these designs. The following introduces some of these scholars. Mohammadian and Faramarzi (2011) have made a significant contribution by proposing a typology based on the geometric and structural organisation of Karbandis, distinguishing between simple and non-simple configurations and offering a systematic framework for categorisation. Their work remains one of the most frequently cited references in categorical studies of Iranian vaulting systems. Further advancing this field, Eskandari *et al.* (2023) have explored the interplay between geometry, structure, and form in traditional Iranian architecture, focusing on notable

examples such as the Soltaniyeh Dome and Azadi Tower. Their research highlighted the governing role of geometric principles in regulating both the structural integrity and spatial organisation of these architectural masterpieces. In parallel, Tadayon *et al.* (2021) have investigated the application of advanced computational techniques, particularly genetic algorithms, for multi-objective optimisation within architectural design processes. Their study has confirmed the considerable potential of such algorithms in optimising building forms and layouts in response to environmental parameters, including solar radiation — thereby enhancing both systematic design thinking and architectural. Despite these advancements, most studies have focused on general Karbandi types, with fewer studies specifically addressing the Akhtari subtype. This highlights a significant scholarly gap, especially in our understanding of how to logically classify Akhtari Karbandis based on their geometric or numerical properties.

Research Gap in Analytical and Non-Graphical Classification of Akhtari Karbandi

Although the geometrical richness of Akhtari Karbandi has attracted attention from architects and historians, most existing studies emphasise traditional, graphical design and classification methods (Bozorgmehri 2010; Sherbaf 2006). These approaches primarily involve visual categories and hand-drawn reconstructions, often guided by the experience and intuition of traditional master builders rather than objective, quantifiable criteria, highlighting a significant research gap in developing mathematical and algorithmic models capable of distinguishing Karbandi types—especially the Akhtari subtype—without relying on visual construction. As pointed out by Mohammadi *et al.* (2020), even with the identification of newer subtypes like the Tarkin pattern, the classification remains largely descriptive, lacking formal, generalisable, and computationally implementable rules. This deficiency is particularly problematic for digital heritage preservation and parametric design, where precise definitions and reproducible rules are essential (Sobh and Samy 2018; Liberotti and Gusella 2023). The current limitations in integrating Akhtari Karbandi into contemporary design tools stem from the lack of a mathematical basis for it. Furthermore, the development of automated systems for recognising and reconstructing historical structures using machine learning or pattern recognition technologies demand numerical criteria for precisely identifying architectural elements (Aoulalay *et al.* 2022). Akhtari Kar-

bandi, generated by systematic rotational connections across divided circles, exhibits a structure ripe for classification using number theory. Concepts, such as common divisors, modular arithmetic, and cyclic sequences could provide a rigorous, scalable, and objective classification model. However, this potential remains largely unexplored, indicating a significant gap in current research and a missed opportunity to leverage number theory's power for Akhtari Karbandi analysis.

This research aims to bridge this gap by introducing a number-theoretic framework that correlates the number of base circle divisions (n) and the connection distance (d) to predict the Karbandi type. This approach not only enhances the analytical clarity of classification but also opens new avenues for automation and digital modelling in architectural design.

Importance of the Study and Its Contribution to the Field

The current research holds both theoretical significance and practical applicability, making it a valuable addition to the fields of architectural history, digital heritage, and computational design. By proposing a method to classify Akhtari Karbandi using mathematical relationships rather than visual construction, this study introduces a novel approach that transforms an age-old artistic process into a logic-based framework. From a theoretical perspective, the research bridges the gap between traditional architectural knowledge—often transmitted orally or visually—and modern mathematical logic. It establishes a correspondence between the number of base circle division (n), connection distance (d), and the resulting Karbandi type, offering a formalised system grounded in number theory. Such a system demystifies the classification process and provides a rigorous platform for further academic study, especially for those interested in geometry, topology, or modular design in Islamic architecture (Sobh and Samy 2018; Liberotti and Gusella 2023). From a practical standpoint, this method has the potential to revolutionise the way historical patterns are recognised, reconstructed, and reinterpreted. For example, in architectural conservation, practitioners can use this method to identify missing or degraded sections of Karbandi with greater accuracy. In parametric and algorithmic designs, the framework allows for rule-based generation of traditional motifs in contemporary digital workflows, making Islamic geometry more accessible to architects and designers using tools like Grasshopper or Rhino. Furthermore, in machine learning and pattern recognition,

