

NON-FRAGILE GUARANTEED COST CONTROL OF T-S FUZZY TIME-VARYING DELAY SYSTEMS WITH LOCAL BILINEAR MODELS

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ABSTRACT. This paper focuses on the non-fragile guaranteed cost control problem for a class of T-S fuzzy time-varying delay systems with local bilinear models. The objective is to design a non-fragile guaranteed cost state feedback controller via the parallel distributed compensation (PDC) approach such that the closed-loop system is delay-dependent asymptotically stable and the closed-loop performance is no more than a certain upper bound in the presence of the additive controller gain perturbations. A sufficient condition for the existence of such non-fragile guaranteed cost controllers is derived via the linear matrix inequality (LMI) approach and the design problem of the fuzzy controller is formulated in term of LMIs. The simulation examples show that the proposed approach is effective.

1. Introduction

In recent years, T-S (Takagi- Sugeno) model-based fuzzy control has attracted wide attention, essentially because the fuzzy model is an effective and flexible tool for control of nonlinear systems [1, 5, 11, 22, 24, 27, 35, 37, 38]. The T-S fuzzy model is employed to represent or approximate a nonlinear system, which is described by a family of fuzzy IF-THEN rules that represent local linear input-output relations of the system. The overall fuzzy model of the system is achieved by smoothly blending these local linear models together through membership functions. Therefore, it has a convenient dynamic structure so that some well-established linear systems theories can be easily applied for theoretical analysis and design of the overall closed-loop controlled system. The control design is carried out based on the fuzzy model via the so-called parallel distributed compensation (PDC) scheme [1, 11, 27, 38]. The idea is that for each local linear model, a linear feedback control is designed and the resulting overall controller, which is nonlinear in general, is fuzzy blending of each individual linear controller. Just because of this, T-S fuzzy model has been paid considerable attention and is widely used to the control design of nonlinear systems.

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In practical applications, time-delay often occur in many dynamic systems such as biological systems, network systems, and so on. It is shown that the existence of delays usually becomes the source of instability and deteriorating performance of systems. In recent years, some authors have paid their attention to control of nonlinear systems with time-delay by using T-S fuzzy models [1, 5, 11, 35, 38]. The existing results can be classified into two types: delay-independent results [38] and delay-dependent results [1, 5, 11, 13, 27, 35]. The former is irrespective of the delay size, whereas the latter usually contains the delay information. When some information about the time-delay is known, for example, an upper bound of time-delay, or upper bounds of both time-vary delay and its derivative are known, delay-dependent results are more appropriated and generally performance better than delay-independent conditions [1, 5, 6, 11, 27, 35]. It is worth pointing out that to derive delay-dependent results, some model transformations were usually performed to the original system, and thus, an inequality was inevitably employed to bound the inner product between two vectors, which gave rise to possible conservatism. It should be pointed out that all the aforementioned works did not take into account the effect of the control input delays on the systems. The results therein are not applicable to systems with input delay. Recently, some controller design approaches have been presented for systems with input delay, see [4, 18, 32, 2, 3] for fuzzy T-S systems and [33, 15, 8] for non-fuzzy systems and the references therein. All of these results are required to know the exact delay values in the implementation.

On the other hand, imprecision in controller implementation caused by finite word length in any digital systems or additional turning of parameters in the final controller implementation is often unavoidable. In this case, the controllers are very sensitive, or fragile, with respect to errors in the controllers' coefficients [14]. Since the controller fragility is basically the performance deterioration of a feedback control system due to inaccuracies in controller implementation, non-fragile control problem has been important issues. Recently, the research of non-fragile control has been paid a lot of attention and a series of productions have been obtained [29, 30, 31, 34, 28, 33].

It is known that bilinear models can describe many physical systems and dynamical processes in engineering fields [9, 21]. There are two main advantages of the bilinear system. One is that it provides a better approximation to a nonlinear system than a linear one. Another one is that many real physical processes may be appropriately modeled as bilinear systems when the linear models are inadequate. A good example of a bilinear system is the population of biological species described by $\frac{d\theta}{dt} = \theta v$, where v is the birth rate minus death rate, and θ denotes the population. It is impossible to approximate the aforementioned equation by a linear model [21].

Most of the existing results focus on the stability analysis and synthesis based on T-S fuzzy model with linear local model. However, when a nonlinear system has complex nonlinearities, the constructed T-S model will have to consist of a number of fuzzy local models. This will lead to very heavy computational burden. Considering the advantages of bilinear systems and T-S fuzzy control, the fuzzy control based on the T-S fuzzy model with bilinear rule consequence was attracted

the interest of researchers [19, 20, 25, 17]. The T-S fuzzy bilinear model may be suitable for some classes of nonlinear plants [19]. The robust stabilization for continuous-time fuzzy system with local bilinear model was studied in [19], and then the result was extended to the fuzzy system with time-delay only in the state [25]. The problem of robust stabilization for discrete-time fuzzy system with local bilinear model was investigated in [20]. In [17], we extended the idea in [19] to multiple input fuzzy bilinear systems with uncertainties, and proposed a robust H-infinity control strategy. Very recently, a class of nonlinear systems is described by T-S fuzzy models with nonlinear local models in [7], and a new fuzzy control scheme with local nonlinear feedbacks is proposed, and the corresponding control synthesis conditions are given in terms of solutions to a set of linear matrix inequalities (LMIs). In contrast to the existing methods for fuzzy control synthesis, the new proposed control design method is based on fewer fuzzy rules and less computational burden. However, in [7], there is not considering the time-delay effects on the system. The paper [25] is only considered the fuzzy system with the delay in the state and the derivatives of time-delay, $\dot{d}(t) < 1$ is required. So far, the problem of non-fragile guaranteed cost control for fuzzy system with local bilinear model which has time-delay in both state and input has not been discussed.

Motivated by the above observation, in this paper, the problem of delay-dependent non-fragile guaranteed cost control is studied for the fuzzy time-varying state and input delay systems with local bilinear model. Based on the PDC scheme, new delay-dependent stabilization conditions for the closed-loop fuzzy systems are derived. No model transformation is involved in the derivation. The merit of the proposed conditions lies in its reduced conservatism, which is achieved by circumventing the utilization of some bounding inequalities for the cross product between two vectors as in [25]. ***The three main contributions of this paper are the following: 1) a non-fragile guaranteed cost controller is presented for the fuzzy system with time-varying delay in both state and input; 2) some free weighting matrices are introduced in the derivation process, which the constraint of the derivatives of time-delay, $\dot{d}(t) < 1$ is eliminated; 3) the delay-dependent stability conditions for the fuzzy system are described by LMIs.*** Finally, simulation examples are given to illustrate the effectiveness of the obtained results.

The paper is organized as follows. Section 2 introduces the fuzzy delay system with local bilinear model, and non-fragile control law for such system is designed based on the parallel distributed compensation approach in section 3. Results of non-fragile guaranteed cost control are given in section 4. Two simulation examples are used to illustrate the effectiveness of the proposed method in section 5, which is followed by conclusions in section 6.

