ATANASSOV'S INTUITIONISTIC FUZZY GRADE OF I.P.S. HYPERGROUPS OF ORDER LESS THAN OR EQUAL TO 6

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Abstract. In this paper we determine the sequences of join spaces and Atanassov's intuitionistic fuzzy sets associated with all i.p.s. hypergroups of order less than or equal to 6, focusing on the calculation of their lengths.

1. **Introduction**

The concept of algebraic hyperstructure is a natural generalization of algebraic structure. This theory has been introduced by Marty [27] in 1934 at the 8th Congress of Scandinavian Mathematicians. In the last decades, many connections between hyperstructures and other branches of mathematics were considered leading to applications in geometry, hypergraphs, binary relations, combinatorics, artificial intelligence, automata, rough and fuzzy sets. The books [6, 14, 22, 31] are surveys of the theory of algebraic hyperstructures and their applications.

After the introduction of the concept of fuzzy set by Zadeh [32], various connections between fuzzy sets and algebraic structures have been established and studied. In 1971, Rosenfeld [29] defined the fuzzy subgroup of a group, and this paper led to a new area in the abstract algebra and its applications. Ameri and Zahedi in [3] have extended this connection for constructing a new hyperstructure associated with a group G, which under suitable conditions is a hypergroup and moreover a join space. Another topic on the relations between hyperstructures and fuzzy sets was introduced by Corsini [9, 10] and it was later continued by Corsini, Cristea, Leoreanu-Fotea, Iranmanesh (for example see [1, 2, 15, 16, 17, 30]). In [20], Davvaz applied the concept of fuzzy sets to the theory of algebraic hyperstructures and defined the notion of fuzzy subhypergroup, with the intent to generalize the concept of Rosenfeld's fuzzy subgroup of a group, also see [5, 19, 21, 24, 25]. Atanassov's intuitionistic fuzzy sets [4] are an intuitively straightforward extension of Zadeh's fuzzy sets and therefore many researchers study intuitionistic fuzzification of the concepts from the algebraic hyperstructure theory, such as Atanassov's intuitionistic (S, T)-fuzzy n-ary subhypergroups [23], etc.

A connection between hypergroupoids and Atanassov's intuitionistic fuzzy sets has been presented by Cristea and Davvaz [18], where the notion of fuzzy grade of a hypergroup has been extended through Atanassov's intuitionistic fuzzy sets. Moreover, it has been proved that, with every hypergroupoid H endowed with

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an Atanassov's intuitionistic fuzzy set, it is possible to associate two sequences of Atanassov's intuitionistic fuzzy sets and join spaces. Now, in this paper we determine the sequences of join spaces and Atanassov's intuitionistic fuzzy sets associated with all i.p.s. hypergroups of order less than or equal to 6, focusing on the calculation of their lengths.

The paper is organized as follows: in section 2 we present some fundamental definitions in the theory of hypergroups, and in section 3 we recall the construction of the sequence of join spaces and Atanassov's intuitionistic fuzzy sets associated with a hypergroupoid. In section 4 we determine the sequence of join spaces and Atanassov's intuitionistic fuzzy sets associated with all non-isomorphic i.p.s. hypergroups of order less than or equal to 6. Finally we indicate some conclusions and future work covered in the last section of the paper.

2. **Preliminaries**

In this section we recall some basic notions about hypergroups for the sake of completeness of our study; for a comprehensive overview of this area, the reader is reffered to [6, 14].

Let H be a non-empty set and $\mathcal{P}^*(H)$ be the set of all nonempty subsets of H. A set H endowed with a hyperoperation $\circ : H^2 \to \mathcal{P}^*(H)$ is called a *hypergroupoid*. The image of the pair $(x, y) \in H \times H$ is denoted by $x \circ y$. Moreover, for any $x \in H$ and non-empty subsets A, B of H, by $A \circ B$, $A \circ x$ and $x \circ B$ we mean $A \circ B = \bigcup_{\substack{a\in A\\ b\in B}} a \circ b$, $A \circ x = A \circ \{x\}$ and $x \circ B = \{x\} \circ B$, respectively.

If the hyperoperation satisfies the conditions:

(i) For any $(a, b, c) \in H^3, (a \circ b) \circ c = a \circ (b \circ c)$ (the associativity),

(ii) For any $a \in H, H \circ a = a \circ H = H$ (the reproducibility),

then the hypergroupoid $\langle H, \circ\rangle$ is a *hypergroup*. A hypergroup $\langle H, \circ\rangle$ is called a *total hypergroup* if, for any $(x, y) \in H^2, x \circ y = H$.

For each pair $(a, b) \in H^2$, we denote:

$$a/b = \{x \in H \mid a \in x \circ b\} \quad \text{and} \quad b/a = \{y \in H \mid a \in b \circ y\}.$$

A commutative hypergroup $\langle H, \circ\rangle$ is called a *join space* if, for any four elements $a, b, c, d \in H$, such that $a/b \cap c/d \neq \emptyset$, it follows that $a \circ d \cap b \circ c \neq \emptyset$. The notion of join space introduced by Prenowitz, has been studied by Prenowitz and Jantosciak [28] to reconstruct, from an algebraic point of view, the projective, the descriptive and the spherical geometries.

Let $\langle H, \circ\rangle$ and $\langle H', \circ'\rangle$ be two hypergroupoids and $f : H \to H'$ be a mapping from H in H'. We say:

(i) f is a *homomorphism* if, for all $(x, y) \in H^2$, $f(x \circ y) \subseteq f(x) \circ' f(y)$;

(ii) f is a *good homomorphism* if, for all $(x, y) \in H^2$, $f(x \circ y) = f(x) \circ' f(y)$.

We say that the two hypergroups are *isomorphic* if there is a good homomorphism between them which is also a bijection.

We say that a hypergroup $\langle H, \circ \rangle$ is *canonical* if

(i) it is commutative;

(ii) it has a scalar identity;

(iii) every element $a \in H$ has a unique inverse $a^{-1} \in H$;

(iv) it is reversible, that is, for any $(x, y, z) \in H^3$, the following condition is satisfied: if $y \in a \circ x$, then $x \in a^{-1} \circ y$.

A canonical hypergroup $\langle H, \circ \rangle$ is called *strongly canonical* [26] if it satisfies the following conditions:

(i) for any $(x, a) \in H^2$, if $x \in x \circ a$, then $x = x \circ a$;

(ii) $(x \circ y) \cap (z \circ w) \neq \emptyset$ implies that $x \circ y \subset z \circ w$ or $z \circ w \subset x \circ y$.

An *i.p.s. hypergroup* (a hypergroup with partial scalar identities) $\langle H, \circ \rangle$ is a canonical hypergroup such that, for any $(x, a) \in H^2$, if $x \in x \circ a$, then it follows that $x = x \circ a$.

3. Atanassov's Intuitionistic Fuzzy Grade of Hypergroups

In this section we recall the construction of the sequence of join spaces and Atanassov's intuitionistic fuzzy sets associated with a hypergroupoid H.

As a generalization of the notion of fuzzy set in a nonempty set X, Atanassov [4] has introduced the concept of intuitionistic fuzzy set in X, as an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, where, for any $x \in X$, the *degree of membership* of x (namely $\mu_A(x)$) and the *degree of non-membership* of x (namely $\lambda_A(x)$) verify the relation $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For the sake of simplicity, we use the symbol $A = (\mu, \lambda)$ for the Atanassov's intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$.

For any hypergroupoid $\langle H, \circ \rangle$, Cristea and Davvaz [18] defined an Atanassov's intuitionistic fuzzy set $A = (\bar{\mu}, \bar{\lambda})$ in the following way: for any $u \in H$, we consider:

$$\bar{\mu}(u) = \frac{\displaystyle\sum_{(x,y)\in Q(u)} \frac{1}{|x \circ y|}}{n^2}, \quad \bar{\lambda}(u) = \frac{\displaystyle\sum_{(x,y)\in \bar{Q}(u)} \frac{1}{|x \circ y|}}{n^2} \tag{1}$$

where $Q(u) = \{(a, b) \in H^2 \mid u \in a \circ b\}$, $\bar{Q}(u) = \{(a, b) \in H^2 \mid u \notin a \circ b\}$. If $Q(u) = \emptyset$, we set $\bar{\mu}(u) = 0$ and similarly, if $\bar{Q}(u) = \emptyset$ we set $\bar{\lambda}(u) = 0$. Moreover, it is clear that, for any $u \in H$, $0 \leq \bar{\mu}(u) + \bar{\lambda}(u) \leq 1$.