this classification can serve as labelled training data for automated pattern detection of Karbandi in historical documentation or 3D models (Aoulalay *et al.* 2022). Identifying conditions such as the presence or absence of common divisors between n and d , this study reveals intrinsic geometric logic behind each pattern, allowing scholars to classify Akhtari Karbandi without needing to render the full form. This mathematical abstraction reduces time and error while facilitating automated recognition and analysis at various scales. In conclusion, this research makes a pioneering contribution by moving beyond descriptive categories toward quantitative classification models, setting a foundation for further studies that merge architecture, mathematics, and computational tools in meaningful and scalable ways.

The Role of Karbandi in Islamic Architecture

Karbandi, a hallmark of traditional Islamic and Persian architecture, represents a unique synthesis of structural logic, artistic sophistication, and mathematical harmony. Functionally, Karbandis were designed to transition between different architectural geometries, such as from square bases to circular domes, and to distribute structural loads. Beyond function, they serve as intricate decorative systems, highlighting architectural skill. Historically, they have been extensively applied in the construction of mosques, bazaars, caravanserais, and traditional houses, particularly during the Seljuk, Il-Khanid, Timurid, and Safavid periods. These structural patterns not only fulfilled architectural needs but also embodied symbolic meanings, with their complex interlacing geometries reflecting cosmic order and divine unity—concepts deeply rooted in Islamic philosophy and cosmology. In architectural literature, Karbandi is frequently examined alongside other geometric elements, like muqarnas, girih, and rasmi-bandi as components of a larger ornamental geometric system in architecture. Karbandi, as noted by Sherbaf (2006), is among the most complete and complicated examples of geometric patterning in architecture, that often utilise rotational and reflective symmetries. Pirnia (2010) further emphasised that Karbandi reveals the architect's understanding of geometry and spatial organisation, requiring both aesthetic appreciation and mastery of mathematical division and construction logic. This design relies heavily on geometric principles such as circle division, modularity, and the application of radial and axial symmetry. Traditional artisans, using only a compass and straightedge, developed highly advanced patterns through intuitive yet pre-

cise geometric operations. Today, these historical techniques are being revisited and reanalysed using digital tools and mathematical logic to preserve, reconstruct, and reinterpret this valuable architectural heritage.

Summary of the Theoretical Framework

The theoretical research framework is grounded in both the traditional understanding of Karbandi structures and contemporary advancements in digital modelling. Historically, Karbandis—especially the Akhtari type—have been studied from a geometrical and categorical perspective, by scholars like Pirnia and Sherbaf, who have offered foundational methods for classification and drawing. These frameworks have defined Akhtari Karbandis based on their visual patterns and generative processes, distinguishing between continuous, discrete, and Tarkin (rotational) types. However, the reliance on visual or manual drawing methods has hampered the development of quick, formal identification techniques. This gap motivated the current study exploring the mathematical relationships—particularly between the number of base circle divisions and connection distances—as a means to classify Akhtari Karbandi without the need for drawing. The proposed logic-driven framework based on common divisors and arithmetic structure provides a scalable and objective method for type recognition. Moreover, the increasing application of digital and algorithmic tools in architecture opens new possibilities for both the study and practice of Karbandi design. Parametric modelling platforms, algorithmic generation techniques, and even machine learning have been successfully used to recreate, analyse, and classify Islamic geometric patterns. These methods emphasise the need for formal mathematical descriptors—such as those introduced in this study—to support digital heritage efforts, reverse engineering, and computational design workflows. Therefore, this theoretical foundation connects the historical wisdom of traditional craftsmanship with modern computational capabilities, enabling a more holistic, efficient, and accurate approach to recognising and analysing Akhtari Karbandi. It positions the current research at the intersection of heritage, mathematics, and digital innovation—offering a meaningful contribution to both academic scholarship and practical architectural applications.

