Iranian Journal of Fuzzy Systems
Volume 23, Number 1, (2026), pp. 163-181
Original paper

A Choquet-based multi-expert decision-making methodology with N-soft sets

J. C. R. Alcantud[1], M. Akram[2], G. Santos-García[3] and W. Ding[4]

[1] *Unidad de Excelencia Gestión Económica para la Sostenibilidad GECOS and IME, University of Salamanca, Salamanca, Spain*
[2] *Institute of Mathematics, University of the Punjab, New Campus, Lahore, Pakistan*
[3] *BORDA Research Unit and IME, University of Salamanca, Salamanca, Spain*
[4] *School of Artificial Intelligence and Computer Science, Nantong University, Nantong, China*
[4] *Faculty of Data Science, City University of Macau, Macau, China*

jcr@usal.es, m.akram@pucit.edu.pk, jcr@usal.es, dwp9988@163.com

Abstract

The objective of this paper is to provide advanced multi-expert decision-making techniques using N-soft sets as a referential framework. For the first time, the primary analytical tool for achieving this goal is the Choquet integral. First, the application of this aggregation operator within the context of a set $\{0, 1, 2, \ldots, N\}$, representing the available ratings, is investigated. A straightforward formulation of the Choquet integral tailored to this specific set, followed by a detailed presentation of its computational implementation, is presented. Then, the practical implications of these constructions in the realm of N-soft set theory are shown. They encompass the computation of new scores for the assessment of alternatives in N-soft sets (both in individual and multi-agent cases), and aggregation of data that comes in the form of N-soft sets. Ultimately, we demonstrate how these innovative tools enhance multi-expert decision-making methodologies within the framework of N-soft sets. Three different approaches are discussed. Examples and comparisons with existing methodologies are provided too.

Keywords: Choquet integral, aggregation operator, N-soft set, score, decision making, *Mathematica*.

1 Introduction

The general aim of this paper is to provide advanced multi-expert decision-making techniques for N-soft information with the help of intermediate tools that take advantage of the Choquet integral. Let us review these two main components of this article.

The Choquet integral was developed in 1954 and has become a prominent example of a nonlinear aggregation operator [18]. It was later applied in decision making under uncertainty since the pioneering work of Schmeidler [34], who proved its most popular characterization. However, it was not used for multi-criteria decision aid (MCDA) till the decade of the 1990s (for example, [24, 25] dwelt on this subject). Nowadays, many generalizations exist [21], the topic is the subject of intense research and development in many areas [12, 14, 22, 31, 39, 45]. New handbooks on and the topic continue to be published regularly [13].

The Choquet integral is fundamentally built upon a special category of non-additive set functions known as capacities, or fuzzy measures. They are at the root of a generalization of measure theory with multiple applications [11]. Initially, capacities were not intended to address general forms of uncertainty; rather, their primary applications were rooted in potential theory. Nevertheless, they quickly found relevance in various other fields, such as stochastic processes. In our framework, two key perspectives on the values provided by a capacity are particularly useful. The first perspective views capacities as indicators of the significance of elements within a set, such as criteria in MCDA

Corresponding Author: J. C. R. Alcantud
Received: August 2025; Revised: December 2025; Accepted: February 2026.
https://doi.org/10.22111/ijfs.2026.52870.9342

or experts in group decision-making. The second perspective considers capacities in the context of event uncertainty. Here, the value assigned to a set reflects the degree of uncertainty regarding whether the set contains the outcome of a particular event (e.g., an experiment).

In this paper, we are particularly driven by the utilization of the Choquet integral with sets of the form $\{0, 1, 2, \ldots, N\}$, because they are the available ratings in an N-soft set. We focus on this model because it has become very popular since its introduction by Fatimah *et al.* [23], as the recent survey [6] proves (see Section 2 below for more evidence, specifically, for the provision of real situations that justify their investigation). Their advantages with respect to other uncertainty theories are: simplicity (because numerical assignments must not be fine-tuned); implementability (because users are accustomed to grading their opinions on real issues, and in fact, many tests and surveys are conducted in forms that are fully compatible with this model); and flexibility (because the granularity of the model can be adapted via the N parameter). However, an inspection of the literature shows that the applicability of Choquet integrals to problems such as aggregation within this model still needs to be exploited and argued.

All these facts are the motivation for our research, which is aimed at improving group decision-making techniques for N-soft data. There are few papers on this topic since the pioneering Alcantud *et al.* [7]. Neither of these papers has taken into account the possible existence of interactions among the individual variables (e.g., relative values of the opinions of the experts, or of the characteristics of the alternatives). Hence, our main goal is the production of reasonable multi-agent decision-making methodologies in an N-soft framework. A secondary objective is to provide a systematic presentation of the fundamental techniques involved in the solution of this problem.

Consequently, our specific targets can be summarized as follows:

1. We prove a simple expression of the Choquet integral associated with capacities on $\{0, 1, 2, \ldots, N\}$ that dispenses by sorting the entry vector, and then we show an implementation of this particular formula with Wolfram *Mathematica*.

2. With the help of this restricted form of the Choquet operator, we produce two types of tools for the practical analysis of N-soft sets:

    (a) The computation of scores of N-soft sets is a natural application of the restricted form of the Choquet integral described above. We accomplish this task both in individual and multi-agent problems.

    (b) We apply Choquet integrals to the aggregation of N-soft sets. In fact, antecedents of its utilization within N-soft set theory exists under restricted conditions, because Yager's OWA operator is a particular example of the Choquet integral that has been used to aggregate N-soft sets recently [7].

3. Ultimately, we study how these innovative tools can enhance group decision-making methodologies within the framework of N-soft sets. Three methodologies are proposed. We evaluate the suitability of each approach based on the specific characteristics of the problems under consideration. Examples and comparisons with existing methodologies are provided, too.

The research strategy of this study is mostly analytical. It is based on a comprehensive analysis of the existing literature about multi-criteria decision making and models of soft computing, conducive to the identification of a research gap in the field of multi-expert decision-making. Computational programming provides a reliable tool for numerical experimentation and testing with datasets within the scope of the article.

The rest of this article is organized as follows. Section 2 reviews related literature about both Choquet integrals and N-soft sets. Section 3 gives technical information on Choquet integration and the model that will be used for applications. Section 4 presents the mathematical and computational exercises concerning the restricted application of the Choquet integral that is needed in this article. Section 5 produces three applications to the computation of scores (both in single- and multi-agent contexts) and aggregation. Section 6 introduces three flexible algorithms for multi-agent decision-making with N-soft sets. These procedures owe to various strategies: either score-based (section 6.1), or adhering to either a *merge-then-decide* strategy (section 6.2) or a *decide-then-merge* strategy (section 6.3). A summary of their characteristics, with advantages and disadvantages, is given in section 6.4. Section 7 gives concluding comments and lines for future inspection.

2 Literature review

Within the broad fields of optimization and soft computing, in recent years the Choquet integral has been in use for a wide variety of purposes in addition to the situations mentioned in the previous section. Beliakov and James [12] provide a non-symmetric function that satisfies the Pigou-Dalton principle, and characterize which Choquet integrals are consistent with this principle from welfare economics through the antibuoyancy property of a fuzzy measure. Their study dispenses with a property –symmetry– that is unavoidable in welfare measurement, a field where Aristondo *et al.* [9] have suggested that OWA (Ordered Weighted Averaging) operators with an additional property –antibuoyancy of the weighting vectors– are reasonable welfare functions. OWA operators were introduced by Yager [43], and they are discrete Choquet integrals with respect to symmetric fuzzy measures. Xu [42] investigated Choquet-integrals of weighted intuitionistic fuzzy information. Choquet based fuzzy rough sets have been defined recently too [36]. The multi-hesitant fuzzy linguistic-based Choquet integral has been utilized to evaluate renewable energy sources [29]. And Wang *et al.* [41] used the Choquet integral for granular structure reduction.

The identification of fuzzy measures became a cornerstone in the process of aggregation of interacting inputs using the Choquet integral and its generalizations [24]. This problem is quite challenging because of the exceedingly large complexity of the polytope of capacities. To provide assistance with simulations (and algorithms of probabilistic optimization), Beliakov [10] studied random generation of capacities with the supermodular or submodular properties. Now, a Python module exists that learns fuzzy measures from empirical data [40]. Updated information about the utilization of Python can be found in Beliakov *et al.* [13, Appendix A]. To mention but a few of the recent developments, Torra [37] defined Υ-values as a novel way to assess the individual importance of the elements of the set, and Ontkovičová and Torra [32] developed new strategies for calculating Choquet integrals for continuous functions. Importantly, Torra [38] pioneered the study of privacy-preserving mechanisms for the disclosure of Choquet evaluations, which has been extended by Alcantud.

Another topic that we need to review is the benchmark model that we investigated in this article. We have witnessed how many models have extended the multi-belongingness spirit of soft sets to either more general situations or hybrid structures [1, 3, 4, 5, 15]. In this general avenue of research, the N-soft set model appeared as a simple yet natural generalization of that basic idea. Section 2.2 of the recent survey of Alcantud *et al.* [6] summarizes four specific reasons making this model superior to soft sets. This survey also recalls that since their introduction by Fatimah *et al.* [23], multiple articles have justified the use of N-soft sets in real situations [7, 27]. Now their semantics are well understood. A proof of their success is that nowadays many extensions and hybrid models exist in the literature, and they have been applied to a wide range of fields. First off, Akram *et al.* merged the model with fuzzy sets, and the fuzzy N-soft set model that stems from has been applied, e.g., to prevent cyber harassment on social media platforms [28]. Likewise, Akram *et al.* [2] first combined the model with hesitancy. Bipolarity within N-soft sets was introduced by [27]. Other sophisticated models have extended the original formulation and presented applications to various fields [19, 44]. Demir [20] defined N-soft mappings and exploited the notion in medical diagnosis. Kamacı [26] first introduced N-soft structures. As mentioned above, Alcantud *et al.* [7] pioneered the investigation of aggregation for multi-agent decisions with N-soft sets.