Notation 1: Throughout this paper, a real symmetric matrix $P > 0$ ($P \geq 0$) denotes P being a positive definite (or positive semi-definite) matrix. In symmetric block matrices, we use an asterisk (*) to represent a term that is induced by symmetry and $diag(\dots)$ stands for a block-diagonal matrix. The notion $\sum_{i,j=1}^s$

means $\sum_{i=1}^s \sum_{j=1}^s$. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. System Description and Assumptions

In this section, we introduce the T-S fuzzy time-delay system with local bilinear model. The i -th rule of the fuzzy system is represented by the following form

$$\begin{aligned}
 & \text{Plant Rule } i : \\
 & \text{IF } \vartheta_1(t) \text{ is } F_{i1} \text{ and } \dots \text{ and } \vartheta_v(t) \text{ is } F_{iv} \\
 & \text{THEN } \dot{x}(t) = A_i x(t) + A_{di} x(t-d(t)) + B_i u(t) \\
 & \quad + B_{di} u(t-d(t)) + N_i x(t) u(t) + N_{di} x(t-d(t)) u(t-d(t)) \\
 & \quad x(t) = \varphi(t), \quad t \in [-\tau, 0] \quad i = 1, 2, \dots, s
 \end{aligned} \tag{1}$$

where F_{ij} is the fuzzy set, s is the number of fuzzy rules, $x(t) \in R^n$ is the state vector, and $u(t) \in R$ is the control input, $\vartheta_1(t), \vartheta_2(t), \dots, \vartheta_v(t)$ are the premise variables. It is assumed that the premise variables do not depend on the input $u(t)$. $A_i, A_{di}, N_i, N_{di} \in R^{n \times n}$, $B_i, B_{di} \in R^{n \times 1}$ denote the system coefficient matrices with appropriate dimensions. $d(t)$ is a time-varying differentiable function that satisfies $0 \leq d(t) \leq \tau$, where τ is a real positive constant as the upper bound of the time-varying delay. It is also assumed that $\dot{d}(t) \leq \sigma$ and σ is a known constant. The initial condition $\varphi(t)$ is a continuous function of t , $t \in [-\tau, 0]$.

Remark 2.1. The fuzzy system with time-varying delay in both state and input will be investigated in this paper, which is different from the system in [25]. In [25], only state time-varying delay is considered. And also, here, we assume that the derivative of time-varying delay is less than or equal to a known constant that may be greater than 1, this assumption is relaxed the assumption on time-varying delay in [25].

By using singleton fuzzifier, product inferred, and weighted defuzzifier, the fuzzy system can be expressed by the following globe model:

$$\begin{aligned}
 \dot{x}(t) = \sum_{i=1}^s h_i(\vartheta(t)) [& A_i x(t) + A_{di} x(t-d(t)) + B_i u(t) + B_{di} u(t-d(t)) \\
 & + N_i x(t) u(t) + N_{di} x(t-d(t)) u(t-d(t))]
 \end{aligned} \tag{2}$$

where $h_i(\vartheta(t)) = \omega_i(\vartheta(t)) / \sum_{i=1}^s \omega_i(\vartheta(t))$, $\omega_i(\vartheta(t)) = \prod_{j=1}^v \mu_{ij}(\vartheta(t))$, $\mu_{ij}(\vartheta(t))$ is the grade of membership of $\vartheta_i(t)$ in F_{ij} . In this paper, it is assumed that $\omega_i(\vartheta(t)) \geq 0$, $\sum_{i=1}^s \omega_i(\vartheta(t)) > 0$ for all t . Then, we have the following conditions $h_i(\vartheta(t)) \geq 0$, $\sum_{i=1}^s h_i(\vartheta(t)) = 1$ for all t .

In consequent, we use abbreviation $h_i, h_{di}, x_d(t), u_d(t)$ to replace $h_i(\vartheta(t)), h_i(\vartheta(t-d(t))), x(t-d(t)), u(t-d(t))$ respectively, for convenience.

The objective of this paper is to design a state-feedback non-fragile guaranteed cost control law for the fuzzy system (2).

3. Non-fragile Guaranteed Cost Controller Design

Extending the design concept in [25], we give the following non-fragile fuzzy control law in the presence of the additive controller gain perturbations:

$$\begin{aligned}
& \text{IF } \vartheta_1(t) \text{ is } F_1^i \text{ and ... and } \vartheta_v(t) \text{ is } F_v^i \\
& \text{THEN } u(t) = \frac{\rho(K_i + \Delta K_i(t))x(t)}{\sqrt{1+x^T(K_i + \Delta K_i(t))^T(K_i + \Delta K_i(t))x}} \\
& = \rho \sin \theta_i = \rho \cos \theta_i (K_i + \Delta K_i(t))x(t)
\end{aligned} \tag{3}$$

where $\rho > 0$ is a scalar to be assigned, and $K_i \in R^{1 \times n}$ is a local controller gain to be determined. $\Delta K_i(t)$ represents the additive controller gain perturbations of the form $\Delta K_i(t) = H_i F_i(t) E_{ki}$ with H_i and E_{ki} being known constant matrices, and $F_i(t)$ the uncertain parameter matrix satisfying $F_i^T(t) F_i(t) \leq I$. $\sin \theta_i = \frac{\bar{K}_i x(t)}{\sqrt{1+x^T \bar{K}_i^T \bar{K}_i x}}$, $\cos \theta_i = \frac{1}{\sqrt{1+x^T \bar{K}_i^T \bar{K}_i x}}$, $\theta_i \in [-0.5\pi, 0.5\pi]$, $i = 1, 2, \dots, s$, $\bar{K}_i = K_i + \Delta K_i(t) = K_i + H_i F_i(t) E_{ki}$.

The overall fuzzy control law can be represented by

$$u(t) = \sum_{i=1}^s h_i \frac{\rho \bar{K}_i x(t)}{\sqrt{1+x^T \bar{K}_i^T \bar{K}_i x}} = \sum_{i=1}^s h_i \rho \sin \theta_i = \sum_{i=1}^s h_i \rho \cos \theta_i \bar{K}_i x(t) \tag{4}$$

When there exists an input delay $d(t)$, we have that

$$u_d(t) = \sum_{i=1}^s h_{di} \rho \sin \phi_i = \sum_{i=1}^s h_{di} \rho \cos \phi_i \tilde{K}_i x_d(t) \tag{5}$$

where $\sin \varphi_i = \frac{\tilde{K}_i x_d(t)}{\sqrt{1+x_d^T \tilde{K}_i^T \tilde{K}_i x_d}}$, $\cos \varphi_i = \frac{1}{\sqrt{1+x_d^T \tilde{K}_i^T \tilde{K}_i x_d}}$, $\varphi_i \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $i = 1, 2, \dots, s$,

$$\tilde{K}_i = K_i + \Delta K_i(t - d(t)) = K_i + H_i F_i(t - d(t)) E_{ki}.$$

So, it is natural and necessary to make an assumption that the functions $h_i(\vartheta(t))$ are well defined for all $t \in [-\tau, 0]$, and satisfy the following properties

$$h_i(\vartheta(t - d(t))) \geq 0, \text{ for } i=1, 2, \dots, s, \text{ and } \sum_{i=1}^s h_i(\vartheta(t - d(t))) = 1.$$

By substituting (4-5) into (2), the closed-loop system can be given by

$$\dot{x}(t) = \sum_{i,j,l=1}^s h_i h_j h_{dl} (\Lambda_{ij} x(t) + \Lambda_{dil} x_d(t)) \tag{6}$$

where $\Lambda_{ij} = A_i + \rho \sin \theta_j N_i + \rho \cos \theta_j B_i \bar{K}_j$, $\Lambda_{dil} = A_{di} + \rho \sin \phi_l N_{di} + \rho \cos \phi_l B_{di} \tilde{K}_l$.