Now, let $A = (\bar{\mu}, \bar{\lambda})$ be an Atanassov's intuitionistic fuzzy set on H. We may associate with H two join spaces $\langle {}_0H, \circ_{\bar{\mu}\wedge\bar{\lambda}} \rangle$ and $\langle {}^0H, \circ_{\bar{\mu}\vee\bar{\lambda}} \rangle$, where, for any fuzzy set α on H, the hyperproduct "$\circ_\alpha$", introduced by Corsini [10] is defined by

$$x \circ_\alpha y = \{u \in H \mid \alpha(x) \wedge \alpha(y) \leq \alpha(u) \leq \alpha(x) \vee \alpha(y)\}.$$

Corsini [10] has proved that the associated hypergroup $\langle H, \circ_\alpha \rangle$ is a join space.

By using the same procedure as in (1), from the join space $\langle {}_0H, \circ_{\bar{\mu}\wedge\bar{\lambda}} \rangle$ we can construct the Atanassov's intuitionistic fuzzy set $\bar{A}_1 = (\bar{\mu}_1, \bar{\lambda}_1)$ as in (1); we associate again the join space $\langle {}_1H, \circ_{\bar{\mu}_1\wedge\bar{\lambda}_1} \rangle$, we determine, like in (1), its Atanassov's intuitionistic fuzzy set $\bar{A}_2 = (\bar{\mu}_2, \bar{\lambda}_2)$ and we construct the join space $\langle {}_2H, \circ_{\bar{\mu}_2 \wedge \bar{\lambda}_2} \rangle$ and so on. We obtain the sequence $({}_iH = \langle {}_iH, \circ_{\bar{\mu}_i \wedge \bar{\lambda}_i} \rangle; \bar{A}_i = (\bar{\mu}_i, \bar{\lambda}_i))_{i \geq 0}$ of join spaces and Atanassov's intuitionistic fuzzy sets associated with H. Similarly, we may construct the second sequence $({}^iH = \langle {}^iH, \circ_{\bar{\mu}_i \vee \bar{\lambda}_i} \rangle; \bar{A}_i = (\bar{\mu}_i, \bar{\lambda}_i))_{i \geq 0}$.

We call the lower, and respectively, the upper Atanassov's intuitionistic fuzzy grade of H the length of the two corresponding sequences associated with H, more exactly:

Definition 3.1. [18]. A set H endowed with an Atanassov's intuitionistic fuzzy set $A = (\mu, \lambda)$ has the *lower Atanassov's intuitionistic fuzzy grade* $m, m \in N^*$, and we write $l.i.f.g.(H) = m$ if, for any $i, 0 \leq i < m-1$, the join spaces $\langle {}_iH, \circ_{\bar{\mu}_i \wedge \bar{\lambda}_i} \rangle$ and $\langle {}_{i+1}H, \circ_{\bar{\mu}_{i+1} \wedge \bar{\lambda}_{i+1}} \rangle$ associated with H are not isomorphic (where ${}_0H = \langle {}_0H, \circ_{\bar{\mu} \wedge \bar{\lambda}} \rangle$) and for any $s, s \geq m$, ${}_sH$ is isomorphic with ${}_{m-1}H$.

Definition 3.2. [18]. A set H endowed with an Atanassov's intuitionistic fuzzy set $A = (\mu, \lambda)$ has the *upper Atanassov's intuitionistic fuzzy grade* $m, m \in N^*$, and we write $u.i.f.g.(H) = m$ if, for any $i, 0 \leq i < m-1$, the join spaces $\langle {}^iH, \circ_{\bar{\mu}_i \wedge \bar{\lambda}_i} \rangle$ and $\langle {}^{i+1}H, \circ_{\bar{\mu}_{i+1} \vee \bar{\lambda}_{i+1}} \rangle$ associated with H are not isomorphic (where ${}^0H = \langle {}^0H, \circ_{\bar{\mu} \vee \bar{\lambda}} \rangle$) and for any $s, s \geq m$, ${}^sH$ is isomorphic with ${}^{m-1}H$.

If we start the construction of the above sequences with a hypergroupoid $\langle H, \circ \rangle$, and not with a set H endowed with an Atanassov's intuitionistic fuzzy set, then we obtain only one sequence of join spaces because the join spaces $\langle {}_0H, \circ_{\bar{\mu} \wedge \bar{\lambda}} \rangle$ and $\langle {}^0H, \circ_{\bar{\mu} \vee \bar{\lambda}} \rangle$ are isomorphic (see [18]). In order to explain this situation, we introduce a new concept.

Definition 3.3. [18]. We say that a hypergroupoid H has the Atanassov's intuitionistic fuzzy grade $m, m \in N^*$, and we write $i.f.g.(H) = m$, if $l.i.f.g.(H) = m$.

It is important to know when the sequence of join spaces associated with a hypergroupoid H doesn't stop, that is, when two consecutive join spaces are not isomorphic. In order to resolve this problem one introduces some notations. Let $({}_iH = \langle {}_iH, \circ_{\bar{\mu}_i \wedge \bar{\lambda}_i} \rangle; \bar{A}_i = (\bar{\mu}_i, \bar{\lambda}_i))_{i \geq 0}$ be the sequence of join spaces and Atanassov's intuitionistic fuzzy sets with a hypergroupoid H. Then, for any i, there are r, namely $r = r_i$ and a partition $\Pi = \{{}^iC_j\}_{j=1}^r$ of ${}_iH$ such that, for any $j \geq 1, x, y \in {}^iC_j \iff \bar{\mu}_i(x) \wedge \bar{\lambda}_i(x) = \bar{\mu}_i(y) \wedge \bar{\lambda}_i(y)$. For $x \in H$, we denote $\lambda(x) = i_j$, when $x \in {}^iC_j$. On the set of classes $\{{}^iC_j\}_{j=1}^r$ we define the following ordering relation:

$i_j < i_k$ if for elements $x \in {}^iC_j$ and $y \in {}^iC_k$,

$$\bar{\mu}_i(x) \wedge \bar{\lambda}_i(x) < \bar{\mu}_i(y) \wedge \bar{\lambda}_i(y) \text{ (therefore } \lambda(x) < \lambda(y)).$$

With any ordered chain $({}^iC_{j_1}, {}^iC_{j_2}, \ldots, {}^iC_{j_r})$ we may associate an ordered r-tuple $(k_{j_1}, k_{j_2}, \ldots, k_{j_r})$, where $k_{j_l} = |{}^iC_{j_l}|$, for all $l, 1 \leq l \leq r$.

Theorem 3.4. [13]. *Let ${}^iH$ and ${}^{i+1}H$ be the join spaces associated with H determined by the membership functions $\tilde{\mu}_i$ and $\tilde{\mu}_{i+1}$, where* ${}^iH = \bigcup_{l=1}^{r_1} C_l$, ${}^{i+1}H =$

$\bigcup_{l=1}^{r_2} C'_l$ *and* $(k_1, k_2, \ldots, k_{r_1})$ *is the* r_1*-tuple associated with* ${}^iH$, $(k'_1, k'_2, \ldots, k'_{r_2})$ *is the* r_2*-tuple associated with* ${}^{i+1}H$. *The join spaces* ${}^iH$ *and* ${}^{i+1}H$ *are isomorphic if and only if* $r_1 = r_2$ *and* $(k_1, k_2, \ldots, k_{r_1}) = (k'_1, k'_2, \ldots, k'_{r_1})$ *or* $(k_1, k_2, \ldots, k_{r_1}) = (k'_{r_1}, k'_{r_1-1}, \ldots, k'_1)$.

