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Prediction of the Local Particle Velocity in the Spout Region of Cylindrical Spouted Beds Using an Artificial Neural Network

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ABSTRACT

The local particle velocity is an important hydrodynamic parameter in the design and optimization of gas-solid systems, such as the spouted beds. Predicting the particle velocity in the spout region is often complex, and developing a simple and reliable method for estimating this parameter is both valuable and highly beneficial for engineers and system designers. In the present study, the intelligent predictive approach of a multi-layer perceptron neural network (MLP-NN) is used for the first time to investigate the feasibility of measuring the local particle velocity within the spout region of spouted beds. Accordingly, 224 measured data points are collected, covering a broad range of factors such as bed diameter, inlet diameter, static bed height, cone angle, minimum spouting velocity, and inlet gas velocity for Geldart D particles with varying densities and diameters. The data are then used for training, validating, and testing the intelligence-based model. The MLP-NN-based model shows a strong predictive capability for estimating particle velocity in the spouted bed, with an Average Absolute Relative Error (AARE) of 13.32%, an R^2 value of 0.9842, and a Root Mean Squared Error (RMSE) of 0.033 for the test dataset. In addition, for all data, the model achieves an AARE of 6.30%, an RMSE of 0.0098, and an R^2 value of 0.9942. Furthermore, in the present study, a sensitivity analysis is conducted on the data to determine the degree of influence of the input parameters on the target function.

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1. Introduction

Spouted beds have been introduced as an alternative technique for fluidizing Geldart D particles, which are extremely dense and coarse [1]. The application of these beds has expanded across different industries, such as pharmaceutical, petrochemical, combustion, agricultural, nuclear fuel particles, and the food industry [2–5]. To achieve better control over the heat and mass transfer rates between gas and solid particles, it is crucial to understand the particle velocity at different levels within the spouted beds. Several experimental techniques have been proposed and used for this purpose, such as particle image velocimetry (PIV), Laser Doppler Anemometry (LDA), and fiber-optic probe (FOP). However, these techniques are often limited by the system geometries, scales, and operating conditions. Besides, the experimental set-ups are typically expensive, particularly at real-scale applications.

Today, computational fluid dynamics (CFD) has significantly progressed as an alternative and accurate method for providing valuable insight concerning particle behavior, aiding in the optimization, design, and scale-up of spouted beds [6,7]. Two broadly used approaches embedded in most CFD software are used for predicting the particles' behavior in gas-solid systems: the Eulerian and Lagrangian approaches. It should be noted that CFD modeling not only includes numerous adjustable parameters that can significantly affect the results, but also consumes substantial computational time [8,9].

An alternative method for overcoming the mentioned limitations is to develop a nonlinear correlation between the target function and the influential parameters in the form of dimensionless groups. In this regard, several empirical correlations have been established to forecast some important design factors, such as minimum spouting velocity [10,11], pick and operating pressure drop [12], as well as other spouted bed factors, namely, the particle cycle times [13,14], heat transfer (Nusselt number), and mass transfer (Sherwood number) in spouted beds [15,16]. It should be emphasized that the minimum spouting velocity is determined as the minimum gas inlet velocity required to maintain a spouting regime [17].

Recently, it has been reported that the intelligence-based models are capable of predicting the minimum spouting velocity with accuracy greater than that of the explicit correlation built by the least-squares fitting methods (LSFM) in conical spouted beds with vegetable biomasses [18]. Therefore, intelligence-based predictive models are widely used rather than those developed by LSFM in terms of accuracy and covering a large volume of measured data (big data).

Although several studies using intelligence-based models have been reported in the open literature [18–20] to estimate important design factors, there is a lack of machine learning-based models to estimate the local particle velocity within the spout section of the cylindrical spouted beds. Thus, this study aims to develop a machine learning method based on the well-known approach of a multi-layer perceptron neural network (MLP-NN) to predict the local particle velocity through the cylindrical spouted beds. In addition, to assess the importance of each input parameter on the local particle velocity in cylindrical spouted beds. In addition, a sensitivity analysis is conducted to assess the importance of each input parameter on the local particle velocity.

