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Original paper

Solving optimal control problems under interval uncertainty using robust model predictive controls

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Abstract

In this paper, as a novel control strategy that is implemented online, we use model predictive control. In natural phenomena, some parameters may have uncertainty in which their exact amount is unknown, but we know that they are in a specific range. In this case, the predictive control formulation can be changed so that the control system becomes resistant to these parameters. In this article, two methods for solving optimal control problems with uncertainty will be proposed using model predictive control. In first one, we introduce the robust control, in which discretize the continuous time dynamic model and apply the predictive control algorithm of the resilient model to a typical system. In the second one, when the range of parameters has a particular uncertainty, it is appropriate that the solutions are in the form of interval values. So, we introduce the interval model predictive control and solve two sub-models for optimal control problems. According to the simulation results, it can be conclude that even though the dynamic of the system has severe uncertainty and the behavior of the system changes randomly between stable and unstable modes, the predictive controller of the interval optimal control problems is well able to converge the state variables.

Keywords: Optimal control problem, model predictive control, interval uncertainty, robust control, interval optimal control.

1 Introduction

Classical optimal control problems play an important and extensive role in the design of modern control systems. In such issues, the control signals must be calculated in such a way that a cost function is minimized. According to the type of process, the cost function can include the minimization of energy consumption, the minimization of pollutants, or the maximization of product production profit [1]. Furthermore, in determining the optimal control signals, a set of physical constraints governing the system should also be considered. In other words, control signals are not arbitrary values and must be reproducible in practice. For example, in controlling the speed of a 12V DC motor, the control signal can only be in the range of −12V to +12V. Therefore, if the controller applies voltages outside this range to the motor, the probability of a motor fault will increase. Optimal control theory provides alternative problem formulations, and these may be exploited in order to reduce the complexity of constrained performance optimization in Pontryagin's theorem [22]. But their most important disadvantage is that they will be solved offline, which limits their application in absolute control systems. For example, in DC motor speed control, optimum control signals should be generated by analyzing the motor output signals at the same time as the motor is rotating. The so-called controller must be online. Considering the physical constraints also makes the solution of Pontryagin's problems more challenging.

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As a result in this article, a novel control strategy that is implemented online, model predictive control (MPC) will be presented [24]. MPC is an advanced control tool that originates for approximating the optimal control of linear and nonlinear systems [8, 14, 23]. This method approximates optimal solutions to computationally intractable infinite-horizon optimal control problems by iteratively solving finite-horizon problems, called the receding horizon strategy. In conventional approaches to MPC, the predicted state variables are eliminated from the problem of optimizing predicted performance using the plant model. The MPC optimization is an unconstrained minimization of a continuous piecewise quadratic function. Typically, this leaves the variables parameterizing predicted input trajectories as optimization variables. The receding horizon optimization solved online is then a quadratic programming problem in a number of variables that depends linearly on the horizon length. Solving this problem requires matrix factorizations with computational complexity that increases cubically with the horizon length. As a result, the computational burden of constrained receding horizon control can be prohibitive for systems with fast sampling, even when the plant model is linear. At present, because of the fact that MPC has good prediction ability, the introduction of MPC to overcome the shortcomings of traditional controllers is a meaningful job. For example, a stable fuzzy MPC is proposed to solve the temperature control of a power plant [26]. A control method is derived and applied in the medical equipment, by combination of fuzzy theory and MPC, which successfully solves the anesthesia injection quantity control and ensures the injected volume at a safe set-point [18]. A robust MPC architecture for power converter systems with parametric uncertainties has developed in [15, 16]. Authors in [19] present a data-driven distributionally robust MPC for unknown discrete-time linear time-invariant systems affected by unknown and possibly unbounded additive uncertainties. In the literature, many strategies have been proposed the robust MPC for the industrial processes with uncertainty [17, 27].

On the other hand, in real systems, some parameters may have uncertainty. One of the most critical uncertainties is fuzzy uncertainty. Fuzzy sets were first introduced in 1965 by Lotfizadeh [29]. After the emergence of fuzzy sets, subsequent extensive studies led to the emergence of systems influenced by fuzzy logic [4, 11, 13, 28]. Now, we briefly discuss some preliminaries of fuzzy and interval logic.

Definition 1.1. [29] *A fuzzy set $\tilde{x}$ of X (universal set) is defined by its membership function*

$$\tilde{x} : X \to [0, 1].$$

Definition 1.2. [5] *A triangular fuzzy number is a fuzzy number $\tilde{x} = (x_1, x_2, x_3)$, where its membership function is given by:*

$$\tilde{x}(y) = \begin{cases} \dfrac{y - x_1}{x_2 - x_1}, & x_1 \le y \le z_2 \\ \dfrac{y - x_3}{x_2 - x_3}, & x_2 \le y \le x_3 \\ 0, & x_1 \ge y \text{ or } y \ge x_3. \end{cases}$$

Definition 1.3. [9] *Suppose that $\varepsilon \in \mathbb{R}$ and $I = \{[a^-, a^+] | a^-, a^+ \in \mathbb{R},\ a^- \le a^+\}$. If $v^\pm = [v^-, v^+]$ and $w^\pm = [w^-, w^+] \in I$, then the following rules hold:*

$$v^\pm + w^\pm = [v^-, v^+] + [w^-, w^+] = [v^- + w^-, v^+ + w^+],$$

$$\varepsilon v^\pm = \varepsilon[v^-, v^+] = \begin{cases} [\varepsilon v^-, \varepsilon v^+], & \varepsilon \ge 0 \\ [\varepsilon v^+, \varepsilon v^-], & \varepsilon < 0, \end{cases}$$

and

$$v^\pm w^\pm = \left[min\{v^-w^-, v^-w^+, v^+w^-, v^+w^+\}, max\{v^-w^-, v^-w^+, v^+w^-, v^+w^+\}\right].$$

Definition 1.4. [12] *The $\alpha-$cut form of the triangular fuzzy number $\tilde{v} = (v_1, v_2, v_3)$ is the interval $\tilde{v}_\alpha = [v_\alpha^-, v_\alpha^+]$, in which $v_\alpha^- = v_1 + \alpha(v_2 - v_1)$ and $v_\alpha^+ = v_3 + \alpha(v_2 - v_3)$, $0 < \alpha \le 1$.*

Due to the characteristics of the fuzzy concept, it was also given a lot of attention in control systems. The problem of fuzzy optimal control has been considered as one of the most critical applications of these concepts in many studies. In [7], the problem of fuzzy optimal control has been considered as one of the most important applications of these concepts in many studies. In [20], an alpha cut set was used to solve fuzzy linear control systems. In [5, 6], the necessary conditions for fuzzy optimal control problems with an alpha cutting strategy were proposed through Euler-Lagrange equations [25].

The purpose of the current study is to propose a new interval MPC control that inherits the merits of fuzzy and predictive control. Using prior information, the forecasting model is established as a basic model to predict the process dynamics, and the error between the output measurement and the predicted process output is used as the information predict the uncertainty. Considering that in real systems, some parameters may have uncertainty, that is, their exact value is not known, but we know that they are in a particular range. In this case, the predictive control formulation can be changed so that the control system becomes resistant to that parameters. On the other hand, for times when the parameter range has a particular uncertainty, it is appropriate that the solutions are in the form of interval values. In this paper, the optimal control problem for input-constrained linear systems with interval uncertainty is formulated. As a new strategy, we present two methods for solving these problems using model predictive control. In the first method, the common optimization problem in model predictive control was expressed as a two-stage $min - max$ optimization problem. The worst impact of the uncertainty parameter on the cost function is calculated, and then it will be minimized using the control signal. Unlike the analytical solution of the Hamiltonian equations, which only calculates the optimal control signal within the defined interval for the cost function and may cause the system to diverge outside of it, the robust predictive control will often lead to stabilizing controllers, which caused this control strategy has been widely used in industrial systems. In the second method, when the parameter range has a certain uncertainty, it is appropriate that the solutions are in the form of interval values. Here, we compute the solution of interval optimal control problems via two sub-interval models. In fact, we solve these problems once for the lower bound, and then for the upper bound of the interval parameter using the model predictive control strategy. Then, the lowest value at each moment is considered as the lower limit of the solution, and the highest value at each moment is considered as the upper limit of the solution. The presented interval MPC has a robust performance against the uncertainties and disturbances introduced into the system in such a way that by changing the parameters of the model from the nominal value, the controller minimizes the tracking error well. The transient response is also favorable.

The structure of the paper is as follows. In the second part, the predictive control strategy of the model, its advantages, and disadvantages are reported. In the third section, predictive control of a robust model, as an algorithm to solve optimal control problems with uncertainty, is proposed. Then the proposed algorithm has been applied to a typical system, and the results have been compared with Pontryagin's method. In the fourth section, two methods for optimal control problems with uncertainty using interval MPC are presented. The proposed methods are simulated for some typical and applicable systems and the results are compared with the quantitative performance metrics, in the fifth section. The last part is devoted to conclusions, summary and future directions.

