Iranian Journal of Fuzzy Systems **Vol. 10, No. 1,** (2013) pp. 61-74

FUZZY GOAL PROGRAMMING TECHNIQUE TO SOLVE MULTIOBJECTIVE TRANSPORTATION PROBLEMS WITH SOME NON-LINEAR MEMBERSHIP FUNCTIONS

M. ZANGIABADI AND H. R. MALEKI

Abstract. The linear multiobjective transportation problem is a special type of vector minimum problem in which constraints are all equality type and the objectives are conflicting in nature. This paper presents an application of fuzzy goal programming to the linear multiobjective transportation problem. In this paper, we use a special type of nonlinear (hyperbolic and exponential) membership functions to solve multiobjective transportation problem. It gives an optimal compromise solution. The obtained result has been compared with the solution obtained by using a linear membership function. To illustrate the methodology some numerical examples are presented.

1. Introduction

The transportation problem is one of the earliest applications of linear programming problems. The basic transportation problem was originally developed by Hitchcock [17]. Subsequently, several kinds of transportation problems have appeared in the literature [4, 19, 22, 27]. Efficient methods of solution derived from the simplex algorithm were developed in 1947, primarily by Dantzig [9] and then by Charnes et al. [7]. The transportation problem can be modeled as a standard linear programming problem, which can then be solved by the simplex method. Lee et al. [22] have studied the optimization of transportation problems with multiple objectives. Diaz [12] and Isermann [18] have developed algorithms for identifying all of the nondominated solutions for a linear multiobjective transportation problem. Current et al. [8] have done a review of multiobjective design of transportation networks. Diaz [11] presents an alternative procedure to generate all nondominated solutions to the multiobjective transportation problem. This approach depends upon specifying a priori measure of the closeness of any compromise solution to the ideal solution. Edwards [13] advocates the use of an additive multiattribute utility function for the linear multiobjective problem. An additive linear function may provide a good initial solution for the linear multiobjective problem. Ringuest et al. [27] have developed two interactive algorithms for the linear multiobjective transportation problem. Bit et al. [3] applied the fuzzy programming technique with linear membership function to solve multiobjective transportation problem

Received: February 2011; Revised: December 2011; Accepted: May 2012

Key words and phrases: Multiobjective decision making, Goal programming, Transportation problem, Membership function, Fuzzy programming.

(MOTP), and obtained efficient solutions for MOTP as well as an optimal compromise solution. Li et al. [23] presented a fuzzy approach to the MOTP. Verma et al. [30] applied a fuzzy programming technique to solve MOTP with some nonlinear membership functions. Other research works in this realm presented in [20, 26].

In 1961, goal programming introduced by Charnes and Cooper [6]. Aenaida and Kwak [2] applied goal programming to find a solution for MOTP. Recently, the authors used the fuzzy goal programming approach to solve MOTP [32]. Other authors used fuzzy goal programming technique to solve different types of multiobjective linear programming problems [1, 14, 15, 16, 24, 25, 29, 31].

Leberling [21] used hyperbolic membership function for the multiobjective linear programming problem. He showed that solutions obtained by fuzzy linear programming with this type of nonlinear membership function are always efficient. Dhingra and Moskowitz [10] defined other types of the nonlinear (exponential, quadratic and logarithmic) membership functions and applied them to an optimal design problem. Peidro and Vasant [26] adopted a nonlinear membership function known as the modified S-curve. They used modified S-curve membership function to apply an interactive fuzzy approach to solve the multi-objective transportation planning decision problem for the purpose of finding a preferred compromise solution. We apply the fuzzy goal programming technique with some nonlinear (hyperbolic and exponential) membership functions to solve multiobjective transportation problems. The paper has the following structure. Section 2 reviews the problem formulation. In section 3, we study some nonlinear (hyperbolic and exponential) membership functions. Section 4 uses fuzzy goal programming approach with some nonlinear (hyperbolic and exponential) membership functions to solve MOTP. In section 5, some examples are presented.

2. Problem Formulation

In the real-world situations, the transportation problem usually involves multiple, incommensurable and conflicting objective functions. This kind of problem is called multiobjective transportation problem. Similar to a typical transportation problem in a MOTP a product is to be transported from m sources to n destinations and their capacities are $a_1, a_2, ..., a_m$ and $b_1, b_2, ..., b_n$, respectively. In addition, there is a penalty c_{ij} associated with transporting a unit of product from source i to destination j. The penalty may be cost or delivery time or safety of delivery or etc. A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j. A mathematical model of MOTP can be writhen as follows:

$$\begin{aligned}
\min\ & Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{r} x_{ij}, \quad r = 1, 2, \cdots, k, \\
\text{s.t.} & \\
& \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \cdots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \cdots, n, \\
& x_{ij} \geq 0, \quad \text{for all } i, j.
\end{aligned} \tag{1}$$

The subscript on Z_r and superscript on c_{ij}^r are related to the rth penalty criterion. Without loss of generality, it will be assumed that $a_i \geq 0$ for all i, $b_j \geq 0$ for all j and the equilibrium condition $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ is satisfied.

We denote by **S** the set of all feasible solutions of the MOTP, i.e.,

$$\mathbf{S} = \{\mathbf{x} \in \mathbb{R}^{m\times n} \mid \sum_{j=1}^{n} x_{ij} = a_i,\ \sum_{i=1}^{m} x_{ij} = b_j,\ x_{ij} \geq 0,$$
$$i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n\}.$$

We review the concept of optimality for MOTP as usual manner.

