Trans. Phenom. Nano Micro Scales, 1(2): 124-137, Summer – Autumn 2013
DOI: 10.7508/tpnms.2013.02.006

ORIGINAL RESEARCH PAPER

MHD Nanofluid Flow Analysis in a Semi-Porous Channel by a Combined Series Solution Method

R. Nouri[1], D. D. Ganji[1], M. Hatami[1,2,*]
1. Babol University of Technology, Department of Mechanical Engineering, Babol, Iran.
2. Esfarayen Faculty of Industrial and Engineering, Department of Mechanical Engineering, Esfarayen, North Khorasan, Iran

Abstract

In this paper, Least Square Method (LSM) and Differential Transformation Method (DTM) are used to solve the problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field. Due to existence some shortcomings in each method, a novel and efficient method named LS-DTM is introduced which omitted those defects and has an excellent agreement with numerical solution. In the present study, the effective thermal conductivity and viscosity of nanofluid are calculated by Maxwell–Garnetts (MG) and Brinkman models, respectively. The influence of the three dimensionless numbers: the nanofluid volume friction, Hartmann number and Reynolds number on non-dimensional velocity profile are considered. The results show that velocity boundary layer thickness decrease with increase of Reynolds number and nanoparticle volume friction and it increases as Hartmann number increases.

Keywords: *Least Square Method (LSM); Nanofluid; Semi-porous channel; Uniform magnetic*

1. Introduction

Most scientific problems in fluid mechanics and heat transfer problems are inherently nonlinear. All these problems and phenomena are modelled by ordinary or partial nonlinear differential equations. Most of these described physical and mechanical problems are with a system of coupled nonlinear differential equations. For an example heat transfer by natural convection which frequently occurs in many physical problems and engineering applications such as geothermal systems, heat exchangers, chemical catalytic reactors and nanofluid flow in a semi-porous channel has a system of coupled nonlinear differential equations for temperature or velocity distribution equations.

The flow problem in porous tubes or channels has been under considerable attention in recent years because of its various applications in biomedical engineering, for example, in the dialysis of blood in artificial kidney, in the flow of blood in the capillaries, in the flow in blood oxygenators as well as in many other engineering areas such as the design of filters, in transpiration cooling boundary layer control [1] and gaseous diffusion [2]. In 1953, Berman [3] described an exact solution of the Navier-Stokes equation for steady two-dimensional laminar flow of a viscous, incompressible fluid in a channel with parallel, rigid

*Corresponding author
Email Address: m.hatami2010@gmail.com

	Nomenclature		**Greek Symbols**
A^*, B^*	Constant parameter	υ	Kinematic viscosity
P	Fluid pressure	σ	Electrical conductivity
q	Mass transfer parameter	ε	Aspect ratio h/Lx
x_k	General coordinates	μ	Dynamic viscosity
f	Velocity function	υ	Kinematic viscosity
$\bar{k}$	Fluid thermal conductivity	ρ	Fluid density
n	Power law index in temperature distribution		
Re	Reynolds number		**Subscripts**
Ha	Hartmann number	∞	Condition at infinity
u,v	Dimensionless components velocity in x and y directions, respectively	nf	Nanofluid
u^*,v^*	Velocity components in x and y directions respectively	f	Base fluid
x, y	Dimensionless horizontal, vertical coordinates respectively	s	Nano-solid-particles
x^*, y^*	Distance in x,y directions parallel to the plates		

porous walls driven by uniform, steady suction or injection at the walls. This mass transfer is paramount in some industrial processes. More recently, Sheikholeslami et al. [4] analyzed the effects of a magnetic field on the nanofluid flow in a porous channel through weighted residual methods called Galerkin method. Nanofluid, which is a mixture of nano-sized particles (nanoparticles) suspended in a base fluid, is used to enhance the rate of heat transfer via its higher thermal conductivity compared to the base fluid. Soleimani et al. [5] studied natural convection heat transfer in a semi-annulus enclosure filled with nanofluid using the Control Volume based Finite Element Method. They found that the angle of turn has an important effect on the streamlines, isotherms and maximum or minimum values of local Nusselt number. Natural convection of a non-Newtonian copper-water nanofluid between two infinite parallel vertical flat plates is investigated by Domairry et al. [6]. They conclude that as the nanoparticle volume fraction increases, the momentum boundary layer thickness increases, whereas the thermal boundary layer thickness decreases. Sheikholeslami et al. [7] performed a numerical analysis for natural convection heat transfer of Cu-water nanofluid in a cold outer circular enclosure containing a hot inner sinusoidal circular cylinder in presence of horizontal magnetic field using the Control Volume based Finite Element Method.

There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square are examples of the WRMs. Stern and Rasmussen [8] used collocation method for solving a third order linear differential equation. Vaferi et al. [9] have studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami [10] used Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently Least square method is introduced by A. Aziz and M.N. Bouaziz [11] and is applied for a predicting the performance of a longitudinal fin [12]. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan [13] developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations. The concept of differential transformation method (DTM) was first introduced by Zhou [14] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. This method can be applied directly for linear and nonlinear differential equation without requiring linearization, discretization, or perturbation and this is the main benefit of this method. S. Ghafoori et al. [15] used the DTM for solving the nonlinear oscillation equation.

The main aim of this paper is to investigate the problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field using Least Square (LSM) and Differential Transformation Methods (DTM). Also a novel and combined method from these two methods is introduced as LS-DTM which is very accurate and efficient. The effects of the nanofluid volume friction, Hartmann number and Reynolds number on velocity profile are considered. Furthermore velocity profiles for different structures of nanofluid (copper and silver nanoparticles in water or ethylene glycol) are investigated.

2. Problem Description

Consider the laminar two-dimensional stationary flow of an electrically conducting incompressible viscous fluid in a semi-porous channel made by a long rectangular plate with length of L_x in uniform translation in x^* direction and an infinite porous plate.

The distance between the two plates is h. We observe a normal velocity q on the porous wall. A uniform magnetic field B is assumed to be applied towards direction y^* (Fig. 1).