2 Model predictive control

One of the essential controllers that has received much attention in recent years, especially in the industry, is MPC. This controller is a multivariable and constrained strategy that determines the optimal control signals by online optimization of a cost function. Therefore, by using this controller, it is possible to consider the physical restrictions governing the system, such as the allowable voltage range that can be applied to the DC motor or the permissible range of the solenoid valve stem position. The most important reasons for the popularity of this controller are:

a) Being optimal: in fact, the predictive control strategy of the model can be considered as an optimal control algorithm, in which the optimization step has been work online. So, for example, it is possible to minimize the energy consumption in the control system or maximize the profit of the production of products.

b) Considering physical constraints: It is clear that in all industrial and real plants, there are limitations in control signals as well as state variables. By using MPC, these physical constraints can be considered directly, while the constraints cannot be considered in classical optimal controllers that are based on Pontryagin.

c) Easy generalization to multivariable systems: MPC strategies are generalized without much complexity to multi-variable systems with any number of inputs and outputs. Adjusting the parameters of proportional integral derivative (PID) controllers for multivariable systems is much more complicated than for univariate systems. In the multivariable mode, in the optimization step of the cost function, only the computational load will increase, and the optimizer algorithm must be selected correctly.

Figure 1 and Figure 2 show the predictive control strategy of the model. In this controller, based on the mathematical model of the plant and its past signals measured which recorded by the sensors, the prediction of the system states in future times (along the forecast horizon) is calculated. Here, it should be noted that the Pontryagin's principle of optimization will be performed simultaneously for the entire time period of system performance. But in the MPC, a shorter future called the predictive horizon will be considered. In fact, if we assume that the duration of the system's operation is 1 second, using Pontryagin's principle, the control signal will be calculated simultaneously over the entire duration of 1 second. But in the predictive control of the model (Assuming a forecast horizon of 0.1 seconds), the cost function will be minimized in $0.1-$second intervals. The shorter horizon in MPC makes it possible to obtain online with good accuracy the same control signals as the Pontryagin's solution.

With this, as will be mentioned later, we do not need to solve the cost function optimization theory, and it will be done by an online optimizer. Predictions are obtained as a multivariate function of control signals during the control horizon. The control horizon is the defined interval for the control signal. For example, suppose that the prediction horizon is equal to 0.1 second and the control horizon is equal to 0.05 second. This means that we want to push the system output to desired values in the time interval t to $t+0.1$ seconds with control signals in the time interval t to $t+0.05$ seconds. According to the Figure 1, pay attention to the fact that the control signal is assumed to be constant between $t+0.05$ and $t+0.1$. Also, because the control signal will be produced by a digital system in practice, we define it as a constant interval. It is assumed to be steady in the intervals between the sampling times of the system.

Now, a cost function is considered, which is often the sum of squares of the tracking error and the control signal, as a multivariate function of the control signals along the control horizon. In order to make the predictions as close as possible to the reference outputs, the sum of squares of the tracking error is often used in the cost function. The i step forward tracking error is defined as follows:

$$e(k+i) = y^*_{k+i} - y_{k+i},$$

where i varies from 1 to p and y^*_{k+i} is the value of the reference signal. Using an optimization algorithm such as gradient descent or genetic algorithm, considering the physical constraints governing the system, the cost function relative to the control signals is minimized during the control horizon. After the optimal values have been determined, according to the principle of the reducing horizon, the first sample of the control signal is applied to the plant, and this process will be repeated at later times. The reason for this is to compensate for the effects of uncertainty in disturbances and random signals. Because the uncertainties and disturbances in the system are often random in nature and cannot be estimated during the forecast horizon. Hence, the last recorded perturbation will be considered as the perturbation during the forecast horizon. Then, optimization calculations are performed with this assumption to determine the control signal at the current time. After applying it to the system, in the next time sample, all the cost function optimization steps will be repeated, considering the new measurements.

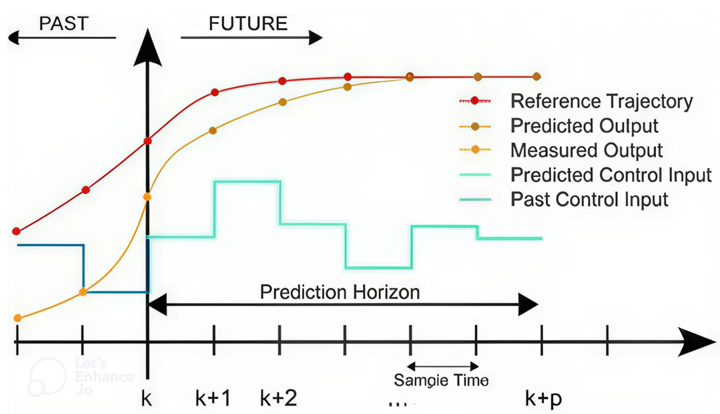

Figure 1: Model predictive control strategy[2]

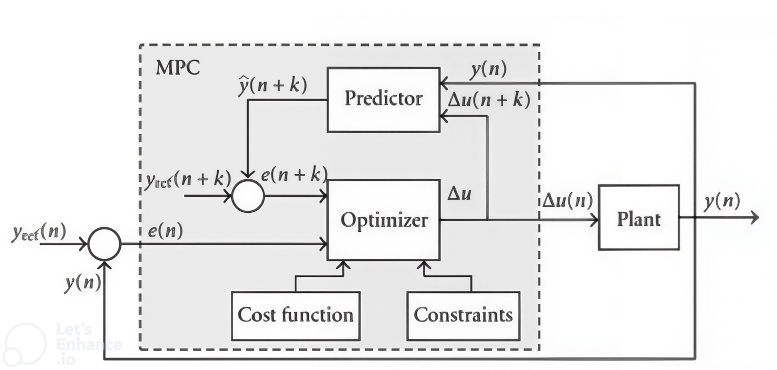

Figure 2: Block diagram of model predictive control strategy [10]

Another critical point is that the MPC strategy is implemented in discrete time. Therefore, we suggest that the continuous time dynamic model of the system be discretized using the Euler approximation or the fourth order Runge-Kutta method [3]. The continuous-time model of the system can be expressed as follows:

$$\dot{x}^*(t) = h_c(x(t), u(t), p, t). \tag{1}$$

It will be converted into a discrete time model in the following form:

$$x_{k+1} = x_k + T_s h_c(x_k, u_k, p, t) = h(x_k, u_k, p, t). \tag{2}$$

In the above relationship, h_c represents the continuous time dynamics of the system, T_s is the sampling time (step length of numerical solution of differential equations), and k represents the sample number or measurement at time kT_s. The smaller T_s is chosen, the discretization accuracy will be better, but the volume of calculations will be larger. According to the nature of the system and the rate of change of its signals, a suitable value should be chosen for it. It is known that there are various parameters in real systems, some of which may have uncertainty, and others are precisely known. In the Equation (2), we denote the set of uncertain parameters with p. Therefore, the exact value of this parameters is not known, and we assume them in the range of $[p^-, p^+]$ in which p^- is the lower limit and p^+ is the upper limit of the parameter p.

Along with the dynamic equation of the system, a static model for its outputs should be defined. We denote the output vector by z, and in the general case, an arbitrary function of the system state variables can be defined as follows:

$$y = f(x_k), \tag{3}$$

for an easier understanding of the predictive control strategy of the model, assume that the predictive horizon and the control horizon are equal to 2 samples. This means that with the control signals u_k and u_{k+1}, we want to make the output signals y_{k+1} and y_{k+2} as close as possible to their reference values, i.e. y^*_{k+1} and y^*_{k+2}. It is also assumed that all information of the system exists up to sample k (k represents the present tense). Now, one step forward prediction of the output y_{k+1} is obtained as follows:

$$y_{k+1} = f(x_{k+1}) = f(h(x_k, u_k, p)). \tag{4}$$

In Equation (4), considering that all of the information is known up to time k, then the only unknown parameter is u_k. By successive placement of two-step forward prediction, y_{k+2} is obtained as follows:

$$y_{k+2} = f(x_{k+2}) = f(h(h(x_k, u_k, p), u_{k+1}, p)). \tag{5}$$

Therefore, according to the above equation, the two-step forward prediction depends on the past signals and the future control signals during the control horizon, i.e., u_k and u_{k+1}. Now, to determine the optimal values of the control signals, the following cost function is defined:

$$Cost = \sum_{i=1}^{2} \alpha_i (y^*_{k+i} - y_{k+i})^2 + \beta_i u^2_{k+i-1}. \tag{6}$$