2. Materials and methods

2.1. The developed machine learning model

The MLP-NN with Levenberg-Marquardt backpropagation is used because it is a non-linear multi-variable system capable of establishing strong correlations between the input and output variables. Due to its suitability for

normalization within the $[0, 1]$ range and its stable convergence during the training stage, the logistic sigmoid function was chosen rather than other activation functions such as ReLU and tanh. This is as follows:

$$f(x) = \frac{1}{(1 + \exp(-x))} \quad (1)$$

where x is the sum of the inputs of the neuron and $f(x)$ is in the range $[0, 1]$, and an identity function is used in the output layer. Although alternative configurations, such as two-hidden-layer networks with varying neuron counts, were examined, they did not significantly improve model accuracy and resulted in increased training time compared to a single-hidden-layer network. Consequently, consistent with the previous research [21], the current MLP-NN utilizes a single hidden layer, achieving comparable results with reduced computational time. The model is constructed with seven neurons for the input layer, seven neurons for the hidden layer, and one neuron for the output layer. The input layer receives the relevant information regarding the independent variables from the dataset. Fig. 1 illustrates the scheme of the developed MLP-NN, including seven neurons in the input layer, seven neurons in the hidden layer, and one neuron in the output layer, for estimating the local particle velocity through the spouted beds.

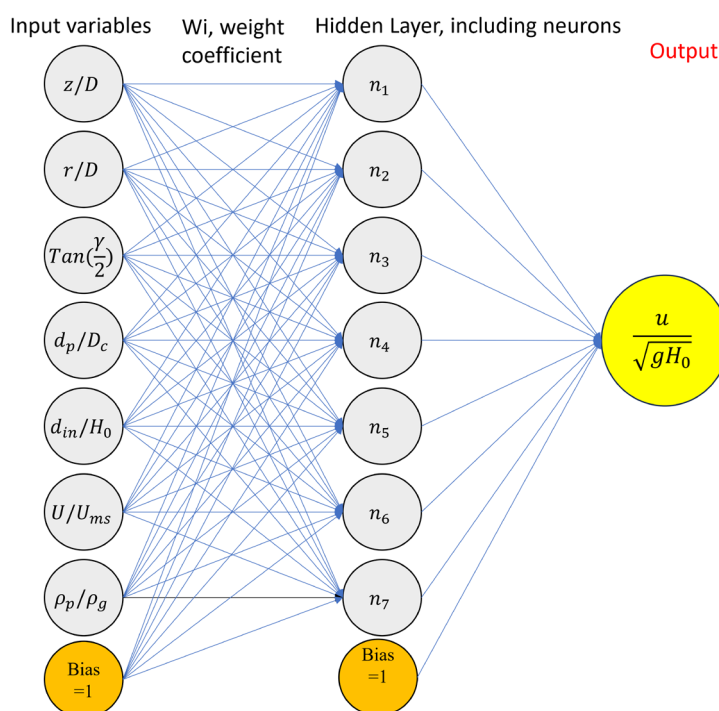


Fig. 1. The developed MLP-NN model for the predicting of the local particle velocity

A random data-splitting method was employed in the present study, which is an acceptable and widely used approach in engineering applications [18–20]. The number of measured data points is 224, which are randomly divided into 70% (156 data points) for training the model, 15% (34 data points) for model validation, and the remaining data (34 data points), *i.e.*, unseen data points, for the test dataset. The network is trained based on the configuration mentioned previously. Fig. 2 illustrates the mean squared error (MSE) against epoch for the training, testing, and validation data. It should be noted that the training data points are also classified into two subsets: training and validation. Furthermore, the validation graph (green line) can define the optimal epoch. As observed at 82 iterations, the MSE of both the validation and test stages reaches its minimum value. It is important to note that, to enhance the convergence of the MLP-NN, the input variables have been normalized, as each input variable was scaled to the range of $[0, 1]$ by subtracting the minimum value and dividing by the range ($\max - \min$).

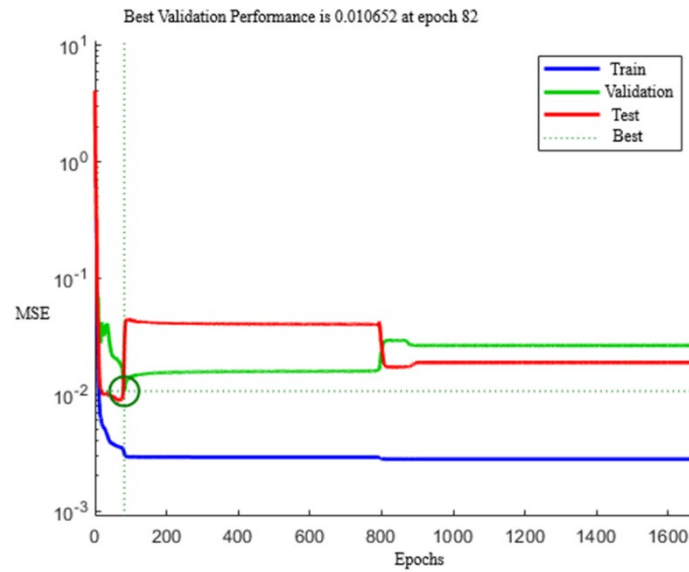


Fig. 2. Mean squared error versus number of epochs for train, validation, and test sets