In the case of a short circuit to neglect the electrical field and perturbations to the basic normal field and without any gravity forces, the governing equations are:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \tag{1}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial x^*}+ \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)-u^*\frac{\sigma_{nf}B^2}{\rho_{nf}}, \tag{2}$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial y^*} +\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right), \tag{3}$$

The appropriate boundary conditions for the velocity are:

$$y^*=0: u^*=u_0^*, v^*=0, \tag{4}$$

$$y^*=h: u^*=0,\ v^*=-q, \tag{5}$$

Calculating a mean velocity U by the relation:

$$y^*=0: u^*=u_0^*, v^*=0, \tag{6}$$

We consider the following transformations:

$$x=\frac{x^*}{L_x};\ y=\frac{y^*}{h}, \tag{7}$$

$$u=\frac{u^*}{U};\ v=\frac{v^*}{q}, P_y=\frac{P^*}{\rho_f . q^2} \tag{8}$$

Then, we can consider two dimensionless numbers: the Hartman number Ha for the description of magnetic forces [16] and the Reynolds number Re for dynamic forces:

$$Ha=Bh\sqrt{\frac{\sigma_f}{\rho_f . \upsilon_f}}, \tag{9}$$

$$\mathrm{Re}=\frac{hq}{\mu_{nf}}\rho_{nf}. \tag{10}$$

where the effective density(ρ_{nf}) is defined as [12]:

$$\rho_{nf}=\rho_f(1-\phi)+\rho_s\phi \tag{11}$$

Where ϕ is the solid volume fraction of nanoparticles. The dynamic viscosity of the nanofluids given by Brinkman [12] is

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{12}$$

the effective thermal conductivity of the nanofluid can be approximated by the Maxwell–Garnetts (MG) model as [12]:

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)} \tag{13}$$

The effective electrical conductivity of nanofluid was presented by Maxwell [17] as below

$$\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi} \quad (14)$$

The thermo physical properties of the nanofluid are given in Table 1.
Introducing Eqs. (6) and (10) into Eqs. (1) and (3) leads to the dimensionless equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (15)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\varepsilon^2\frac{\partial P_y}{\partial x}+ \frac{\mu_{nf}}{\rho_{nf}}\frac{1}{hq}\left(\varepsilon^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-u\frac{Ha^2}{\mathrm{Re}}\frac{B^*}{A^*}, \quad (16)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P_y}{\partial x}+\frac{\mu_{nf}}{\rho_{nf}}\frac{1}{hq}\left(\varepsilon^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \quad (17)$$

where A^* and B^* are constant parameters:

$$A^*=(1-\phi)+\frac{\rho_s}{\rho_f}\phi,\quad B^*=1+\frac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi} \quad (18)$$

Quantity of ε is defined as the aspect ratio between distance h and a characteristic length L_x of the slider. This ratio is normally small. Berman's similarity transformation is used to be free from the aspect ratio of ε:

$$v=-V(y);u=\frac{u^*}{U}=u_0U(y)+x\frac{dV}{dy}. \quad (19)$$

Introducing Eq. (19) in the second momentum equation (17) shows that quantity $\partial P_y/\partial y$ does not depend on the longitudinal variable x. With the first momentum equation, we also observe that $\partial^2 P_y/\partial x^2$ is independent of x.

We omit asterisks for simplicity. Then a separation of variables leads to [16]:

$$V'^2-VV''-\frac{1}{\mathrm{Re}}\frac{1}{A^*(1-\phi)^{2.5}}V''' +\frac{Ha^2}{\mathrm{Re}}\frac{B^*}{A^*}V'=\varepsilon^2\frac{\partial^2 P_y}{\partial x^2}=\varepsilon^2\frac{1}{x}\frac{\partial P_y}{\partial x}, \quad (20)$$

$$UV'-VU'=\frac{1}{\mathrm{Re}}\frac{1}{A^*(1-\phi)^{2.5}} \times\left[U''-Ha^2B^*(1-\phi)^{2.5}U\right]. \quad (21)$$

The right-hand side of Eq. (20) is constant. So, we derive this equation with respect to x. This gives:

$$V^{IV}=Ha^2B^*(1-\phi)^{2.5}V''+ \mathrm{Re}\,A^*(1-\phi)^{2.5}\left[V'V''-VV'''\right], \quad (22)$$

Where primes denote differentiation with respect to y and asterisks have been omitted for simplicity. The dynamic boundary conditions are:

$$\begin{cases} y=0:\ U=1;V=0;V'=0,\\ y=1:\ U=0;V=1;V'=0.\end{cases} \quad (23)$$

3. Analytical Methods

3. 1 Least Square Method (LSM)

Suppose a differential operator *D* is acted on a function *u* to produce a function *p*:

$$D(u(x))=p(x) \quad (24)$$

It is considered that *u* is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$u\cong\tilde{u}=\sum_{i=1}^{n}c_i\varphi_i \quad (25)$$

Now, when substituted into the differential operator, D, the result of the operations generally isn't *p(x)*. Hence an error or residual will exist:

$$R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{26}$$

The notion in WRMs is to force the residual to zero in some average sense over the domain. That is:

$$\int_X R(x)\, W_i(x) = 0 \qquad i = 1, 2, ..., n \tag{27}$$

Where the number of weight functions W_i is exactly equal the number of unknown constants c_i in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants c_i. If the continuous summation of all the squared residuals is minimized, the rationale behind the name can be seen. In other words, a minimum of

$$S = \int_X R(x)R(x)dx = \int_X R^2(x)dx \tag{28}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is,

$$\frac{\partial S}{\partial c_i} = 2\int_X R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{29}$$

Comparing with Eq. (27), the weight functions are seen to be

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{30}$$

However, the "2" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the Least Squares Method are just the derivatives of the residual with respect to the unknown constants

$$W_i = \frac{\partial R}{\partial c_i} \tag{31}$$

Because trial functions must satisfy the boundary conditions in Eq. (23), so they will be considered as,

$$\begin{cases} U(y) = 1 - y + c_1(y - y^2) \\ +c_2(y - y^3) \\ V(y) = c_3(\dfrac{y^2}{2} - \dfrac{y^3}{3}) \\ +c_4(\dfrac{y^2}{2} - \dfrac{y^4}{4}) + c_5(\dfrac{y^2}{2} - \dfrac{y^5}{5}) \end{cases} \tag{32}$$