Remark 3.1. In time-delay fuzzy model (2), we take the identical time-varying delay in both state and input. In fact, the fuzzy non-fragile control law (4) is readily extended to the case of the fuzzy system with different time-varying delays in state and input. In this case, we say the input time-delay is $g(t)$, where $g(t)$ is a time-varying differentiable function that satisfies $0 \leq g(t) \leq \varpi$, where ϖ is a real positive constant as an upper bound of the time-varying delay. It is also assumed that $\dot{g}(t) \leq v$ and v is a known constant. We can modify the time-delay control term in (3) as follows

$$u_g(t) = \frac{\rho(K_i + \Delta K_i)x_g(t)}{\sqrt{1+x_g^T(K_i + \Delta K_i)^T(K_i + \Delta K_i)x_g}} = \rho \sin \varphi_i = \rho \cos \varphi_i (K_i + \Delta K_i)x_g(t)$$

where $\sin \varphi_i = \frac{(K_i + \Delta K_i)x_g(t)}{\sqrt{1+x_g^T(K_i + \Delta K_i)^T(K_i + \Delta K_i)x_g}}$, $\cos \varphi_i = \frac{1}{\sqrt{1+x_g^T(K_i + \Delta K_i)^T(K_i + \Delta K_i)x_g}}$,

and $u_g(t)$, $x_g(t)$ stand for $u(t-g(t))$, $x(t-g(t))$, respectively.

Remark 3.2. In the paper, we only consider the presence of the additive controller gain perturbations in the non-fragile control law. It can be easily extended to multiplicative controller gain variations case via minor modification [29, 33].

Remark 3.3. In system model (1), for statement in brief and symbol simplicity, we only consider the single input case. In [17], a robust H-infinity control strategy for multiple inputs T-S fuzzy bilinear systems with uncertainties has proposed, a sufficient condition in term of LMIs is derived to guarantee the robust global stability of the overall fuzzy system. Similar to [17], the proposed approach in this paper can be easily extended to the multiple input case.

Given positive-definite symmetric matrix $S \in R^{n \times n}$ and $W > 0$, we consider the cost function

$$J = \int_0^{\infty} [x^T(t)Sx(t) + Wu^2(t)]dt \quad (7)$$

Definition 3.4. Consider the system (2). If there exists a fuzzy non-fragile control law (4) and a scalar J_0 such that the closed-loop system is asymptotically stable and the closed-loop value of the cost function (7) satisfies $J \leq J_0$, then J_0 is said to be a guaranteed cost and the control law $u(t)$ is said to be a non-fragile guaranteed cost control law for (2).

4. Analysis of Stability for the Closed-loop System

First, the following lemmas are presented which will be used in the paper.

Lemma 4.1. [5] *Given any matrices M and N with appropriate dimensions such that $\varepsilon > 0$, we have $M^T N + N^T M \leq \varepsilon M^T M + \varepsilon^{-1} N^T N$.*

Lemma 4.2. [7] *Given constant matrices G, E and a symmetric constant matrix S of appropriate dimensions. The inequality $S + GFE + E^T F^T G^T < 0$ holds, where $F(t)$ satisfies $F^T(t)F(t) \leq I$ if and only if, for some $\varepsilon > 0$, $S + \varepsilon GG^T + \varepsilon^{-1} E^T E < 0$.*

The following theorem gives the sufficient conditions for the existence of the non-fragile guaranteed cost controller for system (6) with additive controller gain perturbations.

Theorem 4.3. *Consider the system (6) associated with cost function (7). For given controller gain factor $\rho > 0$, upper bound of the time-varying delay $\tau > 0$ and upper bound of its derivative $\sigma > 0$, if there exist matrices $P > 0$, $Q > 0$, $R > 0$, K_i , $i = 1, 2, \dots, s$, X_1, X_2, X_3 , Y_1, Y_2, Y_3 and scalar $\varepsilon > 0$ satisfying the inequalities (8), the system (6) is asymptotically stable and the control law (4) is a fuzzy non-fragile guaranteed cost control law for any $0 \leq d(t) \leq \tau$, and $\dot{d}(t) \leq \sigma$. Moreover, $J \leq J_0 = x^T(0)Px(0) + \int_{-d(0)}^0 x^T(s)Qx(s)ds + \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s)R\dot{x}(s)d\theta ds$.*

$$\begin{bmatrix} T_{ijl} & * \\ \tau X^T & -\tau R \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s \quad (8)$$

where

$$\begin{aligned} \mathbf{T}_{ijl} &= \begin{bmatrix} \mathbf{T}_{11,ij} & * & * \\ \mathbf{T}_{21,i} & \mathbf{T}_{22,il} & * \\ \mathbf{T}_{31,i} & \mathbf{T}_{32,i} & \mathbf{T}_{33} \end{bmatrix}, \\ \mathbf{T}_{11,ij} &= Q + X_1 + X_1^T + Y_1 A_i + A_i^T Y_1^T + S + 2\varepsilon \rho^2 Y_1 Y_1^T \\ &\quad + 3\varepsilon^{-1} N_i^T N_i + 3\varepsilon^{-1} (B_i \bar{K}_j)^T (B_i \bar{K}_j) + \rho^2 \bar{K}_i^T W \bar{K}_i, \\ \mathbf{T}_{21,i} &= -X_1^T + X_2^+ Y_2 A_i + A_{di}^T Y_1^T, \quad \mathbf{T}_{31,i} = P + X_3^+ Y_3 A_i^- Y_1^T, \\ \mathbf{T}_{22,il} &= (\sigma - 1)Q - X_2 - X_2^T + Y_2 A_{di} + A_{di}^T Y_2^T + 2\varepsilon \rho^2 Y_2 Y_2^T \\ &\quad + 3\varepsilon^{-1} N_{di}^T N_{di} + 3\varepsilon^{-1} (B_{di} \tilde{K}_l)^T (B_{di} \tilde{K}_l), \\ \mathbf{T}_{32,i} &= -X_3^+ Y_3 A_{di} - Y_2^T, \quad \mathbf{T}_{33} = -\tau R - Y_3 - Y_3^T + 2\varepsilon \rho^2 Y_3 Y_3^T. \end{aligned} \quad (9)$$

Proof. Take Lyapunov function candidate as

$$V(x(t), t) = x^T(t) P x(t) + \int_{t-d(t)}^t x^T(s) Q x(s) ds + \int_{-\tau}^0 \int_{t+\theta}^t \dot{x}^T(s) R \dot{x}(s) ds d\theta \quad (10)$$

The time derivatives of $V(x(t), t)$, along the trajectory of the system (6) is given by

$$\begin{aligned} \dot{V}(x(t), t) &= 2x^T(t) P \dot{x}(t) + x^T(t) Q x(t) - (1 - \dot{d}(t)) x_d^T(t) Q x_d(t) \\ &\quad + \tau \dot{x}^T(t) R \dot{x}(t) - \int_{t-\tau}^t \dot{x}^T(s) R \dot{x}(s) ds \end{aligned} \quad (11)$$

Define the free weighting matrices as $X = [X_1^T \quad X_2^T \quad X_3^T]^T$, $Y = [Y_1^T \quad Y_2^T \quad Y_3^T]^T$, where $X_k \in R^{n \times n}$, $Y_k \in R^{n \times n}$, $k = 1, 2, 3$ will be determined in later.