To assess the accuracy of the particle velocity estimated by the developed MLP-NN model compared to the measured data, the metrics of RMSE, AARE, and R^2 are used. They are defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (u_{Exp.} - u_{cal.})^2}{n}} \quad (2)$$

$$AARE = \frac{1}{n} \sum_{i=1}^n \left| \frac{u_{Exp.} - u_{cal.}}{u_{Exp.}} \right| \times 100 \quad (3)$$

$$R^2 = \left(1 - \frac{\sum (u_{cal.} - u_{Exp.})^2}{\sum (u_{cal.} - \bar{u}_{Exp.})^2} \right) \quad (4)$$

where $u_{Exp.}$ is the experimental value of particle velocity and $u_{cal.}$ is the corresponding value estimated by the MLP-NN model, and n refers to the number of data points.

2.2. Data collection and model variables

In contrast to fluidized beds, the minimum velocity for spouting particles in cylindrical spouted beds is affected not only by various parameters, such as the acceleration of gravity and gas and particle properties, but also by design parameters like the bed cone angle, gas inlet diameter, static bed height, and bed diameter. In addition to these parameters, the minimum spouting gas velocity and the inlet gas velocity influence the particle velocity. Therefore, to achieve a general correlation between the independent variables and the local particle velocity for cylindrical spouted beds, the aforementioned parameters are arranged into dimensionless groups. The experimental data were gathered from [22–25] for training and testing of the MLP-NN model. The bed diameters in the cylindrical portion, *i.e.*, D_c , are 0.152 and 0.076 m; cone angles, γ , are 30°, 45°, and 60°. The inlet diameters, D_i , are 0.0095, 0.019, and 0.03 m. The static bed heights, H_0 , are 0.16, 0.14, 0.20, 0.28, and 0.323 m. Moreover, the particle diameters within the beds are 0.00109, 0.00133, 0.00184, 0.00254, and 0.004 m, and the particle densities are 2420, 2450, 2485, and 2493 kg/m³. The relationship between the variables has been incorporated into the MLP-NN as follows:

$$\frac{u_p}{\sqrt{gH_0}} = f\left(\frac{r}{D_c}, \frac{z}{D_c}, \frac{d_p}{D_c}, \frac{D_i}{H_0}, \tan(\gamma/2), \frac{U}{U_{ms}}, \frac{\rho_p}{\rho_g}\right) \quad (5)$$

where u_p is the time-averaged local particle velocity, and the target function, *i.e.*, U , is the superficial gas velocity, and U_{ms} is the minimum spouting velocity, ρ is the material density, and subscripts p and g refer to the particle and gas phases. To capture the complex relationship between the variables, an artificial neural network is utilized here.

3. Results and discussion

According to Eq. (5), the intelligent approach of MLP-NN with a hidden layer containing 7 neurons is employed to estimate the local particle velocity for cylindrical spouted beds using Geldart D particles, utilizing a back-propagation algorithm.

Table 1 presents the different error metrics for evaluating the developed MLP-NN model based on the training, validation, and testing datasets. As can be seen, the model was perfectly trained with R^2 , AARE, and RMSE of 0.9974, 4.33%, and 7.6×10^{-5} , which confirms that the model is capable of providing a suitable relationship between the variables. In addition, the results of the validation stage from the training process also reveal that the model accurately provides correlations between the input and output results. Furthermore, the testing stage on the unseen 34 measured data yielded R^2 , AARE, and RMSE values of 0.9842, 13.32%, and 0.033, respectively. The model results are also excellent across all data points. These results demonstrate that the present MLP-NN model can predict the local particle velocity in various locations with high accuracy. In addition, these error metrics confirm the reliability and generality of the model in estimating this important hydrodynamic parameter in gas-solid cylindrical spouted beds. The analysis of relative error percentages reveals that 85% of all examined data (including training, validation, and testing data points) have an error percentage lower than 9.6%, which affirms the accuracy of the proposed model.

