GLOBAL ROBUST STABILITY CRITERIA FOR T-S FUZZY SYSTEMS WITH DISTRIBUTED DELAYS AND TIME DELAY IN THE LEAKAGE TERM

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Abstract. The paper is concerned with robust stability criteria for Takagi-Sugeno (T-S) fuzzy systems with distributed delays and time delay in the leakage term. By exploiting a model transformation, the system is converted to one of the neutral delay system. Global robust stability result is proposed by a new Lyapunov-Krasovskii functional which takes into account the range of delay and by making use of some inequality techniques. Based on the interval time-varying delays, new stability criteria are obtained in terms of linear matrix inequalities (LMIs). Finally, three numerical examples and their simulations are given to show the effectiveness and advantages of our results.

1. Introduction

In the last two decades, the fuzzy model has been extensively studied because T-S fuzzy model can provide an effective representation of complex nonlinear systems, see for example [28, 29, 4, 2] and references therein. The phenomena of time delays often occur in many dynamic systems such as chemical processes, metallurgical processes, biological systems, and mechanics. Furthermore, the existence of time-delays is usually a source of instability and deteriorated performance. Recently stability criteria for T-S fuzzy system has been widely investigated by many authors, see for example [30, 32, 21, 8, 3, 18, 33, 10, 15, 16, 17, 23]. Additionally, in many practical cases, the delay may typically exist in an interval $0 \leq h_1 \leq \tau(t) \leq h_2$, that is, the range of delay varies in an interval for which the lower bound is not restricted to 0, see for example [20, 11, 35, 24]. Recently the problem of an aggregated fuzzy reliability index for slope stability analysis is investigated in [19]. In [27], potential energy based stability analysis of fuzzy linguistic systems is discussed and the authors in [5] deal with exact and approximate solutions of fuzzy lr linear systems: new algorithms using a least squares model and the abs approach.

In practice, uncertainty in mathematical modelling is unavoidable because it is very difficult to obtain an exact mathematical model due to environmental noise, uncertain or slowly varying parameters, etc. Therefore, considerable amounts of efforts have been done to the robust stability for uncertain systems, see for example [12, 34, 9, 22, 31].

Received: July 2010; Revised: November 2010; Accepted: April 2011

Key words and phrases: Delay-dependent stability, Linear matrix inequality, Lyapunov–Krasovskii functional, T-S fuzzy systems.

Initially Gopalsamy [7] has investigated the bidirectional associative memory (BAM) neural networks with constant delays in the leakage term. Li et al [14] have been investigated the stability of nonlinear systems. Li et al [13] have been discussed existence, uniqueness and stability analysis of recurrent neural networks with time delay in the leakage term under impulsive perturbations. Such time delay in leakage term has also great impact on dynamical behavior of systems. To the best of authors knowledge, so far, no result on the robust stability criteria for T-S fuzzy systems with time delay in the leakage and distributed delays is available in the existing literature. This motivates our research work.

Motivated by the above discussion, the global robust stability criteria for T-S fuzzy systems with distributed delays and time delay in the leakage term is consider in this paper. By constructing a new Lyapunov-Krasovskii functional and employing some analysis techniques, sufficient conditions are derived from the considered T-S fuzzy systems in terms of LMI, which can be easily calculated by MATLAB LMI control toolbox. Numerical examples are given to illustrate the effectiveness of the proposed method.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ will denote, respectively, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices. The superscript T denotes the transposition and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I_n is the $n \times n$ identity matrix. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. The notation $*$ always denotes the symmetric block in one symmetric matrix. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Preliminaries

Consider a T-S fuzzy system with a time-varying delay, which is represented by a T-S fuzzy model composed of a set of fuzzy implications, and each implication is expressed by a linear system model. The ith rule of this T-S fuzzy model is of the following form

Rule i: If $\Theta_1(t)$ is μ_{i1} and $\ldots$ $\Theta_p(t)$ is μ_{ip} then

$$\begin{aligned} \dot{x}(t) &= (A_i + \Delta A_i(t))x(t-\sigma) + (B_i + \Delta B_i(t))x(t-\tau(t)) + (C_i + \Delta C_i(t)) \int_{t-\eta(t)}^{t} x(s)ds \\ x(t) &= \phi(t), \quad t \in [-\tau^*, 0],\ i = 1, \ldots, r, \quad \tau^* = max\{\sigma, h_2, \bar{\eta}\}. \end{aligned} \tag{1}$$