Using the Leibniz-Newton formula and system equation (6), we have the following identical equations

$$\begin{aligned} [x^T(t) X_1 + x_d^T(t) X_2 + \dot{x}^T(t) X_3] [x(t) - x_d(t) - \int_{t-d(t)}^t \dot{x}(s) ds] &\equiv 0 \\ \sum_{i,j,l=1}^s h_i h_j h_{dl} [x^T(t) Y_1 + x_d^T(t) Y_2 + \dot{x}^T(t) Y_3] \\ \times [\Lambda_{ij} x(t) + \Lambda_{dil} x_d(t) - \dot{x}(t)] &\equiv 0 \end{aligned} \quad (12)$$

Then, substituting (12) into (11), yields

$$\begin{aligned} \dot{V}(x(t), t) &= 2x^T(t) P \dot{x}(t) + x^T(t) Q x(t) - (1 - \dot{d}(t)) x_d^T(t) Q x_d(t) \\ &\quad + \tau \dot{x}^T(t) R \dot{x}(t) - \int_{t-\tau}^t \dot{x}^T(s) R \dot{x}(s) ds \\ &\quad + 2\eta^T(t) X [x(t) - x_d(t) - \int_{t-d(t)}^t \dot{x}(s) ds] \\ &\quad + 2\eta^T(t) Y \sum_{i,j,l=1}^s h_i h_j h_{dl} [\Lambda_{ij} x(t) + \Lambda_{dil} x_d(t) - \dot{x}(t)] \\ &\leq 2x^T(t) P \dot{x}(t) + x^T(t) Q x(t) - (1 - \sigma) x_d^T(t) Q x_d(t) + \tau \dot{x}^T(t) R \dot{x}(t) \\ &\quad - \int_{t-d(t)}^t \dot{x}^T(s) R \dot{x}(s) ds + 2\eta^T(t) X [x(t) - x_d(t) - \int_{t-d(t)}^t \dot{x}(s) ds] \\ &\quad + 2\eta^T(t) Y \sum_{i,j,l=1}^s h_i h_j h_{dl} [\Lambda_{ij} x(t) + \Lambda_{dil} x_d(t) - \dot{x}(t)] + x^T(t) S x(t) \\ &\quad + \sum_{i,j=1}^s h_i h_j \rho^2 x^T(t) \bar{K}_i^T \cos \theta_i W \bar{K}_j \cos \theta_j x(t) - [x^T(t) S x(t) + W u^2(t)] \end{aligned} \quad (13)$$

where $\eta(t) = [x^T(t), x_d^T(t), \dot{x}^T(t)]^T$.

Applying Lemma 4.1, we have the following inequalities

$$\begin{aligned}
2x^T(t)Y_1\Lambda_{ij}x(t) &\leq 2x^T(t)Y_1A_ix(t) + \varepsilon\rho^2x^T(t)Y_1Y_1^Tx(t) \\
&\quad + \varepsilon^{-1}x^T(t)(N_i^TN_i + (\bar{K}_jB_i)^T(B_i\bar{K}_j))x(t), \\
2x^T(t)Y_1\Lambda_{di}x_d(t) &\leq 2x^T(t)Y_1A_{di}x_d(t) + \varepsilon\rho^2x^T(t)Y_1Y_1^Tx(t) \\
&\quad + \varepsilon^{-1}x_d^T(t)(N_{di}^TN_{di} + (\bar{K}_lB_{di})^T(B_{di}\bar{K}_l))x_d(t), \\
2x_d^T(t)Y_2\Lambda_{ij}x(t) &\leq 2x_d^T(t)Y_2A_ix(t) + \varepsilon\rho^2x_d^T(t)Y_2Y_2^Tx_d(t) \\
&\quad + \varepsilon^{-1}x^T(t)(N_i^TN_i + (\bar{K}_jB_i)^T(B_i\bar{K}_j))x(t), \\
2x_d^T(t)Y_2\Lambda_{di}x_d(t) &\leq 2x_d^T(t)Y_2A_{di}x_d(t) + \varepsilon\rho^2x_d^T(t)Y_2Y_2^Tx_d(t) \\
&\quad + \varepsilon^{-1}x_d^T(t)(N_{di}^TN_{di} + (\bar{K}_lB_{di})^T(B_{di}\bar{K}_l))x_d(t), \\
2\dot{x}^T(t)Y_3\Lambda_{ij}x(t) &\leq 2\dot{x}^T(t)Y_3A_ix(t) + \varepsilon\rho^2\dot{x}^T(t)Y_3Y_3^T\dot{x}(t) \\
&\quad + \varepsilon^{-1}x^T(t)(N_i^TN_i + (\bar{K}_jB_i)^T(B_i\bar{K}_j))x(t), \\
2\dot{x}^T(t)Y_3\Lambda_{di}x_d(t) &\leq 2\dot{x}^T(t)Y_3A_{di}x_d(t) + \varepsilon\rho^2\dot{x}^T(t)Y_3Y_3^T\dot{x}(t) \\
&\quad + \varepsilon^{-1}x_d^T(t)(N_{di}^TN_{di} + (\bar{K}_lB_{di})^T(B_{di}\bar{K}_l))x_d(t).
\end{aligned} \tag{14}$$

Substituting (14) into (13), results in

$$\begin{aligned}
\dot{V}(x(t), t) &\leq \sum_{i,j,l=1}^s h_i h_j h_{dl} \eta^T(t) \tilde{\Gamma}_{ijl} \eta(t) - \int_{t-d(t)}^t \dot{x}^T(s) R \dot{x}(s) ds \\
&\quad - 2\eta^T(t) X \int_{t-d(t)}^t \dot{x}(s) ds - [x^T(t) S x(t) + W u^2(t)] \\
&\leq \sum_{i,j,l=1}^s h_i h_j h_{dl} \eta^T(t) (\tilde{\Gamma}_{ijl} + \tau X R^{-1} X^T) \eta(t) - \int_{t-d(t)}^t (\eta^T(t) X + \dot{x}^T(s) R) \\
&\quad \times R^{-1} (\eta^T(t) X + \dot{x}^T(s) R)^T ds - [x^T(t) S x(t) + W u^2(t)] \\
&\leq \sum_{i,j,l=1}^s h_i h_j h_{dl} \eta^T(t) (\tilde{\Gamma}_{ijl} + \tau X R^{-1} X^T) \eta(t) - [x^T(t) S x(t) + W u^2(t)] \tag{15}
\end{aligned}$$

$$\text{where } \tilde{\Gamma}_{ijl} = \begin{bmatrix} \tilde{\Gamma}_{11,ij} & * & * \\ \Gamma_{21,i} & \Gamma_{22,il} & * \\ \Gamma_{31,i} & \Gamma_{32,i} & \Gamma_{33} \end{bmatrix}, \quad \tilde{\Gamma}_{11,ij} = \Gamma_{11,ij} + \rho^2 \bar{K}_i^T \cos \theta_i W \bar{K}_j \cos \theta_j - \rho^2 \bar{K}_i^T W \bar{K}_i.$$