Table 1. The metric errors obtained by the MLP-NN model for the training, validation, and testing stages

Dataset	RMSE,	AARE	R^2
Train	7.6×10^{-5}	4.33	0.99745
Validation	0.0314	8.31	0.99279
Test	0.0330	13.32	0.98425
All	0.0098	6.30	0.99421

Fig. 3 compares the particle velocity through the bed from the MLP-NN model to the corresponding measured data for the training, testing, and validation stages, as well as for all data combined. This figure clearly confirms the reliability and superiority of the MLP-NN model for all training, testing, and validation stages, as the proximity of the estimated data to the best-fit line is considerable for all the mentioned datasets. Therefore, the constructed MLP-NN model can be used with high accuracy for describing the key hydrodynamic parameter of local particle velocity. Moreover, due to the similarity of the testing stage results to those for the training data in terms of accuracy and trend, it can be concluded that the developed model does not exhibit overfitting or underfitting. The best line (named by Fit) closely tracking the predicted data is nearly identical to the experimental data (best-fit named by dashed line) for the training and all data cases, which affirms the robustness of the developed model in predicting particle velocity in spouted beds.

The performance of the MLP-NN model in demonstrating the physical behavior of particle velocity in the spout region is also examined. The results of this analysis are presented in Figs. 4 and 5.

Fig. 4 compares the predicted radial distribution of particle velocity at different levels with the measured data extracted by Olazar et al. [22] under the same conditions. A close agreement between the experimental findings and MLP-NN outcomes is observed, particularly in the central region at lower levels, *i.e.*, $Z/D = 0.13$ and 0.66 . However, as the flow reaches the annular region, where the particles move downward, the model results slightly deviate from the measured data. In addition, at higher levels, *i.e.*, $Z/D = 0.92$, the MLP-NN model shows a considerable error in predicting the particle velocity in the central region. The particle velocities at the bed axis ($r/R = 0$) are at their maximum values and decrease to approximately zero at the spout-annulus boundary. Overall, a similar trend between the MLP-NN model results and the real data is observed in terms of longitudinal vertical particle velocity.

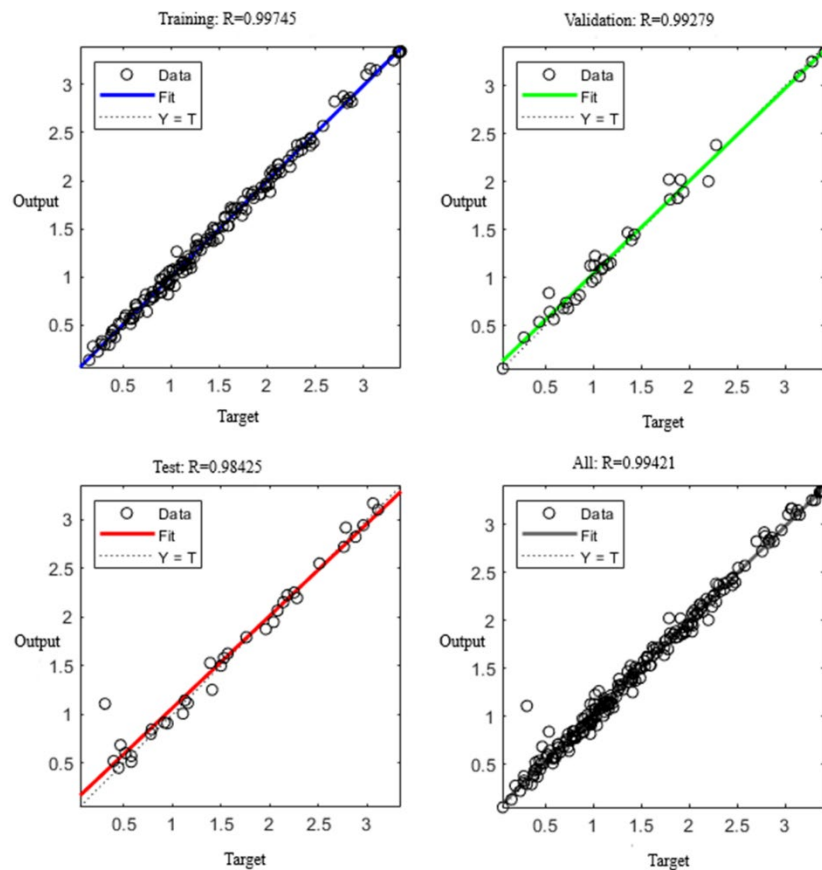


Fig. 3. Particle velocity in the bed predicted by the MLP-NN model compared to the corresponding measured data for the training, validation, and testing stages

Fig. 5 compares the predicted particle velocity along the bed axis with the measured data extracted by Olazar et al. [22] under the same conditions. As can be seen, the particles reach their maximum velocity and then gradually decelerate. This sharp decrease continues to zero at the fountain region. An excellent consistency between the model and experimental results is observed, which reveals the robustness of the model in predicting the physical behavior of particle velocity in the spout region for beds with different cone angles (30° and 45°).