where $\mu_{ij}, i = 1, 2, \ldots, r, j = 1, \ldots, p$ is the fuzzy set; $x(t) \in \mathbb{R}^n$ is the state vector; A_i, B_i and C_i are constant real matrices with appropriate dimensions; r is the number of IF-Then rules; $\Theta_1(t), \Theta_2(t), \ldots,$
$\Theta_p(t)$ are the premise variables. The time-varying delays $\tau(t)$ satisfy

$$0 \leq h_1 \leq \tau(t) \leq h_2, \quad \dot{\tau}(t) \leq \mu,$$

where h_1, h_2 and μ are constants and $\eta(t)$ represent the distributed delay of systems with $0 \leq \eta(t) \leq \bar{\eta}$. $\sigma \geq 0$ is the leakage delay. We shall consider model (1) with the initial condition $x(t) = \phi(t),\ t \in [-\tau^*, 0]$, and the norm is defined by $\|\phi\|_{\tau^*} = max\{sup_{\tau^* \leq s \leq 0}\|\phi(s)\|,\ sup_{-\tau^* \leq s \leq 0}\|\dot{\phi}(s)\|\}$.

Next we address the uncertainty, suppose that matrices A_i, B_i and C_i have parameter perturbations $\Delta A_i(t)$, $\Delta_i B(t)$ and $\Delta C_i(t)$, which are of the form

$$\begin{bmatrix}\Delta A_i(t) & \Delta B_i(t) & \Delta C_i(t)\end{bmatrix} = H_i F_i(t)\begin{bmatrix}E_{1i} & E_{2i} & E_{3i}\end{bmatrix} \tag{2}$$

where E_{1i}, E_{2i} and E_{3i} are known constant matrices. and $F_i(t)$ is unknown matrix function satisfying

$$F_i^T(t)F_i(t) \leq I. \tag{3}$$

By fuzzy blending, the overall fuzzy model is inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \frac{\sum_{i=1}^r \omega_i(\theta(t))\Big[(A_i+\Delta A_i(t))x(t-\sigma)+(B_i+\Delta B_i(t))x(t-\tau(t))+(C_i+\Delta C_i(t))\int_{t-\eta(t)}^t x(s)ds\Big]}{\sum_{i=1}^r \omega_i(\theta(t))}\\ &= \sum_{i=1}^r \rho_i(\theta(t))\Big[(A_i+\Delta A_i(t))x(t-\sigma)+(B_i+\Delta B_i(t))x(t-\tau(t))+(C_i+\Delta C_i(t))\int_{t-\eta(t)}^t x(s)ds\Big]\\ x(t) &= \phi(t), \quad t\in[-\tau^*,0], \quad i=1,\ldots,r. \end{aligned}\tag{4}$$

Equation (4) can be written as

$$\begin{aligned}\dot{x}(t) &= A_i(t)x(t-\sigma)+B_i(t)x(t-\tau(t))+C_i(t)\int_{t-\eta(t)}^t x(s)ds\\ x(t) &= \phi(t), \quad t\in[-\tau^*,0], \quad i=1,\ldots,r,\end{aligned}\tag{5}$$

where $\theta = [\theta_1,\theta_2,\ldots,\theta_p]$; $\omega_i : \mathbb{R}^p \longrightarrow [0,1], i=1,\ldots,r$ is the membership function of the system with respect to the plant rule i; $\rho_i(\theta(t)) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^r \omega_i(\theta(t))}$; $A_i(t) = \sum_{i=1}^r \rho_i(\theta(t))(A_i+\Delta A_i(t))$, $B_i(t) = \sum_{i=1}^r \rho_i(\theta(t))(B_i+\Delta B_i(t))$ and $C_i(t) = \sum_{i=1}^r \rho_i(\theta(t))$ $(C_i+\Delta C_i(t))$. It is obvious that the fuzzy weighting functions $\rho_i(\theta(t))$ satisfy $\rho_i(\theta(t)) \geq 0$, $\sum_{i=1}^r \rho_i(\theta(t)) = 1$. to obtain the main results of this paper, the following lemmas will be essential.