The first term is equal to the sum of the squares of the tracking error of the reference signals, and the second term is equal to the sum of the squares of the control signal or the energy of the control signal. Coefficients α_i and β_i are respectively the weights of the tracking error in the i-step forward prediction and the control signal at time $k + i - 1$. These weights are defined so that they can be used to determine the relative importance between different terms and signals. For example, a one step forward tracking error may be much more critical than a two step forward tracking error. Therefore, the weight of α_1 should be chosen much larger than α_2. Or the amplitude of the control signal may not be important, so the weights of β_i can be considered zero. In general, if each of the weights has a larger range, the corresponding term is more important, and the optimization algorithm tries to minimize that term more.Therefore, the optimal control signal during the control horizon is obtained by solving the following optimization problem:

$$u^*_k = min_{u_k, u_{k+1} \in U} \sum_{i=1}^{2} \alpha_i (y^*_{k+i} - y_{k+i})^2 + \beta_i u^2_{k+i-1}. \tag{7}$$

Of course, it should be noted that by solving the above optimization problem, the optimal values of u^*_k and u^*_{k+1} are obtained. But as mentioned, according to the reducing horizon principle, the first sample of the vector of optimal control signals is applied to the system during the control horizon, i.e., u_K^*, and then all the previous steps are repeated for the $(k+1)$th sample. The set of U in Equation (7) represents the set of physical constraints governing the system. For example, in speed control of a $12VDC$ motor, the control signal can only be in the range of $-12V$ to $+12V$. Of course, in addition to the mentioned restrictions, other restrictions may also be considered in order to improve the performance of the controller. For example, the output may need to be precisely zero at one second. Therefore, when optimizing the cost function, this equality constraint must also be satisfied. In general, the predictive control strategy of the model is implemented as follows:

a) Explicit use of the process model to predict its future behavior.

b) Determining the vector of future control signals based on the optimization of a cost function, the cost function of the weighted sum of squares of the tracking error and the control signal is often used.

c) Applying the first sample of the control signal to the process and repeating the above steps in subsequent times.

3 A novel solution of optimal control problems using robust MPC

The formulation mentioned in the previous section can be used when all system parameters are constant and have no uncertainty. But as mentioned before, it is assumed that some parameters of the system are uncertain and have uncertainty. Therefore, we rewrite the optimization Problem (7) in the following form:

$$u_k^* = min_{u_k, u_{k+1} \in U} \quad max_{p \in [p^-, p^+]} \sum_{i=1}^{2} \alpha_i (y_{k+i}^* - y_{k+i})^2 + \beta_i u_{k+i-1}^2. \tag{8}$$

In fact, first, the maximum value of the cost function will be calculated in the defined interval for the parameters with uncertainty. Then the maximum value will be minimized using control signals. Note that in the above formulation, it is assumed that the prediction horizon and the control horizon are both equal to 2. Still, it can be easily generalized to the desired state. To evaluate the predictive controller performance of the robust model, the following first-order model is considered:

$$\dot{x}(t) = u(t) - px(t), \tag{9}$$

in which parameter p is the same parameter with uncertainty and vagueness. In the Pontryagin's method, assuming that this parameter has changes in the range of 0 to 3 and the boundary condition of the state variable x at the beginning and end of the time interval is equal to 1 and 0, respectively. The upper bound of the control signal is obtained by solving the Pontryagin's optimality equations in the following form:

$$u^*(t) = -\frac{(3-2\alpha)e^{-(T-t)(3-2\alpha)}}{sinh(3-2\alpha)}, \tag{10}$$

where T represents the horizon of the cost function and α is chosen between 0 and 1. Of course, note that in this problem, the cost function is assumed in the following form:

$$Cost = \int_0^T u^2(t)dt. \tag{11}$$

According to Equation (10), the control signal is determined simultaneously for the entire period time from 0 to T seconds. But its prerequisite is to solve the Hamiltonian equation. For this simple problem, we are able to determine the exact form of the solutions, but if the dynamics of the system are nonlinear and complex, it is not possible to solve these equations. Therefore, MPC should be used. Indeed, another reason to switch to MPC is to achieve the same control signal as (11) but in an online form. In order to make the problem more challenging, it is assumed that the parameter p varies randomly in the range of 0 to 3. Therefore, the equation of the system will be in the following form:

$$\dot{x}(t) = u(t) - p(t)x(t),$$

where $p(t)$ can change with a uniform probability distribution in the interval from 0 to 3. In such a case, we are not able to determine the control signal by solving the Hamiltonian equation. Because in practice, the exact value of Parameters p is not known, and we will only have its probability density function. Therefore, we have to use the predictive control strategy of the resilient model. Because in this strategy, first, the maximum of the cost function will be determined with respect to the parameters p in the range of 0 to 3, and then this maximum value will be minimized with the appropriate selection of control signals. Figure 3 shows the diagram of the state variable and the control signal using the Pontryagin's method. For the numerical solution of differential equations, the fourth order Runge-kutta method has been used. It is clear that according to the form of the problem, the state variable moves from the initial state 1 at time 0 to the final state 0 at time 1 second. During this simulation, it is assumed that the parameter p is constant and equal to 2, and α is equal to 0.5. One of the fundamental problems in Hamiltonian equation is that the control signal may cause instability of the closed-loop system at times outside the horizon of the cost function. In other words, the control signal obtained in Equation (10) is only optimal for minimizing the cost function (11) in the range of 0 to T seconds, and for times after T it may not be optimal and may even cause the system to diverge. In Figure 4, the simulation of the same controller is done for a period of 2 seconds. It is clear that the state variable of the system is diverging and will tend to minus infinity with the passage of time. Therefore, if we use the Pontryagin's method, in

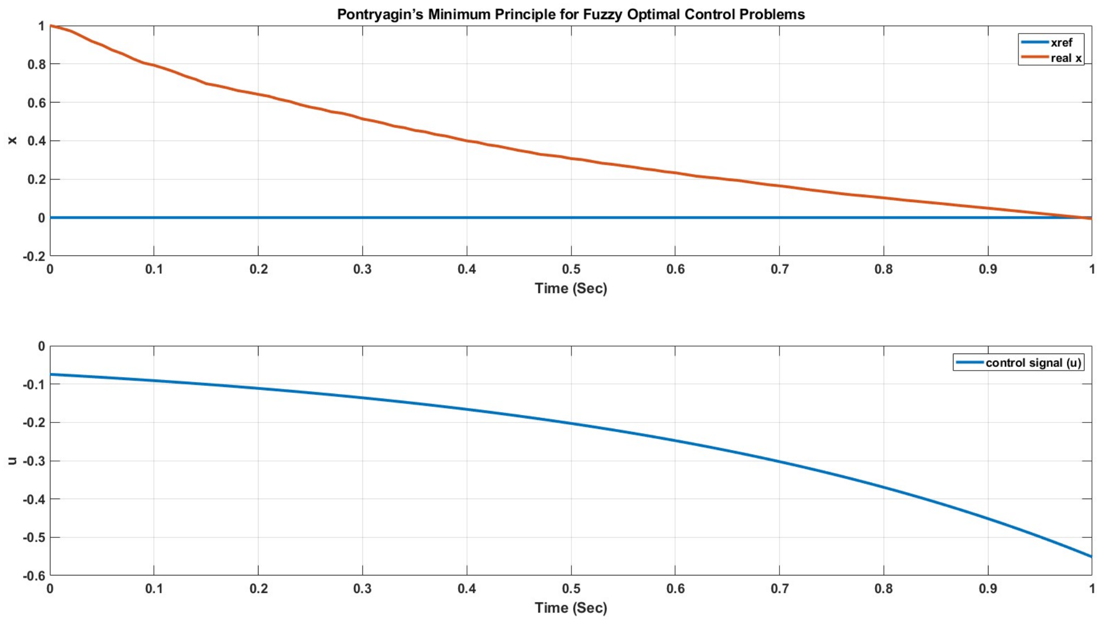

Figure 3: State variable diagram and control signal by Pontryagin's method

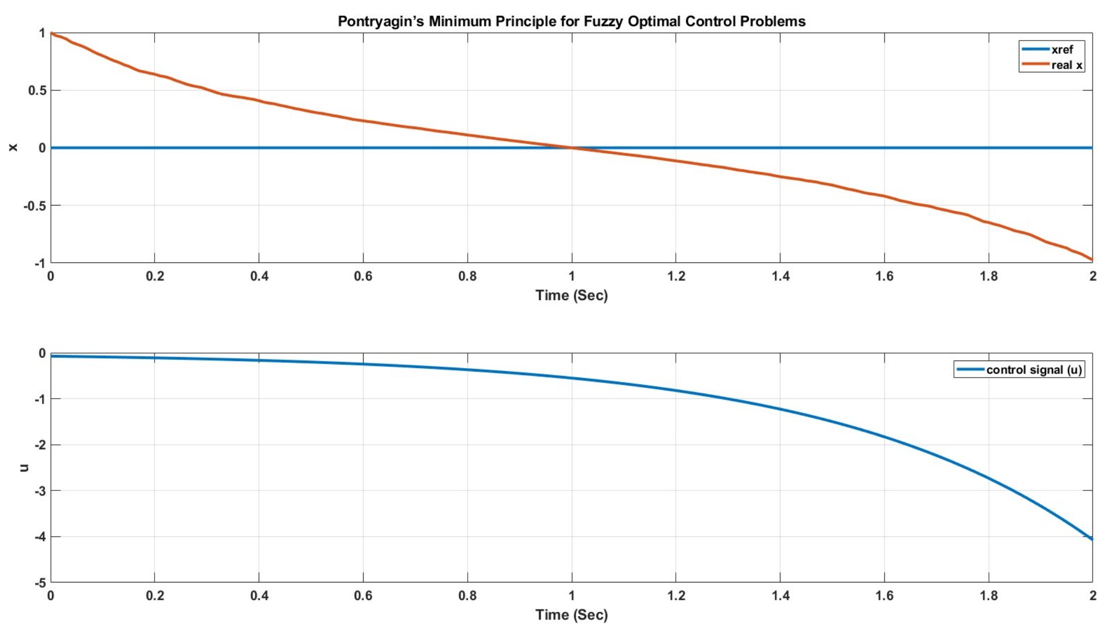

Figure 4: State variable diagram and control signal by Pontryagin's method-Simulate for two seconds