In this problem, we have two coupled equations (Eqs. (21) and (22)), so two residual functions will be appeared as,

$$\begin{cases} R_1(c_1,c_2,c_3,c_4,c_5,y) = (1-y+c_1(y-y^2)+c_2(y-y^3))(c_3(y-y^2)+c_4(y-y^3)+c_5(y-y^4)) - (c_3\left(\dfrac{y^2}{2}-\dfrac{y^3}{3}\right)+c_4\left(\dfrac{y^2}{2}-\dfrac{y^4}{4}\right)+ \\ c_5\left(\dfrac{y^2}{2}-\dfrac{y^5}{5}\right))(-1+c_1(1-2y)+c_2(1-3y^2)) - \dfrac{-2c_1-6c_2y-Ha^2\left(1+\dfrac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}\right)(1-\phi)^{2.5}(1-y+c_1(y-y^2)+c_2(y-y^3))}{\mathrm{Re}\left(1-\phi+\dfrac{\rho_s\phi}{\rho_f}\right)(1-\phi)^{2.5}} \\ R_2(c_1,c_2,c_3,c_4,c_5,y) = -6c_4-24c_5y-Ha^2\left(1+\dfrac{3\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}\right)(1-\phi)^{2.5}(c_3(1-2y)+c_4(1-3y^2)+c_5(1-4y^3)) + \mathrm{Re}\left(1-\phi+\dfrac{\rho_s\phi}{\rho_f}\right)(1-\phi)^{2.5} \\ (c_3(y-y^2)+c_4(y-y^3)+c_5(y-y^4))(c_3(1-2y)+c_4(1-3y^2)+c_5(1-4y^3)) - \left(c_3\left(\dfrac{y^2}{2}-\dfrac{y^3}{3}\right)+c_4\left(\dfrac{y^2}{2}-\dfrac{y^4}{4}\right)+c_5\left(\dfrac{y^2}{2}-\dfrac{y^5}{5}\right)\right)(-2c_3-6c_4y-12c_5y^2) \end{cases} \tag{33}$$

By substituting the residual functions, $R_1(c_1,c_2,c_3,c_4,c_5, y)$ *and* $R_2(c_1,c_2,c_3,c_4,c_5, y)$, into Eq. (29), a set of equation with five equations will appear and by solving this system of equations, coefficients c_1-c_5 will be determined. For example, Using Least Square Method for a water-copper nanofluid with Re=0.5, Ha=0.5 and ϕ=0.05. U(y) and V(y) are as follows:

$$\begin{cases} U(y)=1-1.334953917y+ \\ 0.3461783819y^2-0.01122446534y^3 \\ V(y)=1.8703229y^2+3.1584693y^3- \\ 6.9279074y^4+2.8991152y^5 \end{cases} \quad (34)$$

3.2 Differential Transformation Method (DTM)

In this section the fundamental basic of the Differential Transformation Method is introduced. For understanding method's concept, suppose that *x(t)* is an analytic function in domain *D*, and $t=t_i$ represents any point in the domain. The function *x(t)* is then represented by one power series whose center is located at t_i . The Taylor series expansion function of *x(t)* is in form of:

$$x(t)=\sum_{k=0}^{\infty}\frac{(t-t_i)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \qquad \forall t\in D \quad (35)$$

The Maclaurin series of *x(t)* can be obtained by taking $t_i=0$ in Eq. (35) expressed as:

$$x(t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \qquad \forall t\in D \quad (36)$$

As explained in [14] the differential transformation of the function *x(t)* is defined as follows:

$$X(k)=\sum_{k=0}^{\infty}\frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \quad (37)$$

Where *X(k)* represents the transformed function and *x(t)* is the original function. The differential spectrum of *X(k)* is confined within the interval $t\in[0,H]$, where H is a constant value. The differential inverse transform of *X(k)* is defined as follows:

$$x(t)=\sum_{k=0}^{\infty}\left(\frac{t}{H}\right)^k X(k) \quad (38)$$

It is clear that the concept of differential transformation is based upon the Taylor series expansion. The values of function *X(k)* at values of argument *k* are referred to as discrete, i.e. *X(0)* is known as the zero discrete, *X(1)* as the first discrete, etc. The more discrete available, the more precise it is possible to restore the unknown function. The function *x(t)* consists of the *T*-function *X(k)*, and its value is given by the sum of the *T*-function with *(t/H)k* as its coefficient. In real applications, at the right choice of constant *H*, the larger values of argument k the discrete of spectrum reduce rapidly. The function *x(t)* is expressed by a finite series and Eq. (38) can be written as:

$$x(t)=\sum_{k=0}^{n}\left(\frac{t}{H}\right)^k X(k) \quad (39)$$

Some important mathematical operations performed by differential transform method are listed in Table 2. Now we apply Differential Transformation Method (DTM) from Table 2 in to Eqs. (21) and (22) for finding $U(y)$ and $V(y)$.

$$\begin{aligned}&\sum_{l=0}^{k}(k+1-l).\bar{U}(l).\bar{V}(k+1-l)\\ &-\sum_{l=0}^{k}(k+1-l).\bar{V}(l).\bar{U}(k+1-l)-\frac{1}{\mathrm{Re}}.\frac{1}{A.(1-\phi)^{2.5}}\\ &\times\left((k+1).(k+2).\bar{U}(k+2)-Ha^2.B.(1-\phi)^{2.5}.\bar{U}(k)\right)=0\end{aligned} \quad (40)$$

$$\begin{aligned}&(k+1)(k+2)(k+3)(k+4)\bar{V}(k+4)-\\ &Ha^2.B.(1-\phi)^{2.5}(k+1)(k+2)\bar{V}(k+2)-\\ &\mathrm{Re}.A.(1-\phi)^{2.5}\times\left(\sum_{l=0}^{k}(l+1)\bar{V}(l+1)(k-l)(k+1-l)\right.\\ &\left.-\sum_{l=0}^{k}\bar{V}(k)(k+1-l)(k+2-l)(k+3-l)=0\right)\end{aligned} \quad (41)$$

Where $\bar{U}$ and $\bar{V}$ represent the DTM transformed form of U and V respectively. The transformed form of boundary conditions can be written as:

$$\begin{cases}\bar{V}(0)=0, \quad \bar{V}(1)=0, \quad \bar{V}(2)=a, \quad \bar{V}(3)=b\\ \bar{U}(0)=1, \quad \bar{U}(1)=c.\end{cases} \quad (42)$$