3 Preliminary concepts

Henceforth, let X^k represent the Cartesian product of k copies of a non-empty set X, and let $\mathscr{P}(X)$ denote the collection of all subsets of X. Also, $\mathscr{P}^*(X)$ refers to the set of all non-empty subsets of X.

The key definitions and concepts required to present our contribution are drawn from two distinct areas. First, in subsection 3.1, we provide a brief overview of the Choquet integral. Next, in subsection 3.2, we revisit some essential aspects of N-soft sets.

3.1 Choquet integrals: The discrete case

Let $X = \{1, \dots, n\}$. Depending on the application, this set X can represent n distinct criteria (in the context of Multi-Criteria Decision Analysis) or a group of experts (in group decision-making scenarios). Alternatively, it may signify the possible outcomes of an event with n potential results. However, this paper will not focus on this latter interpretation. In cases such as the analysis in section 5.1, $X = \{x_1, \dots, x_n\}$ will be used instead for clarity.

The next concept is important to develop the discrete Choquet integral:

Definition 3.1. *A discrete fuzzy measure (also called a capacity) is* $\mu : 2^X \longrightarrow [0, 1]$*, a set function that satisfies monotonicity (the property that* $\mu(T) \leqslant \mu(S)$ *when* $T \subseteq S \subseteq X$*),* $\mu(\varnothing) = 0$*, and* $\mu(X) = 1$*.*

There are many interesting properties that one might want to have in a discrete fuzzy measure. The capacity μ is symmetric if $\mu(T) = \mu(S)$ when $T, S \subseteq N$ are such that $|T| = |S|$. And it is additive if for any disjoint $T, S \subseteq X$, $\mu(T \cup S) = \mu(T) + \mu(S)$. Any additive set function is completely identified by $\{\mu(1), ..., \mu(n)\}$. And any additive normalized (i.e., with the property $\mu(X) = 1$, that is sometimes dropped) capacity is a probability measure.

We define the existence of a synergy between T and S (where T and S are disjoint subsets of X) with respect to μ if $\mu(T\cup S) > \mu(T)+\mu(S)$. Conversely, T and S exhibit redundancy when $\mu(T\cup S) < \mu(T)+\mu(S)$. If $\mu(T\cup S) = \mu(T)+\mu(S)$, then there is no interaction between T and S. If for a given capacity there is synergy between any possible pair of non-empty and disjoint sets we say that it is superadditive. Subadditivity holds when there is redundancy between any possible pair of non-empty and disjoint sets.

In the case of non-negative real inputs, the discrete Choquet integral is defined as follows:[1]

Definition 3.2. *The capacity μ defines a discrete Choquet integral $C^{\mu} : \mathbb{R}^n_+ \longrightarrow \mathbb{R}_+$ as follows:*

$$C^{\mu}(a_1,\ldots,a_n) = \sum_{i=1}^{n} \left(a_{(i)} - a_{(i-1)}\right) \mu(H_i), \text{ for all } \boldsymbol{a} = (a_1,\ldots,a_n) \in \mathbb{R}^n_+. \quad (1)$$

In this expression, we represent a non-decreasing permutation of $\boldsymbol{a}$ by $\boldsymbol{a}_{\nearrow} = (a_{(1)},\ldots,a_{(n)})$, and we also let $a_{(0)} = 0$. Besides, $H_i = \{(i),\ldots,(n)\}$ represents the set of indices of the $n-i+1$ largest components of $\boldsymbol{a}$.

The Choquet integral defined above is idempotent, continuous with respect to the Euclidean topology, and compensative, which means that for every $\mathbf{a} = (a_1,\ldots,a_n) \in \mathbb{R}^n_+$:

$$\min\{a_1,\ldots,a_n\} \leqslant C^{\mu}(a_1,\ldots,a_n) \leqslant \max\{a_1,\ldots,a_n\}.$$

When μ is symmetric, Equation (1) defines an OWA operator [43]. To define this operator, consider a weighting vector $\mathbf{w} = (w_1,\ldots,w_k) \in [0,1]^k$, i.e., $\mathbf{w}$ satisfies $\sum_{i=1}^{k} w_i = 1$. Then $\mathbf{w}$ defines an OWA operator $F^{\mathbf{w}} : \mathbb{R}^k \longrightarrow \mathbb{R}$ such that

$$F^{\mathbf{w}}(r_1,\ldots,r_k) = \sum_{i=1}^{k} w_i\, b_i \text{ for each } (r_1,\ldots,r_k) \in \mathbb{R}^k. \quad (2)$$

In this expression, b_i is the i-th largest member of $\{r_1,\ldots,r_k\}$. If C^{μ} is a Choquet integral associated with a symmetric capacity, defining $w_i = \mu(H_{n-i+1}) - \mu(H_{n-i})$, one has $C^{\mu} = F^{\mathbf{w}}$ on the restricted domain of non-negative entries.

Finally, when the capacity μ is additive, Definition 3.2 produces the weighted arithmetic average associated with weights $w_i = \mu(\{i\})$. So symmetric and additive capacities generate a Choquet integral that coincides with the simple arithmetic mean.

3.2 N-soft sets

Most of the contribution in this article is concerned with N-soft sets. The N-soft set model is a generalization of soft sets, which consist of the case $N = 2$. To define them, the following common elements are fixed for this section: O is a universe of options, X is a set of characteristics (some authors add that $X \subseteq P$, taking P as a 'universal' list of characteristics), and $G = \{0,1,\ldots,N-1\}$ are grades for some $N \in \{2,3,\ldots\}$.[2]

With these elements, N-soft sets can be defined in the following manner:

Definition 3.3. [23] *The tuple (F,X,N) defines an N-soft set on O when F is a mapping from X to $\mathscr{P}(O \times G)$ and it satisfies that for any $x \in X$, $o \in O$, a unique $(o,g_x) \in O \times G$ exists for which $(o,g_x) \in F(x)$, $g_x \in G$.*

Henceforth, the collection formed by all N-soft sets on O is denoted by $\mathcal{NS}(O)$.

The interpretation of N-soft sets has been described in e.g., Alcantud *et al.* [6, 23]. Instead of a binary description of the alternatives by their characteristics, they provide multinary parameterized descriptions. Specifically, Definition 3.3 captures that for every characteristic $x \in X$, every alternative o is associated with a unique assessment from G, the list of grades. This assessment is the only value g_x with $(o,g_x) \in F(x)$. Normally, we simply write $F(x)(o) = g_x$ instead of $(o,g_x) \in F(x)$.

In many practical situations, we work with $O = \{o_i : i = 1,2,\ldots,p\}$ and $X = \{x_j : j = 1,2,...,n\}$, i.e., both sets O and X are finite. Then the N-soft set (F,X,N) can be characterized by a table or a matrix. We present in Table 1 this codification. The table displays all values $r_{ij} = F(x_j)(o_i) \in G$ so that they can be adequately identified. From an operational viewpoint, this expression coincides with a matrix representation of the well-known approach to multi-criteria decisions. Nonetheless, to see the main differences between these approaches, the reader is addressed to Alcantud [7], who discussed the semantics of Definition 3.3.

Two extreme cases of N-soft sets can be pinpointed:

[1] When working with real integrands, either symmetric or asymmetric Choquet integral should be used [24].

[2] We choose the numbers $0,1,\ldots,N-1$ for notational convenience, although they are just ordered symbols. Examples show that they can belong to another scale, for example, $A_1, A_2, B_1, B_2, C_1, C_2$ in the Common European Framework of Reference for Languages (CEFR), or stars in rating hotels or movies [2].

Table 1: The tabular form of an N-soft set.

(F, X, N)	x_1	x_2	$\ldots\ldots$	x_n
o_1	r_{11}	r_{12}	$\ldots\ldots$	r_{1q}
o_2	r_{21}	r_{22}	$\ldots\ldots$	r_{2q}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_p	r_{p1}	r_{p2}	$\ldots\ldots$	r_{pq}

1. The N-soft set $\mathbf{0}_{X,N}$ is such that $(o, 0) \in F(x)$, for each $o \in O$, $x \in X$. When all attributes in X are of benefit type, $\mathbf{0}_{X,N}$ is the worst possible attainment: all elements are rated with the lowest grade in every attribute.
2. The N-soft set $\mathbf{1}_{X,N}$ is such that $(o, N-1) \in F(x)$, for each $o \in O$, $x \in X$. It captures the best possible attainment, when all attributes in X are of benefit type.

The next remark will be useful to unify contexts with several N-soft sets.

Remark 3.4. *If we work with* (F_1, X, N_1) *that is an* N_1*-soft set, ...,* (F_k, X, N_k) *that is an* N_k*-soft set, then we can embed each of them in a common* N*-soft set modelization, with* $N = \max\{N_1, \ldots, N_k\}$*: see Fatimah* et al. [23, Remark 2].

To quantify how valuable the options are in an N-soft set (F, X, N) as in Table 1, Fatimah *et al.* [23] defined a proxy called extended weighted choice value (abbreviated as EWCV). It is weighted because it uses $\mathbf{w} = (w_1, \ldots, w_n)$, a vector of weights associated with the characteristics in the problem, so $w_i \geqslant 0$ for all $i \in X$, $w_1 + \ldots + w_n = 1$. We can assess the value of o_i by its EWCV which is

$$\sigma_i^w = \sum_{j=1}^{n} w_j r_{ij}. \tag{3}$$

Note that the utilization of this proxy should be limited to cases when we only care about the independent value of each property. And the same is true in the case of the OWA scores, which can be defined as follows: for each $o_i \in O$, its OWA score is

$$O_i^w = F^{\mathbf{w}}(r_{i1}, \ldots, r_{in}). \tag{4}$$

Now, an implicit assumption is made, namely, all attributes are treated equally (because OWAs are symmetric aggregation operators).