Determining the importance of the input variables as dimensionless groups in controlling the local particle velocity in the spout region can be a useful task for designers. Fig. 6 presents the relevance factors between the model's input parameters and local particle velocity in the spout region. From the figure, the dimensionless groups that directly affect the local particle velocity are d_p/D_c and D_i/H_0 . However, Z/D , r/D , $\tan(\gamma)$, U/U_{ms} , and ρ_p/ρ_g have an inverse relationship with the local particle velocity at their given ranges. For instance, Figs. 4 and 5, which are provided for

two cone angles of 45 and 30, respectively, under the same other conditions, demonstrate two different particle velocities of 3.41 and 3.06 m/s at $Z/D = 0.13$ (highlighted points) that the sensitivity analysis presented here. On the other hand, Z/D has the highest impact on the u_p , which is followed by r/D , D_i/H_0 , U/U_{ms} , d_p/D_c , ρ_p/ρ_g and $\tan(\gamma)$.

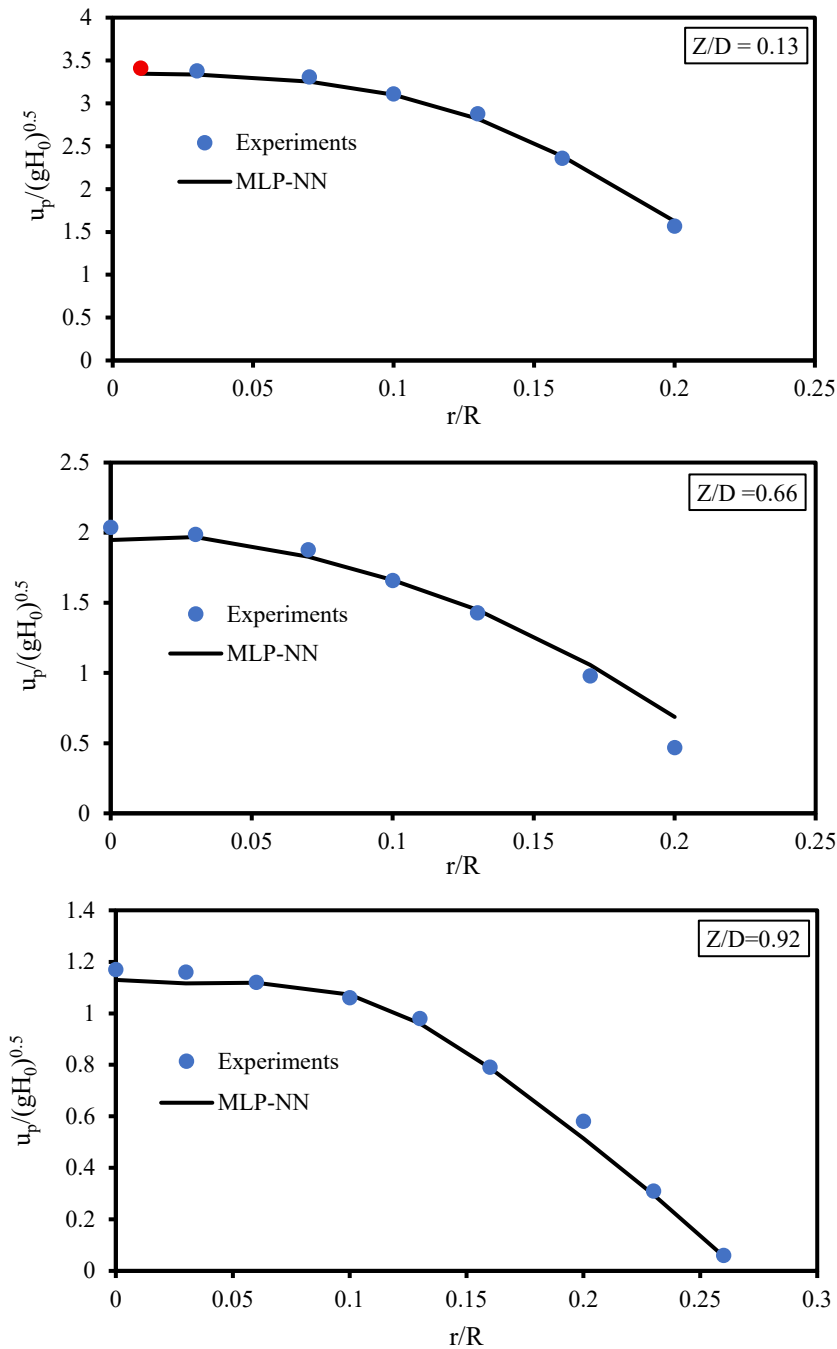


Fig. 4. The radial distribution of particle velocity at different levels for the spouted bed operated by Olazar et al. [22] with the characteristics and operating conditions of ($D_c=0.152$ m, $D_i = 0.03$ m, $\gamma = 45^\circ$, $H_0=0.2$ m, $\rho_p = 2420$ kg/m³ and $d_p = 4$ mm, $U_{ms} = 1.36$ m/s, $U=1.39$ m/s)

4. Conclusion