Definition 2.1. A feasible solution $\mathbf{x}^* = \{x_{ij}{}^*\} \in \mathbf{S}$ is an efficient (nondominated) solution for MOTP if and only if there does not exist another $\mathbf{x} = \{x_{ij}\} \in \mathbf{S}$ such that $Z_r(\mathbf{x}) \leq Z_r(\mathbf{x}^*),\ r = 1, 2, ..., k$, and $Z_l(\mathbf{x}) \neq Z_l(\mathbf{x}^*)$, for some $l, 1 \leq l \leq k$.

Definition 2.2. A feasible solution $\mathbf{x}^* = \{x_{ij}{}^*\} \in \mathbf{S}$ is a weak efficient solution for MOTP if and only if there does not exist another $\mathbf{x} = \{x_{ij}\} \in \mathbf{S}$ such that $Z_r(\mathbf{x}) < Z_r(\mathbf{x}^*),\ r = 1, 2, ..., k$.

Definition 2.3. [3] A feasible solution $\mathbf{x}^* = \{x_{ij}{}^*\} \in \mathbf{S}$ is an optimal compromise solution for MOTP if it is preferred by DM to all other feasible solutions, taking into consideration all criteria contained in the multiobjective functions.

Let E and E^w denote the set of all efficient solutions and all weak efficient solutions for MOTP, respectively, then $E \subseteq E^w$. Note that an optimal compromise solution of MOTP has to be a weak efficient solution of MOTP, according to the definition of weak efficient solutions.

3. Membership Functions

One of the major assumptions in solving fuzzy mathematical programming problems in the literature involves the use of linear membership functions for all fuzzy sets involved in a decision making process. A linear approximation is most commonly used because of its simplicity and is defined by fixing two points, the upper and lower levels of acceptability. If fuzzy set theory is to be considered a purely formal theory, such an assumption is acceptable, even though some kind of formal justification of this assumption would be desirable. If, however, fuzzy set theory is used to model real decision making processes, and an assertion is made that the resulting models are true models of reality, then some kind of empirical justification for this assumption is necessary. In view of this, several other (nonlinear) shapes for membership functions, such as concave or convex shaped membership functions are analyzed to determine their impact on the overall design process.

Let L_r and U_r be the aspired level of achievement and the highest acceptable level of achievement for the r-th objective function, respectively. In the following three subsections we study different membership functions.

3.1. **Linear Membership Function.** A linear membership function can be defined as follows.

$$\mu_r(Z_r(\mathbf{x})) = \begin{cases} 1 & \text{if } Z_r \le L_r, \\ 1 - \frac{Z_r - L_r}{U_r - L_r} & \text{if } L_r < Z_r < U_r, \\ 0 & \text{if } Z_r \ge U_r. \end{cases} \quad (2)$$

3.2. **Exponential Membership Function.** An exponential membership function is defined by

$$\mu_r^E(Z_r(\mathbf{x})) = \begin{cases} 1 & \text{if } Z_r \le L_r, \\ \frac{e^{-s\psi_r(x)} - e^{-s}}{1 - e^{-s}} & \text{if } L_r \le Z_r \le U_r, \\ 0 & \text{if } Z_r \ge U_r, \end{cases} \quad (3)$$

where $\psi_r(x) = (Z_r - L_r)/(U_r - L_r),\ r = 1, 2, \ldots, k$ and s is a non-zero parameter prescribed by the decision maker. Figure 1 depicts a possible shape of $\mu_r^E(Z_r(\mathbf{x}))$ with respect to $Z_r(\mathbf{x})$ [10].

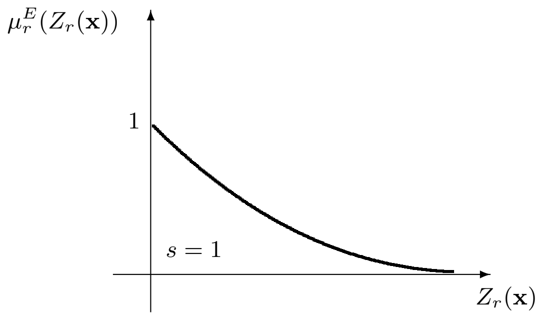

FIGURE 1. Exponential Membership Function

3.3. **Hyperbolic Membership Function.** The hyperbolic function [28] is convex over a part of the objective function values and is concave over the remaining part. The rationale for such a shape has been discussed in [5] for utility functions and in our problem context is as follows: When the decision maker is worse off with respect to a goal, the decision maker tends to have a higher marginal rate of satisfaction with respect to that goal. A convex shape captures that behavior in the membership function. On the other hand, when one is better off with respect to a goal, one tends to have a smaller marginal rate of satisfaction. Such behavior is modeled using the concave portion of the membership function. The complete function is as follows:

$$\mu_r^H(Z_r(\mathbf{x})) = \begin{cases} 1 & \text{if } Z_r \le L_r, \\ \frac{1}{2}\tanh(\frac{U_r + L_r}{2} - Z_r(\mathbf{x})\}\alpha_r) + \frac{1}{2} & \text{if } L_r \le Z_r \le U_r, \\ 0 & \text{if } Z_r \ge U_r, \end{cases} \quad (4)$$

where $\alpha_r = \frac{6}{U_r - L_r}$.