Definition 3.3 admits fuzzy and hesitant extensions, cf. Akram *et al.*. These articles explained the alternative functional representations of fuzzy N-soft sets and hesitant N-soft sets (HNSSs), as well as their respective tabular representations under finiteness.

Figure 1 presents a flowchart with the contents of the rest of this paper.

4 An expression for the Choquet integral

This section is concerned with the operations of our main analytical tool in this paper on the set of ratings that are available during the utilization of N-soft sets. Consequently, henceforth in this section μ denotes a capacity on a finite set $X = \{1, \ldots, n\}$. We shall explore the expression of the restriction of the Choquet integral defined from μ to $\mathbf{N} = \{0, 1, 2, \ldots, N\}$, i.e., $C^\mu : \mathbf{N}^n \longrightarrow \mathbb{R}_+$. Section 4.1 gives the mathematical proof of the new expression, and then section 4.2 presents its implementation with *Mathematica*. Applications to N-soft set theory will follow from these contributions in the next sections.

4.1 The Choquet integral on $\{0, 1, 2, \ldots, N\}$

To define $C^\mu : \mathbf{N}^n \longrightarrow \mathbb{R}_+$, Definition 3.2 uses two types of elements: the evaluations of the capacity on some subsets of X, and the values of the successive $a_{(i)} - a_{(i-1)}$, which in our restricted setting are values from $\mathbf{N} = \{0, 1, 2, \ldots, N\}$ too. Based on this observation, our next result produces an explicit expression that provides a way of computing the Choquet integral for integer entries without sorting the entry vector. We point out that other (less efficient) expressions of the discrete Choquet integral can be used for that purpose in more general settings, for example, the one based on the Möbius transform of the fuzzy measure.

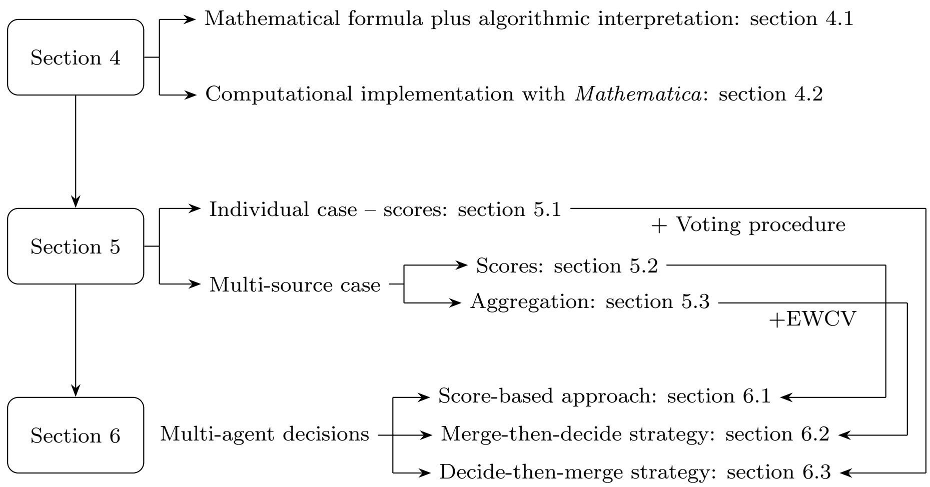

Figure 1: Flowchart with the goals of the following sections.

Proposition 4.1. *For each* $\boldsymbol{a} = (a_1, \ldots, a_n) \in \mathbf{N}^n$*, define* $\mathcal{F}_{\boldsymbol{a}} = \{X_i\}_{i=1}^k \subseteq \mathcal{P}(X)$ *such that* $X_i \in \mathcal{F}_{\boldsymbol{a}}$ *if and only if* $X_i = \{j \in X | a_j \geqslant t\}$ *for some* $t \in \{1, \ldots, N\}$*. Define also* $v_i = |\{t \in \{1, \ldots, N\} | X_i = \{j \in X | a_j \geqslant t\}\}|$*. Then*

$$C^\mu(a_1, \ldots, a_n) = \sum_{i=1}^{k} v_i \cdot \mu(X_i). \tag{5}$$

The expression in Equation (5) uses for each $a \in \mathbf{N}^n$, the different "level sets" associated with values $t \in \{1, \ldots, N\}$. They form the family $\mathcal{F}_{\mathbf{a}}$. Due to the possibility of repetitions (i.e., it may happen that $\{j \in X | a_j \geqslant t\} = \{j \in X | a_j \geqslant t+1\}$), the number of these "level sets" is represented as $k \leqslant N$, which actually depends on $\mathbf{a}$. We omit this dependence for notational convenience.

Proof. Because $\mathbf{N}^n \subseteq \mathbb{R}^n_+$, we can obviously rewrite Equation (1) as

$$C^\mu(a_1, \ldots, a_n) = \sum \left\{ \left(a_{(i)} - a_{(i-1)}\right) \mu(H_i) \,|\, a_{(i)} > a_{(i-1)}, i \in \{1, \ldots, n\} \right\},$$

where $a_{(0)} = 0$ by convention, and $\mathbf{a}_{\nearrow} = (a_{(1)}, \ldots, a_{(n)})$ and $H_i = \{(i), \ldots, (n)\}$ are as in Definition 3.2.

Under our assumptions, one has $H_i = \{t \in \{1, \ldots, n\} | a_t \geqslant a_{(i)}\}$ and we can relabel the indices and replace $a_{(i)} = j \in \mathbf{N}$. Then we can write

$$C^\mu(a_1, \ldots, a_n) = \sum \left\{ \left(a_{(i)} - a_{(i-1)}\right) \mu(A_j) \,|\, j \in \mathbf{N}, j = a_{(i)} > a_{(i-1)}, \text{ for some } i \in \{1, \ldots, n\} \right\},$$

with $A_j = \{t \in \{1, \ldots, n\} | a_t \geqslant j\}$ when $j \in \mathbf{N}$.

Note that if $j = a_{(i)} > a_{(i-1)} = j'$, then $A_j = A_{j-1} = \ldots = A_{j'+1}$.

Therefore when $j \in \mathbf{N}$, $j = a_{(i)} > a_{(i-1)}$, and $i \in \{1, \ldots, n\}$, one has $a_{(i)} - a_{(i-1)} = |\{t \in \mathbf{N} | A_j = A_t\}|$.

From these considerations, the thesis follows with the appropriate relabelling of elements that appear in the formula. □

By inspection of the formula in Proposition 4.1, the procedure for computing the Choquet integral on vectors from $\mathbf{N}^n$ described in Algorithm 1 can be readily determined.

The next example illustrates the computations required to implement the alternative procedure presented above in Algorithm 1.

Example 4.2. *Suppose* $n = 3$*, hence* $X = \{1, 2, 3\}$*. Here* $1, 2, 3$ *represent properties or attributes. We want to apply the formula proven in Proposition 4.1 when their importances are such that* $\mu(\{1\}) = \mu(\{2\}) = 0.2$*,* $\mu(\{3\}) = 0.25$*,* $\mu(\{1, 2\}) = 0.7$*,* $\mu(\{2, 3\}) = \mu(\{1, 3\}) = 0.4$*, and* $\mu(\{1, 2, 3\}) = 1$ *(i.e., the capacity is normalized).*

We suppose $N = 10$*. And we need to compute the value of the Choquet integral at* $\boldsymbol{a} = (4, 8, 7)$*.*

Definition 3.2 produces $C^\mu(4, 8, 7) = 4 \cdot \mu(\{1, 2, 3\}) + 3 \cdot \mu(\{2, 3\}) + 1 \cdot \mu(\{2\}) = 5.4$*.*

Algorithm 1: Computing the Choquet integral on $\mathbf{N} = \{0, 1, 2, \ldots, N\}$

Input: A capacity μ on $X = \{1, \ldots, n\}$.
A vector $\mathbf{a} = (a_1, \ldots, a_n) \in \mathbf{N}^n$.

1: Compute $A_i = \{j \in X | a_j \geqslant i\}$ for each $i = 1, \ldots, N$.
This step produces a list of N possibly repeated subsets of X, namely, $(A_1, \ldots, A_N)$.
Let k be the number of distinct subsets in this list.

2: Define $(X_1, \ldots, X_k)$ and $(v_1, \ldots, v_k)$ such that:
$(X_1, \ldots, X_k)$ contains all the subsets in $(A_1, \ldots, A_N)$ without repetition, and
$(v_1, \ldots, v_k)$ is such that v_i is the number of times that X_i appears in $(A_1, \ldots, A_N)$.

Output: $C^\mu(\mathbf{a}) = \sum_{i=1}^{k} v_i \cdot \mu(X_i)$.

To apply either Proposition 4.1 or Algorithm 1 we need the following elements:

• $X_1 = \{1, 2, 3\} = A_4 = \{j \in X = \{1, 2, 3\} | a_j \geqslant 4\} = A_3 = \{j \in X | a_j \geqslant 3\} = A_2 = \{j \in X | a_j \geqslant 2\} = A_1 = \{j \in X | a_j \geqslant 1\}$ *hence* $v_1 = 4$.

• $X_2 = \{2, 3\} = A_7 = \{j \in X = \{1, 2, 3\} | a_j \geqslant 7\} = A_6 = \{j \in X | a_j \geqslant 6\} = A_5 = \{j \in X | a_j \geqslant 5\}$ *hence* $v_2 = 3$.

• $X_3 = \{2\} = A_8 = \{j \in X = \{1, 2, 3\} | a_j \geqslant 8\}$ hence $v_3 = 1$.

Therefore, Proposition 4.1 produces the same computations leading to

$$C^\mu(4, 8, 7) = 4 \cdot \mu(\{1, 2, 3\}) + 3 \cdot \mu(\{2, 3\}) + 1 \cdot \mu(\{2\}) = 5.4.$$

4.2 *Mathematica* code

This section discusses the implementation of Algorithm 1 with *Mathematica*. We shall present the notebook that computes the result for the data in Example 4.2.