4. Atanassov's Intuitionistic Fuzzy Grade of i.p.s. Hypergroups

The i.p.s. hypergroups were investigated in [7, 8] and it was proved that if their order is less than 9, they are strongly canonical, also see [11, 12].

Theorem 4.1. (i) *There is only one i.p.s. hypergroup* H *of order 3 and* $i.f.g.(H) = 1$.

(ii) *There are three non isomorphic i.p.s. hypergroups* H_1, H_2, H_3 *of order 4 and* $i.f.g.(H_1) = 1$, $i.f.g.(H_2) = i.f.g.(H_3) = 2$.

(iii) *There are eight non isomorphic i.p.s. hypergroups of order 5: four of them have* $i.f.g.(H) = 1$, *three of them have* $i.f.g.(H) = 2$ *and only for one of them we find* $i.f.g.(H) = 3$.

(iv) *There are nineteen non isomorphic i.p.s. hypergroups of order 6: ten of them have* $i.f.g.(H) = 1$, *eight of them have* $i.f.g.(H) = 2$ *and only for one of them we find* $i.f.g.(H) = 3$.

Proof. We will denote, in the following tables, for any $s \in H$, $A_s = H \setminus \{s\}$ and for simplicity we take $H = \{1, 2, \ldots, n\}$, with $n \in \mathbb{N}$, $3 \le n \le 6$.

(i) The unique i.p.s. hypergroup H of order 3 is represented by the following table

$\circ$	0	1	2
0	0	1	2
1		A_1	1
2			0

,

then

$$\bar{\mu}(0) = \bar{\mu}(2) = 5/18, \quad \bar{\mu}(1) = 8/18,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = 12/18, \quad \bar{\lambda}(1) = 9/18.$$

Therefore, the associated join space ${}_0H$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2
0	A_1	H	A_1
1		1	H
2			A_1

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(2) = 10/27, \quad \bar{\mu}_1(1) = 7/27,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(2) = 3/27, \quad \bar{\lambda}_1(1) = 6/27.$$

Then, for any $r \ge 1$, ${}_rH \simeq_0 H$ and therefore $i.f.g.(H) = 1$.

(ii) Let us suppose H of order 4. We denote the three non-isomorphic i.p.s. hypergroups of order 3 by H_1, H_2, H_3.

(a)

$\circ$	0	1	2	3
0	0	1	2	3
1		2	0	3
2			1	3
3				A_3

,

then

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = 5/24, \quad \bar{\mu}(3) = 9/24,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = 18/24, \quad \bar{\lambda}(3) = 14/24.$$

Therefore, the associated join space ${}_0(H_1)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3
0	A_3	A_3	A_3	H
1		A_3	A_3	H
2			A_3	H
3				3

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(2) = 9/32, \quad \bar{\mu}_1(3) = 5/32,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(2) = 2/32, \quad \bar{\lambda}_1(3) = 6/32.$$

Then, for any $r \geq 1$, ${}_rH \simeq_0 (H_1)$ and so $i.f.g.(H_1) = 1$.

(b) Set H_2 the following i.p.s. hypergroup

$\circ$	0	1	2	3
0	0	1	2	3
1		{0,2}	1	3
2			0	3
3				A_3

,

then

$$\bar{\mu}(0) = \bar{\mu}(2) = 17/96, \quad \bar{\mu}(1) = 26/96, \quad \bar{\mu}(3) = 36/96,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = 72/96, \quad \bar{\lambda}(1) = 63/96, \quad \bar{\lambda}(3) = 53/96.$$

Therefore, the associated join space ${}_0(H_2)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3
0	{0,2}	A_3	{0,2 }	H
1		1	A_3	{1,3}
2			{0,2}	H
3				3

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(2) = 13/48, \quad \bar{\mu}_1(3) = 9/48,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(2) = 9/48, \quad \bar{\lambda}_1(3) = 13/48.$$

Therefore, the associated join space ${}_1(H_2)$ is as follows:

$\circ_{\bar{\mu_1}\wedge\bar{\lambda_1}}$	0	1	2	3
0	H	H	H	H
1	H	H	H	H
2	H	H	H	H
3	H	H	H	H

,

Therefore, ${}_1(H_2)$ is a total hypergroup of order 4 and $i.f.g.(H_2) = 2$.

(c) The third i.p.s. hypergroup H_3 has the table

$\circ$	0	1	2	3
0	0	1	2	3
1		2	{0,3}	1
2			1	2
3				0

,

then

$$\bar{\mu}(0) = \bar{\mu}(3) = 3/16, \quad \bar{\mu}(1) = \bar{\mu}(2) = 5/16,$$
$$\bar{\lambda}(0) = \bar{\lambda}(3) = 12/16, \quad \bar{\lambda}(1) = \bar{\lambda}(2) = 10/16.$$

Therefore, the associated join space ${}_0(H_3)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3
0	{0,3}	H	H	{0,3}
1		{1,2}	{1,2}	H
2			{1,2}	H
3				{0,3}

,

From this we have: for all $u \in H, \bar{\mu}_1(u) = 2/8, \bar{\lambda}_1(u) = 1/8$, that is, ${}_1(H_3)$ is a total hypergroup of order 4 and $i.f.g.(H_3) = 2$.

(iii) We now consider the i.p.s. hypergroups of order 5, denoted by $H_1, \ldots, H_8$. □

(a_1) Let us consider the first i.p.s. hypergroup H_1

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		{0,2}	1	4	3
2			0	3	4
3				{0,2}	1
4					{0,2}

,

then

$$\bar{\mu}(0) = \bar{\mu}(2) = 7/50, \quad \bar{\mu}(1) = \bar{\mu}(3) = \bar{\mu}(4) = 12/50,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = 40/50, \quad \bar{\lambda}(1) = \bar{\lambda}(3) = \bar{\lambda}(4) = 35/50.$$

Therefore, the associated join space ${}_0(H_1)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4
0	{0,2}	H	{0,2}	H	H
1		{1,3,4}	H	{1,3,4}	{1,3,4}
2			{0,2}	H	H
3				{1,3,4}	{1,3,4}
4					{1,3,4}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(2) = 22/125, \quad \bar{\mu}_1(1) = \bar{\mu}_1(3) = \bar{\mu}_1(4) = 27/125,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(2) = 15/125, \quad \bar{\lambda}_1(1) = \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = 10/125.$$

Then, for any $r \geq 1$, ${}_rH \simeq_0 (H_1)$ and therefore $i.f.g.(H_1) = 1$.

(a_2) The second i.p.s. hypergroup H_2 is the following one

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		{0,2}	1	3	4
2			0	3	4
3				A_3	3
4					{0,1,2}

,

then

$$\bar{\mu}(0) = \bar{\mu}(2) = 37/300, \quad \bar{\mu}(1) = 55/300, \quad \bar{\mu}(3) = 96/300, \quad \bar{\mu}(4) = 75/300,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = 240/300, \quad \bar{\lambda}(1) = 222/300, \quad \bar{\lambda}(3) = 181/300, \quad \bar{\lambda}(4) = 202/300.$$

Therefore, the associated join space ${}_0(H_2)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4
0	{0,2}	{0,1,2}	{0,2}	H	A_3
1		1	{0,1,2}	{1,3,4}	{1,4}
2			{0,2}	H	A_3
3				3	{3,4}
4					4

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(2) = 77/375, \quad \bar{\mu}_1(1) = 87/375, \quad \bar{\mu}_1(3) = 52/375, \quad \bar{\mu}_1(4) = 82/375,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(2) = 85/375, \quad \bar{\lambda}_1(1) = 75/375, \quad \bar{\lambda}_1(3) = 110/375, \quad \bar{\lambda}_1(4) = 80/375.$$

Therefore, the associated join space ${}_1(H_2)$ is as follows:

$\circ_{\bar{\mu}_1\wedge\bar{\lambda}_1}$	0	1	2	3	4
0	{0,2}	{0,1,2}	{0,2}	A_4	{0,2,4}
1		1	{0,1,2}	{1,3}	A_3
2			{0,2}	A_4	{0,2,4}
3				3	H
4					4

,

then

$$\bar{\mu}_2(0) = \bar{\mu}_2(2) = 197/750, \quad \bar{\mu}_2(1) = 157/750, \quad \bar{\mu}_2(3) = 102/750, \quad \bar{\mu}_2(4) = 97/750,$$
$$\bar{\lambda}_2(0) = \bar{\lambda}_2(2) = 120/750, \quad \bar{\lambda}_2(1) = 160/750, \quad \bar{\lambda}_2(3) = 215/750, \quad \bar{\lambda}_2(4) = 220/750.$$

Then, for any $r \geq 2$, ${}_rH \simeq_1 (H_2)$ and therefore $i.f.g.(H_2) = 2$.