addition to the fact that there may not be analytical solutions, it is possible that the control signal for times outside the horizon of the cost function causes the system to diverge. Figure 5 shows the state variable changes and control signal using robust predictive control. The boundary conditions are the same as the previous example. The prediction and control horizons are assumed to be 3 and 2, respectively. According to this figure, it is clear that the state variable has moved from the position 1 at time 0 to the position nearly 0 at time 1 and has remained constant at the same position for times after 1 second. At the same time, Pontryagin's algorithm was different. So, the critical result is that the robust predictive control is a stable controller in addition to driving the system state variable to zero in the same time period of 1 second. To further evaluate the robust controller, assume that the parameters p changes in a wide

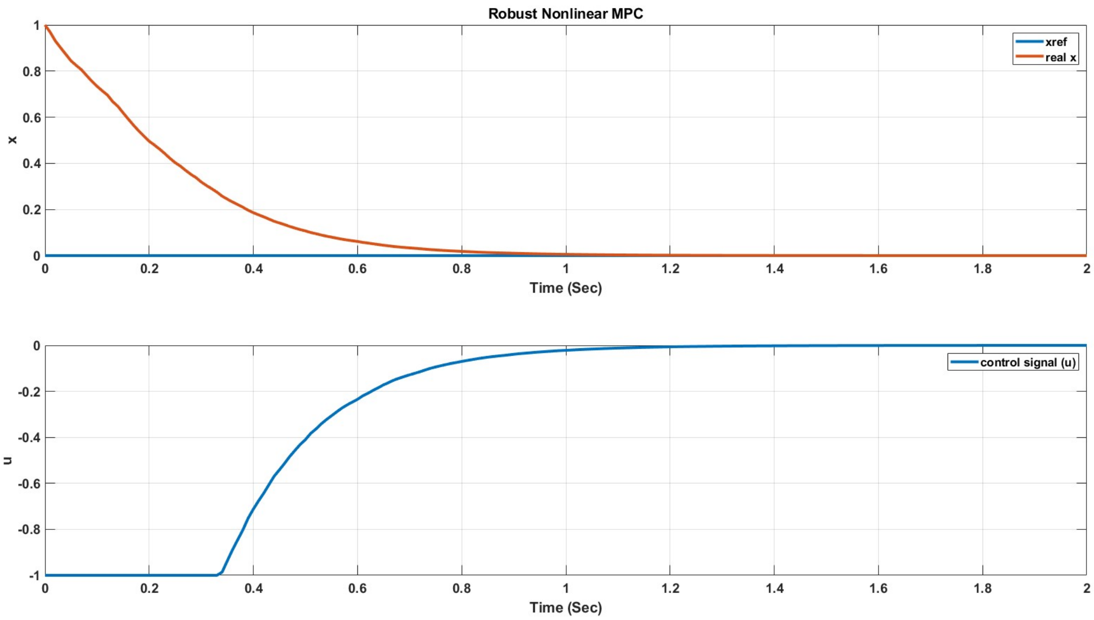

Figure 5: State variable diagram and control signal with robust predictive control algorithm

range of -5 to $+5$. In the previous state, the system itself was always stable because p is always positive. But in this case, the open loop system is stable for the distance from 0 to $+5$ and unstable for the distance from -5 to 0. The changes of p are assumed to be random and with a uniform distribution in the range of -5 to $+5$. The change diagram of the state variable and control signal is shown in Figure 6. It is clear that in this case, the controller has been able to converge the state variable of the random system to zero within 1 second. Figure 7 shows the effect of the prediction horizon on the performance of the closed-loop control system for different values of 2 to 5, assuming a control horizon of 2. According to this figure, we can conclude that as the forecasting horizon increases, the controller will become faster and the control signal u will be determined so that the system state variable converges to zero as quickly as possible. Similarly, Figure 8 shows the influence of the control horizon on the performance of the closed-loop control system for different values from 1 to 4, with the assumption of the forecasting horizon of 5. The control horizon actually shows the number of decision variables in the cost function optimization. Therefore, the higher the control horizon, the more freedom the system has to be controlled.

4 Interval MPC for interval optimal control problems

The formulation mentioned in Section 2 can be used when all system parameters are constant and have no uncertainty. It is known that there are various parameters in real systems, some of which may have uncertainty, and others are precisely known. In Equation (2), we denote the set of uncertain parameters with $p^{\pm}$, which leads to the intervalization of the optimal control problem. Therefore, we define the interval optimal control problem as follows:

$$minimize\ J^{\pm}(u^{\pm}) = \int_{t_0}^{t_1} (g^{\pm}(x^{\pm}(t), u^{\pm}(t), t))dt, \tag{12}$$

$$s.t. \quad \dot{x}^{\pm}(t) = h^{\pm}(x^{\pm}(t), u^{\pm}(t), p^{\pm}, t) \tag{13}$$

$$x^{\pm}(t_0) = x_0^{\pm}, x^{\pm}(t_1) = x_1^{\pm}. \tag{14}$$

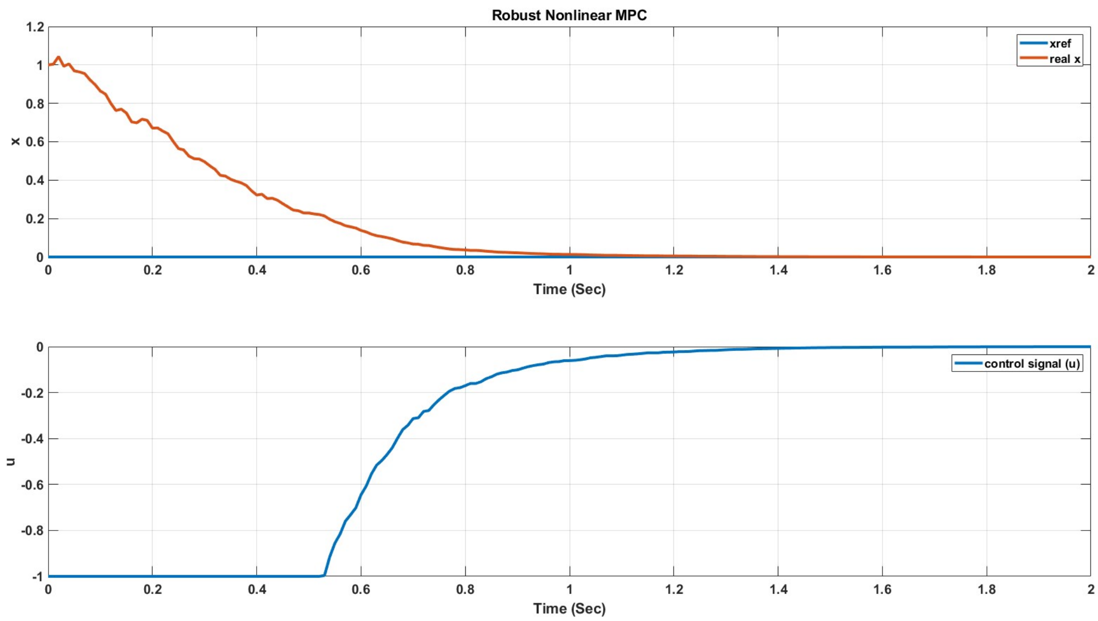

Figure 6: State variable diagram and control signal with robust predictive control algorithm-The second scenario