In fact, we present the *Mathematica* code that solves this exercise using a variation of Algorithm 1. In this variant, we use a vector containing the evaluations of the capacity over all subsets of X, and also a vector with the same dimension that at each component, it has a zero when the position is associated with a subset not in the list $(A_1, \ldots, A_n)$, and it has v_i when the subset is X_i. It is freely available at Wolfram Community: https://community.wolfram.com/groups/-/m/t/3129284.

```
ClearAll;
X = {1, 2, 3};
subsets = Subsets[X]; (* Defines all the subsets of attributes *)
(* Insert the capacity on {1,2,3} *)
 fuzzyMeasure = ConstantArray[0, 2^Length[X]];
 fuzzyMeasure[[1]] = 0; (* value of capacity at empty set *)
 fuzzyMeasure[[2]] = 0.2; (* value of capacity at {1} *)
 fuzzyMeasure[[3]] = 0.2; (* value of capacity at {2} *)
 fuzzyMeasure[[4]] = 0.25; (* value of capacity at {3} *)
 fuzzyMeasure[[5]] = 0.7; (* value of capacity at {12} *)
 fuzzyMeasure[[6]] = 0.4; (* value of capacity at {13} *)
 fuzzyMeasure[[7]] = 0.4; (* value of capacity at {23} *)
 fuzzyMeasure[[8]] = 1; (* value of capacity at X *)
(* Now we insert the vector whose evaluations we want to compute *)
 mylist = {4, 8, 7};
(* Below we generate a vector with 2^3 components -- 3 is the cardinality of X *)
 vector = ConstantArray[0, 2^Length[X]];
 (* Now we loop over all subsets of X and set the corresponding component of the vector to v_i if it is one of X_i *)
 Do[pos = Position[mylist, x_ /; x >= k, 1];
   Print[Union[Flatten[Position[mylist, x_ /; x >= k]]]];
   If[MemberQ[subsets, Union[Flatten[Position[mylist, x_ /; x >= k]]]],
       vector[[Position[subsets,Union[Flatten[Position[mylist, x_ /; x >= k]]]][[1, 1]]]] +=1],
 {k, 1, 10}];
 fuzzyMeasure.vector (* The output *)
```

Listing 1: A *Mathematica* notebook for the calculation of the Choquet integral in a case with 3 attributes, in the framework of Section 4.1.

The next section presents three subsections with respective applications of the Choquet integral on $\{0, 1, 2, \ldots, N\}$ to the field of N-soft set theory. Section 6 below uses these tools for the purpose of decision making from multi-expert N-soft set inputs.

5 Applications of Choquet operators on $\{0, 1, 2, \ldots, N\}$

Subsets of a set U are characterized by their respective characteristic or indicator functions. Soft sets over U extend subsets of U by taking several characteristic functions simultaneously. These characteristic functions are linked to the properties that are relevant to define the alternatives. Fatimah *et al.* [23] defined N-soft sets to enhance the informational ability of soft sets in a realistic manner. In finite environments, any N-soft set can be expressed by Table 1, which contains figures from $\{0, 1, \ldots, N-1\}$. In the case of a soft set, Table 1 only has 0's and 1's.

The next sections explain in which contexts and how we can take advantage of Choquet integrals on $\{0, 1, 2, \ldots, N-1\}$ to improve the toolbox for N-soft set theory. Afterwards, these tools will be implemented to refine decision making in this setting (cf., section 6).

5.1 Application 1: Scores of N-soft sets in the presence of interactions among attributes

Scores produce numerical evaluations of the alternatives in U based on the information provided by Table 1. Section 3.2 explained that the soft set literature produced some basic scores that were extended to N-soft sets in [23]. Alternatives with higher scores are preferred over alternatives whose scores are smaller. Both the extended weighted choice value (EWCV) and the OWA score proposed in Equation (4) uses further information, namely, a vector of weights $(\omega_1, \ldots, \omega_n)$ associated with the properties in X. The utilization of OWA scores should be limited to cases where all attributes are equally valuable. In both cases, it is implicit that the attributes are independent.

However, in multi-attribute decision making or MCDA, it is important to give adequate treatment to the cases where there are synergies or redundancies among attributes with varying influence on the assessment. As shown above, capacities should replace vectors of weights (which define EWCVs and OWA-based scores), and Choquet integrals should take the place of (ordered) weighted average means.

In conclusion, the following new score for N-soft sets on X can be defined that extends both EWCVs and OWA scores:

Definition 5.1. *Let $\mu : 2^X \longrightarrow [0, 1]$ be a capacity defined on a set of attributes X. Then the μ-Choquet score of o_i defined by Table 1 is $S^\mu(o_i) = C^\mu(r_{i1}, \ldots, r_{in})$.*

When the capacity μ is symmetric, this score is an OWA score because the μ-Choquet integral becomes an OWA operator. And in case that the capacity is additive, then this score becomes the EWCV with weights $\omega_i = \mu(\{x_i\})$.

Our next example shows the application of this proxy for the valuation of alternatives defined by N-soft sets:

Example 5.2. *Let us revisit the capacity defined in Example 4.2 (here we identify 1 with x_1, 2 with x_2, and 3 with x_3). We interpret that there is synergy between properties x_1 and x_2, but there are redundancies between both x_1 and x_3, and x_2 and x_3. We shall rank the alternatives whose evaluations are shown in Table 2.*

Table 2: The tabular form of the 11-soft set on $U = \{o_1, o_2, o_3\}$ with properties $X = \{x_1, x_2, x_3\}$ studied in Example 5.2.

	x_1	x_2	x_3
o_1	4	8	7
o_2	10	3	6
o_3	10	6	3

We are ready to apply Definition 5.1 to calculate the μ-Choquet scores of the alternatives corresponding to this situation:

- $S^\mu(o_1) = C^\mu(4, 8, 7) = 4 \cdot \mu(\{1, 2, 3\}) + 3 \cdot \mu(\{2, 3\}) + 1 \cdot \mu(\{2\}) = 4 \cdot 1 + 3 \cdot 0.4 + 0.2 = 5.4,$
- $S^\mu(o_2) = C^\mu(10, 3, 6) = 3 \cdot \mu(\{1, 2, 3\}) + 3 \cdot \mu(\{1, 3\}) + 4 \cdot \mu(\{1\}) = 3 \cdot 1 + 3 \cdot 0.4 + 4 \cdot 0.2 = 5,$
- $S^\mu(o_3) = C^\mu(10, 6, 3) = 3 \cdot \mu(\{1, 2, 3\}) + 3 \cdot \mu(\{1, 2\}) + 4 \cdot \mu(\{1\}) = 3 \cdot 1 + 3 \cdot 0.7 + 4 \cdot 0.2 = 5.9.$

With these proxies, we recommend the prioritization of $o_3 \succ o_1 \succ o_2$.

5.2 Application 2: Aggregation of N-soft sets in the presence of interactions among the experts' opinions

In this section, we consider the case of a group of k experts, $X = \{a_1, \ldots, a_k\}$, who give their opinions in the form described in section 3.2. Table 3 summarizes the information they provide. It consists of (F_1, A, N), ..., (F_k, A, N), where (F_i, A, N) is the N-soft set on U submitted by agent i, and A is the set of attributes that characterize the alternatives in U.

Table 3: The tabular form of k N-soft sets on $U = \{o_1, \ldots, o_p\}$ with a common set of attributes $A = \{x_1, \ldots, x_n\}$. Table i contains the information submitted by a_i (agent i).

Agent 1	x_1	......	x_n
o_1	r^1_{11}	......	r^1_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_p	r^1_{p1}	......	r^1_{pn}

......

Agent k	x_1	......	x_n
o_1	r^k_{11}	......	r^k_{1n}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_p	r^k_{p1}	......	r^k_{pn}

Alcantud *et al.* [7] pioneered the study of this problem. The solution proposed in their work relied on an expression based on the OWA operator. Two implicit assumptions were made, namely, the independence of the assessments given for the attributes and by the experts, and their equal value. For this reason, Alcantud *et al.* [7, section 5] raised the question of whether these assumptions can be removed. Indeed, here we continue to explore the possibility of combining N-soft sets when there is a dependency among the experts' evaluations, which are no longer required to be equally valuable.

In the current framework, a Choquet-based aggregator is recommended. The modeling when we have information about the relative importances of the agents' opinions, and also of all possible groups of agents' opinions, is in terms of a capacity μ on the set of agents X. Then, for each possible i and j, the experts' opinions about how alternative i is rated with respect to attribute j can be combined by the expression defined in section 4.1.

But this is not the end of the story if we aim to produce an aggregated N-soft set, because the result of these computations will not necessarily be one of the numbers in $\{0, 1, 2, \ldots, N-1\}$. Section 5.4 reports on the solution provided by Alcantud *et al.* [7, section 3.1]. Alternatively, since the result of the computations will always lie in the interval $[0, N-1]$ because the Choquet integral is compensative, two strategies come to mind easily in a more direct manner. One is pessimistic: the floor function is applied after the computation of $C^\mu(r^1_{ij}, \ldots, r^k_{ij})$. The other is optimistic, and then the ceiling function is employed after the computation of $C^\mu(r^1_{ij}, \ldots, r^k_{ij})$. Recall that when applied to a real number a, the floor function returns the greatest integer less than or equal to a, and the ceiling function returns the least integer greater than or equal to a. We show the respective outputs in Table 4.

Table 4: The respective tabular forms of the pessimistic and optimistic μ-Choquet aggregated N-soft sets computed from the data in Table 3, and the capacity μ defined on the set of agents X.

	x_1	...	x_n
o_1	$\lfloor C^\mu(r^1_{11}, \ldots, r^k_{11}) \rfloor$	...	$\lfloor C^\mu(r^1_{1n}, \ldots, r^k_{1n}) \rfloor$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_p	$\lfloor C^\mu(r^1_{p1}, \ldots, r^k_{p1}) \rfloor$	...	$\lfloor C^\mu(r^1_{pn}, \ldots, r^k_{pn}) \rfloor$

	x_1	...	x_n
o_1	$\lceil C^\mu(r^1_{11}, \ldots, r^k_{11}) \rceil$	...	$\lceil C^\mu(r^1_{1n}, \ldots, r^k_{1n}) \rceil$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_p	$\lceil C^\mu(r^1_{p1}, \ldots, r^k_{p1}) \rceil$	...	$\lceil C^\mu(r^1_{pn}, \ldots, r^k_{pn}) \rceil$

Algorithm 2 implements these ideas.