This membership function has the following formal properties [33]:

(1) $\mu_r^H(Z_r(\mathbf{x}))$ is strictly monotonously decreasing function with respect to $Z_r(\mathbf{x})$;

(2) $\mu_r^H(Z_r(\mathbf{x})) = \frac{1}{2} \Leftrightarrow Z_r(\mathbf{x}) = \frac{1}{2}(U_r + L_r)$;

(3) $\mu_r^H(Z_r(\mathbf{x}))$ is strictly convex for $Z_r(\mathbf{x}) \geq \frac{1}{2}(U_r + L_r)$ and strictly concave for $Z_r(\mathbf{x}) \leq \frac{1}{2}(U_r + L_r)$;

(4) $\mu_r^H(Z_r(\mathbf{x}))$ satisfies $0 < \mu_r^H(Z_r(\mathbf{x})) < 1$ for $L_r < Z_r(\mathbf{x})) < U_r$ and approaches asymptotically $\mu_r^H(Z_r(\mathbf{x})) = 0$ and $\mu_r^H(Z_r(\mathbf{x})) = 1$ as $Z_r(\mathbf{x}) \to \infty$ and $-\infty$, respectively.

Figure 2 depicts a possible shape of $\mu_r^H(Z_r(\mathbf{x}))$ with respect to $Z_r(\mathbf{x})$ [28].

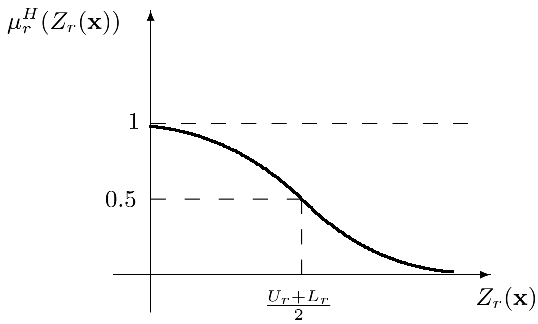

FIGURE 2. Hyperbolic Membership Function

4. Fuzzy Goal Programming Approach for Solving MOTP

Mohamed in [24] has used linear membership functions, where he introduced fuzzy goal programming approach for solving multiobjective linear programming problem. In [32], mohamed's approach was adopted to present a fuzzy goal programming approach for solving multiobjective transportation problems.

Let L_r and U_r be the aspired level of achievement and the highest acceptable level of achievement for the r-th objective function, respectively.

To solve MOTP problem based on the fuzzy goal programming technique [32], one can use the following steps:

Step 1: Solve the multiobjective transportation problem as a single objective transportation problem, taking each time only one objective as objective function and ignoring all others.

Step 2: Compute the value of each objective function at each solution derived in Step 1.

Step 3: From Step 2, find for each objective the best (L_r) and the worst (U_r) values corresponding to the set of solutions. Recall that L_r and U_r are the aspired level of achievement and the highest acceptable level of achievement for the r-th objective function, respectively.

Step 4: Define a membership functions μ_r (linear μ_r^L, hyperbolic μ_r^H or exponential μ_r^E) for the rth objective function.

If we use the linear membership function as defined in (2) then an equivalent linear model for the model (1) can be formulated as:

$$\begin{aligned}
&\min : \phi,\\
&\text{s.t.}\\
&\frac{U_r - Z_r}{U_r - L_r} + d_r^- - d_r^+ = 1,\\
&\phi \geq d_r^-, \quad r = 1, 2, \cdots, k,\\
&d_r^+ d_r^- = 0,\\
&\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \cdots, m,\\
&\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \cdots, n,\\
&d_r^+, \ d_r^- \geq 0,\\
&\phi \leq 1, \ \phi \geq 0,\\
&x_{ij} \geq 0, \quad \text{for all } i, j,
\end{aligned}$$

where the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

If we use the exponential membership function as defined in (3), then an equivalent nonlinear model for the model (1) can be formulated as:

$$\begin{aligned}
&\min : \phi,\\
&\text{s.t.}\\
&\frac{e^{-s\psi_r(x)} - e^{-s}}{1 - e^{-s}} + d_r^- - d_r^+ = 1,\\
&\phi \geq d_r^-, \quad r = 1, 2, \cdots, k,\\
&d_r^+ d_r^- = 0,\\
&\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \cdots, m,\\
&\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \cdots, n,\\
&d_r^+, \ d_r^- \geq 0,\\
&\phi \leq 1, \ \phi \geq 0,\\
&x_{ij} \geq 0, \quad \text{for all } i, j,
\end{aligned}$$

where the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

If we use the hyperbolic membership function as defined in (4) then an equivalent nonlinear model for the model (1) can be formulated as:

$$\min : \phi,$$
s.t.
$$\frac{1}{2}+\frac{1}{2}\frac{e^{\{\frac{(U_r+L_r)}{2}-Z_r\}\alpha_r}-e^{-\{\frac{(U_r+L_r)}{2}-Z_r\}\alpha_r}}{e^{\{\frac{(U_r+L_r)}{2}-Z_r\}\alpha_r}+e^{-\{\frac{(U_r+L_r)}{2}-Z_r\}\alpha_r}}+d_r^- - d_r^+ = 1,$$
$$\phi \geq d_r^-, \quad r = 1, 2, \cdots, k,$$
$$d_r^+ d_r^- = 0,$$
$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \cdots, m,$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \cdots, n,$$
$$d_r^+, \ d_r^- \geq 0,$$
$$\phi \leq 1, \ \phi \geq 0,$$
$$x_{ij} \geq 0, \quad \text{for all } i, j,$$

where the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

Step 5: Solve the equivalent crisp model obtained in Step 4.
The solution obtained in Step 5 will be the optimal compromise solution of MOTP model [30].