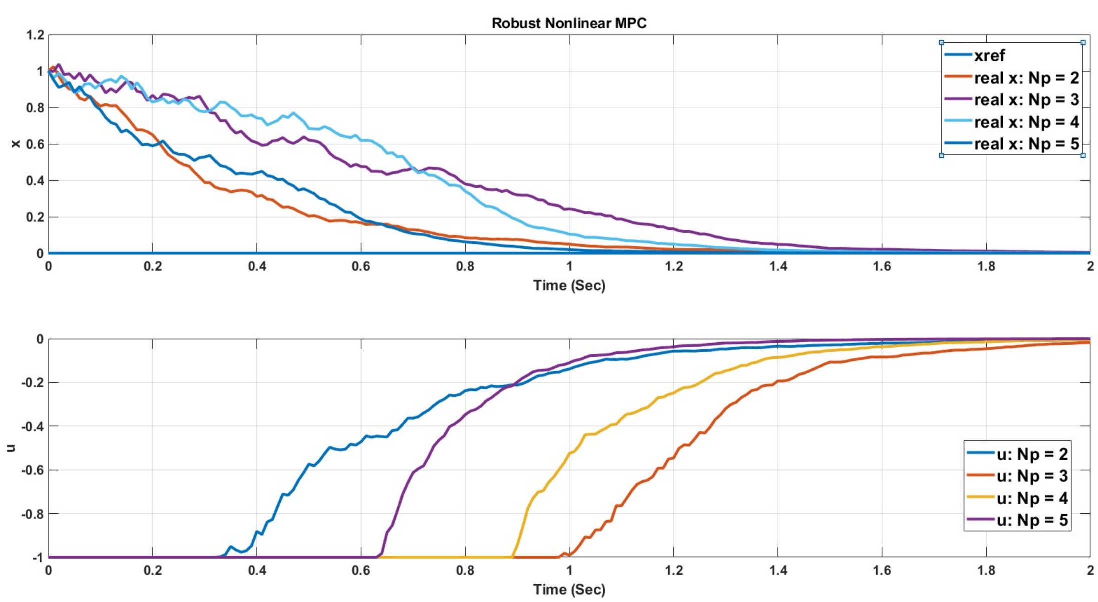

Figure 7: Diagram of state variable and control signal for different values of prediction horizon

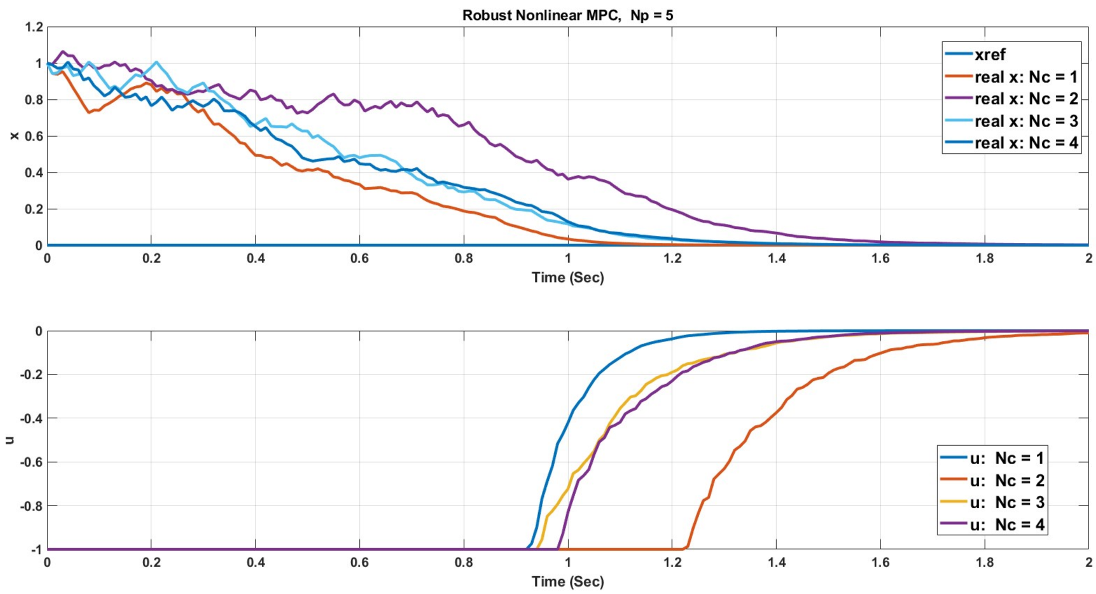

Figure 8: Diagram of state variable and control signal for different values of control horizon

Now, for solving problem (12)-(14) we have two strategy as follow.

Method I (actual solution): The MPC strategy is implemented in discrete time. Therefore, we suggest to discretize the continuous time dynamic model of the system using the fourth order Runge-kutta method. So, the continuous time system (13) will be converted into a discrete-time system as follows:

$$x^{\pm}_{k+1}(t) = x^{\pm}_{k}(t) + T_s h^{\pm}_{c}(x^{\pm}_{k}(t), u^{\pm}_{k}(t), p^{\pm}, t) = h^{\pm}(x^{\pm}_{k}(t), u^{\pm}_{k}(t), p^{\pm}, t). \tag{15}$$

In the above relationship, $h^{\pm}_{c}$ represents the continuous time dynamics of the system, T_s is the sampling time (Step length of numerical solution of differential equations), and k shows the sample number or measurement at time kT_s. When T_s is sufficiently enough small, the discretization accuracy will be better, but the volume of calculations will be larger. According to the nature of the system and the rate change of its signals, a suitable value should be chosen for it. It is known that there are various parameters in real systems, some of which may have uncertainty, and others are precisely known. In Equation (15), we denote the set of uncertain parameters with $p^{\pm}$. Therefore, the exact value of this parameters is not known and we assume them in the range of $[p^{-}, p^{+}]$.

Along with the dynamic equation of the system, a static model for its outputs should be defined. We denote the output vector by $z^{\pm}$, and in the general case, an arbitrary function of the system state variables can be defined as follows:

$$y^{\pm} = f(x^{\pm}_{k}). \tag{16}$$

Now, to determine the optimal values of the control signals, we define the following cost function:

$$u^{\pm *}_{k} = min_{u_k, u_{k+1} \in U} \sum_{i=1}^{2} \alpha_i (y^{\pm *}_{k+i} - y^{\pm}_{k+i})^2 + \beta_i u^{\pm 2}_{k+i-1}, \tag{17}$$

in which the first term is equal to the sum of the squares of the tracking error of the reference signals and the second term is equal to the sum of the squares of the control signals or the energy of the control signasl. Also, the cost function uses the square root to penalize significant errors and trivialize minor mistakes.

We solve the optimal control Problem (12)-(14) for all values of the interval $p^{\pm} = [p^{-}, p^{+}]$ using the presented method in Section 2. Then, the actual solutions are obtained as follows:

$$x^{-*}_{j}(t) = min_i \ \{x_{ij}(t)\},$$

$$x^{+*}_{j}(t) = max_i \ \{x_{ij}(t)\},$$

$$u^{-*}_{j}(t) = min_i \ \{u_{ij}(t)\},$$

$$u_j^{+*}(t) = max_i \ \{u_{ij}(t)\},$$

in which $u_j^*(t) = [u_j^{-*}(t), u_j^{+*}(t)]$ and $x_j^*(t) = [x_j^{-*}(t), x_j^{+*}(t)]$, $t \in [t_0, t_1]$, are the solution of Problem (12)-(14).

Method II: We solve the optimal control Problem (12)-(14) using the presented method in Section 2. So, we construct the following two sub-interval problems:

$$minimize \ J(u) = \int_{t_0}^{t_1} (g(x(t), u(t), t))dt, \tag{18}$$

$$s.t. \quad \dot{x}(t) = h(x(t), u(t), p^-, t), \tag{19}$$

$$x(t_0) = x_0, x(t_1) = x_1, \tag{20}$$

and

$$minimize \ J(u) = \int_{t_0}^{t_1} (g(x(t), u(t), t))dt, \tag{21}$$

$$s.t. \quad \dot{x}(t) = h(x(t), u(t), p^+, t), \tag{22}$$

$$x(t_0) = x_0, x(t_1) = x_1. \tag{23}$$

We assume that $(\hat{x}_j(t), \hat{u}_j(t))$ is the solution of Problem (18)-(20) and $(\widehat{x}_j(t), \widehat{u}_j(t))$ is the solution of Problem (21)-(23). Then, we have:

$$x_j^{-*}(t) = min\{\hat{x}_j(t), \widehat{x}_j(t)\},$$

$$x_j^{+*}(t) = max\{\hat{x}_j(t), \widehat{x}_j(t)\},$$

$$u_j^{-*}(t) = min\{\hat{u}_j(t), \widehat{u}_j(t)\},$$

$$u_j^{+*}(t) = max\{\hat{u}_j(t), \widehat{u}_j(t)\},$$

in which $u_j^*(t) = [u_j^{-*}(t), u_j^{+*}(t)]$ and $x_j^*(t) = [x_j^{-*}(t), x_j^{+*}(t)]$, $t \in [t_0, t_1]$, are the solution of Problem (12)-(14).