Now we present a streamlined example illustrating the utilization of this strategy for the aggregation of N-soft sets:

Example 5.3. *Consider the following capacity defined on $X = \{a_1, a_2, a_3\}$, that represents a set of three experts: $\mu(\{a_1\}) = \mu(\{a_2\}) = 0.2$, $\mu(\{a_3\}) = 0.25$, $\mu(\{a_1, a_2\}) = 0.7$, $\mu(\{a_2, a_3\}) = \mu(\{a_1, a_3\}) = 0.4$, and $\mu(\{a_1, a_2, a_3\}) = 1$.*

Thus similarly to the case of Example 5.2, with this assignment we capture the fact that there are redundancies between both a_1 and a_3, and a_2 and a_3 (for example, because these pairs of experts worked together previously), but there are synergies between the opinions of experts a_1 and a_2 (for example, because their skills complement each other).

The experts are asked to rate two alternatives, $U = \{o_1, o_2\}$, on two attributes, namely, $A = \{x_1, x_2\}$. They can use the ordered degrees $\{0, 1, \ldots, 10\}$ hence $N = 11$. The assessments provided by them are given in Table 5.

Algorithm 2: Computing an aggregate N-soft set associated with Table 3

Input: A list of N-soft sets on $U=\{o_1,\dots,o_p\}$ with a common set of attributes $A=\{x_1,\dots,x_n\}$. Their fomal structure is as in Table 3.
A capacity μ on $X=\{a_1,\dots,a_k\}$, the set of experts or agents.
Pessimistic or optimistic attitude.

1: **for** $i=1$ to p **do**
2:   **for** $j=1$ to n **do**
3:     Compute $v_{ij}=C^{\mu}(r^1_{ij},\dots,r^k_{ij})$.
4:     **if** we take the pessimistic attitude **then**
5:       $a_{ij}=\lfloor v_{ij}\rfloor$
6:     **else**
7:       $a_{ij}=\lceil v_{ij}\rceil$
8:     **end if**
9:   **end for**
10: **end for**

Output: μ-Choquet aggregated N-soft set defined by a_{ij}, for each i and j.

Table 5: Tabular form of the 11-soft sets on $U=\{o_1,o_2\}$ with attributes $A=\{x_1,x_2\}$ studied in Example 5.3.

Expert 1	x_1	x_2
o_1	3	10
o_2	4	7

Expert 2	x_1	x_2
o_1	5	3
o_2	8	6

Expert 3	x_1	x_2
o_1	4	6
o_2	7	3

We shall aggregate these 11-soft sets with the information available about the agents' relative importances and interactions. To use Algorithm 2, first we apply the μ-Choquet integral to the table cell by cell. This produces the following values for the v_{ij}'s in Step 3:

- $v_{11}=C^{\mu}(3,5,4)=3\cdot\mu(\{a_1,a_2,a_3\})+1\cdot\mu(\{a_2,a_3\})+1\cdot\mu(\{a_2\})=3\cdot 1+0.4+0.2=3.6$,
- $v_{12}=C^{\mu}(10,3,6)=3\cdot\mu(\{a_1,a_2,a_3\})+3\cdot\mu(\{a_1,a_3\})+4\cdot\mu(\{a_1\})=3\cdot 1+3\cdot 0.4+4\cdot 0.2=5$,
- $v_{21}=C^{\mu}(4,8,7)=4\cdot\mu(\{a_1,a_2,a_3\})+3\cdot\mu(\{a_2,a_3\})+1\cdot\mu(\{a_2\})=4\cdot 1+3\cdot 0.4+0.2=5.4$,
- $v_{22}=C^{\mu}(7,6,3)=3\cdot\mu(\{a_1,a_2,a_3\})+3\cdot\mu(\{a_1,a_2\})+1\cdot\mu(\{a_1\})=3\cdot 1+3\cdot 0.7+0.2=5.3$.

Finally, these figures are used to produce the two aggregate 11-soft sets (one with a pessimistic attitude, the other with an optimistic attitude) presented in Table 6.

Table 6: Tabular form of the aggregate 11-soft sets produced in Example 5.3 using Algorithm 2.

Pessimistic	x_1	x_2
o_1	3	5
o_2	5	5

Optimistic	x_1	x_2
o_1	4	5
o_2	6	6

5.3 Scores of alternatives defined from multi-source information

Now we turn our attention to another issue inspired by the framework of section 5.2. From information about a set of alternatives in the form of Table 3, Alcantud *et al.* [7] defined a WAOWA score of the alternatives that extends the single-expert rationale of EWCVs to multi-expert cases. This idea unifies scores for N-soft sets with the help of OWA aggregation operators:

Definition 5.4. [7] *When* $\boldsymbol{w} = (w_1, \ldots, w_k) \in [0,1]^k$ *and* $\omega = (\omega_1, \ldots, \omega_n) \in [0,1]^n$ *are weighting vectors such that* $\sum_{i=1}^k w_i = 1$ *and* $\sum_{i=1}^n \omega_i = 1$, *the WAOWA score of* $o_j \in U$ *defined by Table 3 is*

$$\Upsilon^{\boldsymbol{w},\omega}(o_j) = \sum_{i=1}^{n} \omega_i \cdot F^{\boldsymbol{w}}(r_{ji}^1, \ldots, r_{ji}^k). \tag{6}$$

Definition 5.4 acts in the following manner to evaluate alternative o_j. Firstly, o_j is assessed for every attribute x_i using an OWA operator with weights $\mathbf{w}$. This aggregator merges the opinions expressed by each agent on this pair alternative-attributes. It is therefore implicit that the opinions of the experts are all equally important. Afterwards the values computed for each attribute are combined by a weighted arithmetic average that produces the final evaluation of o_j. The latter operation is inspired by EWCV defined by Equation (3), which is a particular case: note that if $k = 1$, i.e., if there is one agent only, then Equation (6) produces the EWCV of o_j associated with ω.

In the current section we extend this score so that it can operate in situations of different values of the experts' opinions, and also take into account the interactions among the individual expertises of the agents involved in the evaluation. In this way we can define a novel WAChoquet score for the assessment of alternatives defined by various N-soft sets.

Definition 5.5. *When* $\mu : 2^X \longrightarrow [0,1]$ *is a capacity on the set* $X = \{a_1, \ldots, a_k\}$ *of experts, and* $\omega = (\omega_1, \ldots, \omega_n) \in [0,1]^n$ *is a weighting vector (i.e.,* $\sum_{i=1}^n \omega_i = 1$*), the WAChoquet score of* $o_j \in U$ *defined by Table 3 is*

$$C^{\mu,\omega}(o_j) = \sum_{i=1}^{n} \omega_i \cdot C^{\mu}(r_{ji}^1, \ldots, r_{ji}^k). \tag{7}$$

Of course, when the fuzzy measure is symmetric, the WAChoquet score becomes a WAOWA score. And when there is one agent only, then it becomes the μ-Choquet score presented in Definition 5.1.

The next example explains the utilization of the WAChoquet score on an N-soft set:

Example 5.6. *Reconsider the case of Example 5.3. In addition to the information provided there, let us assume that we know that the second attribute* x_2 *is twice as important as the first one, therefore we fix* $\omega = (\frac{1}{3}, \frac{2}{3}) \in [0,1]^2$.

Now we are ready to compute the WAChoquet score of the two alternatives:

- $C^{\mu,\omega}(o_1) = \frac{1}{3}C^{\mu}(3,5,4) + \frac{2}{3}C^{\mu}(10,3,6) = \frac{1}{3}3.6 + \frac{2}{3}5 \approx 4.53$,
- $C^{\mu,\omega}(o_2) = \frac{1}{3}C^{\mu}(4,8,7) + \frac{2}{3}C^{\mu}(7,6,3) = \frac{1}{3}5.4 + \frac{2}{3}5.3 \approx 5.33$.

5.4 Discussion

N-soft set theory has proven to be an effective tool for managing the uncertainty associated with multivariate information in a crisp format. Current research indicates that aggregation within N-soft set theory offers a broader scope of practical applications compared to traditional soft sets.

In a single-agent situation, section 5.1 has shown how the family of μ-Choquet scores generalizes both OWA based scores (introduced in section 3.2) and EWCVs. They enable us to evaluate the alternatives when the assessments made for each attribute have unequal importance, and we can measure the synergies and/or redundancies among them.

Moving to a multi-agent environment, Alcantud *et al.* [7] have imported OWA operators into this framework. Alcantud *et al.* [7, section 3.1] guarantee that the OWA case produces another N-soft set when we use distributive weighting vectors. It is therefore possible to take advantage of this tool to produce a natural variation of Algorithm 2 in section 5.2.

The elements considered in section 5.3 can be applied directly to N-soft set based multi-agent decision making: alternatives with higher scores are preferred over alternatives whose scores are smaller. For example, the computation of scores in Example 5.6 recommends selecting the second alternative o_2. The next section explores this and other decision-making mechanisms.

6 Decision making from multi-expert N-soft set inputs

This section presumes a framework with a set of alternatives U and (F_1, X, N), …, (F_k, X, N), so we have a group of k agents each submitting an N-soft set on U. A common set of properties X characterizes the alternatives in U.

Thus our input is k tabular forms (one for each N-soft set submitted by an agent). They are given in Table 3, which summarizes every possible $r_{ij}^{a} = F_a(x_j)(o_i) \in G$ in this input.