The present study established an intelligence-based model to estimate the particle velocity in the spout region of cylindrical spouted beds. Therefore, the predictive machine learning approach of a multi-layer perceptron neural network (MLP-NN) was used to analyze the feasibility of calculating the local particle velocity through the spouted beds. About 224 experimental data points, including variations in bed diameter, inlet diameter, static bed height, cone angle, minimum spouting velocity, and inlet gas velocity for Geldart D particles with different densities and

diameters, were gathered and subsequently used for training, validating, and testing the proposed model. The MLP-NN-based model obtained the proper results in the prediction of the particle velocity in the bed with the $R^2 = 0.9842$, AARE = 13.32%, and RMSE = 0.33 for the test data. In addition, R^2 , AARE, and RMSE for all data were 0.9942, 6.30%, and 0.0098, respectively. In addition, the developed model was evaluated for its ability to predict the trend of particle velocity in the longitudinal and axial directions, and a strong agreement was observed with the measured data.

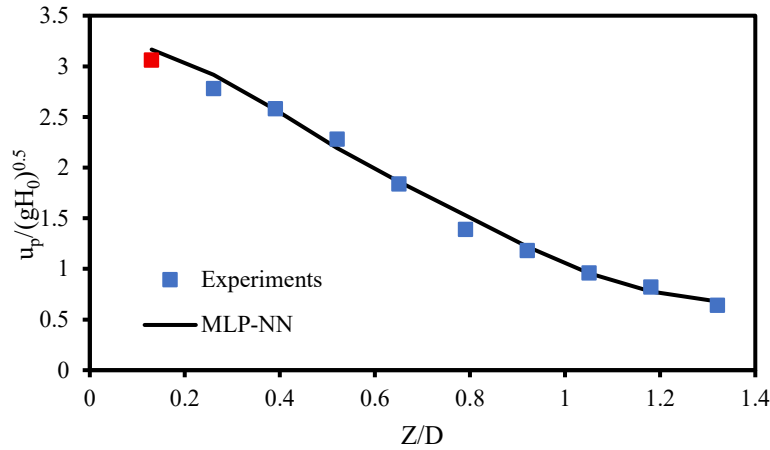


Fig. 5. The particle velocity along the bed axis for the spouted bed operated by Olazar et al. [22] with the characteristics and operating conditions of ($D_c=0.152$ m, $D_i=0.03$ m, $\gamma=30^\circ$, $H_0=0.2$ m, $\rho_p=2420$ kg/m³ and $d_p=4$ mm, $U_{ms}=1.36$ m/s, $U=1.39$ m/s)

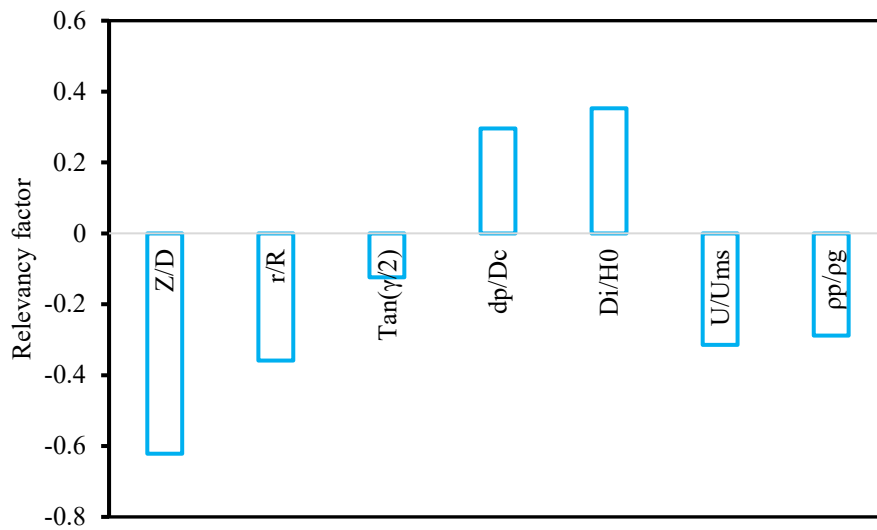


Fig. 6. Relevancy of the input factors with the local particle velocity in the spouted bed