In the light of the inequality

$$\bar{K}_i^T \cos \theta_i W \bar{K}_j \cos \theta_j + \bar{K}_j^T \cos \theta_j W \bar{K}_i \cos \theta_i \leq \bar{K}_i^T W \bar{K}_i \cos^2 \theta_i + \bar{K}_j^T W \bar{K}_j \cos^2 \theta_j,$$

and from (15), we obtain

$$\begin{aligned}
\dot{V}(x(t), t) &\leq \sum_{i,j,l=1}^s h_i h_j h_{dl} \eta^T(t) (\Gamma_{ijl} + \tau X R^{-1} X^T) \eta(t) \\
&\quad - [x^T(t) S x(t) + W u^2(t)] \tag{16}
\end{aligned}$$

Applying the Schur complement to (8) yields

$$\Gamma_{ijl} + \tau X R^{-1} X^T < 0.$$

Therefore, it follows from (16) that

$$\dot{V}(x(t), t) \leq -[x^T(t) S x(t) + W u^2(t)] < 0 \tag{17}$$

which implies that the system (6) is asymptotically stable.

Integrating (17) from 0 to T , produces

$$\int_0^T [x^T(t)Sx(t) + Wu^2(t)]dt \leq -V(x(T), T) + V(x(0), 0) \leq V(x(0), 0) \quad (18)$$

since $V(x(t), t) \geq 0$ and $\dot{V}(x(t), t) < 0$, we have $\lim_{T \rightarrow \infty} V(x(T), T) = c$, where c is a nonnegative constant. thus, noting (3.3) and (4.3), the following inequality can be obtained

$$J \leq x^T(0)Px(0) + \int_{-d(0)}^0 x^T(s)Qx(s)ds + \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s)R\dot{x}(s)dsd\theta \quad (19)$$

This completes the proof. \square

Remark 4.4. In the derivation of Theorem 4.3, the free weighting matrices $X_k \in R^{n \times n}$, $Y_k \in R^{n \times n}$, $k = 1, 2, 3$ are introduced, the purpose of which is to reduce conservatism in the existing delay-dependent stabilization conditions, see [25].

In the following section, we shall turn the conditions given in Theorem 4.3 into linear matrix inequalities (LMIs). Under the assumption that Y_1, Y_2, Y_3 are non-singular, we can define the matrix $Y_i^{-T} = \lambda Z$, $i = 1, 2, 3$, $Z = P^{-1}$, $\lambda > 0$. Pre- and post-multiply (4.3) with $\Theta = \text{diag}\{Y_1^{-1}, Y_2^{-1}, Y_3^{-1}, Y_3^{-1}\}$ and $\Theta^T = \text{diag}\{Y_1^{-T}, Y_2^{-T}, Y_3^{-T}, Y_3^{-T}\}$, respectively, and letting $\bar{Q} = Y_1^{-1}QY_1^{-T}$,

$\bar{R} = Y_3^{-1}RY_3^{-T}$, $\bar{X}_i = Y_i^{-1}X_iY_i^{-T}$, $i = 1, 2, 3$, we obtain the following inequalities (20) is equivalent to (8)

$$\begin{bmatrix} \bar{T}_{11,ij} & * & * & * \\ \bar{T}_{21,i} & \bar{T}_{22,il} & * & * \\ \bar{T}_{31,i} & \bar{T}_{32,i} & \bar{T}_{33} & * \\ \tau\bar{X}_1 & \tau\bar{X}_2 & \tau\bar{X}_3 & -\tau\bar{R} \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s \quad (20)$$

where

$$\begin{aligned} \bar{T}_{11,ij} &= \bar{Q} + \bar{X}_1 + \bar{X}_1^T + \lambda A_i Z + \lambda Z A_i^T + \lambda^2 Z S Z + 2\varepsilon \rho^2 I \\ &+ 3\varepsilon^{-1} \lambda^2 Z N_i^T N_i Z + 3\varepsilon^{-1} \lambda^2 (B_i \bar{K}_j Z)^T (B_i \bar{K}_j Z) + \rho^2 \lambda^2 Z \bar{K}_i^T W \bar{K}_i Z, \\ \bar{T}_{21,i} &= -\bar{X}_1^T + \bar{X}_2 + \lambda A_i Z + \lambda Z A_{di}^T, \\ \bar{T}_{31,i} &= \lambda^2 Z + \bar{X}_3 + \lambda A_i Z - \lambda Z, \\ \bar{T}_{22,il} &= -(1 - \sigma)\bar{Q} - \bar{X}_2 - \bar{X}_2^T + \lambda A_{di} Z + \lambda Z A_{di}^T + 2\varepsilon \rho^2 I \\ &+ 3\varepsilon^{-1} \lambda^2 Z N_{di}^T N_{di} Z + 3\varepsilon^{-1} \lambda^2 (B_{di} \bar{K}_l Z)^T (B_{di} \bar{K}_l Z), \\ \bar{T}_{32,i} &= -\bar{X}_3 + \lambda A_{di} Z - \lambda Z, \\ \bar{T}_{33} &= -\tau \bar{R} - 2\lambda Z + 2\varepsilon \rho^2 I. \end{aligned} \quad (21)$$

Applying the Schur complement to (20), results in

$$\Gamma_{ijl} = \begin{bmatrix} \bar{\bar{T}}_{11,i} & * & * & * & * \\ \bar{\bar{T}}_{21,i} & \bar{\bar{T}}_{22,i} & * & * & * \\ \bar{\bar{T}}_{31,i} & \bar{\bar{T}}_{32,i} & \bar{\bar{T}}_{33} & * & * \\ \tau\bar{X}_1 & \tau\bar{X}_2 & \tau\bar{X}_3 & -\tau\bar{R} & * \\ \lambda Z & 0 & 0 & 0 & -S^{-1} \\ \lambda N_i Z & 0 & 0 & 0 & 0 \\ \lambda B_i \bar{K}_j Z & 0 & 0 & 0 & 0 \\ \rho \lambda \bar{K}_i Z & 0 & 0 & 0 & 0 \\ 0 & \lambda B_{di} \bar{K}_l Z & 0 & 0 & 0 \\ 0 & \lambda N_{di} Z & 0 & 0 & 0 \end{bmatrix} \quad (22)$$

$$\begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ -\frac{\varepsilon}{3}I & * & * & * & * \\ 0 & -\frac{\varepsilon}{3}I & * & * & * \\ 0 & 0 & -W^{-1} & * & * \\ 0 & 0 & 0 & -\frac{\varepsilon}{3}I & * \\ 0 & 0 & 0 & 0 & -\frac{\varepsilon}{3}I \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s,$$

where $\bar{\bar{T}}_{11,i} = \bar{Q} + \bar{X}_1 + \bar{X}_1^T + \lambda A_i Z + \lambda Z A_i^T + 2\varepsilon\rho^2 I$, and $\bar{\bar{T}}_{22,i} = -(1 - \sigma)\bar{Q} - \bar{X}_2 - \bar{X}_2^T + \lambda A_{di} Z + \lambda Z A_{di}^T + 2\varepsilon\rho^2 I$.