Alcantud *et al.* [7] first studied this problem and contributed with two decision making methodologies, called Algorithms 1 and 2 in that article. Algorithm 2 uses the WAOWA score to evaluate and rank the alternatives directly. Algorithm 1 takes advantage of an OWA based aggregation operator. A different approach to this multi-agent decision making problem led to the proposal in Alcantud *e*t al. [8]. The main difference with respect to Algorithm 1 in Alcantud *e*t al. [7] is that such Algorithm 1 uses a *merge-then-decide* strategy where OWA based aggregation operators act at the first step and then EWCVs decide, whereas the position in Alcantud *e*t al. [8] is the opposite (a *decide-then-merge* strategy that benefits from voting theory to merge individual score based choices).

The tools that we have developed help us to improve these three procedures. Let us give more details about these facts in separate sections, which will serve us to perform a comparative study too. Afterwards we present a systematic summary of techniques with advantages and disadvantages in section 6.4.

6.1 Score based decisions

WAOWA scores are a simplified version of WAChoquet scores (v., Definition 5.5), which are therefore more reliable when we know that synergies or redundancies among agents with different expertises exist, as argued above. For this reason the application of the WAChoquet score improves the performance of the multi-expert decision strategy presented in Alcantud *et al.* [7, Algorithm 2], i.e., of WAOWA based decisions, in the presence of richer information about the agents.

The next comparative example will help us to see the difference and to understand the reasons for the superiority of the new WAChoquet score based decision making strategy:

Example 6.1. *Reconsider the case of Examples 5.3 and 5.6. If we compute the WAOWA score of the alternatives with the weights* $\boldsymbol{w} = (1, 0, 0)$*, we express that for each fixed alternative and attribute, the highest evaluation given by one agent is the aggregate opinion, i.e.,* $F^{\boldsymbol{w}}$ *is the maximum operator. For comparison, note that we are discarding the information about the skills of the agents used in Example 5.6. Then with the current information we obtain:*

- $\Upsilon^{\boldsymbol{w},\omega}(o_1) = \frac{1}{3}F^{\boldsymbol{w}}(3,5,4) + \frac{2}{3}F^{\boldsymbol{w}}(10,3,6) = \frac{1}{3}5 + \frac{2}{3}10 = \frac{25}{3}$,
- $\Upsilon^{\boldsymbol{w},\omega}(o_2) = \frac{1}{3}F^{\boldsymbol{w}}(4,8,7) + \frac{2}{3}F^{\boldsymbol{w}}(7,6,3) = \frac{1}{3}8 + \frac{2}{3}7 = \frac{22}{3}$.

The recommendation using this WAOWA score is that alternative o_1 *should be chosen. Observe that the more complete information about the skills of the agents used in Example 5.6 recommends to select the second alternative* o_2 *instead.*

It is appropriate the highlight that Example 5.2 recalled the utilization of scores in single-agent decision making with N-soft sets.

6.2 Decisions using a *merge-then-decide* strategy

Alcantud *et al.* [7] gave other procedures to recommend an option in the framework posed by this section. Actually, two other classes of flexible procedures were stated to tackle the multi-expert problem herein formulated. Both combine multi-agent data, producing either an aggregate N-soft set or a hesitant N-soft set. Afterwards, they utilize the corresponding decision-making strategy for this result, using solutions from either Fatimah *e*t al. [23] or Akram *e*t al. [2]. before these articles, Çağman *et al.* [17] used a related approach.

In previous sections, tools have been developed that improve the first class of procedures.

Specifically, the application of the WAChoquet score (cf., section 6.1) bears comparison with a similar but different calculation that consists of two steps: the application of Algorithm 2 (which accounts for the *merge* step of the strategy), plus the computation of the EWCV of the resulting N-soft set (which accounts for the *decide* step of the strategy).

For illustration, if we use the information given in Example 5.6, then:

- The EWCVs produced from the pessimistic version of Algorithm 2 are

$$\frac{1}{3}3 + \frac{2}{3}5 = \frac{13}{3} \approx 4.33 \text{ for } o_1, \text{ and } \frac{1}{3}5 + \frac{2}{3}5 = 5 \text{ for } o_2.$$

- The EWCVs produced from the optimistic version of Algorithm 2 are

$$\frac{1}{3}4 + \frac{2}{3}5 = \frac{15}{3} \approx 4.67 \text{ for } o_1, \text{ and } \frac{1}{3}6 + \frac{2}{3}6 = 6 \text{ for } o_2.$$

Both *merge-then-decide* approaches recommend to select the second alternative o_2, in line with the solution proposed in Example 5.6.

6.3 Decisions using a *decide-then-merge* strategy

Alcantud *et al.* [8] explained the ethos of this alternative principle. Namely, in the presence of a multiplicity of N-soft sets on the set of alternatives (the set of characteristics is the same), we can enforce individual rankings using all available information about the value of the attributes (*decide* step), and then produce a final ranking of the alternatives using methodologies from voting theory (*merge* step). Specifically, EWCVs or OWA scores can be utilized at the *decision* step. And for the *merge* step, Alcantud *et al.* [8, section 2.3] referred to adapted versions of the Borda rule, approval voting, and evaluative voting. For example: in the case of the Borda rule, every agent a computes $B_a(o_i)$ for each $o_i \in U$ (the Borda score of o_i for a) as the number of alternatives that are less valuable than o_i, minus the number of alternatives that are more valuable than it (in the opinion of a). This comparison is made with respect to a's individual's ranking. Thus an implicit assumption is made, namely, that even if each ranking is derived from a numerical assessment, the content of their evaluations is purely ordinal. Then a weighted arithmetic average of these numbers (either Borda scores or other proxies) produces the final score of each alternative. Here the weights are designed to capture the agents' relative importance.

To illustrate how the tools developed in section 5 can help us refine this technique, we reconsider the synthetic example in Alcantud *et al.* [8, section 3.3]. It concerns $A = \{a, b, c\}$, a group of agents who submit their opinions on $U = \{o_1, \dots, o_5\}$. They can resort to five common grades to evaluate them with respect to three attributes, hence $X = \{t_1, t_2, t_3\}$. Table 7 summarizes the information that they submit. We are asked to compute a reasonable ranking over U.

Table 7: The 5-soft sets in section 6.3, extracted from Alcantud *et al.* [8, section 3.3].

Agent a	t_1	t_2	t_3
o_1	4	1	2
o_2	1	3	3
o_3	3	1	2
o_4	2	3	0

Agent b	t_1	t_2	t_3
o_1	0	4	2
o_2	3	2	1
o_3	2	2	1
o_4	2	1	2

Agent c	t_1	t_2	t_3
o_1	3	3	2
o_2	2	1	4
o_3	1	2	2
o_4	3	1	2

In the example from Alcantud *et al.* [8, section 3.3], the opinions of the agents were considered equally relevant, and the weights associated with the alternatives were $w = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$. It was therefore assumed that the attributes were independent of each other, because the methodology presented in that paper could not make full utilization of synergies and redundancies among the properties. With the help of the corresponding EWCVs, the agents respectively propose the rankings $o_2 \succ^a o_1 \succ^a o_3 \succ^a o_4$, $o_1 \succ^b o_2 \sim^b o_4 \succ^b o_3$, and $o_2 \succ^c o_1 \succ^c o_4 \succ^c o_3$. The final Borda scores obtained from this information are $B(o_1) = \frac{5}{3}$, $B(o_2) = 2$, $B(o_3) = -\frac{7}{3}$, and $B(o_4) = -\frac{4}{3}$, hence the final recommendation to select the second alternative because $o_2 \succ o_1 \succ o_4 \succ o_3$.

Now Choquet scores allow us to improve EWCVs and OWA scores at the *decide* step when we can assess synergies and/or redundancies among the attributes. Hence, suppose that we can refine the information about the attributes with the information contained in the following capacity: $\mu(\{t_1\}) = \mu(\{t_2\}) = 0.25$, $\mu(\{t_3\}) = 0.5$, $\mu(\{t_1, t_2\}) = \mu(\{t_2, t_3\}) = 0.75$, $\mu(\{t_1, t_3\}) = 0.5$, and $\mu(\{t_1, t_2, t_3\}) = 1$. Table 8 presents the μ-Choquet scores of the alternatives defined above with respect to this capacity (cf., Definition 5.1).

Table 8: The μ-Choquet scores computed in section 6.3.

	Agent a	Agent b	Agent c
o_1	2	2	2.75
o_2	2.5	2	2.5
o_3	1.75	1.75	1.75
o_4	1.75	1.5	1.75

We are ready to combine the individual ranked recommendations of the alternatives that these ratings produce, to obtain their final Borda scores:

$B(o_1) = \frac{1}{3}B_a(o_1) + \frac{1}{3}B_b(o_1) + \frac{1}{3}B_c(o_1) = \frac{1}{3}(1+2+3) = 1$,
$B(o_2) = \frac{1}{3}B_a(o_2) + \frac{1}{3}B_b(o_2) + \frac{1}{3}B_c(o_2) = \frac{1}{3}(3+2+1) = 1$,
$B(o_3) = \frac{1}{3}B_a(o_3) + \frac{1}{3}B_b(o_3) + \frac{1}{3}B_c(o_3) = \frac{1}{3}(-2-1-2) = -\frac{5}{3}$, and
$B(o_4) = \frac{1}{3}B_a(o_4) + \frac{1}{3}B_b(o_4) + \frac{1}{3}B_c(o_4) = \frac{1}{3}(-2-3-2) = -\frac{7}{3}$.

The ranking of U that we obtain from these figures is $o_1 \sim o_2 \succ o_3 \succ o_4$. Therefore, the recommendation is that both o_1 and o_2 are equally good, and they are strictly better than o_3 and o_4.

We confirm that with respect to the methodology presented in Alcantud et al. [8] that we have recalled, a more sophisticated utilization of information about the structure of the problem produces an alternative solution. Therefore we can safely conclude that this outcome is better grounded.