Using transformed boundary condition and Eq. we have,

$$\begin{cases}\bar{U}(2)=0.5Ha^2B\sqrt{1-\phi}-\\ Ha^2B\sqrt{1-\phi}\phi+0.5Ha^2B\sqrt{1-\phi}\phi^2\\ \bar{V}(4)=0.0833Ha^2B\sqrt{1-\phi}a-\\ 0.1667Ha^2B\sqrt{1-\phi}a\phi+0.0833Ha^2B\sqrt{1-\phi}a\phi^2\end{cases}$$

$$\begin{cases}\bar{U}(3)=0.333a\,\mathrm{Re}\,A\sqrt{1-\phi}-0\\ .667a\,\mathrm{Re}\,A\sqrt{1-\phi}\phi+0.333a\,\mathrm{Re}\,A\sqrt{1-\phi}\phi^2\\ +0.1667Ha^2B\sqrt{1-\phi}c-0.333Ha^2B\sqrt{1-\phi}c\phi\\ +0.1667Ha^2B\sqrt{1-\phi}c\phi^2\\ \bar{V}(5)=0.05Ha^2B\sqrt{1-\phi}b-\\ 0.1Ha^2B\sqrt{1-\phi}b\phi+0.05Ha^2B\sqrt{1-\phi}b\phi^2\end{cases} \quad (43)$$

Where *a, b, c* are unknown coefficients that after specifying $U(y)$ and $V(y)$ and applying boundary condition (Eq. (42)) into it, will be determined. For water-copper nanofluid with Re=0.5, Ha=0.5 and ϕ=0.05 following values were determined for *a*, *b and c* coefficients.

$$a=3.011719150,\ b=-2.049532443,\ c=-1.673547080 \quad (44)$$

Finally, U(y) and V(y) are as follows,

$$\begin{cases}U(y)=1-1.673547080y-0.1273175011y^2\\ +0.5462295787y^3\\ V(y)=3.011719150y^2-2.049532443y^3\\ +0.06390742601y^4-0.02609413491y^5\end{cases} \quad (45)$$

3.3 LS- DTM Combined Method

Since LSM and DTM have a little shortcoming in some areas for predicting the V(y) and U(y) (See results section), we combined these two methods as LS-DTM combined method which eliminated those defects and for all areas has an excellent agreement with numerical procedure. For this purpose we selected U(y) from Eq. (32) and V(y) from Eq. (41). By using these two equations four unknown coefficients will be existed: a, b, c_1 and c_2. For finding these coefficients, four equations are needed; two of them are obtained from Eq. (29) for c_1 and c_2, and other two equations are selected from boundary condition for V(y) in Eq. (23). For water-copper nanofluid with Re=0.5, Ha=0.5 and ϕ=0.05 following formula are calculated for U(y) and V(y) by this efficient and novel method,

$$\begin{cases}U(y)=1-1.332674596y+0.3491297634y^2\\ -0.01645516732y^3\\ V(y)=3.011719150y^2-2.049532443y^3\\ +0.06390742601y^4-0.02609413491y^5\end{cases} \quad (46)$$

4. Results and discussion

In the present study LSM and DTM methods are applied to obtain an explicit analytic solution of the laminar nanofluid flow in a semi-porous channel in the presence of uniform magnetic field (Fig. 1).

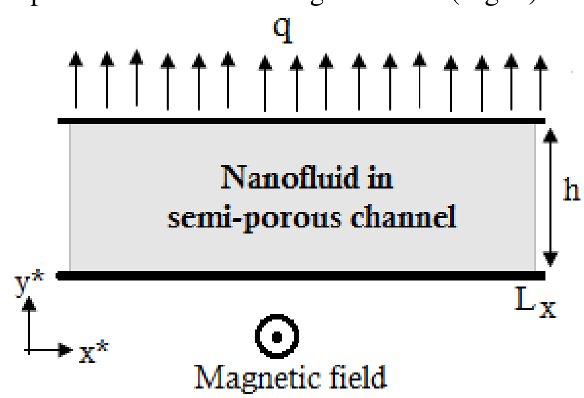

Fig. 1. Schematic of the problem (Nanofluid in a porous media between parallel plates and magnetic field)

For this aim Eqs. (21) and (22) are solved for different nanofluid structures (see Table 1) and comparison between described methods is demonstrated in Figs. 2 and 3. These figures explain that each of these two methods has a shortcoming which cannot completely predict the velocity distribution in a special area. As seen in Figs. 2 and 3, LSM has a defect in predicting the V(y) and DTM has a defect in demonstrating the U(y). For eliminating this shortcoming and achieving to a reliable result, these two methods were combined and introduced as a novel method called LS-DTM combined method.

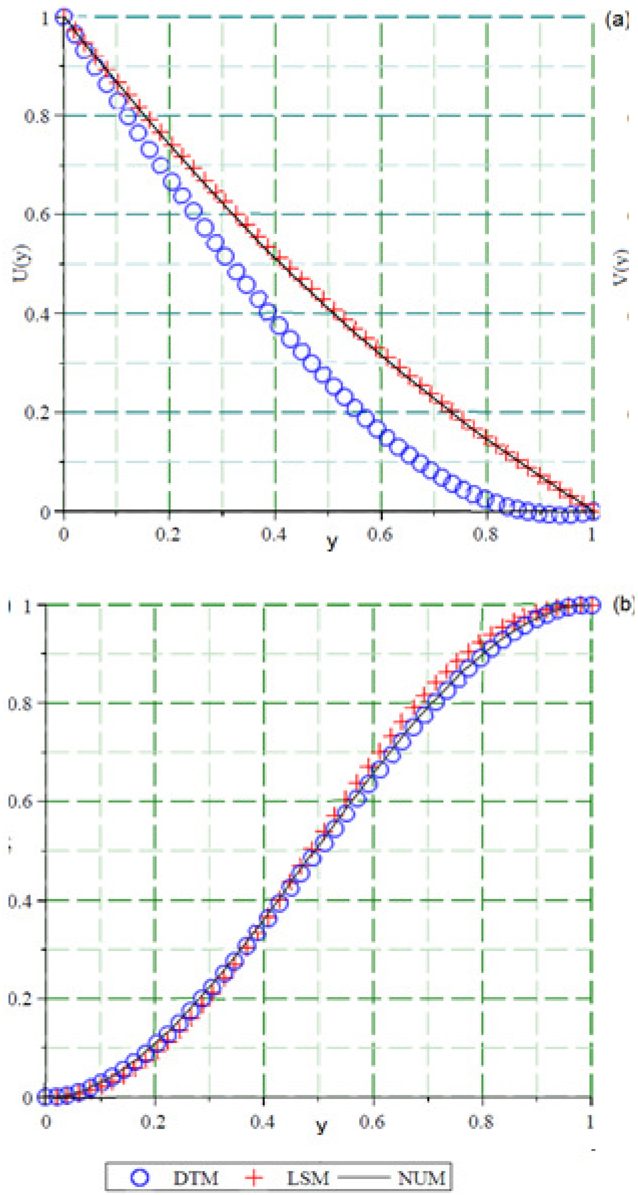

Fig. 2. Comparison of the DTM and LSM results for a) U(y) and b) V(y) for Cu-Water nanofluid when Re=0.5, Ha=0.5 and ϕ=0.05