Lemma 2.1. [34] *For any vectors $x, y \in \mathbb{R}^n$, matrices $A, P \in \mathbb{R}^{n\times m}$ and $F(t) \in \mathbb{R}^{m\times m}$ with $P > 0$, $F^T(t)F(t) \leq I$ and scalar $\epsilon > 0$, the following inequalities hold*

(i) $2x^Ty \leq x^TP^{-1}x + y^TPy$

(ii) $HF(t)N + N^TF^T(t)D^T \leq \epsilon^{-1}DD^T + \epsilon N^TN$

(iii) *If* $P^{-1} - \epsilon^{-1}HH^T > 0$, *then*
$(A+HF(t)N)^TP(A+HF(t)N) \leq A^T(P^{-1}-\epsilon^{-1}HH^T)^{-1}A + \epsilon N^TN.$

Lemma 2.2. [1] *(Schur Complement) Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T = \Omega_1$ and $\Omega_2^T = \Omega_2 > 0$, then*

$$\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$$

if and only if

$$\begin{bmatrix}\Omega_1 & \Omega_3^T\\ * & -\Omega_2\end{bmatrix} < 0 \quad or \quad \begin{bmatrix}-\Omega_2 & \Omega_3\\ * & \Omega_1\end{bmatrix} < 0.$$

Lemma 2.3. *(Jensen's inequality) For any constant matrix $M > 0$, any scalars a and b with $a < b$ and a vector function $x(t) : [a,b] \longrightarrow \mathbb{R}^n$ such that integrals concerned are well defined, then the following inequality holds*

$$\Big[\int_a^b x(s)ds\Big]^T M\Big[\int_a^b x(s)ds\Big] \le (b-a)\Big[\int_a^b x(s)^T Mx(s)ds\Big].$$

3. Main Results

In this section, we derive a new delay-dependent stability analysis of delayed T-S system (5). Using a simple transformation, model (5) has an equivalent form as follows:

$$\frac{d}{dt}\Big[x(t) + A_i(t)\int_{t-\sigma}^t x(s)ds\Big] = A_i(t)x(t) + B_i(t)x(t-\tau(t)) + C_i(t)\int_{t-\eta(t)}^t x(s)ds. \quad (6)$$

Now we state and prove the following theorem without uncertainties.

Theorem 3.1. *For given scalars $h_2 > h_1 \ge 0$, σ, $\bar{\eta}$ and μ, the equilibrium point of system (5) or (6) is globally asymptotically stable if there exist symmetric matrices $P > 0$, $Q_l > 0, l = 1,2,3,4$, $Z_k > 0$, $k = 1,2,3,4,5$, $R_1 > 0$, $R_2 > 0$, $S > 0$, for any matrices N_1, N_2, N_3, M_1, M_2 and M_3 such that the following LMI is feasible*

$$\Pi^i = \begin{bmatrix} \Xi^i & \frac{1}{\sqrt{2}}h_2 N & \sqrt{\frac{1}{2}(h_2^2-h_1^2)}M & \Upsilon^{iT}U \\ * & -R_1 & 0 & 0 \\ * & * & -R_2 & 0 \\ * & * & * & -U \end{bmatrix} < 0, \quad (7)$$

where $\Xi^i = (\Xi^i_{l\times k})_{10\times 10}$ *with*

$$\begin{aligned}
\Xi^i_{11} &= PA_i + A_i^T P + \sigma^2 Z_1 - Z_2 + Q_1 + Q_2 + Q_3 + Q_4 + \bar{\eta}^2 S + h_2 Z_4 + (h_1 - h_2)Z_5 + h_2 N_1 \\
&\quad + h_2 N_1^T + (h_2 - h_1)M_1 + (h_2 - h_1)M_1^T, \ \Xi^i_{12} = PB_i + Z_2 + h_2 N_2^T + (h_2 - h_1)M_2^T, \\
\Xi^i_{13} &= \Xi^i_{14} = 0, \quad \Xi^i_{15} = h_2 N_3^T + (h_2 - h_1)M_3^T, \quad \Xi^i_{16} = A_i^T PA_i, \quad \Xi^i_{17} = PC_i \\
\Xi^i_{18} &= -N_1, \quad \Xi^i_{19} = -N_1 - M_1, \quad \Xi^i_{1,10} = -M_1, \quad \Xi^i_{22} = -Z_2 - Z_3 - Z_3^T - (1-\mu)Q_3, \\
\Xi^i_{23} &= \Xi^i_{24} = Z_3, \quad \Xi^i_{25} = 0, \quad \Xi^i_{26} = B_i^T PA_i, \quad \Xi^i_{27} = 0, \ \Xi^i_{28} = -N_2, \ \Xi^i_{29} = -N_2 - M_2, \\
\Xi^i_{2,10} &= -M_2, \quad \Xi^i_{33} = -Z_3 - Q_1, \quad \Xi^i_{34} = \ldots = \Xi^i_{3,10} = 0, \quad \Xi^i_{44} = -Z_3 - Q_2, \\
\Xi^i_{45} &= \ldots = \Xi^i_{4,10} = 0, \quad \Xi^i_{55} = -Q_4, \quad \Xi^i_{56} = \Xi^i_{57} = 0, \Xi^i_{58} = -N_3, \Xi^i_{59} = -N_3 - M_3, \\
\Xi^i_{5,10} &= -M_3, \quad \Xi^i_{66} = -Z_1, \quad \Xi^i_{67} = A_i^T PC_i, \quad \Xi^i_{68} = \Xi^i_{69} = \Xi^i_{6,10} = 0, \quad \Xi^i_{77} = -S, \\
\Xi^i_{78} &= \Xi^i_{79} = \Xi^i_{7,10} = 0, \quad \Xi^i_{88} = -\frac{1}{h_2}Z_4, \quad \Xi^i_{89} = \Xi^i_{8,10} = 0, \quad \Xi^i_{99} = -\frac{1}{(h_2 - h_1)}(Z_4 + Z_4), \\
\Xi^i_{9,10} &= 0, \quad \Xi^i_{10,10} = -\frac{1}{(h_2 - h_1)}Z_5, \quad U = h_2^2 Z_2 + (h_2 - h_1)^2 Z_3 + \frac{h_2^2}{2}R_1 + \frac{h_2^2 - h_1^2}{2}R_2, \\
N &= [N_1^T \ N_2^T \ 0 \ 0 \ N_3^T \ 0 \ 0 \ 0 \ 0 \ 0]^T, \quad M = [M_1^T \ M_2^T \ 0 \ 0 \ M_3^T \ 0 \ 0 \ 0 \ 0 \ 0]^T.
\end{aligned}$$