5. Application Examples

Example 5.1. To illustrate the efficiency of the proposed method, we consider the following numerical example presented by Verma et al. [30]:

$$\begin{aligned} \min \ Z_1 =& 16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + \\ & 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33}, \\ \min \ Z_2 =& Z_2 = 9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} + \\ & 14x_{23} + 8x_{31} + 20x_{32} + 6x_{33} \end{aligned}$$

s.t.

$$\begin{aligned} & x_{11} + x_{12} + x_{13} = 14, \\ & x_{21} + x_{22} + x_{23} = 16, \\ & x_{31} + x_{32} + x_{33} = 12, \\ & x_{11} + x_{21} + x_{31} = 10, \\ & x_{12} + x_{22} + x_{32} = 15, \\ & x_{13} + x_{23} + x_{33} = 17, \\ & x_{ij} \geq 0, \quad i = 1, 2, 3, \quad j = 1, 2, 3. \end{aligned}$$

In the following the proposed steps of the previous section are presented.

Step 1: The solution of each single objective transportation problem is:

$$X^1 = (x_{11}^1 = 9, x_{12}^1 = 0, x_{13}^1 = 5, x_{21}^1 = 1, x_{22}^1 = 5, x_{23}^1 = 0, x_{31}^1 = 0, x_{32}^1 = 0, x_{33}^1 = 12)',$$
$$X^2 = (x_{11}^2 = 10, x_{12}^2 = 0, x_{13}^2 = 4, x_{21}^2 = 0, x_{22}^2 = 15, x_{23}^2 = 1, x_{31}^2 = 0, x_{32}^2 = 0, x_{33}^2 = 12)'.$$

Step 2: The objective function values are:

$$Z_1(X^1) = 517,\ Z_1(X^2) = 518,\ Z_2(X^1) = 379, Z_2(X^2) = 374.$$

Step 3: The upper and lower bounds of each objective function can be written as follows:

$$\begin{aligned} &517 \le Z_1 \le 518,\ 374 \le Z_2 \le 379,\\ &L_1 = 517,\ U_1 = 518,\ L_2 = 374,\ U_2 = 379. \end{aligned}$$

Step 4: If we use the linear membership function as defined in (2), an equivalent crisp model can be formulated as:

$$\begin{aligned} &\min : \phi \\ &\text{s.t.} \\ &\quad 518 - Z_1 + d_1^- - d_1^+ = 1, \\ &\quad \frac{379 - Z_2}{5} + d_2^- - d_2^+ = 1, \\ &\quad \phi \ge d_r^-, \quad r = 1, 2, \\ &\quad d_r^+ d_r^- = 0, \\ &\quad x_{11} + x_{12} + x_{13} = 14, \\ &\quad x_{21} + x_{22} + x_{23} = 16, \\ &\quad x_{31} + x_{32} + x_{33} = 12, \\ &\quad x_{11} + x_{21} + x_{31} = 10, \\ &\quad x_{12} + x_{22} + x_{32} = 15, \\ &\quad x_{13} + x_{23} + x_{33} = 17, \\ &\quad d_r^+,\ d_r^- \ge 0, \\ &\quad \phi \le 1,\ \phi \ge 0, \\ &\quad x_{ij} \ge 0, \quad \text{for all } i, j, \end{aligned}$$

where $Z_1 = 16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33}$ and $Z_2 = 9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} + 14x_{23} + 8x_{31} + 20x_{32} + 6x_{33}$.

The problem is solved and the results are:

$$\begin{aligned} &x_{11}^* = 9.5,\ x_{13}^* = 4.5,\ x_{21}^* = 0.5, x_{22}^* = 15,\ x_{23}^* = 0.5,\ x_{33}^* = 12, \\ &d_1^- = 0.5,\ d_1^+ = 0,\ d_2^- = 0.5,\ d_2^+ = 0,\ \phi^* = 0.5, \\ &Z_1^* = 517.5,\ Z_2^* = 376.5. \end{aligned}$$

The other variables that are not in the above have a zero value.

If we use the exponential membership function as defined in (3) with the parameter $s = 1$, an equivalent crisp model can be formulated as:

$$
\begin{aligned}
&\min : \phi \\
&\text{s.t.} \\
&\quad \frac{e^{-(Z_1-517)} - e^{-1}}{1-e^{-1}} + d_1^- - d_1^+ = 1, \\
&\quad \frac{e^{-(Z_2-374)/5} - e^{-1}}{1-e^{-1}} + d_2^- - d_2^+ = 1, \\
&\quad \phi \geq d_r^-, \quad r = 1,2, \\
&\quad d_r^+ d_r^- = 0, \\
&\quad x_{11} + x_{12} + x_{13} = 14, \\
&\quad x_{21} + x_{22} + x_{23} = 16, \\
&\quad x_{31} + x_{32} + x_{33} = 12, \\
&\quad x_{11} + x_{21} + x_{31} = 10, \\
&\quad x_{12} + x_{22} + x_{32} = 15, \\
&\quad x_{13} + x_{23} + x_{33} = 17, \\
&\quad d_r^+,\ d_r^- \geq 0, \\
&\quad \phi \leq 1,\ \phi \geq 0, \\
&\quad x_{ij} \geq 0, \quad \text{for all } i, j,
\end{aligned}
$$

where $Z_1 = 16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33}$ and $Z_2 = 9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} + 14x_{23} + 8x_{31} + 20x_{32} + 6x_{33}$.
The problem is solved and the results are:

$$
\begin{aligned}
&x_{11}^* = 9.5,\ x_{13}^* = 4.5,\ x_{21}^* = 0.5, x_{22}^* = 15,\ x_{23}^* = 0.5,\ x_{33}^* = 12, \\
&d_1^- = 0.62,\ d_1^+ = 0,\ d_2^- = 0.62,\ d_2^+ = 0,\ \phi^* = 0.62, \\
&Z_1^* = 517.5,\ Z_2^* = 376.5,
\end{aligned}
$$

and the other variables that are not in the above have a zero value.