Definitions of Karbandi

There are several definitions for Karbandi, four expert-proposed cases of which are addressed in this study. Navaei and Haj Ghasemi define “Rasmi

Karbandi” as a centripetal network formed by arcs of equal length which intersect regularly after rolling around a circle’s centre (Navayi *et al.* 2020: 98). Reiszadeh and Mofid describe “rasmibandi” as an evolved form of cornering. This involves using a single mould to place arches between the four main arches, transforming a square into an octagon. In openings in which the octagonal shape is not suitable for dome covering, the same method was applied to convert chahartaqs into 12-, 14-, 16-, 18-, 24-sided forums, etc. before the dome covering is installed (Reiszadeh and Mofid 2019: 33). Sherbaf defines “rasmibandi” as a rasmi, which is formed by intersecting multiple arcs (rounds), which are called moulds, and distinguishes between plumb and out of plumb variations (Sherbaf 2006: 9). Pirnia and Bozorgmehri characterise rasmibandi as a set of intersecting narrow diagonal arches where the intersections are used to build arches as a cover (Pirnia and Bozorgmehri 2006: 10). Plumb Karbandi: a structure whose mould is vertical in space relative to the ground plane. Out of plumb Karbandi: a structure whose mould is inclined (tilted) in space relative to the ground plane, which is further categorised into two types: 1. Rasmi Karbandi (with types that are outside the scope of the research), and 2. Akhtari Karbandi, classified into two continuous and discrete types (Pirnia and Bozorgmehri 2006: 11). In the later ones, the points dividing the peripheral circle are arbitrarily connected in groups of three, four, or more (without regard to how many divisions of the circle the rectangle’s length includes). This result in a regular shape, entitled Akhtari Karbandi. Discrete Karbandi: refers to the patterns obtained from the intersection of the edges of several polygons (mostly square. Continuous Karbandi: It refers to the patterns formed by arranging polygons in a circular fashion, where the arrangement returns to the starting point on the circumference (Pirnia and Bozorgmehri 2006: 15). The third type of Akhtari Karbandi has been discussed, in an article, as Karbandi geometry in Iranian architecture to answer the challenges related to distinguishing rasmi and Akhtari Karbandis. Tarkin Akhtari Karbandi: this category includes those patterns formed by rotating a straight line around its midpoint (Mohammadi *et al.* 2020: 22).

Karbandi Drawing Methods

In general, two main methods have been proposed for drawing Karbandi: Pirnia’s method (quoted by Zohreh Bozorgmehri) presented in the book entitled “Architectural Geometry” and Professor Sherbaf’s method presented in his book en-

titled “Knots and Karbandi” (Reyhani Hamedani *et al.* 2018: 207). Since the main goal of this research is to identify the types of Akhtari Karbandi, Pirnia’s typology is applied as a framework for classification. In this research, the author has noticed Pirnia’s typology and its associated drawing method.

Pirnia’s Drawing Method

1. Drawing a circle: First, draw the background’s surrounding circle.
2. Dividing the circle: divide the circle into equal parts according to the number of the Karbandis.
3. Connecting the drawing points: connect the points resulting from the division of the circle (one by one, two by two, etc.) to complete the Karbandi.
4. Drawing the background: draw the desired background and remove the extra lines outside the background box.

Research Methodology

This present study is applied research. It begins with a literature review of drawing methods and different categories using the library method. Then, it investigates the mathematical relationship between the number of base circle divisions and the connection distance in the 8th to 28th bases in Karbandi (including 102 Karbandis) using logical thinking and inductive reasoning. The Akhtari Karbandi base was excluded from the study since it doesn’t influence the type of (discrete, continuous) Karbandi. (Figure.1).