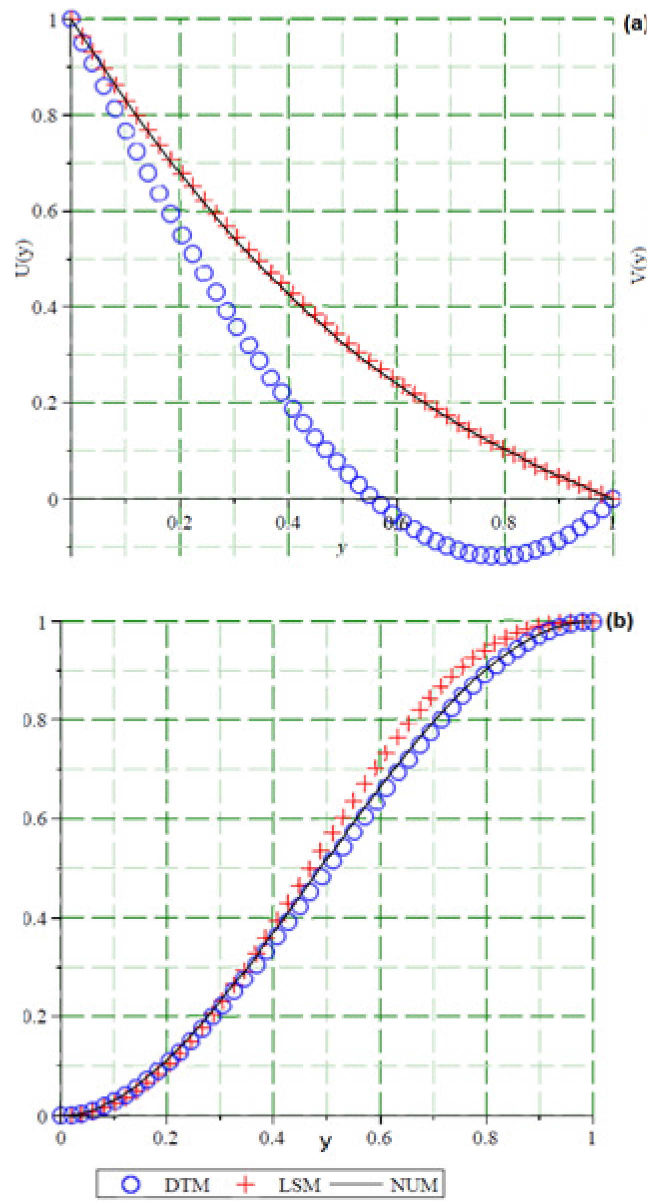

Fig. 3 Comparison of the DTM and LSM results for a) U(y) and b) V(y) for Silver-Ethylene glycol nanofluid when Re=1, Ha=1 and ϕ=0.01

Table 1

Thermo physical properties of nanofluids and nanoparticles

Material	Density(kg / m^3)	Electrical conductivity,σ((Ω.m)$^{-1}$)
Silver	10500	6.30×10^7
Copper	8933	5.96×10^7
Ethylene glycol	1113.2	1.07×10^{-4}
Drinking water	997.1	0.05
Silver	10500	6.30×10^7
Copper	8933	5.96×10^7
Ethylene glycol	1113.2	1.07×10^{-4}

Fig. 4 is improved form of Fig. 3 using LS-DTM combined method. This figure reveals that LS-DTM has an excellent agreement with numerical method in whole areas, and so, it is an accurate and convenient method for solving these kinds of problems.

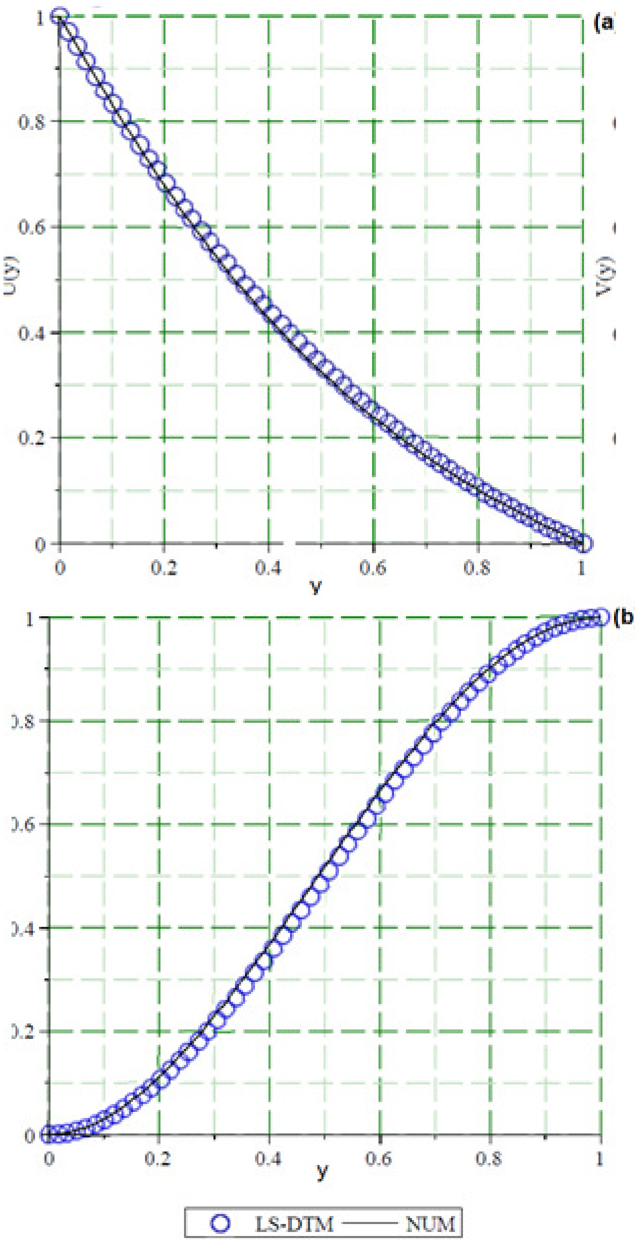

Fig. 4 LS-DTM results compared with numerical procedure for Fig. 3 state.