5 Illustrative examples

Here, we investigate some numerical examples to verify the validity and applicability of the suggested schemes. The associated computations have been performed by MATLAB 2016. The fuzzy numbers are defuzzified using one of several defuzzification strategies, such as centroid (the defuzz command). Also, the continuous-time system is discretized using zero-order hold on the inputs, and then the linear discrete-time dynamic controller is returned (the c2d and ss commands).

The theoretical foundation regarding the convergence behavior of the proposed interval MPC under high uncertainty is based on established Lyapunov stability criteria for interval-based MPC. Under the assumption that uncertainties remain bounded within a defined compact set, we show that the closed-loop system state converges to a robust invariant set, guaranteeing stability despite the bounded uncertainty. Moreover, our approach can be naturally extended to robust MPC frameworks, such as tube-based MPC, which explicitly account for larger uncertainties and disturbances to maintain convergence guarantees. We also present the theoretical results with simulation studies under varying uncertainty levels, which confirm the practical convergence of the controller.

Example 5.1. *To evaluate the performance of the predictive controller of the interval model, we consider the following two-dimensional first-order model:*

$$minimize \int_0^2 ((\tilde{y}^*(t) - \tilde{y}(t))^2 + \tilde{u}^2(t))dt, \tag{24}$$

$$s.t. \quad \dot{\tilde{x}}_1(t) = \dot{\tilde{u}}(t) + \tilde{u}(t) - \tilde{x}_1(t) - (0,1,3)\tilde{x}_2(t,) \tag{25}$$

$$\dot{\tilde{x}}_2(t) = \tilde{x}_1(t), \tag{26}$$

where $p = (0,1,3)$ *is a triangular fuzzy number and can be changed to the interval* $[0,3]$*. To obtain the numerical solutions of Problem* (24)-(26)*, we follow the proposed methods in the previous section.*

method I: *With a step length of* 0.01 *for interval* $[0,3]$*, we solve Problem* (24)-(26) *for* 301 *values. Then we obtain* $x_j^*(t) = [min_{i=1,\dots,301}\{x_{ij}(t)\}, max_{i=1,\dots,301}\{x_{ij}(t)\}]$ *for each* $t \in [0,1]$ *and* $j = 1,2$*. The simulation results with this*

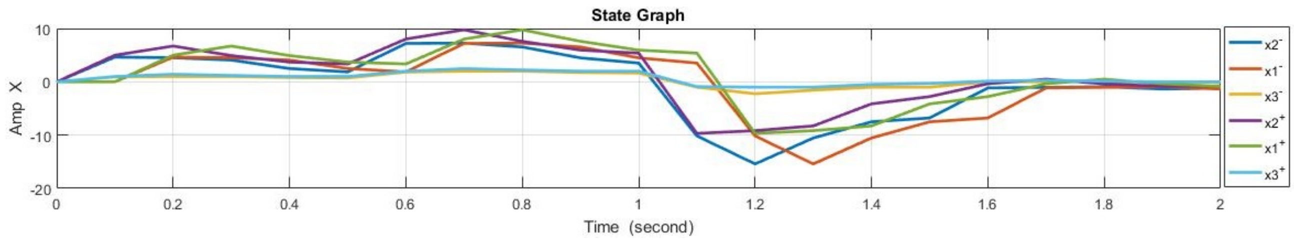

Figure 9: Diagram of state variables for Method I.

method are shown in Figure 9. As can be seen in this figure, the controller of this method has worked very well, and the system has reached stability. Also, numerical results for $t = 0.0, 0.2, 0.4, ..., 2.0$ *are shown in Table 3, where* x_3 *is the auxiliary variable used in solving the problem.*

method II: *We solve the following two sub-models using the presented method in the last section:*

$$minimize \int_0^2 ((y^*(t) - y(t))^2 + u^2(t))dt, \tag{27}$$

$$s.t. \quad \dot{x}_1(t) = \dot{u}(t) + u(t) - x_1(t), \tag{28}$$

$$\dot{x}_2(t) = x_1(t). \tag{29}$$

The optimal solutions of the state variable of the optimal control Problem (27)-(29) *are shown in Table 1.*

Table 1: The state variable of the optimal control Problem (27)-(29).

t	$\hat{x}_1(t)$	$\hat{x}_2(t)$
0.0	0.0000	0.0000
0.2	4.5242	5.0000
0.4	3.7041	4.0937
0.6	8.0327	3.3516
0.8	6.5766	7.2682
1.0	5.3844	5.9507
1.2	−9.1641	−10.1279
1.4	−7.5030	−8.2921
1.6	−1.1429	−6.7890
1.8	−0.9357	−1.0341
2.0	−0.7661	−0.8467

$$minimize \int_0^2 ((y^*(t) - y(t))^2 + u^2(t))dt, \tag{30}$$

$$s.t. \quad \dot{x}_1(t) = \dot{u}(t) + u(t) - x_1(t) - 3x_2(t), \tag{31}$$

$$\dot{x}_2(t) = x_1(t). \tag{32}$$

The optimal solutions of the state variable of the optimal control Problem (27)-(29) *are shown in Table 2:*

We obtain $x_j^*(t) = [min\{\hat{x}_j(t), \widehat{x}_j(t)\}, max\{\hat{x}_j(t), \widehat{x}_j(t)\}]$ *as the optimal solution of the state variable of the optimal control Problem* (24)-(26) *in which* $\hat{x}_j(t)$ *is the solution of Problem* (27)-(29) *and* $\widehat{x}_j(t)$ *is the solution of Problem* (30)-(32), $j = 1, 2$. *Note that for* $0 < p < 3$, *if* $x_2(t) > 0$ *we have:*

$$-3x_2(t) - x_1(t) + u(t) + \dot{u}(t) \leq \tilde{\dot{u}}(t) + \tilde{u}(t) - \tilde{x}_1(t) - (0, 1, 3)\tilde{x}_2(t) \leq \; 0x_2(t) - x_1(t) + u(t) + \dot{u}(t) \Rightarrow$$

$$\dot{x}_1^-(t) \leq \tilde{\dot{u}}(t) + \tilde{u}(t) - \tilde{x}_1(t) - (0, 1, 3)\tilde{x}_2(t) \leq \dot{x}_1^+(t),$$

and if $x_2(t) < 0$ *we deduce:*

$$0x_2(t) - x_1(t) + u(t) + \dot{u}(t) \leq \tilde{\dot{u}}(t) + \tilde{u}(t) - \tilde{x}_1(t) - (0, 1, 3)\tilde{x}_2(t) \leq \; -3x_2(t) - x_1(t) + u(t) + \dot{u}(t) \Rightarrow$$

Table 2: The state variable of the optimal control Problem (30)-(32).

t	$\widehat{x}_1(t)$	$\widehat{x}_2(t)$
0.0	0.0000	0.0000
0.2	6.7140	4.6365
0.4	2.4904	4.9217
0.6	7.2125	1.8626
0.8	7.5943	9.7592
1.0	3.5167	4.4935
1.2	−15.4117	−9.6777
1.4	−4.1390	−10.5411
1.6	−0.3588	−2.7745
1.8	−0.5990	0.2864
2.0	−1.1817	−1.3431

$$\dot{x}_1^+(t) \le \tilde{u}(t) + \tilde{u}(t) - \tilde{x}_1(t) - (0,1,3)\tilde{x}_2(t) \le \dot{x}_1^-(t).$$

So, it is proven that the range of the obtained solution also includes intermediate values. The optimal solutions of the state variable are shown in Table 3.

Table 3: The optimal solutions of the state variables.

	method I		method II	
t	$x_1^{*\pm}(t)$	$x_2^{*\pm}(t)$	$x_1^{*\pm}(t)$	$x_2^{*\pm}(t)$
0.0	$[0.0000, 0.0000]$	$[0.0000, 0.0000]$	$[0.0000, 0.0000]$	$[0.0000, 0.0000]$
0.2	$[4.5241, 6.7140]$	$[4.6364, 5.0000]$	$[4.5242, 6.7140]$	$[4.6365, 5.0000]$
0.4	$[2.4903, 3.7041]$	$[4.0813, 4.9218]$	$[2.4904, 3.7041]$	$[4.0937, 4.9217]$
0.6	$[7.2124, 8.0327]$	$[1.8625, 3.3516]$	$[7.2125, 8.0327]$	$[1.8626, 3.3516]$
0.8	$[6.5411, 7.5943]$	$[7.2682, 9.7592]$	$[6.5766, 7.5943]$	$[7.2682, 9.7592]$
1.0	$[3.5166, 5.3845]$	$[4.4935, 5.9507]$	$[3.5167, 5.3844]$	$[4.4935, 5.9507]$
1.2	$[-15.4118, -9.1641]$	$[-10.1538, -9.6777]$	$[-15.4117, -9.1641]$	$[-10.1279, -9.6777]$
1.4	$[-7.5030, -4.1390]$	$[-10.5411, -8.2921]$	$[-7.5030, -4.1390]$	$[-10.5411, -8.2921]$
1.6	$[-1.1429, -0.3588]$	$[-6.7890, -2.7745]$	$[-1.1429, -0.3588]$	$[-6.7890, -2.7745]$
1.8	$[-0.9357, -0.378]$	$[-1.0341, 0.4927]$	$[-0.9357, -0.599]$	$[-1.0341, 0.2864]$
2.0	$[-1.1930, -0.7633]$	$[-1.3431, -0.8467]$	$[-1.1817, -0.7661]$	$[-1.3431, -0.8467]$

As can be seen in this table, the results of method II are located in the range of the solution of method I which are the actual solutions. Therefore, the predictive control of the interval model works very well in all two methods.