(a_3) Let us see the third i.p.s. hypergroup H_3

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		{0,2}	1	3	4
2			0	3	4
3				4	{0,1,2}
4					3

,

then

$$\bar{\mu}(0) = \bar{\mu}(2) = 19/150, \quad \bar{\mu}(1) = 28/150, \quad \bar{\mu}(3) = \bar{\mu}(4) = 42/150,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = 120/150, \quad \bar{\lambda}(1) = 111/150, \quad \bar{\lambda}(3) = \bar{\lambda}(4) = 97/150.$$

Therefore, the associated join space ${}_0(H_3)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4
0	{0,2}	{0,1,2}	{0,2}	H	H
1		1	{0,1,2}	{1,3,4}	{1,3,4}
2			{0,2}	H	H
3				{3,4}	{3,4}
4					{3,4}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(2) = \bar{\mu}_1(3) = \bar{\mu}_1(4) = 74/375, \quad \bar{\mu}_1(1) = 79/375,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = 65/375, \quad \bar{\lambda}_1(1) = 60/375.$$

Therefore, the associated join space ${}_1(H_3)$ is as follows:

$\circ_{\bar{\mu}_1\wedge\bar{\lambda}_1}$	0	1	2	3	4
0	A_1	H	A_1	A_1	A_1
1		1	H	H	H
2			A_1	A_1	A_1
3				A_1	A_1
4					A_1

,

then

$$\bar{\mu}_2(0) = \bar{\mu}_2(2) = \bar{\mu}_2(3) = \bar{\mu}_2(4) = 28/125, \quad \bar{\mu}_2(1) = 13/125,$$
$$\bar{\lambda}_2(0) = \bar{\lambda}_2(2) = \bar{\lambda}_2(3) = \bar{\lambda}_2(4) = 5/125, \quad \bar{\lambda}_2(1) = 20/125.$$

Then, for any $r \geq 2$, ${}_rH \simeq_1 (H_3)$ and therefore $i.f.g.(H_3) = 2$.

(a_4) The fourth i.p.s. hypergroup H_4 has the following table

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		{0,2,3}	1	1	4
2			{0,3}	2	4
3				0	4
4					A_4

,

then

$$\bar{\mu}(0) = \bar{\mu}(3) = 37/300, \quad \bar{\mu}(1) = 75/300, \quad \bar{\mu}(2) = 55/300, \quad \bar{\mu}(4) = 96/300,$$
$$\bar{\lambda}(0) = \bar{\lambda}(3) = 240/300, \quad \bar{\lambda}(1) = 202/300, \quad \bar{\lambda}(2) = 222/300, \quad \bar{\lambda}(4) = 181/300,$$

Therefore, the associated join space ${}_0(H_4)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4
0	{0,3}	A_4	{0,2,3}	{0,3}	H
1		1	{1,2}	A_4	{1,4}
2			2	{0,2,3}	{1,2,4}
3				{0,3}	H
4					4

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(3) = 77/375, \quad \bar{\mu}_1(1) = 82/375, \quad \bar{\mu}_1(2) = 87/375, \quad \bar{\mu}_1(4) = 52/3775,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(3) = 85/375, \quad \bar{\lambda}_1(1) = 80/375, \quad \bar{\lambda}_1(2) = 75/375, \quad \bar{\lambda}_1(4) = 110/375,$$

Therefore, the associated join space ${}_1(H_4)$ is as follows:

$\circ_{\bar{\mu_1}\wedge\bar{\lambda_1}}$	0	1	2	3	4
0	{0,3}	{0,1,3}	{0,2,3}	{0,3}	A_1
1		1	A_4	{0,1,3}	H
2			2	{0,2,3}	{2,4}
3				{0,3}	A_1
4					4

,

then

$$\bar{\mu}_2(0) = \bar{\mu}_2(3) = 197/750, \quad \bar{\mu}_2(1) = 97/750, \quad \bar{\mu}_2(2) = 157/750, \quad \bar{\mu}_2(4) = 102/750,$$
$$\bar{\lambda}_2(0) = \bar{\lambda}_2(3) = 120/750, \quad \bar{\lambda}_2(1) = 220/750, \quad \bar{\lambda}_2(2) = 160/750, \quad \bar{\lambda}_2(4) = 215/750,$$

Then, for any $r \geq 2$, ${}_rH \simeq_1 (H_4)$ and therefore $i.f.g.(H_4) = 2$.

(a_5) The table of the fifth i.p.s. hypergroup H_5 is the following one

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		{0,2,3}	1	1	4
2			3	0	4
3				2	4
4					A_4

,

then

$$\bar{\mu}(0) = \bar{\mu}(2) = \bar{\mu}(3) = 43/300, \quad \bar{\mu}(1) = 75/300, \quad \bar{\mu}(4) = 96/300,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = \bar{\lambda}(3) = 240/300, \quad \bar{\lambda}(1) = 208/300, \quad \bar{\lambda}(4) = 187/300.$$

Therefore, the associated join space ${}_0(H_5)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4
0	{0,2,3}	A_4	{0,2,3}	{0,2,3}	H
1		1	A_4	A_4	{1,4}
2			{0,2,3}	{0,2,3}	H
3				{0,2,3}	H
4					4

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(2) = \bar{\mu}_1(3) = 57/250, \quad \bar{\mu}_1(1) = 47/250, \quad \bar{\mu}_1(4) = 32/250,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = 30/250, \quad \bar{\lambda}_1(1) = 40/250, \quad \bar{\lambda}_1(4) = 55/250.$$

Then, for any $r \geq 1$, ${}_rH \simeq_0 (H_5)$ and therefore $i.f.g.(H_5) = 1$.

(a_6) For the sixth i.p.s. hypergroup

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		A_1	1	1	1
2			3	{0,4}	2
3				2	3
4					0

,

we obtain

$$\bar{\mu}(0) = \bar{\mu}(4) = 13/100, \quad \bar{\mu}(1) = 32/100, \quad \bar{\mu}(2) = \bar{\mu}(3) = 21/100,$$
$$\bar{\lambda}(0) = \bar{\lambda}(4) = 80/100, \quad \bar{\lambda}(1) = 61/100, \quad \bar{\lambda}(2) = \bar{\lambda}(3) = 72/100.$$

Therefore, the associated join space ${}_0(H_6)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4
0	{0,4}	H	A_1	A_1	{0,4}
1		1	{1,2,3}	{1,2,3}	H
2			{2,3}	{2,3}	A_1
3				{2,3}	A_1
4					{0,4}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(4) = 72/375, \quad \bar{\mu}_1(1) = 47/375, \quad \bar{\mu}_1(2) = \bar{\mu}_1(3) = 92/375,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(4) = 65/375, \quad \bar{\lambda}_1(1) = 90/375, \quad \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = 45/375.$$

Therefore, the associated join space ${}_1(H_6)$ is as follows:

$\circ_{\bar{\mu}_1\wedge\bar{\lambda}_1}$	0	1	2	3	4
0	{0,4}	{0,1,4}	H	H	{0,4}
1		1	{1,2,3}	{1,2,3}	{0,1,4}
2			{2,3}	{2,3}	H
3				{2,3}	H
4					{0,4}

,

then

$$\bar{\mu}_2(0) = \bar{\mu}_2(2) = \bar{\mu}_2(3) = \bar{\mu}_2(4) = 74/375, \quad \bar{\mu}_2(1) = 79/375,$$
$$\bar{\lambda}_2(0) = \bar{\lambda}_2(2) = \bar{\lambda}_2(3) = \bar{\lambda}_2(4) = 65/375, \quad \bar{\lambda}_2(1) = 60/375.$$

We obtain the same join space that we got from the join space associated with ${}_0(H_3)$ in (a_3). Therefore, we have, ${}_rH \simeq {}_2(H_6)$ and $i.f.g.(H_6) = 3$ for any $r \geq 3$.