6.4 A summary of techniques

Table 9 summarizes the techniques for multi-expert N-soft set-based decisions that we have either proposed in this article or recalled from existing literature. The table serves as a reminder of the main characteristic of each proposal, and establishes a methodological classification of solutions to this problem. By doing so, it informs us of the advantages and disadvantages of each approach, too. Table 9 can therefore act as a roadmap of solutions to multi-expert N-soft set-based decisions. We show such a roadmap in a flowchart in Figure 2. For better understanding, abbreviations (M1), . . . , (M7) refer to the methodologies described in Table 9.

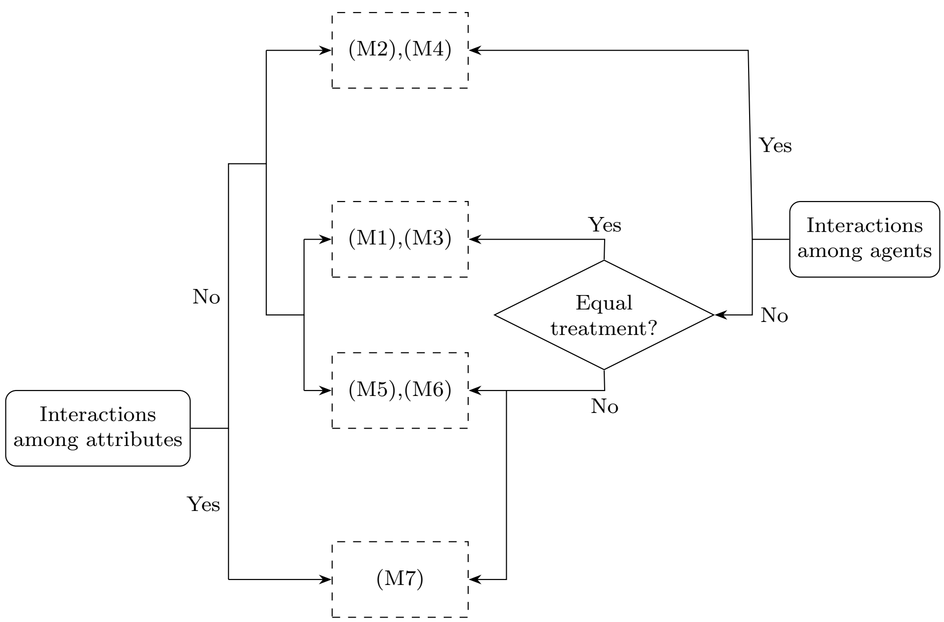

Figure 2: Flowchart with the characteristics of the methodologies summarized in Table 9.

6.5 Advantages and limitations

Table 9 has condensed the main advantages of the approaches presented in this section for decisions with a multiplicity of N-soft sets. They enable us to not only attach different values to the various sources (e.g., agents), but also specify to what extent they are redundant or complementary in a precise manner. This ability guarantees a large degree of flexibility and adaptability. Another remarkable advantage is that now practitioners can use the proposed method with large datasets from real-life situations with the help of well-developed Python packages by Beliakov *et al.* [13, Appendix A]. This computational assistance for all stages of Choquet integration may boost the applicability of the strategy proposed in this work in future studies.

Although we can resort to procedures that consider the existence of interactions among the characteristics of the options, a limitation is that we cannot simultaneously use redundant/complementary opinions. This setback poses an

Table 9: Summary table of solutions for multi-expert N-soft set based decision making.

Approach	Method(s) and tool(s)	References	Characteristics
Score based	**(M1)** WAOWA	Alcantud *et al.* [7, Algorithm 2]	Agents are equally treated Properties may have unequal importance, no measurement of their interactions
	(M2) WAChoquet	This paper	Agents' importances may be unequal, use of measurement of interactions When capacity is symmetric, it reduces to WAOWA
Merge-then-decide	**(M3)** Aggregate by HNSS + techniques from Akram *et al.* [2]	Alcantud *et al.* [7, Algorithm 3]	Disadvantage: we move outside the benchmark framework Agents are equally treated. Properties may have unequal importances, their interactions are disregarded
	(M4) Algorithm 2 produces N-soft set, then WAChoquet	This paper	Agents' importances may be unequal, use of measurement of their interactions Properties may have unequal importances, their interactions are disregarded Flexibility: optimistic/pessimistic positions
Decide-then-merge	**(M5)** EWCV or T-weighted choice value *(decide)* + weighted arithmetic average of Borda scores (or approval voting, evaluative voting, etc.)	Alcantud *et al.* [8, sects 3.1, 3.2]	Agents' importances may be unequal, their interactions are disregarded *(merge)* Properties may have unequal importances, their interactions are disregarded We only retain the ordinal information contained in individual evaluations
	(M6) OWA scores *(decide)* + one of the methods above *(merge)*	This paper	Properties are treated symmetrically, agents' importances as above
	(M7) Choquet scores *(decide)* + one of the methods above *(merge)*	This paper	We can use interactions among properties, agents' importances as above

interesting problem for further investigation. Another disadvantage (that is common to all Choquet-based procedures) is that the elicitation (also called learning, or identification) of capacities is a difficult task [13]. For this reason, in the future it may be interesting to study explicit forms such as k-interactive capacities, which are comprehensively studied in Beliakov *et al.* [13, Section 7.7].

7 Conclusions and future research lines

After Alcantud *et al.* [7] gave the first *multi-agent* approximation to decision making for N-soft set information, only Alcantud *et al.* [8] contributed to this topic with a redesigned approach. Here, we have taken advantage of the main features of the Choquet integral to improve the performance of the methodologies proposed in those papers. The enhancements do not come at the cost of a much larger computational burden, because an advantage of the current framework is that we have developed a specific formula that greatly simplifies the calculations involved in the corresponding Choquet-based algorithms. The upgrades make more accurate assessments thanks to the full utilization of information about the properties and/or the experts who submit their opinions. Examples have illustrated these new procedures and how they compare to more basic methods.

With this perspective, we have been able to organize the methodological approaches to multi-expert N-soft set-based decisions presented in this and previous articles. We have taken on this task in section 6.4. Particularly, Table 9 facilitates qualitative comparative analyses thanks to an exhaustive discussion of the advantages and disadvantages of each approach. The practitioner can better adapt the strategy of solution to the characteristics of the problem with the help of this summary presentation, and Figure 2 gives a clear roadmap for this purpose.

These ideas should pave the way for the utilization of Choquet integrals and other generalizations of this operator in the inspection of decisions when the evaluations of the alternatives take the form of models more general than N-soft sets. This includes the extensions that we discussed in Section 2. We highlight that in such a case, the computational advantages associated with the novel formula presented in section 4 would be lost. In addition, we should bear in mind that more general forms of this fuzzy integral can be applied for similar purposes in the future, inclusive of d-Choquet integrals [16], d_G-Choquet integrals [35], CC-Choquet integrals [30], and many others [33].

A different avenue for future research emerges from concerns about data privacy. The discussion initiated in Torra [38] stimulates the investigation of privacy-preserving mechanisms for the disclosure of Choquet-type scores in the context of N-soft sets or generalized models. The debate can be extended to more general expressions for the computation of scores inspired by the Choquet integral.

Acknowledgement

The authors wish to express their appreciation for several excellent suggestions for improvements in this paper, made by the referees.

Author contributions

José Carlos R. Alcantud: Conceptualization; Visualization; Writing - original draft; Methodology. Muhammad Akram: Conceptualization; Revision of the manuscript; Methodology; Investigation; Validation. Gustavo Santos-García: Conceptualization; Revision of the manuscript; Methodology. Weiping Ding: Supervision; Formal analysis; Funding acquisition; Revision of the manuscript; Project administration.

Code availability

All code for data analysis associated with the current submission is available at this publication. It is also available at Wolfram Community, Staff Picks, https://community.wolfram.com/groups/-/m/t/3129284.

Conflict of interest/Competing interests

All authors declare that they have no conflict of interest or competing interests.

Consent for publication

All authors agree to submit to this journal.

Data availability

Data sharing does not apply to this article as no new data were created or analyzed in this study.

Ethics approval and consent to participate

This study does not involve research on humans or animals.

Funding

Alcantud acknowledges financial support of the Department of Education of the Junta de Castilla y León and FEDER Funds (Reference: CLU-2025-2-03).

References

[1] M. E. M. Abdalla, A. Uzair, A. Ishtiaq, M. Tahir, M. Kamran, *Algebraic structures and practical implications of interval-valued fermatean neutrosophic super hypersoft sets in healthcare*, Spectrum of Operational Research, **2**(1) (2025), 199-218. https://doi.org/10.31181/sor21202523

[2] M. Akram, A. Adeel, J. C. R. Alcantud, *Group decision-making methods based on hesitant N-soft sets*, Expert Systems with Applications, **115** (2019), 95-105. https://doi.org/10.1016/j.eswa.2018.07.060

[3] M. Akram, A. Adeel, A. N. Al-Kenani, J. C. R. Alcantud, *Hesitant fuzzy N-soft ELECTRE-II model: A new framework for decision-making*, Neural Computing and Applications, **33**(13) (2021), 7505-7520. https://doi.org/10.1007/s00521-020-05498-y

[4] M. Akram, G. Ali, J. C. R. Alcantud, F. Fatimah, *Parameter reductions in N-soft sets and their applications in decision-making*, Expert Systems, **38**(1) (2021), e12601. https://doi.org/10.1111/exsy.12601

[5] M. Akram, M. Sultan, J. C. R. Alcantud, *An integrated ELECTRE method for selection of rehabilitation center with m-polar fuzzy N-soft information*, Artificial Intelligence in Medicine, **135** (2023), 102449. https://doi.org/10.1016/j.artmed.2022.102449

[6] J. C. R. Alcantud, A. Z. Khameneh, G. Santos-García, M. Akram, *A systematic literature review of soft set theory*, Neural Computing and Applications, **36** (2024), 8951-8975. https://doi.org/10.1007/s00521-024-09552-x

[7] J. C. R. Alcantud, G. Santos-García, M. Akram, *OWA aggregation operators and multi-agent decisions with N-soft sets*, Expert Systems with Applications, **203** (2022), 117430. https://doi.org/10.1016/j.eswa.2022.117430