Example 5.2. *As a practical example, we consider a two-mass system connected to a spring and a damper (Figure 10), in which, m_1 and m_2 are on a frictionless surface. These two masses are connected by a spring with a stiffness coefficient k and a damper with a damping coefficient of c. The input to the system is the external force u applied to m_1. The output of this system is the position of $m_2(x_2)$ that we want to control and bring it to a certain value of the setpoint. Indeed, the goal of this controller is to adjust the position of x_2 to a desired value (e.g., 1 meter) in a way that minimizes oscillations and noise while, keeping the control input u limited. The equations of motion for this system*

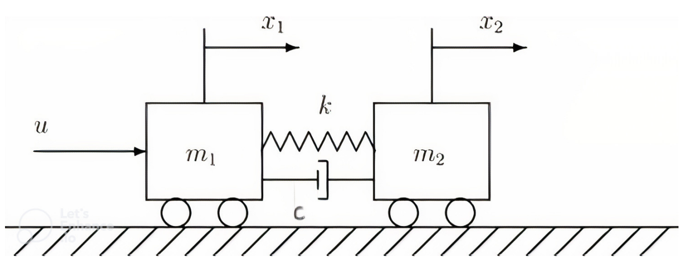

Figure 10: The two-mass spring-damper system.

based on Newton's law in state-space are as follows [21]*:*

$$m_1\ddot{x}_1 = u - k(x_1 - x_2) - c(\dot{x}_1 - \dot{x}_2), \tag{33}$$
$$m_2\ddot{x}_2 = k(x_1 - x_2) + c(\dot{x}_1 - \dot{x}_2),$$

in which, x_1, $\dot{x_1}$, x_2, $\dot{x_2}$ are the position and the velocity of m_1 and m_1 respectively. Assuming $m_1 = m_2 = 1kg$, $k = 1N/m$ and $c = 0.5Ns/m$. Then, we obtain the following matrices for the continuous state-space model (33), which turns it into a linear time-invariant model that is suitable for MPC:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -0.5 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \\ 1 & 0.5 & -1 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = [0\ 0\ 1\ 0], \quad D = 0.$$

The MPC controller works by predicting the future of the system using the model and solving an optimization problem at each time step. Considering a prediction horizon $N_p = 10$ and a control horizon $N_c = 3$, we want to minimize the output error $(y - r)$, where r is the setpoint. So we have:

$$J = \sum_{k=1}^{N_p} (y_k - r_k) Q (y_k - r_k)^T + \sum_{k=0}^{N_c} \Delta u_k R \Delta u_k^T,$$

in which, Q and R are the weight matrices for the state/output error and input changes, respectively. In addition, $\Delta u_k = u_k - u_{k-1}$ which is used to prevent sudden changes in the input signal. Considering a sampling time $T_s = 1$ second, the aforementioned system can be transformed into the following discrete-time state-space model:

$$\begin{aligned} X_{k+1} &= A_d X_k + B_d u_k, \\ y_k &= C_d X_k + D u_k, \end{aligned} \tag{34}$$

which is obtained by using the zero-order holdover from the transformation of the continuous model with the help of the c2d command.

Using MPC, we want to find a control correction Δu that minimize a quadratic programming problem as follows:

$$\min_{\Delta u} J = \sum_{k=1}^{N_p} (y_k - r_k) Q (y_k - r_k)^T + \sum_{k=0}^{N_c} \Delta u_k R \Delta u_k^T, \tag{35}$$

with $Q, R > 0$ and satisfy the constraints

$$\begin{aligned} X_{k+1} &= A_d X_k + B_d u_k, \\ u_k &= u_{k-1} + \Delta u_k, \end{aligned} \tag{36}$$

for $k = 1, \cdots, N_p$. We account this problem by inducing the constraints:

$$\begin{aligned} &-5 \le u \le 5, \\ &-1 \le \Delta u \le 1, \\ &-1 \le x_1, x_2 \le 1. \end{aligned}$$

Summarizing, we obtain:

Algorithm 5.3. *(Linear-quadratic MPC algorithm)*

1. *Let $i = 0$, compute $\Delta u = [\Delta u(0), \Delta u(1), \cdots, \Delta u(N_c - 1)]$ and apply only the first term $(\Delta u(0))$ using the receding principle.*
2. *Compute the solution Δu of Problem (35)-(36) on $[t_i, t_i + \delta t]$.*
3. *Apply the control $u\ |_{[t_i, t_i+\delta t)} := u_{ref}\ |_{[t_i, t_i+\delta t)} + \Delta u$ in $[t_i, t_i + \delta t)$ and predict the state trajectory by solving the initial value Problem (36) in $[t_i, t_i + \delta t]$.*
4. *Let $x_{i+1} = x(t_i + \delta t)$, $\Delta u_{i+1} := u_{i+1} - u_{ref}(t_i + \delta t)$, $i := i + 1$ and go to step 2.*

To create an interval model, we assume that some parameters, such as k and c, are in an interval due to physical uncertainties such as material or temperature variations:

$$k \in [k_{min}, k_{max}] = [0.8, 1.2] N/m, \qquad c \in [c_{min}, c_{max}] = [0.4, 0.6] Ns/m.$$

As before we assume $m_1 = m_2 = 1kg$. *The presence of these uncertainties causes matrix* A *to have interval components as follows:*

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{m_1} & \frac{-c}{m_1} & \frac{k}{m_1} & \frac{c}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & \frac{c}{m_2} & \frac{-k}{m_2} & \frac{-c}{m_2} \end{bmatrix},$$

while matrices B, C, *and* D *remain constant. Let* $k = 0.8$ *and* $c = 0.4$. *So, we have:*

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.8 & -0.4 & 0.8 & 0.4 \\ 0 & 0 & 0 & 1 \\ 0.8 & 0.4 & -0.8 & -0.4 \end{bmatrix},$$

and for values $k = 1.2$ *and* $c = 0.6$, *we obtain:*

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1.2 & -0.6 & 1.2 & 0.6 \\ 0 & 0 & 0 & 1 \\ 1.2 & 0.6 & -1.2 & -0.6 \end{bmatrix},$$

hence, $A_{ij} \in [min(A_{1,ij}, A_{2,ij}), max(A_{1,ij}, A_{2,ij})]$, $i, j = 1, 2, 3, 4$. *For the interval model, we use a min-max MPC that considers the worst case to ensure robust performance against uncertainties. Here, we design a standard controller that operates on a nominal model for the middle of the intervals, i.e. for values* $k = 1$ *and* $c = 0.5$. *The MPC strategy is designed on this nominal model, but the simulation is tested for different values of* $k \in \{0.8, 1.2\}$ *and* $c \in \{0.4, 0.6\}$ *to check its stability and performance. So, the steps of the algorithm are as follows:*

Algorithm 5.4. *(min-max MPC algorithm)*

1. *With the intervals* k *and* c, *we define an interval model.*
2. *Design and apply the MPC to the nominal model.*
3. *Repeat and run the simulation for four cases* $\{c_{min}, c_{max}, k_{min}, k_{max}\}$.
4. *Different shapes related to variables (positions, velocities, input and output) are drawn and examined.*

With the optimal settings, the MPC controller shows good convergence to the setpoint 1.1. *There is a settling time of about* 4 *to* 5 *seconds, with a simulation time of* 15 *seconds and an overshoot of about 10 to 20 percent (depending on the interval). In Figure 11, the positions of* x_1 *and* x_2 *are shown with colored curves for four intervals that all starting from zero. Due to the presence of the initial oscillation in this figure, all the plots converge to 1.1 meter. The case* $k = 0.8$, $c = 0.4$ *(blue) has the largest oscillation and the case* $k = 1.2$, $c = 0.6$ *(red) has the fastest convergence. As can be seen in this figure, for* x_2 *with a setpoint of 1.1, all plots start from zero and reach about 1.15 meter, with an overshoot of about 1.1 to 1.2. The case* $k = 0.8$, $c = 0.4$ *has the slowest tracking error with a settling time of 4 seconds and the case* $k = 1.2$, $c = 0.4$ *has the faster tracking error but with more overshoot.*