Ultimately, a sensitivity analysis was conducted on the data to investigate the impact of each dimensionless independent variable on the target function. The results showed that the more important parameters, in decreasing order of influence, are: Z/D , r/D , D_i/H_0 , U/U_{ms} , d_p/D_c , ρ_p/ρ_g and $\tan(\gamma)$. Developing an intelligent model to predict local particle velocity in the primary regions of spouted beds, such as the spout, annular, and fountain regions, is a topic for future research. Beyond the MLP approach, the applicability of alternative methods like radial basis function (RBF), support vector machine (SVM), and Gaussian process regression (GPR) could also be explored.

References

- [1] Ranade, V. V., 2002. Computational flow modeling for chemical reactor engineering, Published by Academic Press, California, USA.
- [2] Olazar, M., San José, M., Zabala, G., Bilbao, J., 1994. New reactor in jet spouted bed regime for catalytic polymerizations, *Chemical Engineering Science*, 49, 4579–4588.
- [3] Lopes, N.E.C., Moris, V.A.S., Taranto, O.P., 2009. Analysis of spouted bed pressure fluctuations during particle coating, *Chemical Engineering and Processing: Process Intensification*, 48, 1129–1134. <https://doi.org/10.1016/j.ccep.2009.03.003>
- [4] Jono, K., Ichikawa, H., Miyamoto, M., Fukumori, Y., 2000. A review of particulate design for pharmaceutical powders and their production by spouted bed coating, *Powder Technology*, 113, 269–277.
- [5] Alvarez, J., Hooshdaran, B., Cortazar, M., Amutio, M., Lopez, G., Freire, F.B., Haghsheenasfard, M., Hosseini, S.H., Olazar, M., 2018. Valorization of citrus wastes by fast pyrolysis in a conical spouted bed reactor, *Fuel* 224, 111–120. <https://doi.org/10.1016/j.fuel.2018.03.028>
- [6] Hosseini, S.H., Ahmadi, G., Olazar, M., 2014. CFD study of particle velocity profiles inside a draft tube in a cylindrical spouted bed with conical base, *Journal of the Taiwan Institute of Chemical Engineers*, 45, 2140–2149. <https://doi.org/10.1016/J.JTICE.2014.05.027>
- [7] Hosseini, S.H., Zivdar, M., Rahimi, R., 2009. CFD simulation of gas – solid flow in a spouted bed with a non-porous draft tube, *Chemical Engineering and Processing: Process Intensification*, 48, 1539–1548. <https://doi.org/10.1016/j.ccep.2009.09.004>
- [8] Hossein, S.H., Fattahi, M., Ahmadi, G., 2016. CFD Study of hydrodynamic and heat transfer in a 2D spouted bed: Assessment of radial distribution function, *Journal of the Taiwan Institute of Chemical Engineers*, 58, 107–116. <https://doi.org/10.1016/j.jtice.2015.06.027>
- [9] Mahmoodi, B., Hossein, S.H., Olazar, M., Altzibar, H., 2017. CFD-DEM simulation of a conical spouted bed with open-sided draft tube containing fine particles, *Journal of the Taiwan Institute of Chemical Engineers*, 81, 275–287. <https://doi.org/10.1016/j.jtice.2017.09.051>
- [10] Hosseini, S.H., Rezaei, M.J., Bag-Mohammadi, M., Karami, M., Moradkhani, M.A., Panahi, M., Olazar, M., 2019. Estimation of the minimum spouting velocity in shallow spouted beds by intelligent approaches: Study of fine and coarse particles, *Powder Technology* 354, 456–465. <https://doi.org/10.1016/j.powtec.2019.06.025>
- [11] Moradkhani, M.A., Hosseini, S.H., Olazar, M., Altzibar, H., Valizadeh, M., 2021. Estimation of the minimum spouting velocity and pressure drop in open-sided draft tube spouted beds using genetic programming, *Powder Technology*, 387, 363–372. <https://doi.org/10.1016/j.powtec.2021.04.049>