(a_7) Considering the i.p.s. hypergroup H_7

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		A_1	1	1	1
2			3	0	4
3				2	4
4					{0,2,3}

,

we obtain that

$$\bar{\mu}(0) = \bar{\mu}(2) = \bar{\mu}(3) = 43/300, \quad \bar{\mu}(1) = 96/300, \quad \bar{\mu}(4) = 75/300,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = \bar{\lambda}(3) = 240/300, \quad \bar{\lambda}(1) = 187/300, \quad \bar{\lambda}(4) = 208/300.$$

Therefore, the associated join space ${}_0(H_7)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4
0	{0,2,3}	H	{0,2,3}	{0,2,3}	A_1
1		1	H	H	{1,4}
2			{0,2,3}	{0,2,3}	A_1
3				{0,2,3}	A_1
4					4

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(2) = \bar{\mu}_1(3) = 57/250, \quad \bar{\mu}_1(1) = 32/250, \quad \bar{\mu}_1(4) = 47/250,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = 30/250, \quad \bar{\lambda}_1(1) = 55/250, \quad \bar{\lambda}_1(4) = 40/250.$$

Then, ${}_rH \simeq {}_0(H_7)$ for any $r \geq 1$ and therefore $i.f.g.(H_7) = 1$.

(a_8) For the last i.p.s. hypergroup of order 5

$\circ$	0	1	2	3	4
0	0	1	2	3	4
1		A_1	1	1	1
2			4	0	3
3				4	2
4					0

,

one gets

$$\bar{\mu}(0) = \bar{\mu}(2) = \bar{\mu}(3) = \bar{\mu}(4) = 17/100, \quad \bar{\mu}(1) = 32/100,$$
$$\bar{\lambda}(0) = \bar{\lambda}(2) = \bar{\lambda}(3) = \bar{\lambda}(4) = 80/100, \quad \bar{\lambda}(1) = 65/100.$$

We obtain the same join space that we got from the join space associated with H_3 in (a_3). Therefore, we have $i.f.g.(H_8) = 1$.

(iv) Now, we study the i.p.s. hypergroups of order 6, denoted by $H_1, \ldots, H_{19}$.

(b_1) For the first one H_1

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			3	4	5	{0,1}
3				5	{0,1}	2
4					2	3
5						4

,

we find

$$\bar{\mu}(0) = \bar{\mu}(1) = 4/36, \quad \bar{\mu}(2) = \bar{\mu}(3) = \bar{\mu}(4) = \bar{\mu}(5) = 7/36,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = 30/36, \quad \bar{\lambda}(2) = \bar{\mu}(3) = \bar{\lambda}(4) = \bar{\lambda}(5) = 27/36.$$

Therefore, the associated join space ${}_0(H_1)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	H	H	H	H
1		{0,1}	H	H	H	H
2			{2,3,4,5}	{2,3,4,5}	{2,3,4,5}	{2,3,4,5}
3				{2,3,4,5}	{2,3,4,5}	{2,3,4,5}
4					{2,3,4,5}	{2,3,4,5}
5						{2,3,4,5}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = 7/54, \quad \bar{\mu}_1(2) = \bar{\mu}_1(3) = \bar{\mu}_1(4) = \bar{\mu}_1(5) = 10/54,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = 6/54, \quad \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = \bar{\lambda}_1(5) = 3/54.$$

Then, ${}_rH \simeq {}_0(H_1)$ for any $r \geq 1$ and therefore $i.f.g.(H_1) = 1$.

(b_2) For H_2

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	4	3	5
2			{3,4}	5	5	{0,1}
3				1	0	2
4					1	2
5						{3,4}

,

we obtain that

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(3) = \bar{\mu}(4) = 5/36, \quad \bar{\mu}(2) = \bar{\mu}(5) = 8/36,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(3) = \bar{\lambda}(4) = 29/36, \quad \bar{\lambda}(2) = \bar{\lambda}(5) = 26/36.$$

Therefore, the associated join space ${}_0(H_2)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1,3,4}	{0,1,3,4}	H	{0,1,3,4}	{0,1,3,4}	H
1		{0,1,3,4}	H	{0,1,3,4}	{0,1,3,4}	H
2			{2,5}	H	H	{2,5}
3				{0,1,3,4}	{0,1,3,4}	H
4					{0,1,3,4}	H
5						{2,5}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(3) = \bar{\mu}_1(4) = 10/54, \quad \bar{\mu}_1(2) = \bar{\mu}_1(5) = 7/54,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = 3/54, \quad \bar{\lambda}_1(2) = \bar{\lambda}_1(5) = 6/54.$$

Then, ${}_rH \simeq {}_0(H_2)$ for any $r \geq 1$ and therefore $i.f.g.(H_2) = 1$.

(b_3) Set the third i.p.s. hypergroup H_3

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		2	0	3	4	5
2			1	3	4	5
3				{0,1,2}	5	4
4					{0,1,2}	3
5						{0,1,2}

,

for which we find that

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = 2/18, \quad \bar{\mu}(3) = \bar{\mu}(4) = \bar{\mu}(5) = 4/18,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = 15/18, \quad \bar{\lambda}(3) = \bar{\lambda}(4) = \bar{\lambda}(5) = 13/18.$$

Therefore, the associated join space ${}_0(H_3)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1,2}	{0,1,2}	{0,1,2}	H	H	H
1		{0,1,2}	{0,1,2}	H	H	H
2			{0,1,2}	H	H	H
3				{3,4,5}	{3,4,5}	{3,4,5}
4					{3,4,5}	{3,4,5}
5						{3,4,5}

,

From this we have: for any $u \in H, \bar{\mu}_1(u) = 2/12, \bar{\lambda}_1(u) = 1/12$, that is, ${}_1(H_3)$ is a total hypergroup of order 6 and thus $i.f.g.(H_3) = 2$.

(b_4) Let us consider H_4 as

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		2	0	3	4	5
2			1	3	4	5
3				5	{0,1,2}	4
4					5	3
5						{0,1,2}

,

then

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = 2/18, \quad \bar{\mu}(3) = \bar{\mu}(4) = \bar{\mu}(5) = 4/18,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = 15/18, \quad \bar{\lambda}(3) = \bar{\lambda}(4) = \bar{\lambda}(5) = 13/18.$$

whence we obtain the same membership functions as in the previous case. So, $i.f.g.(H_4) = 2$.

(b_5) Let H_5 be the following i.p.s. hypergroup

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		2	3	0	4	5
2			0	1	4	5
3				2	4	5
4					{0,1,2,3}	5
5						A_5

,

then

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = \bar{\mu}(3) = 89/720, \quad \bar{\mu}(4) = 164/720, \quad \bar{\mu}(5) = 200/720,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = \bar{\lambda}(3) = 600/720, \quad \bar{\lambda}(4) = 525/720, \quad \bar{\lambda}(5) = 489/720.$$

Therefore, the associated join space ${}_0(H_5)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	A_5	H
1		{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	A_5	H
2			{0,1,2,3}	{0,1,2,3}	A_5	H
3				{0,1,2,3}	A_5	H
4					4	{4,5}
5						5

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(2) = \bar{\mu}_1(3) = 104/540, \quad \bar{\mu}_1(4) = 74/540, \quad \bar{\mu}_1(5) = 50/540,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = 45/540, \quad \bar{\lambda}_1(4) = 75/540, \quad \bar{\lambda}_1(5) = 99/540.$$

Then, ${}_rH \simeq {}_0(H_5)$ for any $r \geq 1$ and therefore $i.f.g.(H_5) = 1$.