If we use the hyperbolic membership function as defined in (4) then an equivalent crisp model can be formulated as:

$$
\begin{aligned}
&\min : \phi \\
&\text{s.t.} \\
&\quad \frac{1}{2} + \frac{1}{2}\frac{e^{6(517.5-Z_1)} - e^{-6(517.5-Z_1)}}{e^{6((517.5-Z_1)} + e^{-6(517.5-Z_1)}} + d_1^- - d_1^+ = 1, \\
&\quad \frac{1}{2} + \frac{1}{2}\frac{e^{\frac{6}{5}(376.5-Z_2)} - e^{-\frac{6}{5}(376.5-Z_2)}}{e^{\frac{6}{5}(376.5-Z_2)} + e^{-\frac{6}{5}(376.5-Z_2)}} + d_2^- - d_2^+ = 1, \\
&\quad \phi \geq d_r^-, \quad r = 1,2, \\
&\quad d_r^+ d_r^- = 0, \\
&\quad x_{11} + x_{12} + x_{13} = 14, \\
&\quad x_{21} + x_{22} + x_{23} = 16, \\
&\quad x_{31} + x_{32} + x_{33} = 12, \\
&\quad x_{11} + x_{21} + x_{31} = 10, \\
&\quad x_{12} + x_{22} + x_{32} = 15, \\
&\quad x_{13} + x_{23} + x_{33} = 17, \\
&\quad d_r^+,\ d_r^- \geq 0, \\
&\quad \phi \leq 1,\ \phi \geq 0, \\
&\quad x_{ij} \geq 0, \quad \text{for all } i, j,
\end{aligned}
$$

where $Z_1 = 16x_{11} + 19x_{12} + 12x_{13} + 22x_{21} + 13x_{22} + 19x_{23} + 14x_{31} + 28x_{32} + 8x_{33}$ and $Z_2 = 9x_{11} + 14x_{12} + 12x_{13} + 16x_{21} + 10x_{22} + 14x_{23} + 8x_{31} + 20x_{32} + 6x_{33}$.

The problem is solved and the results are:

$$x^*_{11} = 9.5,\ x^*_{13} = 4.5,\ x^*_{21} = 0.5, x^*_{22} = 15,\ x^*_{23} = 0.5,\ x^*_{33} = 12,$$
$$d_1^- = 0.5,\ d_1^+ = 0,\ d_2^- = 0.5,\ d_2^+ = 0,\ \phi^* = 0.5,$$
$$Z_1^* = 517.5,\ Z_2^* = 376.5.$$

The other variables that are not in the above have a zero value.

Example 5.2. This example is adopted from Diaz [11].

$$\begin{aligned}
\min\ Z_1 =& 9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} + 7x_{21} + 3x_{22} + 7x_{23} + 7x_{24} + 5x_{25} \\
&+ 6x_{31} + 5x_{32} + 9x_{33} + 11x_{34} + 3x_{35} + 6x_{41} + 8x_{42} + 11x_{43} + 2x_{44} + 2x_{45}, \\
\min\ Z_2 =& 2x_{11} + 9x_{l2} + 8x_{13} + x_{l4} + 4x_{15} + x_{21} + 9x_{22} + 9x_{23} + 5x_{24} + 2x_{25}, \\
&+ 8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} + 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45}, \\
\min\ Z_3 =& 2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15} + 4x_{21} + 8x_{22} + 4x_{23} + 9x_{24} + 2x_{25} \\
&+ 5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35} + 6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45}, \\
\text{s.t.}& \\
& x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 5, \\
& x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 4, \\
& x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 2, \\
& x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 9, \\
& x_{11} + x_{21} + x_{31} + x_{41} = 4, \\
& x_{12} + x_{22} + x_{32} + x_{42} = 4, \\
& x_{13} + x_{23} + x_{33} + x_{43} = 6, \\
& x_{14} + x_{24} + x_{34} + x_{44} = 2, \\
& x_{15} + x_{25} + x_{35} + x_{45} = 4, \\
& x_{ij} \geq 0, \quad i = 1, 2, 3,4, \quad j = 1, 2, 3, 4, 5.
\end{aligned}$$

In the following the proposed steps of the previous section are presented.

Step 1: The solution of each single objective transportation problem is:

$X^1 = (x^1_{13} = 5, x^1_{22} = 3, x^1_{23} = 1, x^1_{31} = 1, x^1_{32} = 1, x^1_{41} = 3, x^1_{44} = 2, x^1_{45} = 4)'$

$X^2 = (x^2_{11} = 3, x^2_{14} = 2, x^2_{25} = 4, x^2_{32} = 2, x^2_{41} = 1, x^2_{42} = 2, x^2_{43} = 6)'$

$X^3 = (x^3_{11} = 3, x^3_{12} = 2, x^3_{21} = 1, x^3_{23} = 3, x^3_{32} = 2, x^3_{43} = 3, x^3_{44} = 2, x^3_{45} = 4)'$.