Proof. Consider the Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{8}$$

where

$$\begin{aligned}
V_1(t) &= [x(t) + A_i \int_{t-\sigma}^{t} x(s)ds]^T P[x(t) + A_i \int_{t-\sigma}^{t} x(s)ds], \\
V_2(t) &= \int_{t-h_1}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_2}^{t} x^T(s)Q_2x(s)ds \\
&\quad + \int_{t-\tau(t)}^{t} x^T(s)Q_3x(s)ds + \int_{t-\sigma}^{t} x^T(s)Q_4x(s)ds, \\
V_3(t) &= \sigma \int_{-\sigma}^{0}\int_{t+\theta}^{t} x^T(s)Z_1x(s)dsd\theta + h_2\int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\theta \\
&\quad +(h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)Z_3\dot{x}(s)dsd\theta + \int_{-h_2}^{0}\int_{t+\theta}^{t} x^T(s)Z_4x(s)dsd\theta \\
&\quad + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} x^T(s)Z_5x(s)dsd\theta + \bar{\eta}\int_{-\bar{\eta}}^{0}\int_{t+\theta}^{t} x^T(s)Sx(s)dsd\theta, \\
V_4(t) &= \int_{-h_2}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\lambda d\theta + \int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\lambda d\theta.
\end{aligned}$$

Calculating the derivative of $V(t)$ along the solution of (5) or (6), we have

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \tag{9}$$

where

$$\begin{aligned}
\dot{V}_1(t) &= 2\Big[x(t) + A_i\int_{t-\sigma}^{t} x(s)ds\Big]^T P\Big[A_ix(t) + B_ix(t-\tau(t)) + C_i\int_{t-\eta(t)}^{t} x(s)ds\Big], \\
\dot{V}_2(t) &\le x^T(t)Q_1x(t) - x^T(t-h_1)Q_1x(t-h_1) + x^T(t)Q_2x(t) - x^T(t-h_2)Q_2x(t-h_2) \\
&\quad +x^T(t)Q_3x(t) - (1-\mu)x^T(t-\tau(t))Q_3x(t-\tau(t)) + x^T(t)Q_4x(t) - x^T(t-\sigma)Q_4x(t-\sigma), \\
\dot{V}_3(t) &= \sigma^2x^T(t)Z_1x(t) - \sigma\int_{t-\sigma}^{t} x^T(s)Z_1x(s)ds + h_2^2\dot{x}^T(t)Z_2\dot{x}(t) - h_2\int_{t-h_2}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds \\
&\quad +(h_2-h_1)^2\dot{x}^T(t)Z_3\dot{x}(t) - (h_2-h_1)\int_{t-h_2}^{t-h_1} \dot{x}^T(s)Z_3\dot{x}(s)ds + x^T(t)(h_2Z_4 + (h_2-h_1)Z_5)x(t) \\
&\quad -\int_{t-h_2}^{t} x^T(s)Z_4x(s)ds - \int_{t-h_2}^{t-h_1} x^T(s)Z_5x(s)ds + \bar{\eta}^2x^T(t)Sx(t) - \bar{\eta}\int_{t-\eta(t)}^{t} x^T(s)Sx(s)ds, \\
&\le \sigma^2x^T(t)Z_1x(t) - \sigma\int_{t-\sigma}^{t} x^T(s)Z_1x(s)ds + h_2^2\dot{x}^T(t)Z_2\dot{x}(t) - h_2\int_{t-\tau(t)}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds \\
&\quad +(h_2-h_1)^2\dot{x}^T(t)Z_3\dot{x}(t) - (h_2-h_1)\int_{t-h_2}^{t-\tau(t)} \dot{x}^T(s)Z_3\dot{x}(s)ds \\
&\quad -(h_2-h_1)\int_{t-\tau(t)}^{t-h_1} \dot{x}^T(s)Z_3\dot{x}(s)ds + x^T(t)(h_2Z_4 + (h_2-h_1)Z_5)x(t)
\end{aligned}$$