Obviously, the closed-loop fuzzy system (6) is asymptotically stable, if for some scalars $\lambda > 0$, there exist matrices $Z > 0$, $\bar{Q} > 0$, $\bar{R} > 0$ and $\bar{X}_1, \bar{X}_2, \bar{X}_3, K_i, i = 1, 2, \dots, s$ satisfying the inequalities (22).

Theorem 4.5. *Consider the system (6) associated with cost function (7). For given controller gain factor $\rho > 0$, upper bound of the time-varying delay $\tau > 0$ and upper bound of its derivative $\sigma > 0$ and the factor of free weighting matrices $\lambda > 0$, scalar constant $\delta > 0$, if there exist matrices $Z > 0$, $\bar{Q} > 0$, $\bar{R} > 0$ and $\bar{X}_1, \bar{X}_2, \bar{X}_3, M_i \in R^{1 \times n}, i = 1, 2, \dots, s$ and scalar $\varepsilon > 0$ satisfying the following LMIs (23), system (6) is asymptotically stable and the control law (4) is a fuzzy non-fragile guaranteed cost control law.*

$$\begin{bmatrix} \Phi_{1,ijl} & * \\ \Phi_{2,ijl} & \Phi_3 \end{bmatrix} < 0, \quad i, j, l = 1, 2, \dots, s \quad (23)$$

Moreover, the feedback gains are given by

$$K_i = M_i Z^{-1}, i = 1, 2, \dots, s \quad (24)$$

and

$$J \leq J_0 = x^T(0)Z^{-1}x(0) + \int_{-d(0)}^0 x^T(s) \frac{1}{\lambda^2} Z^{-1} \bar{Q} Z^{-1} x(s) ds + \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s) \frac{1}{\lambda^2} Z^{-1} \bar{R} Z^{-1} \dot{x}(s) ds d\theta \quad (25)$$

where

$$\Phi_{1,ijl} = \begin{bmatrix} \bar{T}_{11,i} & * & * & * & * \\ \bar{T}_{21,i} & \bar{T}_{22,i} & * & * & * \\ \bar{T}_{31,i} & \bar{T}_{32,i} & \bar{T}_{33} & * & * \\ \tau \bar{X}_1 & \tau \bar{X}_2 & \tau \bar{X}_3 & -\tau \bar{R} & * \\ \lambda Z & 0 & 0 & 0 & -S^{-1} \\ \lambda N_i Z & 0 & 0 & 0 & 0 \\ \lambda B_i M_j & 0 & 0 & 0 & 0 \\ \rho \lambda M_i & 0 & 0 & 0 & 0 \\ 0 & \lambda B_{di} M_l & 0 & 0 & 0 \\ 0 & \lambda N_{di} Z & 0 & 0 & 0 \end{bmatrix} \quad (26)$$

$$\begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ -\frac{\varepsilon}{3} I & * & * & * & * \\ 0 & -\frac{\varepsilon}{3} I & * & * & * \\ 0 & 0 & -W^{-1} & * & * \\ 0 & 0 & 0 & -\frac{\varepsilon}{3} I & * \\ 0 & 0 & 0 & 0 & -\frac{\varepsilon}{3} I \end{bmatrix},$$

$$\Phi_{2,ijl} = \begin{bmatrix} \lambda E_{kj} Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda E_{ki} Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda E_{kl} Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (B_i H_j)^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho H_i^T & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (B_{di} H_l)^T \end{bmatrix},$$

$$\Phi_3 = \text{diag}\{-\delta I, -\delta I, -\delta I, -\delta^{-1} I, -\delta^{-1} I, -\delta^{-1} I\}.$$

Proof. We prove the inequality (23) implies the inequality (20). Applying the Schur complement to (23), results in

$$\begin{aligned}
& \Phi_{1,ijl} + \delta \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ B_i H_j & 0 \\ 0 & \rho H_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & (B_i H_j)^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho H_i^T & 0 & 0 \end{bmatrix} \quad (27) \\
& + \delta^{-1} \begin{bmatrix} (\lambda E_{kj} Z)^T & (\lambda E_{ki} Z)^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \lambda E_{kj} Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda E_{ki} Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
& + \delta \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ B_{di} H_l \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (B_{di} H_l)^T & 0 \end{bmatrix} \\
& + \delta^{-1} \begin{bmatrix} 0 \\ (\lambda E_{kl} Z)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \times [0 \quad \lambda E_{kl} Z \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] < 0
\end{aligned}$$

Using Lemma 4.2 and noting that $M_i = K_i Z$, by the condition (27), the following inequality, that is, (22) holds

$$\Phi_{1,ijl} + \begin{bmatrix} 0 & * & * & * & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\ \lambda B_i \Delta K_j(t) Z & 0 & 0 & 0 & 0 & 0 & 0 & * & * & * \\ \rho \lambda \Delta K_i(t) Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\ 0 & \lambda B_{di} \Delta K_l(t - d(t)) Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \Gamma_{ijl} < 0. \quad (28)$$

Therefore, it follows from Theorem 4.3 that the system (6) is asymptotically stable and the control law (4) is a fuzzy non-fragile guaranteed cost control law. Thus, we complete the proof. \square

Now, consider the upper bound of cost function (25). Similar to [28], we suppose that there exist positive scalars $\alpha_1, \alpha_2, \alpha_3$ such that $Z^{-1} \leq \alpha_1 I$, $\frac{1}{\lambda^2} P \bar{Q} P \leq \alpha_2 I$, $\frac{1}{\lambda^2} P \bar{R} P \leq \alpha_1 I$. Then define $S_Q = \bar{Q}^{-1}$, $S_R = \bar{R}^{-1}$, by Schur complement lemma, we have the following inequalities

$$\begin{bmatrix} -\alpha_1 I & I \\ I & -Z \end{bmatrix} \leq 0, \quad \begin{bmatrix} -\alpha_2 I & \frac{1}{\lambda} P \\ \frac{1}{\lambda} P & -S_Q \end{bmatrix} \leq 0, \quad \begin{bmatrix} -\alpha_3 I & \frac{1}{\lambda} P \\ \frac{1}{\lambda} P & -S_R \end{bmatrix} \leq 0, \quad (29)$$

$$\begin{bmatrix} Z & I \\ I & P \end{bmatrix} \geq 0, \quad \begin{bmatrix} S_Q & I \\ I & \bar{Q} \end{bmatrix} \geq 0, \quad \begin{bmatrix} S_R & I \\ I & \bar{R} \end{bmatrix} \geq 0$$

Using the idea of the cone complementary linear algorithm in [6], we can obtain the solution of the minimization problem of upper bound of the value of the cost function (25) as follows

$$\begin{aligned} & \text{minimize } \{ \text{trace}(PZ + S_Q \bar{Q} + S_R \bar{R}) + \alpha_1 x^T(0)x(0) \\ & + \alpha_2 \int_{-d(0)}^0 x^T(s)x(s)ds + \alpha_3 \int_{-\tau}^0 \int_{\theta}^0 \dot{x}^T(s)\dot{x}(s)dsd\theta \} \\ & \text{subject to (4.5), (4), } \varepsilon > 0, \quad \bar{Q} > 0, \quad \bar{R} > 0, \quad Z > 0, \quad \alpha_i > 0, \quad i = 1, 2, 3. \end{aligned} \quad (30)$$

Using the following cone complementary linearization (CCL) algorithm [6] can iteratively solve the minimization problem (30).