Step 2: The objective function values are:

$Z_1(X^1) = 102,\ Z_1(X^2) = 157,\ Z_1(X^3) = 134,\ Z_2(X^1) = 141,\ Z_2(X^2) = 72,\ Z_2(X^3) = 116,\ Z_3(X^1) = 94,\ Z_3(X^2) = 86,\ Z_3(X^3) = 64.$

Step 3: The upper and lower bounds of each objective function can be written as follows:

$$102 \leq Z_1 \leq 157,\ 72 \leq Z_2 \leq 141, \text{ and } 64 \leq Z_3 \leq 94,$$

hence $L_1 = 102,\ U_1 = 157,\ L_2 = 72,\ U_2 = 141,\ L_3 = 64$, and $U_3 = 94$.

Step 4: If we use the linear membership function as defined in (2), an equivalent crisp model can be formulated as:

$$\begin{aligned}
&\min : \phi \\
&\text{s.t.} \\
&\quad \frac{157 - Z_1}{55} + d_1^- - d_1^+ = 1, \\
&\quad \frac{141 - Z_2}{69} + d_2^- - d_2^+ = 1, \\
&\quad \frac{94 - Z_3}{30} + d_3^- - d_3^+ = 1, \\
&\quad \phi \ge d_r^-, \quad r = 1, 2, 3, \\
&\quad d_r^+ d_r^- = 0, \\
&\quad x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 5, \\
&\quad x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 4, \\
&\quad x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 2, \\
&x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 9, \\
&x_{11} + x_{21} + x_{31} + x_{41} = 4, \\
&x_{12} + x_{22} + x_{32} + x_{42} = 4, \\
&x_{13} + x_{23} + x_{33} + x_{43} = 6, \\
&x_{14} + x_{24} + x_{34} + x_{44} = 2, \\
&x_{15} + x_{25} + x_{35} + x_{45} = 4, \\
&d_r^+, \ d_r^- \ge 0, \\
&\phi \le 1, \ \phi \ge 0, \\
&x_{ij} \ge 0, \quad \text{for all } i, j,
\end{aligned}$$

where $Z_1 = 9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} + 7x_{21} + 3x_{22} + 7x_{23} + 7x_{24} + 5x_{25} + 6x_{31} + 5x_{32} + 9x_{33} + 11x_{34} + 3x_{35} + 6x_{41} + 8x_{42} + 11x_{43} + 2x_{44} + 2x_{45}$, $Z_2 = 2x_{11} + 9x_{l2} + 8x_{13} + x_{l4} + 4x_{15} + x_{21} + 9x_{22} + 9x_{23} + 5x_{24} + 2x_{25}, +8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} + 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45}$, and $Z_3 = 2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15} + 4x_{21} + 8x_{22} + 4x_{23} + 9x_{24} + 2x_{25} + 5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35} + 6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45}$.

The problem is solved and the results are:

$$\begin{aligned}
&x_{11}^* = 2.737554, \ x_{13}^* = 0.2624456, \ x_{14} = 2.000000, \ x_{22}^* = 2.000000, \ x_{23}^* = 1.842114, \\
&x_{25}^* = 0.1578863, \ x_{32}^* = 2.000000, \ x_{41}^* = 1.262446, \ x_{43}^* = 3.895441, \ x_{45}^* = 3.842114, \\
&d_1^- = 0.4507814, , \ d_1^+ = 0, \ d_2^- = 0.4507814, \ d_2^+ = 0, \ d_3^- = 0.4507814, \ d_3^+ = 0, \ \phi^* = 0.4507814, \\
&Z_1^* = 126.7930, \ Z_2^* = 103.1039, \ Z_3^* = 77.52344,
\end{aligned}$$

and the other variables that are not in the above have a zero value.

If we use the exponential membership function as defined in (3) with the parameter $s = 1$, an equivalent crisp model can be formulated as:

$$\begin{aligned}
&\min : \phi \\
&\text{s.t.} \\
&\quad \frac{e^{-(Z_1 - 102)/55} - e^{-1}}{1 - e^{-1}} + d_1^- - d_1^+ = 1, \\
&\quad \frac{e^{-(Z_2 - 72)/69} - e^{-1}}{1 - e^{-1}} + d_2^- - d_2^+ = 1, \\
&\quad \frac{e^{-(Z_3 - 64)/30} - e^{-1}}{1 - e^{-1}} + d_3^- - d_3^+ = 1, \\
&\phi \ge d_r^-, \quad r = 1, 2, 3, \\
&d_r^+ d_r^- = 0, \\
&x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 5, \\
&x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 4, \\
&x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 2, \\
&x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 9, \\
&x_{11} + x_{21} + x_{31} + x_{41} = 4,
\end{aligned}$$

$$
\begin{aligned}
&x_{12}+x_{22}+x_{32}+x_{42}=4,\\
&x_{13}+x_{23}+x_{33}+x_{43}=6,\\
&x_{14}+x_{24}+x_{34}+x_{44}=2,\\
&x_{15}+x_{25}+x_{35}+x_{45}=4,\\
&d_r^+,\ d_r^-\geq 0,\\
&\phi\leq 1,\ \phi\geq 0,\\
&x_{ij}\geq 0,\quad \text{for all } i,j.
\end{aligned}
$$