(b_6) For the sixth i.p.s. hypergroup H_6 giving by the following table

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	3	2	4	5
2			0	1	4	5
3				0	4	5
4					{0,1,2,3}	5
5						A_5

,

we obtain that

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = \bar{\mu}(3) = 89/720, \quad \bar{\mu}(4) = 164/720, \quad \bar{\mu}(5) = 200/720,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = \bar{\lambda}(3) = 600/720, \quad \bar{\lambda}(4) = 525/720, \quad \bar{\lambda}(5) = 489/720.$$

whence we calculate the same membership functions as in the previous case and thus $i.f.g.(H_6) = 1$.

(b_7) If the seventh i.p.s. hypergroup H_7 is represented by the following table

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		2	3	0	4	5
2			0	1	4	5
3				2	4	5
4					5	{0,1,2,3}
5						4

,

then

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = \bar{\mu}(3) = 9/72, \quad \bar{\mu}(4) = \bar{\mu}(5) = 18/72,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = \bar{\lambda}(3) = 60/72, \quad \bar{\lambda}(4) = \bar{\lambda}(5) = 51/72.$$

Therefore, the associated join space ${}_0(H_7)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	H	H
1		{0,1,2,3}	{0,1,2,3}	{0,1,2,3}	H	H
2			{0,1,2,3}	{0,1,2,3}	H	H
3				{0,1,2,3}	H	H
4					{4,5}	{4,5}
5						{4,5}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(2) = \bar{\mu}_1(3) = 10/54, \quad \bar{\mu}_1(4) = \bar{\mu}_1(5) = 7/54,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = 3/54, \quad \bar{\lambda}_1(4) = \bar{\lambda}_1(5) = 6/54.$$

Then, for any $r \geq 1$, ${}_rH \simeq {}_0(H_7)$ and therefore $i.f.g.(H_7) = 1$.

(b_8) Similarly, for the eight i.p.s. hypergroup H_8

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	3	2	4	5
2			0	1	4	5
3				0	4	5
4					5	{0,1,2,3}
5						4

,

we obtain that

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = \bar{\mu}(3) = 9/72, \quad \bar{\mu}(4) = \bar{\mu}(5) = 18/72,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = \bar{\lambda}(3) = 60/72, \quad \bar{\lambda}(4) = \bar{\lambda}(5) = 51/72.$$

That is, the same membership functions as in the previous case. So, $i.f.g.(H_8) = 1$.

(b_9) Set the ninth i.p.s. hypergroup H_9

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		2	0	3	4	5
2			1	3	4	5
3				4	{0,1,2}	5
4					3	5
5						A_5

,

for which we calculate

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = 58/540, \quad \bar{\mu}(3) = \bar{\mu}(4) = 108/540, \quad \bar{\mu}(5) = 150/540,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = 450/540, \quad \bar{\lambda}(3) = \bar{\lambda}(4) = 400/540, \quad \bar{\lambda}(5) = 358/540.$$

Therefore, the associated join space ${}_0(H_9)$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1,2}	{0,1,2}	{0,1,2}	A_5	A_5	H
1		{0,1,2}	{0,1,2}	A_5	A_5	H
2			{0,1,2}	A_5	A_5	H
3				{3,4}	{3,4}	{3,4,5}
4					{3,4}	{3,4,5}
5						5

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(2) = 114/540, \quad \bar{\mu}_1(3) = \bar{\mu}_1(4) = 119/540, \quad \bar{\mu}_1(5) = 50/540,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(2) = 65/540, \quad \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = 60/540, \quad \bar{\lambda}_1(5) = 129/540.$$

Then, ${}_rH \simeq {}_0(H_9)$ for any $r \geq 1$ and therefore $i.f.g.(H_9) = 1$.

(b_{10}) Let us consider H_{10} as the following hypergroup

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		2	0	3	4	5
2			1	3	4	5
3				{0,1,2}	4	5
4					5	{0,1,2,3}
5						4

,

then

$$\bar{\mu}(0) = \bar{\mu}(1) = \bar{\mu}(2) = 23/216, \quad \bar{\mu}(3) = 39/216, \quad \bar{\mu}(4) = \bar{\mu}(5) = 54/216,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = \bar{\lambda}(2) = 180/216, \quad \bar{\lambda}(3) = 164/216, \quad \bar{\lambda}(4) = \bar{\lambda}(5) = 149/216.$$

Therefore, the associated join space ${}_0(H_{10})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1,2}	{0,1,2}	{0,1,2}	{0,1,2,3}	H	H
1		{0,1,2}	{0,1,2}	{0,1,2,3}	H	H
2			{0,1,2}	{0,1,2,3}	H	H
3				3	{3,4,5}	{3,4,5}
4					{4,5}	{4,5}
5						{4,5}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(2) = 45/216, \quad \bar{\mu}_1(3) = 41/216, \quad \bar{\mu}_1(4) = \bar{\mu}_1(5) = 38/216,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(2) = 26/216, \quad \bar{\lambda}_1(3) = 30/216, \quad \bar{\lambda}_1(4) = \bar{\lambda}_1(5) = 33/216.$$

Then, ${}_rH \simeq {}_0(H_{10})$ for any $r \geq 1$ and therefore $i.f.g.(H_{10}) = 1$.

(b_{11}) For the i.p.s. hypergroup H_{11}

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			{0,1}	4	3	5
3				{0,1}	2	5
4					{0,1}	5
5						A_5

,

we calculate

$$\bar{\mu}(0) = \bar{\mu}(1) = 37/360, \quad \bar{\mu}(2) = \bar{\mu}(3) = \bar{\mu}(4) = 62/360, \quad \bar{\mu}(5) = 100/360,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = 300/360, \quad \bar{\lambda}(2) = \bar{\lambda}(3) = \bar{\lambda}(4) = 275/360, \quad \bar{\lambda}(5) = 237/360.$$

Therefore, the associated join space ${}_0(H_{11})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	A_5	A_5	A_5	H
1		{0,1}	A_5	A_5	A_5	H
2			{2,3,4}	{2,3,4}	{2,3,4}	{2,3,4,5}
3				{2,3,4}	{2,3,4}	{2,3,4,5}
4					{2,3,4}	{2,3,4,5}
5						5

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = 152/1080, \quad \bar{\mu}_1(2) = \bar{\mu}_1(3) = \bar{\mu}_1(4) = 227/1080, \quad \bar{\mu}_1(5) = 95/1080,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = 165/1080, \quad \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = 90/1080, \quad \bar{\lambda}_1(5) = 222/1080.$$

Therefore, the second associated join space ${}_1(H_{11})$ is as follows:

$\circ_{\bar{\mu}_1\wedge\bar{\lambda}_1}$	0	1	2	3	4	5
0	{0,1}	{0,1}	H	H	H	{0,1,5}
1		{0,1}	H	H	H	{0,1,5}
2			{2,3,4}	{2,3,4}	{2,3,4}	{2,3,4,5}
3				{2,3,4}	{2,3,4}	{2,3,4,5}
4					{2,3,4}	{2,3,4,5}
5						5

,

then

$$\bar{\mu}_2(0) = \bar{\mu}_2(1) = 32/216, \quad \bar{\mu}_2(2) = \bar{\mu}_2(3) = \bar{\mu}_2(4) = 39/216, \quad \bar{\mu}_2(5) = 35/216,$$
$$\bar{\lambda}_2(0) = \bar{\lambda}_2(1) = 33/216, \quad \bar{\lambda}_2(2) = \bar{\lambda}_2(3) = \bar{\lambda}_2(4) = 26/216, \quad \bar{\lambda}_2(5) = 30/216.$$

Then, ${}_rH \simeq {}_1(H_{11})$, for any $r \geq 2$ and therefore $i.f.g.(H_{11}) = 2$.

(b_{12}) If we take the i.p.s. hypergroup H_{12} as

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			4	{0,1}	3	5
3				4	2	5
4					{0,1}	5
5						A_5

,

then we obtain the same membership functions as in the previous case and again $i.f.g.(H_{12}) = 2$.