- [12] Saldarriaga, J.F., Atxutegi, A., Aguado, R., Altzibar, H., Bilbao, J., Olazar, M., 2017. Correlations for calculating peak and spouting pressure drops in conical spouted beds of biomass, *Journal of the Taiwan Institute of Chemical Engineers*, 80, 678–685. <https://doi.org/10.1016/j.jtice.2017.09.001>
- [13] Estiati, I., Atxutegi, A., Altzibar, H., Freire, F.B., Aguado, R., Olazar, M., 2021. Multiple-Output Artificial Neural Network to Estimate Solid Cycle Times in Conical Spouted Beds, *Chemical Engineering & Technology*, 44, 542–550. <https://doi.org/10.1002/ceat.202000491>
- [14] Estiati, I., Tellabide, M., Saldarriaga, J.F., Altzibar, H., Freire, F.B., Freire, J.T., Olazar, M., 2019. Comparison of artificial neural networks with empirical correlations for estimating the average cycle time in conical spouted beds, *Particuology*, 42, 48–57. <https://doi.org/10.1016/j.partic.2018.03.010>
- [15] Makibar, J., Fernandez-akarregi, A.R., Alava, I., Cueva, F., Lopez, G., Olazar, M., 2011. Investigations on heat transfer and hydrodynamics under pyrolysis conditions of a pilot-plant draft tube conical spouted bed reactor, *Chemical Engineering and Processing: Process Intensification*, 50, 790–798. <https://doi.org/10.1016/j.ccep.2011.05.013>
- [16] Sukunza, X., Aguado, R., Pablos, A., Tellabide, M., Estiati, I., Olazar, M., 2022. Mass transfer in conical spouted beds equipped with internal devices, *Powder Technology*, 410, 117850. <https://doi.org/10.1016/j.powtec.2022.117850>
- [17] Altzibar, H., Lopez, G., Bilbao, J., Olazar, M., 2013. Minimum spouting velocity of conical spouted beds equipped with draft tubes of different configuration, *Industrial and Engineering Chemistry Research*, 52, 2995–3006. <https://doi.org/10.1021/ie302407f>
- [18] Moradkhani, M.A., Hosseini, S.H., Karami, M., Olazar, M., Saldarriaga, J.F., 2023. Applying conventional and intelligent approaches to model the minimum spouting velocity of vegetable biomasses in conical spouted beds, *Powder Technology*, 418, 118300. <https://doi.org/10.1016/j.powtec.2023.118300>
- [19] Saldarriaga, J.F., Cruz, Y., Estiati, I., Tellabide, M., Olazar, M., 2022. Assessment of pressure drop in conical spouted beds of biomass by artificial neural networks and comparison with empirical correlations, *Particuology*, 70, 1–9. <https://doi.org/10.1016/j.partic.2021.12.004>
- [20] Karimi, M., Vaferi, B., Hosseini, S.H., Olazar, M., Rashidi, S., 2021. Smart computing approach for design and scale-up of conical spouted beds with open-sided draft tubes, *Particuology*, 55, 179–190. <https://doi.org/10.1016/j.partic.2020.09.003>
- [21] Du, K.L., Swamy, M.N.S., 2006. Neural Networks in a Soft computing Framework, Published by Springer, London.
- [22] Olazar, M., San José, M.J., Izquierdo, M.A., De Salazar, A.O., Bilbao, J., 2001. Effect of operating conditions on solid velocity in the spout, annulus and fountain of spouted beds, *Chemical Engineering Science*, 56, 3585–3594.
- [23] Aradhya, S., Taofeeq, H., Al-dahhan, M., 2017. International Journal of Multiphase Flow Evaluation of the dimensionless groups based scale-up of gas – solid spouted beds, 94, 209–218. <https://doi.org/10.1016/j.ijmultiphaseflow.2017.04.006>
- [24] Aradhya, S., Taofeeq, H., Al-dahhan, M., 2016. Chemical Engineering and Processing: Process Intensification, A new mechanistic scale-up methodology for gas-solid spouted beds, *Chemical Engineering and Processing: Process Intensification*, 110, 146–159. <https://doi.org/10.1016/j.ccep.2016.10.005>
- [25] Pianarosa, D.L., Freitas, L.A.P., Lim, C.J., Grace, J.R., Dogan, M., 2000. Voidage and Particle Velocity Profiles in a Spout-Fluid Bed, *The Canadian Journal of Chemical Engineering*, 78, 132–142.