The problem is solved and the results are:

$$
\begin{aligned}
&x_{11}^*=2.737554,\ x_{13}^*=0.2624456,\ x_{14}=2.000000,\ x_{22}^*=2.000000,\ x_{23}^*=1.842114,\\
&x_{25}^*=0.1578863,\ x_{32}^*=2.000000,\ x_{41}^*=1.262446,\ x_{43}^*=3.895441,\ x_{45}^*=3.842114,\\
&d_1^-=0.5740517,\ d_1^+=0,\ d_2^-=0.5740517,\ d_2^+=0,\ d_3^-=0.5740517,\ d_3^+=0,\ \phi^*=0.5740517,\\
&Z_1^*=126.7930,\ Z_2^*=103.1039,\ Z_3^*=77.52344.
\end{aligned}
$$

The other variables that are not in the above have a zero value.

If we use the hyperbolic membership function as defined in (4) then an equivalent crisp model can be formulated as:

$$
\begin{aligned}
&\min:\phi\\
&\text{s.t.}\\
&\frac{1}{2}+\frac{1}{2}\,\frac{e^{\frac{6}{55}(129.5-Z_2)}-e^{-\frac{6}{55}(129.5-Z_2)}}{e^{\frac{6}{55}(129.5-Z_2)}+e^{-\frac{6}{55}(129.5-Z_2)}}+d_1^- - d_1^+=1,\\
&\frac{1}{2}+\frac{1}{2}\,\frac{e^{\frac{6}{69}(106.5-Z_2)}-e^{-\frac{6}{69}(106.5-Z_2)}}{e^{\frac{6}{69}(106.5-Z_2)}+e^{-\frac{6}{69}(106.5-Z_2)}}+d_2^- - d_2^+=1,\\
&\frac{1}{2}+\frac{1}{2}\,\frac{e^{\frac{6}{30}(79-Z_3)}-e^{-\frac{6}{30}(79-Z_3)}}{e^{\frac{6}{30}(79-Z_3)}+e^{-\frac{6}{30}(79-Z_3)}}+d_2^- - d_2^+=1,\\
&\phi\geq d_r^-,\quad r=1,2,3,\\
&d_r^+d_r^-=0,\\
&x_{11}+x_{12}+x_{13}+x_{14}+x_{15}=5,\\
&x_{21}+x_{22}+x_{23}+x_{24}+x_{25}=4,\\
&x_{31}+x_{32}+x_{33}+x_{34}+x_{35}=2,\\
&x_{41}+x_{42}+x_{43}+x_{44}+x_{45}=9,\\
&x_{11}+x_{21}+x_{31}+x_{41}=4,\\
&x_{12}+x_{22}+x_{32}+x_{42}=4,\\
&x_{13}+x_{23}+x_{33}+x_{43}=6,\\
&x_{14}+x_{24}+x_{34}+x_{44}=2,\\
&x_{15}+x_{25}+x_{35}+x_{45}=4,\\
&d_r^+,\ d_r^-\geq 0,\\
&\phi\leq 1,\ \phi\geq 0,\\
&x_{ij}\geq 0,\quad \text{for all } i,j.
\end{aligned}
$$

The problem is solved and the results are:

$$
\begin{aligned}
&x_{11}^*=2.737554,\ x_{13}^*=0.2624456,\ x_{14}=2.000000,\ x_{22}^*=2.000000,\ x_{23}^*=1.842114,\\
&x_{25}^*=0.1578863,\ x_{32}^*=2.000000,\ x_{41}^*=1.262446,\ x_{43}^*=3.895441,\ x_{45}^*=3.842114,\\
&d_1^-=0.3564918,\ d_1^+=0,\ d_2^-=0.3564918,\ d_2^+=0,\ d_3^-=0.3564918,\ d_3^+=0,\ \phi^*=0.3564918,\\
&Z_1^*=126.7930,\ Z_2^*=103.1039,\ Z_3^*=77.52344,
\end{aligned}
$$

and the other variables that are not in the above have a zero value.

6. Conclusion

In this paper, three special types of membership functions have been used to solve the multi-objective transportation problem. The optimal compromise solution does not change if we compare with the solution obtained by the linear membership function. However, if we use the exponential membership function, with different values of s (parameter) then the optimal compromise solution does not change significantly, if we compare with the solution obtained by the linear membership function. Further, we conclude that for a multi-objective probabilistic transportation problem if the demand parameters are gamma random variables, then the deterministic problem becomes non-linear. To solve this type of problem, these nonlinear membership functions can be used. Apart from the transportation problems for the multiobjective nonlinear programming problems, nonlinear membership functions are useful.

Acknowledgements. The first author would like to thank for the financial grant from Shahrekord University. The first author was also partially supported by the Center of Excellence for Mathematics, University of Shahrekord, Shahrekord, Iran.

References

[1] W. F. Abd El-Wahed and S. M. Lee, *Interactive fuzzy goal programming for multi-objective transportation problems*, Omega, **34** (2006), 158-166.

[2] R. S. Aenaida and N. W. Kwak, *A linear goal programming for transshipment problems with flexible supply and demand constraints*, Fuzzy Sets and Systems, **45** (1994), 215-224.

[3] A. K. Bit, M. P. Biswal and S. S. Alam, *Fuzzy programming approach to multicriteria decision making transportation problem*, Fuzzy Sets and Systems, **50** (1992), 135-141.