Effect of Hartman number (Ha) on dimensionless velocities for water with copper is shown in Figs. 5. Generally, when the magnetic field is imposed on the enclosure, the velocity field suppressed owing to the retarding effect of the Lorenz force. For low Reynolds number, as Hartmann number increases $V(y)$ decreases for $y > y_m$ but opposite trend is observed for $y < y_m$, y_m is a meeting point that all curves joint together at this point. When Reynolds number increases this meeting point shifts to the solid wall and it can be seen that $V(y)$ decreases with increase of Hartmann number. Fig. 6-a and b show the effect of nanoparticle volume fraction on U(y) and V(y) for water with copper nanoparticles when Re=1 and Ha=1. Velocity boundary layer thickness decreases with increase of nanoparticle volume fraction.

Effect of Reynolds number (Re) on dimensionless velocities is shown in Figs. 6-c and d. It is worth to mention that the Reynolds number indicates the relative significance of the inertia effect compared to the viscous effect. Thus, velocity profile decreases as Re increases and in turn increasing Re leads to increase in the magnitude of the skin friction coefficient. Contour plots of the effect of Hartman number and nanoparticles volume fraction in wide range of data are depicted in Figs. 7.

Table 2
Some fundamental operations of the differential transform method

Origin function	Transformed function
$x(t) = \alpha f(x) \pm \beta g(t)$	$X(k) = \alpha F(k) \pm \beta G(k)$
$x(t) = \dfrac{d^m f(t)}{dt^m}$	$X(k) = \dfrac{(k+m)!F(k+m)}{k!}$
$x(t) = f(t)g(t)$	$X(k) = \sum_{l=0}^{k} F(l)G(k-l)$
$x(t) = t^m$	$X(k) = \delta(k-m) = \begin{cases} 1, & \text{if } k=m \\ 0, & \text{if } k \neq m \end{cases}$
$x(t) = \exp(t)$	$X(k) = \dfrac{1}{k!}$
$x(t) = \sin(\omega t + \alpha)$	$X(k) = \dfrac{\omega^k}{k!}\sin(\dfrac{k\pi}{2} + \alpha)$
$x(t) = \cos(\omega t + \alpha)$	$X(k) = \dfrac{\omega^k}{k!}\cos(\dfrac{k\pi}{2} + \alpha)$

The effects of the nanoparticle and liquid phase material on velocity's profiles are shown in Tables 3 and 4 for U(y) and V(y), respectively. These tables reveal that when nanofluid includes copper (as nanoparticles) or ethylene glycol (as fluid phase) in its structure, the U(y) and V(y) values are greater than the other structures.

Table 3

U(y) variations for different types of nanofluids and nanoparticles, Re=1, Ha=1 and ϕ=0.04.

y	Water-Copper	Water-Silver	Ethylene glycol-Copper	Ethylene glycol-Silver
0.0	*1.0*	*1.0*	*1.0*	*1.0*
0.1	*0.83570944*	*0.83422691*	*0.8366021*	*0.8352604*
0.2	*0.68526977*	*0.68251844*	*0.6869270*	*0.6844364*
0.3	*0.55072934*	*0.54706012*	*0.5529409*	*0.5496176*
0.4	*0.43267586*	*0.42850606*	*0.4351915*	*0.4314120*
0.5	*0.33056756*	*0.32632363*	*0.3331308*	*0.3292806*
0.6	*0.24305045*	*0.23912555*	*0.2454244*	*0.2418595*
0.7	*0.16826652*	*0.16499404*	*0.1702490*	*0.1672728*
0.8	*0.10414284*	*0.10178554*	*0.1055735*	*0.1034265*
0.9	*0.04864722*	*0.04739956*	*0.0494059*	*0.0482677*
1.0	*0.0*	*0.0*	*0.0*	*0.0*

Table 4

V(y) variations for different types of nanofluids and nanoparticles, Re=1, Ha=1 and ϕ=0.04.

y	Water-Copper	Water-Silver	Ethylene glycol-Copper	Ethylene glycol-Silver
0.0	*0.0*	*0.0*	*0.0*	*0.0*
0.1	*0.02610404*	*0.025997188*	*0.026167546*	*0.026071871*
0.2	*0.097561353*	*0.097211791*	*0.097768963*	*0.097456128*
0.3	*0.204165243*	*0.203542825*	*0.204534660*	*0.203977939*
0.4	*0.335581162*	*0.334740930*	*0.336079488*	*0.335328393*
0.5	*0.48112359*	*0.480180063*	*0.481682731*	*0.480839846*
0.6	*0.62958117*	*0.628678803*	*0.630115463*	*0.629309909*
0.7	*0.769057186*	*0.768337156*	*0.769483116*	*0.768840824*
0.8	*0.88678803*	*0.886349813*	*0.887046994*	*0.886656410*
0.9	*0.96889373*	*0.96874723*	*0.968980208*	*0.968849751*
1.0	*1.0*	*1.0*	*1.0*	*1.0*