Findings

The Karbandis drawn for the 8th to 28th bases and their categories (green=tarkin Karbandi, pink = continuous Karbandi, yellow = regular polygonal discrete Karbandi, blue = star discrete Karbandi) are presented in Figure. 1. In these drawings, the red lines indicate the form continuity (continuous Karbandi), and the grey lines indicate repetitions independent of the original form (discrete Karbandi). Discrete Akhtari Karbandi is divided into two general categories: regular polygon and star. Continuous Akhtari Karbandi can be called the primary (like prime numbers) or mother Karbandi since every continuous Akhtari Karbandi can be represented as a discrete one in Table 1.

Regarding the research results, some families can be defined as discrete Karbandis (i.e. the Karbandis that result from the rotation of the primary (mother) shape) in Tables 2-3.

Mathematical Analysis and Classification of Akhtari Karbandi

The analysis of 102 Akhtari Karbandis constructed over the number of base divisions from 8 to 28 reveals a consistent mathematical relationship between the number of base circle divisions and the connection distance, introducing a new system for Karbandi classification with three main types based on their numerical and graphical properties: discrete polygonal grid, discrete star-like, and continuous. This system bypasses the need for traditional manual drawing. Each type of Karbandi can be mathematically predicted: Continuous (Peyvasteh) Karbandis emerge when the two parameters abovementioned have no common divisors other than one. In such cases, the resulting form is a single continuous loop that traces back to the original point on the circle, indicating a seamless rotational symmetry. This behaviour mirrors the concept of primitive roots in modular arithmetic, where the iterator fully cycles through all positions before returning. Discrete-Star (Setare’i) Karbandis are characterised by having a common divisor between the number of divisions and the connection distance, but this divisor is not a full factor of either. These cases generate multiple overlapping stars or polygonal units, producing visual segmentation within the geometry. Discrete Polygonal Grid (Chandzelli Monazzam) Karbandis arise when the number of divisions is wholly divisible by the connection distance (e.g., 12 divisions and connection every 3rd point). These configurations lead to a regular repetition of polygonal units, and often align with symmetrical motifs found in square-based domes or octagonal plans. The graphical color-coded presentation further enhances this classification: Green for Tarkin; Pink for regular discrete polygonal grids; Blue for star-like discrete forms; Yellow for multiple-segment discrete geometries; Red and grey for continuous patterns. Moreover, based on the recurring ratios in many cases—especially those like (8,2), (12,3), (16,4)—a concept of “parental shapes” (Shakl-e Mader) can be introduced. These parental shapes act as geometric foundations for generating a family of similar Karbandis through rotation and multiplication of the base unit. This hierarchical structure suggests a potential for algorithmic design and parametric modelling of Karbandis based on number theory and modular patterns. Furthermore, this new approach provides not only a classification tool but also a predictive design methodology that can be encoded into digital