5. Simulation Examples

In this section, the proposed approach is applied to the following two examples to verify its effectiveness. In the first example, a pure numerical example is given to show the implement of the proposed method. The second example is a practical application of the Van de Vusse system.

Example 5.1. Consider a fuzzy time-delay system with local bilinear model

$$\begin{aligned}
R^1: & \text{ IF } x_1 \text{ is } L_1 \\
& \text{ THEN } \dot{x}(t) = A_1x(t) + A_{d1}x_d(t) + B_1u(t) + B_{d1}u_d(t) \\
& + N_1x(t)u(t) + N_{d1}x_d(t)u_d(t) \\
R^2: & \text{ IF } x_1 \text{ is } L_2 \\
& \text{ THEN } \dot{x}(t) = A_2x(t) + A_{d2}x_d(t) + B_2u(t) + B_{d2}u_d(t) \\
& + N_2x(t)u(t) + N_{d2}x_d(t)u_d(t)
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
A_1 &= \begin{bmatrix} -45 & 7 \\ 5 & 37 \end{bmatrix}, A_2 = \begin{bmatrix} -32 & 9 \\ 3 & 38 \end{bmatrix}; N_1 = N_2 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}; \\
B_1 &= \begin{bmatrix} 3 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; B_{d1} = B_{d2} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}; A_{d1} = \begin{bmatrix} 10 & 0 \\ 5 & 2 \end{bmatrix}, \\
A_{d2} &= \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}; N_{d1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}, N_{d2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.5 \end{bmatrix}.
\end{aligned}$$

The cost function associated with this system is given by $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $W = 1$.

The controller gain perturbation $\Delta K(t)$ is of the additive form and is given by $H_1 = 0.1$, $H_2 = 0.2$, $E_{k1} = [0.05 \ -0.01]$, $E_{k2} = [0.01 \ 0.01]$. The membership functions are chosen as $\mu_{L_1}(x_1) = \frac{1}{1+\exp(\frac{1}{-2x_1(t)})}$, $\mu_{L_2}(x_1) = 1 - \mu_{L_1}(x_1)$.

We considered the cost function (25) and by letting $\rho = 0.6$, $\tau = 3.5$, $\sigma = 1.5$, $\delta = 0.11$, $\lambda = 2$, solve the optimization problem (30), obtain the feasible solution as follows

$$\begin{aligned}
Z^{-1} = P &= \begin{bmatrix} 8.0143 & -0.7826 \\ -0.7826 & 5.1804 \end{bmatrix}; \alpha_1 = 8.5, \bar{Q} = \begin{bmatrix} 0.9050 & 0.5001 \\ 0.5001 & 0.9879 \end{bmatrix}; \\
K_1 &= [-0.2013, -0.4564]; K_2 = [-1.0313, -0.5312]; \\
\alpha_2 = 15, \bar{R} &= \begin{bmatrix} 0.8457 & -0.6487 \\ -0.6487 & 1.1356 \end{bmatrix}; \alpha_3 = 23, \varepsilon = 1.3646;
\end{aligned}$$

With the time-delay $d(t) = 2 + 1.5 \sin t$ and the initial function $\phi(t) = [0.8 \ -0.6]^T$, $-\tau \leq t \leq 0$, the simulation result on the non-fragile guaranteed cost control is shown in Figure 1 and Figure 2. The state trajectories and control curve are shown in Figure 1 and Figure 2, respectively. With the control law, the closed-loop system is asymptotically stable and an upper bound of cost function (7) of the closed-loop system is $J_0 = 125.3769$.

Example 5.2. Consider the dynamics of an isothermal continuous stirred tank reactor for the Van de Vusse

$$\begin{aligned}
\dot{x}_1 &= -50x_1 - 10x_1^3 + u(10 - x_1) + u(t-d) + u(t-d)(0.5x_1(t-d) \\
& + 0.2x_2(t-d)) + 5x_2(t-d) \\
\dot{x}_2 &= 50x_1 - 100x_2 - u(t-d) + u(t-d)(0.3x_1(t-d) - 0.2x_2(t-d)) \\
& + 10x_2(t-d) - 5x_1(t-d)
\end{aligned} \tag{32}$$

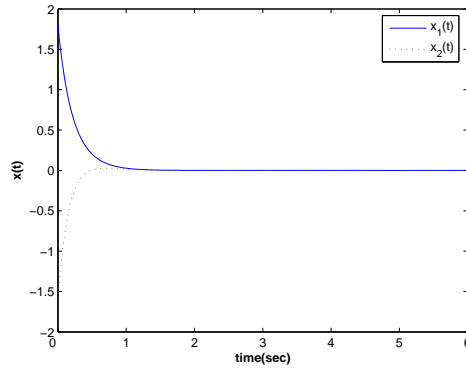


FIGURE 1. State Responses of System

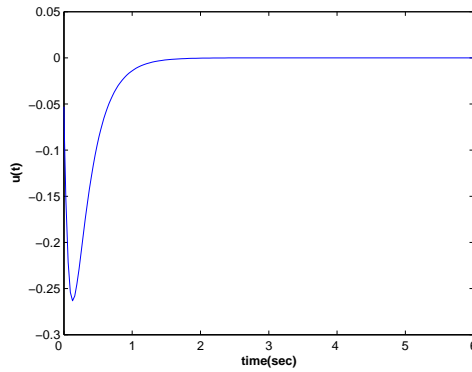


FIGURE 2. Control Trajectory of System

From the system equation (32), some equilibrium points are tabulated in Table 1.