(b_{13}) For the thirteenth i.p.s. hypergroup H_{13}

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			3	{0,1}	4	5
3				2	4	5
4					{0,1,2,3}	5
5						A_5

,

we get

$$\begin{array}{llll} \bar{\mu}(0)=\bar{\mu}(1)=69/720, & \bar{\mu}(2)=\bar{\mu}(3)=109/720, & \bar{\mu}(4)=164/720, & \bar{\mu}(5)=200/720, \\ \bar{\lambda}(0)=\bar{\lambda}(1)=600/720, & \bar{\lambda}(2)=\bar{\lambda}(3)=560/720, & \bar{\lambda}(4)=505/720, & \bar{\lambda}(5)=469/720. \end{array}$$

Therefore, the associated join space $_{0}(H_{13})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2,3}	{0,1,2,3}	A_5	H
1		{0,1}	{0,1,2,3}	{0,1,2,3}	A_5	H
2			{2,3}	{2,3}	{2,3,4}	{2,3,4,5}
3				{2,3}	{2,3,4}	{2,3,4,5}
4					4	{4,5}
5						5

,

then

$$\begin{array}{llll} \bar{\mu}_1(0)=\bar{\mu}_1(1)=82/540, & \bar{\mu}_1(2)=\bar{\mu}_1(3)=117/540, & \bar{\mu}_1(4)=87/540, & \bar{\mu}_1(5)=55/540, \\ \bar{\lambda}_1(0)=\bar{\lambda}_1(1)=110/540, & \bar{\lambda}_1(2)=\bar{\lambda}_1(3)=75/540, & \bar{\lambda}_1(4)=105/540, & \bar{\lambda}_1(5)=137/540. \end{array}$$

Therefore, the second associated join space $_{1}(H_{13})$ is as follows:

$\circ_{\bar{\mu}_1\wedge\bar{\lambda}_1}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2,3}	{0,1,2,3}	{0,1,4}	A_4
1		{0,1}	{0,1,2,3}	{0,1,2,3}	{0,1,4}	A_4
2			{2,3}	{2,3}	A_5	{2,3,5}
3				{2,3}	A_5	{2,3,5}
4					4	H
5						5

,

then

$$\begin{array}{ll} \bar{\mu}_2(0)=\bar{\mu}_2(1)=\bar{\mu}_2(2)=\bar{\mu}_2(3)=109/540, & \bar{\mu}_2(4)=\bar{\mu}_2(5)=52/540, \\ \bar{\lambda}_2(0)=\bar{\lambda}_2(1)=\bar{\lambda}_2(2)=\bar{\lambda}_2(3)=80/540, & \bar{\lambda}_2(4)=\bar{\lambda}_2(5)=137/540. \end{array}$$

We obtain the same join space that we got from the join space associated with H_7 in (b_7). Therefore, we have $_rH \simeq {}_1(H_{13})$, for any $r \geq 2$ and $i.f.g.(H_{13}) = 2$.

(b_{14}) Taking the i.p.s. hypergroup H_{14}

∘	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			3	{0,1}	4	5
3				2	4	5
4					5	{0,1,2,3}
5						4

,

we calculate

$$\bar{\mu}(0) = \bar{\mu}(1) = 7/72, \quad \bar{\mu}(2) = \bar{\mu}(3) = 11/72, \quad \bar{\mu}(4) = \bar{\mu}(5) = 18/72,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = 60/72, \quad \bar{\lambda}(2) = \bar{\lambda}(3) = 56/72, \quad \bar{\lambda}(4) = \bar{\lambda}(5) = 49/72.$$

Therefore, the associated join space ${}_0(H_{14})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2,3}	{0,1,2,3}	H	H
1		{0,1}	{0,1,2,3}	{0,1,2,3}	H	H
2			{2,3}	{2,3}	{2,3,4,5}	{2,3,4,5}
3				{2,3}	{2,3,4,5}	{2,3,4,5}
4					{4,5}	{4,5}
5						{4,5}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = \bar{\mu}_1(4) = \bar{\mu}_1(5) = 8/54, \quad \bar{\mu}_1(2) = \bar{\mu}_1(3) = 11/54,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = \bar{\lambda}_1(4) = \bar{\lambda}_1(5) = 9/54, \quad \bar{\lambda}_1(2) = \bar{\lambda}_1(3) = 6/54.$$

Therefore, the second associated join space ${}_1(H_{14})$ is as follows:

$\circ_{\bar{\mu_1}\wedge\bar{\lambda_1}}$	0	1	2	3	4	5
0	{0,1,4,5}	{0,1,4,5}	H	H	{0,1,4,5}	{0,1,4,5}
1		{0,1,4,5}	H	H	{0,1,4,5}	{0,1,4,5}
2			{2,3}	{2,3}	H	H
3				{2,3}	H	H
4					{0,1,4,5}	{0,1,4,5}
5						{0,1,4,5}

,

then

$$\bar{\mu}_2(0) = \bar{\mu}_2(1) = \bar{\mu}_2(4) = \bar{\mu}_2(5) = 10/54, \quad \bar{\mu}_2(2) = \bar{\mu}_2(3) = 7/54,$$
$$\bar{\lambda}_2(0) = \bar{\lambda}_2(1) = \bar{\lambda}_2(4) = \bar{\lambda}_2(5) = 3/54, \quad \bar{\lambda}_2(2) = \bar{\lambda}_2(3) = 6/54.$$

Then, ${}_rH \simeq {}_1(H_{14})$, for any $r \geq 2$ and so $i.f.g.(H_{14}) = 2$.

(b_{15}) For the i.p.s. hypergroup H_{15}

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			{0,1}	3	4	5
3				{0,1,2}	5	4
4					{0,1,2}	3
5						{0,1,2}

,

we obtain that

$$\bar{\mu}(0) = \bar{\mu}(1) = 7/72, \quad \bar{\mu}(2) = 10/72, \quad \bar{\mu}(3) = \bar{\mu}(4) = \bar{\mu}(5) = 16/72,$$
$$\bar{\lambda}(0) = \bar{\lambda}(1) = 60/72, \quad \bar{\lambda}(2) = 57/72, \quad \bar{\lambda}(3) = \bar{\lambda}(4) = \bar{\lambda}(5) = 51/72.$$

Therefore, the associated join space ${}_0(H_{15})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2}	H	H	H
1		{0,1}	{0,1,2}	H	H	H
2			2	{2,3,4,5}	{2,3,4,5}	{2,3,4,5}
3				{3,4,5}	{3,4,5}	{3,4,5}
4					{3,4,5}	{3,4,5}
5						{3,4,5}

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = 32/216, \quad \bar{\mu}_1(2) = 35/216, \quad \bar{\mu}_1(3) = \bar{\mu}_1(4) = \bar{\mu}_1(5) = 39/216,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = 33/216, \quad \bar{\lambda}_1(2) = 30/216, \quad \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = \bar{\lambda}_1(5) = 26/216.$$

Then, ${}_rH \simeq {}_0(H_{15})$, for any $r \geq 1$ and so $i.f.g.(H_{15}) = 1$.

(b_{16}) Similarly for H_{16}

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			{0,1}	3	4	5
3				5	{0,1,2}	4
4					5	3
5						{0,1,2}

,

we obtain the same membership functions as in the previous case and therefore $i.f.g.(H) = 1$.

(b_{17}) Let us consider H_{17} given by the following table

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			{0,1}	3	4	5
3				{0,1,2}	4	5
4					{0,1,2,3}	5
5						A_5

,

then

$\bar{\mu}(0) = \bar{\mu}(1) = 197/2160$, $\bar{\mu}(2) = 287/2160$, $\bar{\mu}(3) = 387/2160$, $\bar{\mu}(4) = 492/2160$,
$\bar{\lambda}(0) = \bar{\lambda}(1) = 1800/2160$, $\bar{\lambda}(2) = 1710/2160$, $\bar{\lambda}(3) = 1610/2160$, $\bar{\lambda}(4) = 1505/2160$,
$\bar{\mu}(5) = 600/2160$, $\bar{\lambda}(5) = 1397/2160$.