Connection	2	3	4	5	6	7	8	9	10	11	12	13	14
Base circle													
Greatest divisor in the circle 8	(8,2)=2 8/2=4	(8,3)=1	(8,4)=4 8/4=2										
Greatest divisor in the circle 10	(10,2)=2 10/2=5	(10,3)=1	(10,4)=2	(10,5)=5 10/5=2									
Greatest divisor in the circle 12	(12,2)=2 12/2=6	(12,3)=3 12/3=4	(12,4)=4 12/4=3	(12,5)=1	(12,6)=6 12/6=2								
Greatest divisor in the circle 14	(14,2)=2 14/2=7	(14,3)=1	(14,4)=2	(14,5)=1	(14,6)=2	(14,7)=7 14/7=2							
Greatest divisor in the circle 16	(16,2)=2 16/2=8	(16,3)=1	(16,4)=4 16/4=4	(16,5)=1	(16,6)=2	(16,7)=1	(16,8)=8 16/8=2						
Greatest divisor in the circle 18	(18,2)=2 18/2=9	(18,3)=1 18/3=6	(18,4)=2	(18,5)=1	(18,6)=6 18/6=3	(18,7)=1	(18,8)=2	(18,9)=2 18/9=2					
Greatest divisor in the circle 20	(20,2)=2 20/2=10	(20,3)=1	(20,4)=4 20/4=5	(20,5)=5 20/5=4	(20,6)=2	(20,7)=1	(20,8)=4	(20,9)=1	(20,10)=10 20/10=2				
Greatest divisor in the circle 22	(22,2)=2 22/2=11	(22,3)=1	(22,4)=2	(22,5)=1	(22,6)=2	(22,7)=1	(22,8)=2	(22,9)=1	(22,10)=2	(22,11)=11 22/11=2			
Greatest divisor in the circle 24	(24,2)=2 24/2=12	(24,3)=3 24/3=8	(24,4)=4 24/4=6	(24,5)=1	(24,6)=6 24/6=4	(24,7)=1	(24,8)=8 24/8=3	(24,9)=1	(24,10)=2	(24,11)=1	(24,12)=12 24/12=2		
Greatest divisor in the circle 26	(26,2)=2 26/2=13	(26,3)=1	(26,4)=2	(26,5)=1	(26,6)=2	(26,7)=1	(26,8)=2	(26,9)=1	(26,10)=2	(26,11)=1	(26,12)=2	(26,13)=13 26/13=2	
Greatest divisor in the circle 28	(28,2)=2 28/2=14	(28,3)=1	(28,4)=4 28/4=7	(28,5)=1	(28,6)=2	(28,7)=1	(28,8)=4	(28,9)=1	(28,10)=2	(28,11)=1	(28,12)=4	(28,13)=1	(28,14)=14 28/14=2

Figure 1: Drawing of Karbandis for the 8th to 28th bases and their categories (green=tarkin Karbandi, pink = continuous Karbandi, yellow = regular polygonal discrete Karbandi, blue = star discrete Karbandi)

tools for architectural design and restoration. Future research can expand on this by investigating higher-order symmetries, irregular base circles, or applying this framework into AI-based recognition models for historic Karbandis.

Discussion

In an article entitled “Comparison of Geometry and Drawing Method of Karbandi Vault Samples 10,12,14,16- sided Karbandi” in 2017, applying the library, computational techniques, as well as

redrawing different Karbandi drawing methods using AutoCAD software, the differences and similarities between the 10th, 12th, 14th, and 16th Karbandis in the development of drawing methods have been discussed (Ainehchi *et al.* 2019). In another article entitled “Geometric Divisions of Special Karbandis in Uncommon Bases with a Focus on the Mirza Akbar Scrolls”, accomplished applying comparative study, library method, field study, as well as drawing using AutoCAD and 3D Max software, three Karbandi drawing methods

Table 1: Typology of Karbandi Forms Derived from Mathematical Relationships

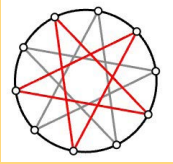
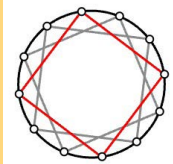
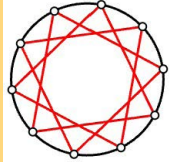
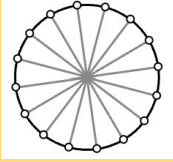
Akhtari Karbandi	Discrete	Star	The common divisor between the number of base circle divisions and the number of connections be equal to a value other than one (the common divisor is the same as the number of rotations).	For instance, the 10 th Karbandi is connected four-by-four. Number of rotations $(4,10)=2$	
		Regular polygon	1. The common divisor between the number of base circle divisions and the number of connections be equal to a value other than one (have common divisor). The value of the common divisor indicates the number of rotations. 2. The number of base circle divisions be divisible by the number of connections (the resulting value indicates the regular polygon)	For instance, the 12 th Karbandi is connected to a three-by-three one. Number of rotations $(3,12)=3$ (regular quadrilateral or square) $12/3=4$	
	Continuous	Requirement: the common divisor between the number of base circle divisions and the number of connections must be equal to one (if they have no common divisor). The value of the common divisor indicates the number of rotations.	For instance, the 10 th Karbandi is connected three by three. $(10,3)=1$		
	Tarkin	Requirement: if the number of connections is half of the number of circle divisions or, in other words, the product of dividing the number of circle divisions by the number of connections is equal to 2. The value of the common divisor also shows the arches in the Karbandi.	For instance, the 16 th Karbandi is connected eight-by-eight. Number of arches $(16,8)=8$ Tarkin Karbandi $16/8=2$		