According to these equilibrium points, $[x_e \ u_e]$, which are also chosen as the desired operating points, $[x'_e \ u'_e]$, we can use the similar modeling method that is described in [19].

x_e^T	$x_{e(t-d)}^T$	u_e	$u_{e(t-d)}$
[2.0422 1.2178]	[2.0422 1.2178]	20.3077	20.3077
[3.6626 2.5443]	[3.6626 2.5443]	77.7272	77.7272
[5.9543 5.5403]	[5.9543 5.5403]	296.2414	296.2414

TABLE 1. Data for Equilibrium Points

Thus, the system (32) can be represented by

$$\begin{aligned}
R^1: & \text{ IF } x_1 \text{ is about } 2.0422 \\
& \text{ THEN } \dot{x}_\delta(t) = A_1 x_\delta(t) + A_{d1} x_{d\delta}(t) + B_1 u_\delta(t) + B_{d1} u_{d\delta}(t) \\
& \quad + N_1 x_\delta(t) u_\delta(t) + N_{d1} x_{d\delta}(t) u_{d\delta}(t) \\
R^2: & \text{ IF } x_1 \text{ is about } 3.6626 \\
& \text{ THEN } \dot{x}_\delta(t) = A_2 x_\delta(t) + A_{d2} x_{d\delta}(t) + B_2 u_\delta(t) + B_{d2} u_{d\delta}(t) \\
& \quad + N_2 x_\delta(t) u_\delta(t) + N_{d2} x_{d\delta}(t) u_{d\delta}(t) \\
R^3: & \text{ IF } x_1 \text{ is about } 5.9543 \\
& \text{ THEN } \dot{x}_\delta(t) = A_3 x_\delta(t) + A_{d3} x_{d\delta}(t) + B_3 u_\delta(t) + B_{d3} u_{d\delta}(t) \\
& \quad + N_3 x_\delta(t) u_\delta(t) + N_{d3} x_{d\delta}(t) u_{d\delta}(t)
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
A_1 &= \begin{bmatrix} -75.2383 & 7.7946 \\ 50 & -100 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -98.3005 & 11.7315 \\ 50 & -100 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} -122.1228 & 8.8577 \\ 50 & -100 \end{bmatrix}; \quad N_1 = N_2 = N_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}; \\
B_1 = B_2 = B_3 &= \begin{bmatrix} 10 \\ 0 \end{bmatrix}; \quad A_{d1} = A_{d2} = A_{d3} = \begin{bmatrix} 0 & 5 \\ 10 & -5 \end{bmatrix}, \\
N_{d1} = N_{d2} = N_{d3} &= \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & -0.2 \end{bmatrix}; \quad B_{d1} = B_{d2} = B_{d3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \\
x_\delta &= x(t) - x'_e, \quad u_\delta = u(t) - u'_e, \quad x_{d\delta} = x(t-d) - x'_{de}, \quad u_{d\delta} = u(t-d) - u'_{de}.
\end{aligned}$$

The cost function associated with this system is given by $S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $W = 1$.

The controller gain perturbation ΔK of the additive form is give with $H_1 = H_2 = H_3 = 0.1$, $E_{k1} = [0.05 \ -0.01]$, $E_{k2} = [0.02 \ 0.01]$, $E_{k3} = [-0.01 \ 0]$.

The membership functions of state x_1 are shown in Figure 3.

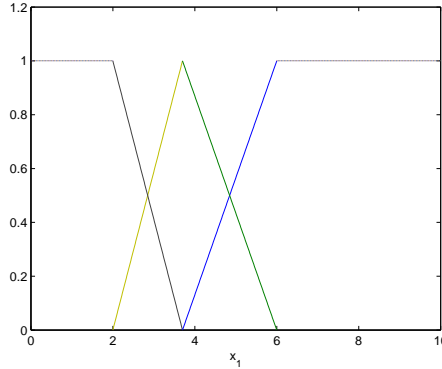


FIGURE 3. Membership Functions

Then, solving LMIs (23) for $\rho = 0.45$, $\lambda = 1.02$ and $\delta = 0.11$, $\tau = 2$, $\sigma = 0$, gives the following feasible solution:

$$P = \begin{bmatrix} 7.5659 & -1.3007 \\ -1.3007 & 6.4906 \end{bmatrix}, Q = \begin{bmatrix} 14.1872 & -1.9381 \\ -1.9381 & 13.0104 \end{bmatrix},$$

$$R = \begin{bmatrix} 8.3691 & -1.3053 \\ -1.3053 & 7.0523 \end{bmatrix}, \varepsilon = 1.8043, K_1 = [-0.4233 \ -0.5031],$$

$$K_2 = [-0.5961 \ -0.7049], K_3 = [-0.4593 \ -0.3874].$$

Figure 4-Figure 6 illustrate the simulation results of applying the non-fragile fuzzy controller to the system (33) with $x'_e = [3.6626 \ 2.5443]^T$ and $u'_e = 77.7272$ under initial condition $\phi(t) = [1.2 \ -1.8]^T$, $t \in [-2, 0]$. It can be seen that with the fuzzy control law the closed-loop system is asymptotically stable and an upper bound of the guaranteed cost is $J_0 = 292.0399$. The simulation results show that the fuzzy non-fragile guaranteed controller proposed in this paper is effective.

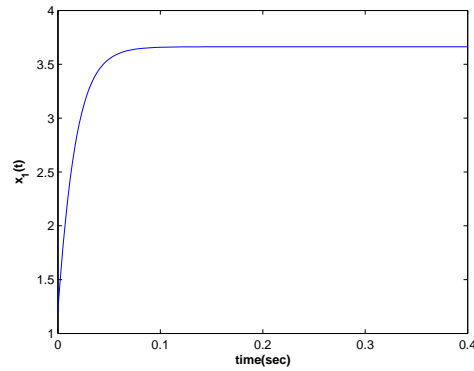


FIGURE 4. State Responses of $x_1(t)$

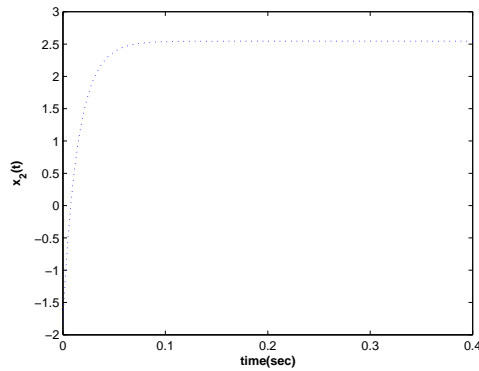


FIGURE 5. State Responses of $x_2(t)$

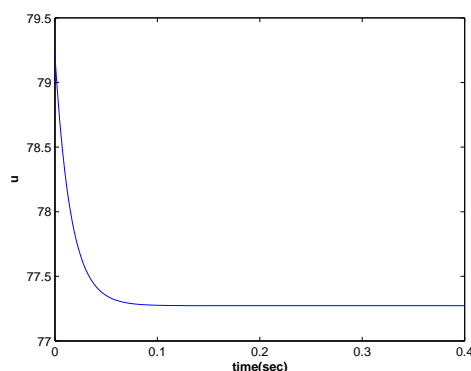


FIGURE 6. Control Trajectory of System

6. Conclusions

In this paper, the problem of non-fragile guaranteed cost control for a class of fuzzy time-varying delay systems with local bilinear models is investigated. The fuzzy controller, which guarantees not only the asymptotically stability of the closed-loop system but also the cost function bound constraint, has been provided in terms of the feasible solutions to the LMIs. Two simulation examples are included to show the effectiveness of the proposed approach. The robust non-fragile guaranteed cost control and robust non-fragile H-infinite control based on Fuzzy bilinear model will further investigate in the future work.

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