[4] J. Brito, J. A. Moreno and J. L. Verdegay, *Transport route planning models based on fuzzy approach*, Iranian Journal of Fuzzy Systems, **9(1)** (2012), 141–158.

[5] S. Chanas and D. Kuchta, *A concept of the optimal solution of the transportation problem with fuzzy cost coefficients*, Fuzzy Sets and Systems, **82** (1996), 299-305.

[6] A. Charnes and W. W. Cooper, *Management models and industrial applications of linear programming*, Wiley, New York, 1961.

[7] A. Charnes, W. W. Cooper and A. Henderson, *An introduction to linear programming*, Wiley, New York, 1953.

[8] J. Current and H. Min, *Multiobjective design of transportation networks: taxonomy and annotation*, European J. Oper. Res., **26** (1986), 187-201.

[9] G. B. Dantzig, *Linear programming and extensions*, Princeton University Press, Princeton, N J, 1963.

[10] A. K. Dhingra and H. Moskowitz, *Application of fuzzy theories to multiple objective decision making in system design*, European J. Oper. Res., **55** (1991), 348-361.

[11] J. A. Diaz, *Solving multiobjective transportation problem*, Ekonom.-Mat. Obzor., **14** (1978), 267-274.

[12] J. A. Diaz, *Finding a complete description of all efficient solutions to a multiobjective transportation problem*, Ekonom.-Mat. Obzor., **15** (1979), 62-73.

[13] W. Edwards, *How to use multiattribute utility measurement for social decision making*, IEEE Trans. Systems Man Cybernet., **7** (1977), 326-340.

[14] A. Gupta and G. W. Evans, *A goal programming model for the operation of closed-loop supply chains*, Engineering Optimization, **41** (2009), 713-735.

[15] E. L. Hannan, *On fuzzy goal programming*, Decision Sci., **12** (1981), 522-531.

[16] H. Hassanpour, H. R. Maleki and M. A. Yaghoobi, *Fuzzy linear regression model with crisp coefficients: a Goal programming appproach*, Iranian Journal of Fuzzy Systems, **7(2)** (2010), 19-39.

[17]] F. L. Hitchcock, *The distribution of a product from several sources to numerous localities*, J. Math. Phys., **20** (1941), 224-230.

[18] H. Isermann, *The enumeration of all efficient solutions for a linear multiobjective transportation problem*, Naval Res. Logist. Quart., **2** (1979), 123-139.

[19] F. Jimenez and J. L. Verdegay, *Solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach*, European Journal of Operational Research, **117** (1999), 485-510.

[20] Amarpreet Kaur and Amit Kumar, *A new method for solving fuzzy transportation problems using ranking function*, Applied Mathematical Modelling, **35** (2011), 5652-5661.

[21]] H. Leberling, *On finding compromise solutions for multicriteria problems using the fuzzy minoperator*, Fuzzy Sets and Systems, **6** (1981), 105-118.

[22] S. M. Lee and L. J. Moore, *Optimizing transportation problems with multiple objectives*, AIEE Transactions, **5** (1973), 333-338.

[23] L. S. Li and K. K. Lai, *A fuzzy approach to the multiobjective transportation problem*, Computers and Operations Research, **27** (2000), 43-57.

[24] R. H. Mohamed, *The relationship between goal programming and fuzzy programming*, Fuzzy Sets and Systems, **89** (1997), 215-222.

[25] B. B. Pal, B. N. Moitra and U. Maulik, *A goal programming procedure for fuzzy multiobjective linear programming problem*, Fuzzy Sets and Systems, **139** (2003), 395-405.

[26] D. Peidro and P. Vasant, *Transportation planning with modified s-curve membership functions using an interactive fuzzy multi-objective approach*, Applied Soft Computing, **11** (2011), 2656-2663.

[27] J. L. Ringuest and D. B. Rinks, *Interactive solutions for the linear multiobjective transportation problem*, European J. Oper. Res., **32** (1987), 96–106.

[28] M. Sakawa, *Fuzzy sets and interactive multiobjective optimization*, Plenum Press, New York, 1993.

[29] R. N. Tiwari, S. Dharmar and J. R. Rao, *Fuzzy goal programming-an additive model*, Fuzzy Sets and Systems, **24** (1987), 27-34.

[30] R. Verma, M. P. Biswal and A. Biswas, *Fuzzy programming technique to solve multi-objective transportation problem with some non-linear membership functions*, Fuzzy Sets and Systems, **91** (1997), 37 43.

[31] M. A. Yaghoobi and M. Tamiz, *A short note on the relationship between goal programming and fuzzy programming for vectormaximum problems*, Iranian Journal of Fuzzy Systems, **2(2)** (1979), 31-36.

[32] M. Zangiabadi and H. R. Maleki, *Fuzzy goal programming for multiobjective transportation problems*, J. Appl. Math. and Computing, **24(1-2)** (2007), 449-460.

[33] H. J. Zimmermann, *Application of fuzzy set theory to mathematical programming*, Information Sciences, **36** (1985), 29-58.

Maryam Zangiabadi*, Department of Applied Mathematics, Faculty of Mathematical Sciences, Shahrekord University, P.O. Box 115, Shahrekord, Iran
E-mail address: zangiabadi-m@sci.sku.ac.ir

Hamid Reza Maleki, Department of Basic Sciences, Shiraz University of Technology, Shiraz, Iran
E-mail address: maleki@sutech.ac.ir

*Corresponding author