Therefore, the associated join space ${}_0(H_{17})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2}	{0,1,2,3}	A_5	H
1		{0,1}	{0,1,2}	{0,1,2,3}	A_5	H
2			2	{2,3}	{2,3,4}	{2,3,4,5}
3				3	{3,4}	{3,4,5}
4					4	{4,5}
5						5

,

then

$\bar{\mu}_1(0)=\bar{\mu}_1(1)=348/2160$, $\bar{\mu}_1(2)=418/2160$, $\bar{\mu}_1(3)=438/2160$, $\bar{\mu}_1(4)=378/2160$,
$\bar{\lambda}_1(0)=\bar{\lambda}_1(1)=530/2160$, $\bar{\lambda}_1(2)=460/2160$, $\bar{\lambda}_1(3)=440/2160$, $\bar{\lambda}_1(4)=500/2160$,
$\bar{\mu}_1(5)=230/2160$, $\bar{\lambda}_1(5)=648/2160$.

Therefore, the associated join space ${}_1(H_{17})$ is as follows:

$\circ_{\bar{\mu}_1\wedge\bar{\lambda}_1}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2,4}	A_5	{0,1,4}	{0,1,5}
1		{0,1}	{0,1,2,4}	A_5	{0,1,4}	{0,1,5}
2			2	{2,3}	{2,4}	A_3
3				3	{2,3,4}	H
4					4	{0,1,4,5}
5						5

,

then

$\bar{\mu}_2(0)=\bar{\mu}_2(1)=231/1080$, $\bar{\mu}_2(2)=186/1080$, $\bar{\mu}_2(3)=114/1080$, $\bar{\mu}_2(4)=211/1080$,
$\bar{\lambda}_2(0)=\bar{\lambda}_2(1)=200/1080$, $\bar{\lambda}_2(2)=245/1080$, $\bar{\lambda}_2(3)=317/1080$, $\bar{\lambda}_2(4)=220/1080$,
$\bar{\mu}_2(5)=107/1080$, $\bar{\lambda}_2(5)=324/1080$.

Then, ${}_rH \simeq {}_1(H_{17})$, for any $r \geq 2$ and therefore $i.f.g.(H_{17}) = 2$.

(b_{18}) Let us consider the following i.p.s. hypergroup H_{18}

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			{0,1}	3	4	5
3				4	{0,1,2,5}	3
4					3	4
5						{0,1,2}

,

for which we find

$\bar{\mu}(0)=\bar{\mu}(1)=20/216$, $\bar{\mu}(2)=29/216$, $\bar{\mu}(3)=\bar{\mu}(4)=54/216$, $\bar{\mu}(5)=39/216$,
$\bar{\lambda}(0)=\bar{\lambda}(1)=180/216$, $\bar{\lambda}(2)=171/216$, $\bar{\lambda}(3)=\bar{\lambda}(4)=146/216$, $\bar{\lambda}(5)=161/216$.

Therefore, the associated join space ${}_0(H_{18})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2}	H	H	{0,1,2,5}
1		{0,1}	{0,1,2}	H	H	{0,1,2,5}
2			2	{2,3,4,5}	{2,3,4,5}	{2,5}
3				{3,4}	{3,4}	{3,4,5}
4					{3,4}	{3,4,5}
5						5

,

then

$$\bar{\mu}_1(0)=\bar{\mu}_1(1)=\bar{\mu}_1(3)=\bar{\mu}_1(4)=17/108, \quad \bar{\mu}_1(2)=\bar{\mu}_1(5)=20/108,$$
$$\bar{\lambda}_1(0)=\bar{\lambda}_1(1)=\bar{\lambda}_1(3)=\bar{\lambda}_1(4)=22/108, \quad \bar{\lambda}_1(2)=\bar{\lambda}_1(5)=19/108.$$

We obtain the same join space that we got from the join space associated with H in (b_2). Therefore, we have, ${}_rH \simeq {}_1(H_{18})$, for any $r \geq 2$ and $i.f.g.(H_{18})=2$.

(b_{19}) Finally, for the i.p.s. hypergroup H_{19}

$\circ$	0	1	2	3	4	5
0	0	1	2	3	4	5
1		0	2	3	4	5
2			{0,1}	3	4	5
3				4	{0,1,2}	5
4					3	5
5						A_5

,

we calculate

$\bar{\mu}(0)=\bar{\mu}(1)=101/1080$, $\bar{\mu}(2)=146/1080$, $\bar{\mu}(3)=\bar{\mu}(4)=216/1080$, $\bar{\mu}(5)=300/1080$,
$\bar{\lambda}(0)=\bar{\lambda}(1)=900/1080$, $\bar{\lambda}(2)=855/1080$, $\bar{\lambda}(3)=\bar{\lambda}(4)=785/1080$, $\bar{\lambda}(5)=701/1080$.

Therefore, the associated join space ${}_0(H_{19})$ is as follows:

$\circ_{\bar{\mu}\wedge\bar{\lambda}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2}	A_5	A_5	H
1		{0,1}	{0,1,2}	A_5	A_5	H
2			2	{2,3,4}	{2,3,4}	{2,3,4,5}
3				{3,4}	{3,4}	{3,4,5}
4					{3,4}	{3,4,5}
5						5

,

then

$$\bar{\mu}_1(0) = \bar{\mu}_1(1) = 168/1080, \quad \bar{\mu}_1(2) = 193/1080, \quad \bar{\mu}_1(3) = \bar{\mu}_1(4) = 223/1080,$$
$$\bar{\lambda}_1(0) = \bar{\lambda}_1(1) = 215/1080, \quad \bar{\lambda}_1(2) = 190/1080, \quad \bar{\lambda}_1(3) = \bar{\lambda}_1(4) = 160/1080,$$
$$\bar{\mu}_1(5) = 105/1080, \quad \bar{\lambda}_1(5) = 278/1080.$$

Therefore, the associated join space ${}_1(H_{19})$ is as follows:

$\circ_{\bar{\mu_1}\wedge\bar{\lambda_1}}$	0	1	2	3	4	5
0	{0,1}	{0,1}	{0,1,2}	{0,1,3,4}	{0,1,3,4}	A_2
1		{0,1}	{0,1,2}	{0,1,3,4}	{0,1,3,4}	A_2
2			2	A_5	A_5	H
3				{3,4}	{3,4}	{3,4,5}
4					{3,4}	{3,4,5}
5						5

,

then

$$\bar{\mu}_2(0) = \bar{\mu}_2(1) = \bar{\mu}_2(3) = \bar{\mu}_2(4) = 109/540, \quad \bar{\mu}_2(2) = \bar{\mu}_2(5) = 52/540,$$
$$\bar{\lambda}_2(0) = \bar{\lambda}_2(1) = \bar{\lambda}_2(3) = \bar{\lambda}_2(4) = 80/540, \quad \bar{\lambda}_2(2) = \bar{\lambda}_2(5) = 137/540.$$

We obtain the same join space that we got from the join space associated with H in (b_2). Therefore, we have, , ${}_rH \simeq {}_2(H_{19})$, for any $r \geq 3$ and thus $i.f.g.(H_{19}) = 3$.

5. **Conclusions and Future Work**

Corsini [9] has associated with any hypergroupoid H a sequence of join spaces and fuzzy sets. The length of this sequence is called the (strong) fuzzy grade of H. Similarly, Cristea and Davvaz [18] have introduced and studied the notion of intuitionistic fuzzy grade of a hypergroup. It is interesting to do a comparison between these two grades for special classes of hypergroups: for example we have investigated the i.p.s. hypergroups of order less than or equal to 6. Corsini and Cristea [11] have determined the fuzzy grade of these hypergroups and in this paper we have calculated their Atanassov's intuitionistic fuzzy grade.

It is obvious that the greater the (intuitionistic) fuzzy grade of a hypergroup is, the more interesting its structure is. We can conclude that there exist more i.p.s. hypergroups of order $4, 5$ or 6 with Atanassov's intuitionistic fuzzy grade greater than 1, than those with the fuzzy grade greater than 1, as we can observe in the following table:

card H	number of the i.p.s. hypergroups with s.f.g ≥ 2	number of the i.p.s. hypergroups with i.f.g ≥ 2
3	0	0
4	1	2
5	1	4
6	5	9

In a future work we will extend our research to the case of i.p.s. hypergroups of order 7 and 8 and to other classes of hypergroups, such as: the complete hypergroups or the non complete 1-hypergroups.

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