Table 2: Relationships between Karbandi types based on similar classifications in terms of the number of divisions

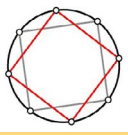

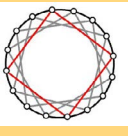

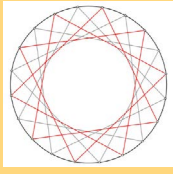
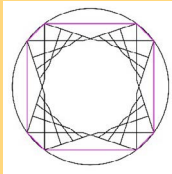

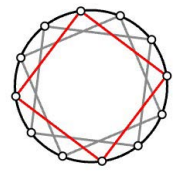
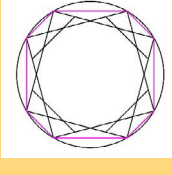

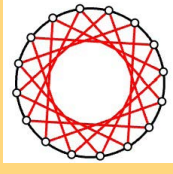
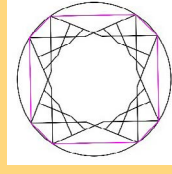

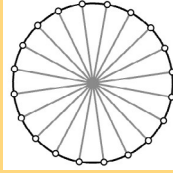
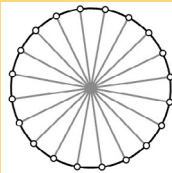

	8 th Karbandi; it is a connected 2-by-2 pattern. $(8, 2)=2$ is a discrete Karbandi. $8/2=4$ is a discrete regular polygon Karbandi and is formed by the rotation of a regular quadrilateral or square.
	12 th Karbandi; it is a connected 3-by-3 pattern. $(12, 3)=3$ is a discrete Karbandi. $12/3=4$ is a discrete regular polygon karbandi and is formed by the rotation of a regular quadrilateral or square..
	16 th Karbandi; it is a connected 4-by-4 pattern. $(16, 4)=4$ is a discrete Karbandi. $16/4=4$ is a discrete regular polygon Karbandi and is formed by the rotation of a regular quadrilateral or square.
	20 th Karbandi; it is a connected 5-by-5 pattern. $(20, 5)=5$ is a discrete Karbandi. $20/5=4$ is a discrete regular polygon Karbandi and is formed by the rotation of a regular quadrilateral or square.

Table 3: Examples of different types of Karbandi implemented in various architectural contexts

Akhtari Karbandi	Discrete	Star	For instance, the 20 th Karbandi is connected six-by-six. Number of rotations $(6,20)=2$			 Sahraqueh Church of Marand
		regular polygon	For instance, the 12 th Karbandi is connected to a three-by-three one. Number of rotations $(3,12)=3$ (regular quadrilateral or square) $12/3=4$			 Tabriz Bazaar
	Continuous	For instance, the 16 th Karbandi is connected five by five. $(16,5)=1$				 Tabriz Jame Mosque
	Tarkin	For instance, the 20 th Karbandi is connected ten-by-ten. Number of arches $(20,10)=10$ Tarkin Karbandi $20/10=2$				 Tabriz Bazaar

in the case of irregular polygons have been defined (Ainehchi *et al.* 2020). In another study entitled “Novel Drawing Method for Innovative Design of Sample Karbandi”, applying the field study, logical reasoning, as well as computer modelling, the requirements of drawing regular and irregular quadrilateral bases in the Karbandi have been discussed (Asefi *et al.* 2019). Further, an article entitled “Geometry of Karbandi in Iranian Architecture” has discussed the challenges of rasmi and akhtari Karbandis. It introduced different categories of Karbandi, highlighting their differences, through logical reasoning, and by integrating field studies, objective observation, library surveys, and computer modelling (Asefi *et al.* 2019). In an article titled “Design of Persian Karbandi: The Problem of Dividing the Base from a Mathematical Viewpoint,” the Karbandi has been presented as a mathematical challenge. The article has discussed how to express this problem using drawings. Furthermore, a precise scientific tool has been provided for the designers by writing a program in Maple language to investigate the