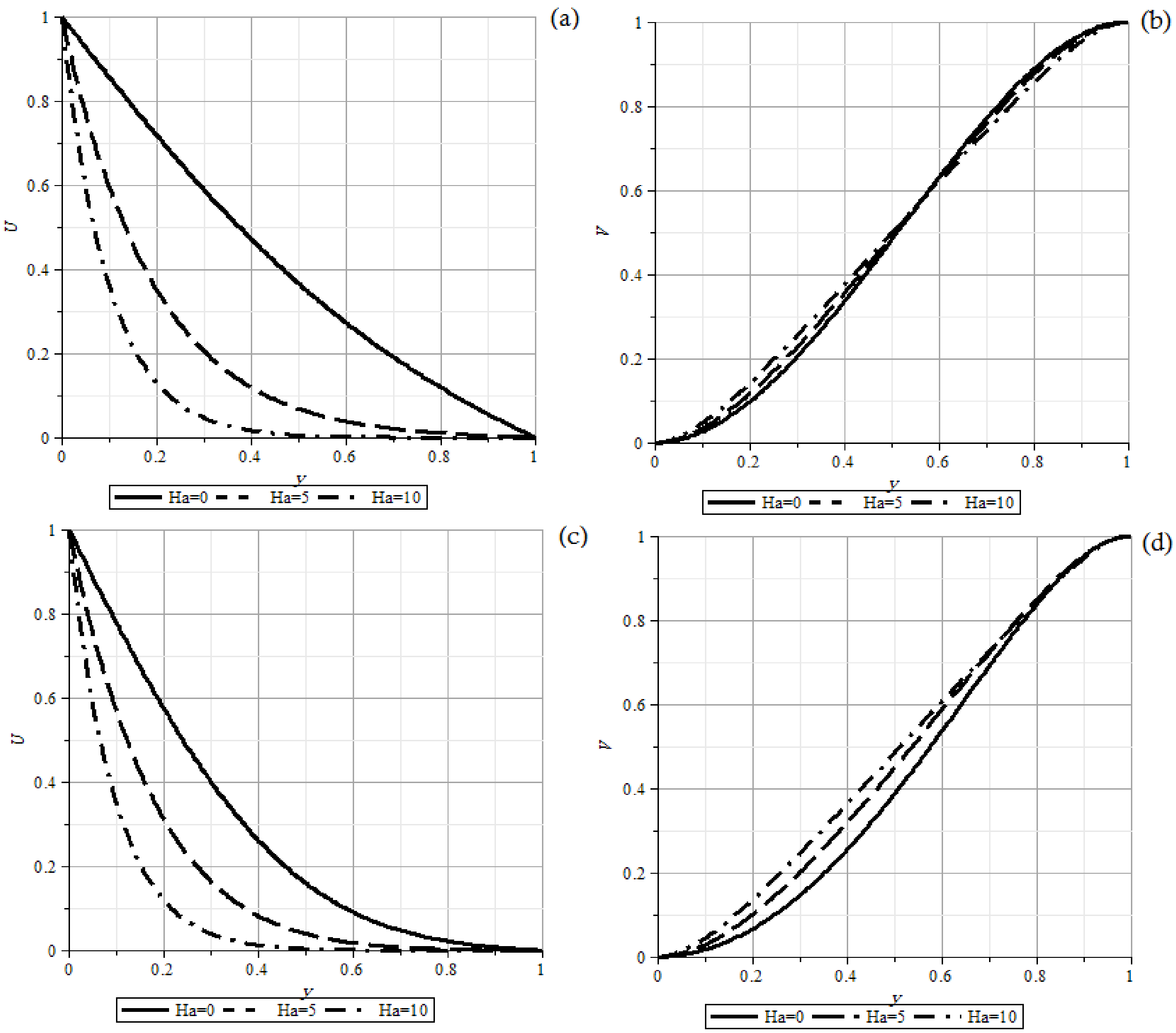

Fig. 5. Effect of Hartman number (Ha) on dimensionless velocities for water with copper nanoparticles, ϕ=0.04, a) U(y), Re=1, b) V(y), Re=1, c) U(y), Re=5 and d) V(y), Re=5.

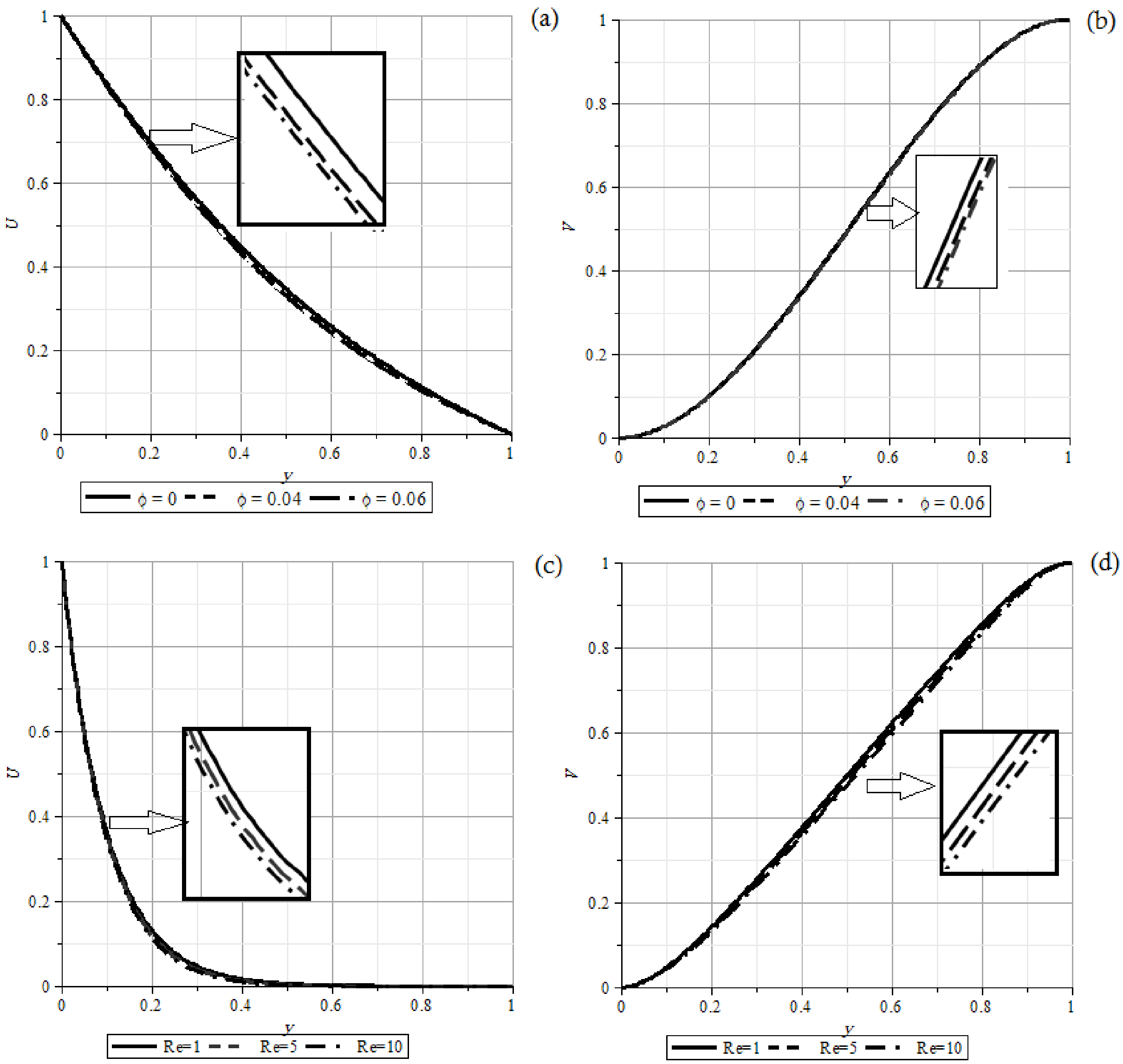

Fig. 6. Effect of nanoparticle volume fraction, ϕ, on a) U(y) and b) V(y), for Cu-Water when Re=1 and Ha=1. Effect of Reynolds number (Re) on c) U(y) and d) V(y) when Ha=10 ϕ=0.04.