Figure 12 shows the velocity v_1 *starting from zero and overshooting to 1.5 m/s and converging to zero (the steady state velocity is zero). Also, it can be seen in this figure that the fluctuations are higher at* $k = 1.2$, $c = 0.4$ *(due to the high stiffness). In addition, this figure shows the velocity* v_2 *similar to* v_1 *but with a smaller overshoot, which clearly shows that the convergence to zero occurred faster and the least fluctuation is observed in the range* $k = 0.8$, $c = 0.6$ *(high damping).*

In Figure 13, the control signal u *with colored curves and starting from point 1, has positive and negative peaks at the beginning of the simulation and converges to zero in the range* $[-5, 5]$. *Also, the variations of* Δu *are limited to the interval* $[-1, 1]$. *In addition, in this figure, the output signal* y *is similar to* x_2 *in Figure 11, but focusing on the output where all curves converge to 1.1, showing a settling time of less than 4 seconds and an overshoot of about 1.2. Therefore, the responses in terms of transient response, overshoot and settling time, tracking error, steady state error, continuous response and stability are very good and the interval MPC has been able to show a strong and appropriate performance in different intervals.*

Now, we analyze the performance of the interva MPC for system (33) *with a simulation time of 25 seconds. The IAE and ISE performance metrics are calculated for four cases* $\{c_{min}, c_{max}, k_{min}, k_{max}\}$, *in such a way that:*

$$IAE = \int_0^T |e(t)|dt,$$

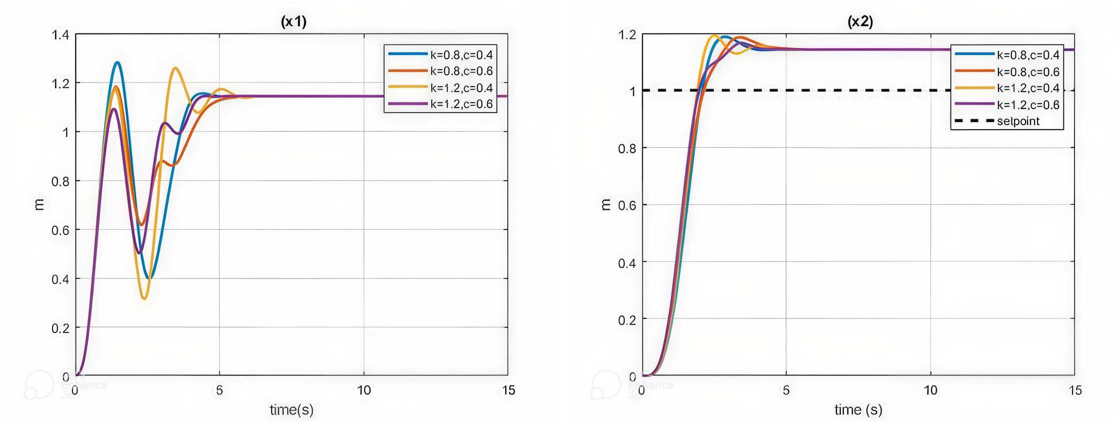

Figure 11: The positions of x_1 and x_2 with colored curves for different intervals.

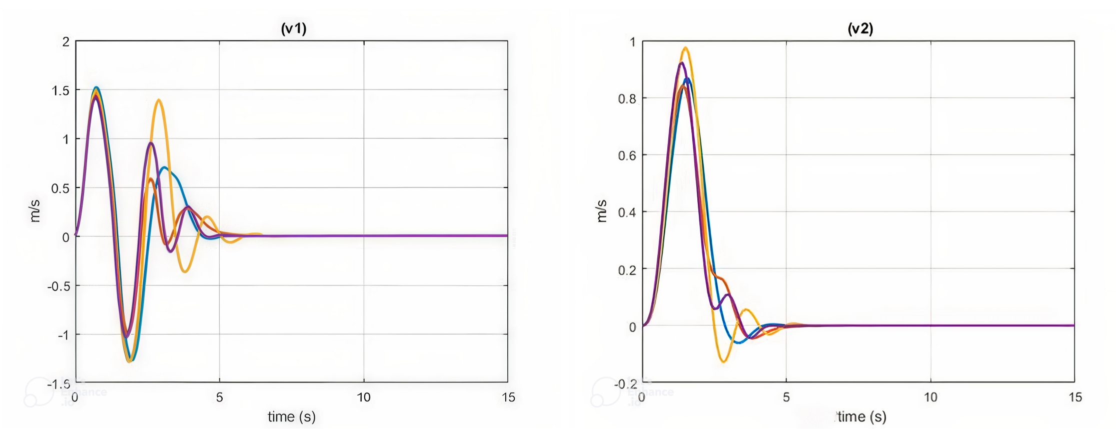

Figure 12: The velocities v_1 and v_2 for different intervals.

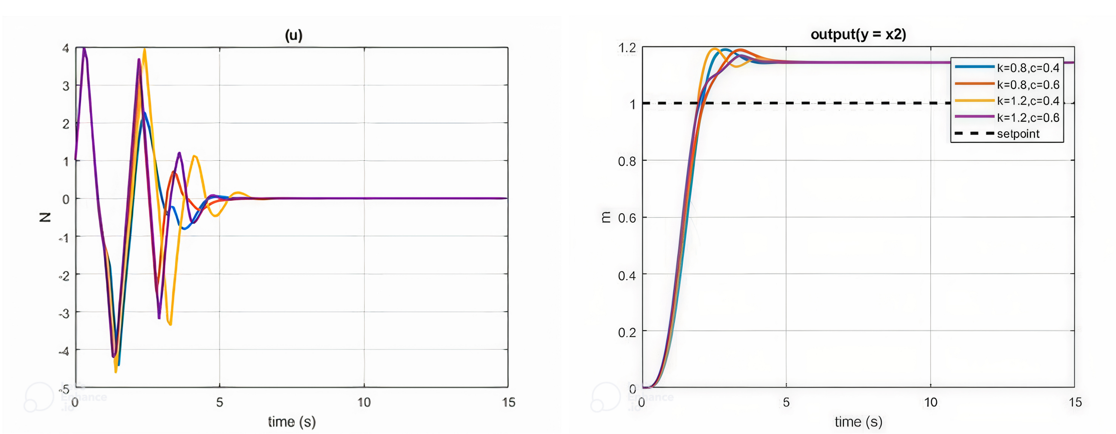

Figure 13: The control signal u and the output signal y with different colored curves.

is the integral absolute error and measures the cumulative absolute deviation of the system's output from the desired setpoint over time. Also,

$$ISE = \int_0^T |e(t)|^2 dt,$$

denotes the integral squared error which computes the cumulative squared deviation of the system's output from the setpoint, giving more weight to larger errors. The calculations are performed on an Intel Pentium (Core i5-2.4 GHZ) 10th generation computer and Windows 11 operating system. The average time for each execution step is $0.0015 - 0.002$ *seconds and the maximum execution time is* $0.004 - 0.005$ *seconds. Also, the average simulation time for* 1000 *execution steps is* 1.75 *seconds and the maximum simulation time is 4.5 seconds. Numerical results are shown in Table 4. At the*

Table 4: Comparing the performance criteria of the designed controller for system (33).

$\{k, c\}$	IAE	ISE
$k = 0.8, c = 0.4$	8.3202	3.1356
$k = 0.8, c = 0.6$	8.9928	3.6696
$k = 1.2, c = 0.4$	11.7818	6.0928
$k = 1.2, c = 0.6$	11.4962	5.7839

simulation time of 25 seconds, IAE is between 8.3202 *and* 11.7818 *and ISE is between* 3.1356 *and* 6.0928*, indicating the stable performance of the interval controller. The higher values of hardness at* $k = 1.2$ *are due to the increased fluctuations in the system. These values have decreased significantly compared to the simulation time of 15 seconds, where IAE is about* 13.40*, indicating the improvement of convergence with increasing simulation time. To further improve the simulation responses, it is suggested to increase the weight* Q *to* 500 *or increase the simulation time to* 50 *seconds to further reduce IAE and ISE and achieve faster convergence.*

6 Conclusions

In order to solve the optimal control problems with uncertainty when it is not possible to solve the Hamiltonian equations analytically, the predictive control strategy of the model is introduced. In this control technique, a cost function is optimized online considering the physical constraints governing the system to calculate the control signal in the current time. After applying it to the system and recording new measurements, two optimization methods will be performed from the beginning to obtain new control signals. Indeed, in this work, we introduce robust MPC with a min-max formulation and interval MPC using extreme parameter values for optimal control problems with interval uncertainty. According to the simulation results, it can be said that even though the dynamics of the system has severe uncertainty and the behavior of the system changes randomly between stable and unstable, but the predictive controller of the interval model is well able to converge the state variables of system. Also, the obtained solutions indicate that the predictive control of the interval model works very well in these proposed methods. Although this study has conducted initial investigations in theoretical modeling and experimental validation, future work could focus on designing an interval stochastic MPC by incorporating noise issues with different variances, thereby enriching the theoretical framework and practical applications of this field.

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