Karbandi design methods in the works of three Iranian traditional architecture professors and teachers named Pirnia, Sherbaf, and Larzadeh (Pour Ahmadi and Sohrabi 2019). The mentioned articles have mostly discussed how to draw the Karbandi and its bases, while the present research has investigated the typology of Akhtari Karbandi and presented a method for identifying its different types (discrete, continuous, Tarkin) based on the mathematical relationships.

Digital and Algorithmic Approaches to Karbandi Design

In recent years, the increasing integration of digital technologies in architecture has spurred renewed interest in traditional geometric patterns, including Karbandi. These technologies enable the systematic study, analysis, and reconstruction of such patterns using computational and parametric tools, thus offering new avenues for their preservation and reinterpretation. Parametric modelling is one of the most powerful tools in this domain. Using platforms like Grasshopper for Rhino, researchers have

been able to recreate traditional vaulting systems by defining geometric parameters such as base circle divisions, radial connections, and curvature constraints (Liberotti and Gusella. 2023). These models are especially useful in studying structural behaviours, optimising materials, and testing variations that would be difficult to explore through manual drawing. Furthermore, algorithmic design—rooted in geometry and number theory—allows for rule-based generation of complex Islamic patterns. This approach is often based on mathematical logic such as modular arithmetic, symmetry groups, and recursive subdivision, enabling the precise replication and variation of historical patterns in new architectural contexts (Sobh *et al.* 2018; Moyano *et al.* 2022). Recent advancements in digital heritage and computer vision have also paved the way for automated recognition of Karbandi types in photographs, architectural surveys, or 3D scans. Machine learning models trained on labelled pattern data have shown promise in identifying and classifying Karbandi elements in historical monuments, particularly when paired with datasets annotated using mathematical classification rules like those proposed in the current study (Aoulalay *et al.* 2022; Ranjazmay Azari *et al.* 2023).

Moreover, reverse engineering techniques allow digital craftsmen to extract design logic from existing structures by mapping key geometric points, reconstructing missing parts, and evaluating alternative interpretations. These techniques are not only beneficial in academic research but also essential in architectural restoration, where understanding the original design logic is crucial for structural integrity and authenticity (Mohammed Ahmed 2022).

This growing body of research underscores the relevance and necessity of formal mathematical models—such as the one developed in this study—for integration with computational tools. By providing a quantitative framework for classifying Akhtari Karbandi, this research directly contributes to the broader effort of digitising and preserving architectural heritage through reliable, rule-based systems.

Conclusion

Karbandi is recognised as one of the widest and most intricate roofing techniques in Iranian Islamic architecture, distinguished by its rich diversity in form and structure. It presents various configurations, including plumb, out-of-plumb, and mixed types that ingeniously combine both systems. In this research,

through the process of drawing and analysing 102 examples of karbandi patterns, the mathematical relationships between the number of base circle divisions and the ways these divisions connect to one another were identified and examined. The findings of this study reveal that Akhtari karbandi patterns can be systematically classified into three general categories based on their mathematical characteristics: discrete, continuous, and Tarkin. This classification is primarily derived from the principles of divisibility and the presence or absence of a common divisor among the divisions. Such an approach not only offers a clearer understanding of the geometrical logic underpinning these structures but also contributes to a more organised framework for categorising the diverse types of karbandis. It should be noted that karbandi encompasses a wide range of types and variations, many of which remain unexplored within the scope of this research. Only a limited segment of these designs has been discussed here. Future studies could expand upon this foundation by investigating additional types of karbandi, their mathematical structures, and their architectural and symbolic functions across various historical periods and regional traditions. Such endeavours would further enrich the academic understanding of Islamic architectural geometry and potentially inspire contemporary applications in design and conservation.

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