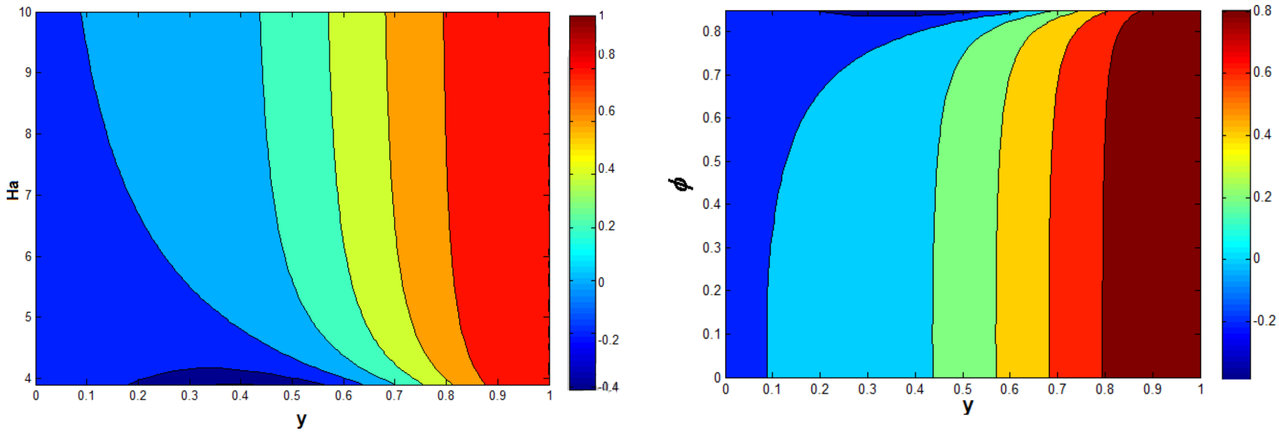

Fig. 7 Contour plot for showing the effect of Hartman number (Left) and nanoparticles fraction (Right) on V(y) distribution

6. Conclusion

In this paper, Least Square and Differential Transformation Methods are combined to eliminate the shortcoming of each method for solving the problem of laminar nanofluid flow in a semi-porous channel in the presence of uniform magnetic field. Outcomes reveal that this method is an accurate, powerful and convenient approach compared by numerical method for solving this problem. The results indicate that velocity boundary layer thickness decrease with increase of Reynolds number and nanoparticles volume fraction and it increases as Hartmann number increases. Choosing copper (as nanoparticles) and ethylene glycol (as fluid phase) leads to maximum increment in velocity.

References

[1] Y. H. Andoh, B. Lips, Prediction of porous walls thermal protection by effusion or transpiration cooling, An analytical approach, Applied Thermal Engineering 23 (2003) 1947-1958.

[2] A. Runstedtler, On the modified Stefan–Maxwell equation for isothermal multicomponent gaseous diffusion, Chemical Engineering Science 61 (2006) 5021-5029.

[3] A.S. Berman, Journal of Applied Physics, 24 (1953) 1232.

[4] M. Sheikholeslami, M.Hatami, D. D. Ganji, Analytical investigation of MHD nanofluid flow in a Semi-Porous Channel, Powder Technology10(2013).

[5] S. Soleimani, M. Sheikholeslami, D.D. Ganji and M. Gorji-Bandpay, Natural convection heat transfer in a nanofluid filled semi-annulus enclosure, International Communications in Heat and Mass Transfer 39 (2012) 565–574.

[6] D. Domairry, M. Sheikholeslami, H. R. Ashorynejad,Rama S. R. Gorla and M. Khani, Natural convection flow of a non-Newtonian nanofluid between two vertical flat plates, Proc IMechE Part N:J Nanoengineering and Nanosystems.225(3) (2012)115–122

[7] M. Sheikholeslami , S. Soleimani, M. Gorji-Bandpy, D.D. Ganji, S.M. Seyyedi, Natural convection of nanofluids in an enclosure between a circular and a sinusoidal cylinder in the presence of magnetic field, International Communications in Heat and Mass Transfer, Doi.org/10.1016/j.icheatmasstransfer (2012)7-026.

[8] R. H. Stern, H. Rasmussen, Left ventricular ejection: Model solution by collocation, an approximate analytical method, Comput. Boil. Med 26 (1996) 255-261.

[9] B. Vaferi, V. Salimi, D. Dehghan Baniani, A. Jahanmiri, S. Khedri, Prediction of transient pressure response in the petroleum reservoirs using orthogonal collocation, J. of Petrol. Sci. and Eng 10(2012) 4-23.

[10] F.A. Hendi, A.M. Albugami, Numerical solution for Fredholm–Volterra integral equation of the second kind by using collocation and Galerkin methods 22 (2010) 37–40.

[11] M. N. Bouaziz, A. Aziz, Simple and accurate solution for convective-radiative fin with temperature dependent thermal conductivity using double optimal linearization, Energy Convers. Manage. 51 (2010) 76–82.

[12] A. Aziz, M. N. Bouaziz, A least squares method for a longitudinal fin with temperature dependent internal heat generation and thermal conductivity, Energ. Convers. and Manage 52 (2011) 2876–2882.

[13] G. Shaoqin, D. Huoyuan, negative norm least-squares methods for the incompressible magneto-hydrodynamic equations, Act. Math. Sci. 28 B (3) (2008) 675–684.

[14] J.K. Zhou, Differential Transformation Method and its Application for Electrical Circuits, Hauzhang University press, China, 1986.

[15] S. Ghafoori, M. Motevalli, M.G. Nejad, F. Shakeri, D.D. Ganji, M. Jalaal, Efficiency of differential transformation method for nonlinear oscillation: Comparison with HPM and VIM, Current Appl. Phys 11 (2011) 965-971.

[16] A. Desseaux, Influence of a magnetic field over a laminar viscous flow in a semi-porous channel. Int J Eng Sci 37(1999)1781–94.

[17] J.C. Maxwell, A Treatise on Electricity and Magnetism, second ed. Oxford University Press, Cambridge (1904) 435-441.