[8] J. C. R. Alcantud, G. Santos-García, M. Akram, *A novel methodology for multi-agent decision-making based on N-soft sets*, Soft Computing, (2023). https://doi.org/10.1007/s00500-023-08522-0

[9] O. Aristondo, J. L. García-Lapresta, C. Lasso de la Vega, R. A. Marques Pereira, *Classical inequality indices, welfare and illfare functions, and the dual decomposition*, Fuzzy Sets and Systems, **228** (2013), 114-136. https://doi.org/10.1016/j.fss.2013.02.001

[10] G. Beliakov, *On random generation of supermodular capacities*, IEEE Transactions on Fuzzy Systems, **30**(1) (2022), 293-296. https://doi.org/10.1109/TFUZZ.2020.3036699

[11] G. Beliakov, T. Cao, V. Mak-Hau, *Aggregation of interacting criteria in land combat vehicle selection by using fuzzy measures*, IEEE Transactions on Fuzzy Systems, **30**(9) (2022), 3979–3989, https://doi.org/10.1109/TFUZZ.2021.3135972

[12] G. Beliakov, S. James, *Choquet integral-based measures of economic welfare and species diversity*, International Journal of Intelligent Systems, **37**(4) (2022), 2849–2867, https://doi.org/10.1002/int.22609

[13] G. Beliakov, S. James, J. Wu, *Choquet capacities and fuzzy integrals*, Springer Nature, Switzerland, (2026). https://doi.org/10.1007/978-3-031-97070-2

[14] A. Bilbao-Terol, V. Cañal Fernández, C. González-Pérez, *Evaluating energy security using choquet integral: Analysis in the southern E.U. countries*, Annals of Operations Research, **355** (2025), 721-763. https://doi.org/10.1007/s10479-023-05748-x

[15] S. Biswas, S. Bhattacharjee, B. Biswas, K. Mitra, N. Khawas, *An expert opinion-based soft computing framework for comparing nanotechnologies used in agriculture*, Spectrum of Operational Research, **4**(1) (2025), 1-39. https://doi.org/10.31181/sor4156

[16] H. Bustince, R. Mesiar, J. Fernández, M. Galar, D. Paternain, A. Altalhi, G. Dimuro, B. Bedregal, Z. Takáč, *d-Choquet integrals: Choquet integrals based on dissimilarities*, Fuzzy Sets and Systems, **414** (2021), 1-27. https://doi.org/10.1016/j.fss.2020.03.019

[17] N. Çağman, F. Çıtak, S. Enginoğlu, *Fuzzy parameterized fuzzy soft set theory and its applications*, Turkish Journal of Fuzzy Systems, **1**(1) (2010), 21-35.

[18] G. Choquet, *Theory of capacities*, Annales de l'institut Fourier, **5** (1954), 131-295. https://doi.org/10.5802/aif.53

[19] A. K. Das, C. Granados, *IFP-intuitionistic multi fuzzy N-soft set and its induced IFP-hesitant N-soft set in decision-making*, Journal of Ambient Intelligence and Humanized Computing, **14** (2023), 10143-10152. https://doi.org/10.1007/s12652-021-03677-w

[20] I. Demir, *N-soft mappings with application in medical diagnosis*, Mathematical Methods in the Applied Sciences, **44**(8) (2021), 7343-7358. urlhttps://doi.org/10.1002/mma.7266

[21] G. P. Dimuro, J. Fernández, B. Bedregal, R. Mesiar, J. A. Sanz, G. Lucca, H. Bustince, *The state-of-art of the generalizations of the choquet integral: From aggregation and pre-aggregation to ordered directionally monotone functions*, Information Fusion, **57** (2020), 27-43. https://doi.org/10.1016/j.inffus.2019.10.005

[22] Y. Even, E. Lehrer, *Decomposition-integral: Unifying choquet and the concave integrals*, Economic Theory, **56** (2014), 33-58. https://doi.org/10.1007/s00199-013-0780-0

[23] F. Fatimah, D. Rosadi, R. B. F. Hakim, J. C. R. Alcantud, *N-soft sets and their decision making algorithms*, Soft Computing, **22** (2018), 3829-3842. https://doi.org/10.1007/s00500-017-2838-6

[24] M. Grabisch, *A new algorithm for identifying fuzzy measures and its application to pattern recognition*, In Proceedings of 1995 IEEE International Conference on Fuzzy Systems, **1** (1995), 145-150. https://doi.org/10.1109/FUZZY.1995.409673

[25] M. Grabisch, C. Labreuche, *A decade of application of the Choquet and Sugeno integrals in multi-criteria decision aid*, Annals of Operations Research, **6** (2008), 144. https://doi.org/10.1007/s10288-007-0064-2

[26] H. Kamacı, *Introduction to N-soft algebraic structures*, Turkish Journal of Mathematics, **44**(6) (2020), 2356-2379. https://doi.org/10.3906/mat-1907-99

[27] H. Kamacı, S. Petchimuthu, *Bipolar N-soft set theory with applications*, Soft Computing, **24** (2020), 16727-16743. https://doi.org/10.1007/s00500-020-04968-8

[28] E. Korkmaz, M. Riaz, M. Deveci, S. Kadry, *A novel approach to fuzzy N-soft sets and its application for identifying and sanctioning cyber harassment on social media platforms*, Artificial Intelligence Review, **57** (2024), 1-14. https://doi.org/10.1007/s10462-023-10640-y

[29] R. Krishankumar, D. Pamucar, M. Deveci, M. Aggarwal, K. S. Ravichandran, *Assessment of renewable energy sources for smart cities' demand satisfaction using multi-hesitant fuzzy linguistic based choquet integral approach*, Renewable Energy, **189** (2022), 1428-1442. https://doi.org/10.1016/j.renene.2022.03.081

[30] G. Lucca, J. A. Sanz, G. P. Dimuro, B. Bedregal, M. J. Asiain, M. Elkano, H. Bustince, *CC-integrals: Choquet-like Copula-based aggregation functions and its application in fuzzy rule-based classification systems*, Knowledge-Based Systems, **119** (2017), 32-43. https://doi.org/10.1016/j.knosys.2016.12.004

[31] E. A. Ok, L. Zhou, *The choquet bargaining solutions*, Games and Economic Behavior, **33**(2) (2000), 249-264. https://doi.org/10.1006/game.1999.0778

[32] Z. Ontkovičová, V. Torra, *Computation of choquet integrals: Analytical approach for continuous functions*, Information Sciences, **679** (2024), 121105. https://doi.org/10.1016/j.ins.2024.121105

[33] J. Sartori, G. Lucca, T. Asmus, H. Santos, E. Borges, B. Bedregal, H. Bustince, G. P. Dimuro, *d-CC integrals: Generalizing CC-integrals by restricted dissimilarity functions with applications to fuzzy-rule based systems*, In Naldi, M. C. Bianchi, R. A. C., editors, Intelligent Systems, volume 14195, Springer Nature, Switzerland, (2023), 243-258. https://doi.org/10.1007/978-3-031-45368-7_16

[34] D. Schmeidler, *Integral representation without additivity*, Proceedings of the American Mathematical Society, **97**(2) (1986), 255-261. https://doi.org/10.2307/2046508

[35] Z. Takáč, M. Uriz, M. Galar, D. Paternain, H. Bustince, *Discrete IV dG-Choquet integrals with respect to admissible orders*, Fuzzy Sets and Systems, **441** (2022), 169-195. https://doi.org/10.1016/j.fss.2021.09.013

[36] A. Theerens, O. U. Lenz, C. Cornelis, *Choquet-based fuzzy rough sets*, International Journal of Approximate Reasoning, **146** (2022), 62-78. https://doi.org/10.1016/j.ijar.2022.04.006

[37] V. Torra, *ϒ-values: Power indices à la orness for non-additive measures*, IEEE Transactions on Fuzzy Systems, **32**(7) (2024), 4099-4108. https://doi.org/10.1109/TFUZZ.2024.3392268

[38] V. Torra, *Differentially private choquet integral: Extending mean, median, and order statistics*, International Journal of Information Security, **24** (2025), 68. https://doi.org/10.1007/s10207-025-00984-7

[39] V. Torra, Y. Narukawa, *Numerical integration for the choquet integral*, Information Fusion, **31** (2016), 137-145. https://doi.org/10.1016/j.inffus.2016.02.007

[40] E. Türkarslan, V. Torra, *Measure identification for the choquet integral: A python module*, International Journal of Computational Intelligence Systems, **15**(1) (2022), 89. https://doi.org/10.1007/s44196-022-00146-w

[41] J. Wang, X. Zhang, J. Dai, J. Zhan, *TI-fuzzy neighborhood measures and generalized choquet integrals for granular structure reduction and decision making*, Fuzzy Sets and Systems, **465** (2023), 108512. https://doi.org/10.1016/j.fss.2023.03.015

[42] Z. Xu, *Choquet integrals of weighted intuitionistic fuzzy information*, Information Sciences, **180**(5) (2010), 726-736. https://doi.org/10.1016/j.ins.2009.11.011

[43] R. R. Yager, *On ordered weighted averaging aggregation operators in multi-criteria decision making*, IEEE Transactions on Systems, Man and Cybernetics, **18**(1) (1988), 183-190. https://doi.org/10.1109/21.87068

[44] H. Zhang, T. Nan, Y. He, *q-Rung orthopair fuzzy N-soft aggregation operators and corresponding applications to multiple-attribute group decision making*, Soft Computing, **26**(13) (2022), 6087-6099. https://doi.org/10.1007/s00500-022-07126-4

[45] X. Zhang, J. Wang, J. Zhan, J. Dai, *Fuzzy measures and choquet integrals based on fuzzy covering rough sets*, IEEE Transactions on Fuzzy Systems, **30**(7) (2022), 2360-2374. https://doi.org/10.1109/TFUZZ.2021.3081916