$$
\begin{aligned}
& -\int_{t-\tau(t)}^{t} x^T(s)Z_4x(s)ds - \int_{t-h_2}^{t-\tau(t)} x^T(s)(Z_4+Z_5)x(s)ds - \int_{t-\tau(t)}^{t-h_1} x^T(s)Z_5x(s)ds \\
& +\bar{\eta}^2 x^T(t)Sx(t) - \Big[\int_{t-\eta(t)}^{t} x(s)ds\Big]^T S\Big[\int_{t-\eta(t)}^{t} x(s)ds\Big], \\
\dot{V}_4((t)) \quad = \quad & \frac{1}{2}h_2^2\dot{x}^T(t)R_1\dot{x}(t) - \int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \frac{1}{2}(h_2^2-h_1^2)\dot{x}^T(t)R_2\dot{x}(t) \\
& -\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta.
\end{aligned}
$$

From Lemma 2.3, we have

$$
-\sigma\int_{t-\sigma}^{t} x^T(s)Z_1x(s)ds \quad \leq \quad -\Big[\int_{t-\sigma}^{t} x(s)ds\Big]^T Z_1\Big[\int_{t-\sigma}^{t} x(s)ds\Big] \tag{10}
$$

$$
-h_2\int_{t-\tau(t)}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds \quad \leq \quad -\Big[x(t)-x(t-\tau(t))\Big]^T Z_2\Big[x(t)-x(t-\tau(t))\Big] \tag{11}
$$

$$
-(h_2-h_1)\int_{t-h_2}^{t-\tau(t)} \dot{x}^T(s)Z_3\dot{x}(s)ds \quad \leq \quad -\Big[x(t-\tau(t))-x(t-h_2)\Big]^T Z_3\Big[x(t-\tau(t))-x(t-h_2)\Big] \tag{12}
$$

$$
-(h_2-h_1)\int_{t-\tau(t)}^{t-h_1} \dot{x}^T(s)Z_3\dot{x}(s)ds \quad \leq \quad -\Big[x(t-h_1)-x(t-\tau(t))\Big]^T Z_3\Big[x(t-h_1)-x(t-\tau(t))\Big] \tag{13}
$$

$$
-\int_{t-\tau(t)}^{t} x^T(s)Z_4x(s)ds \quad \leq \quad -\frac{1}{h_2}\Big[\int_{t-\tau(t)}^{t} x(s)ds\Big]^T Z_4\Big[\int_{t-\tau(t)}^{t} x(s)ds\Big], \tag{14}
$$

$$
-\int_{t-h_2}^{t-\tau(t)} x^T(s)(Z_4+Z_5)x(s)ds \quad \leq \quad -\frac{1}{h_2-h_1}\Big[\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big]^T (Z_4+Z_5)\Big[\int_{t-h_2}^{t-\tau(t)} x(s)ds\Big], \tag{15}
$$

$$
-\int_{t-\tau(t)}^{t-h_1} x^T(s)Z_5x(s)ds \quad \leq \quad -\frac{1}{h_2-h_1}\Big[\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big]^T Z_5\Big[\int_{t-\tau(t)}^{t-h_1} x(s)ds\Big]. \tag{16}
$$

Further, we can see that the following equations are true for any matrices N_1, N_2, N_3, M_1, M_2, and M_3 with appropriate dimensions

$$
\begin{aligned}
0 \quad = \quad & 2[x^T(t)N_1 + x^T(t-\tau(t))N_2 + x^T(t-\sigma)N_3]\Big[h_2x(t) - \int_{t-\tau(t)}^{t} x(s)ds - \int_{t-h_2}^{t-\tau(t)} x(s)ds \\
& -\int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{x}(s)dsd\theta\Big],
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
0 \quad = \quad & 2[x^T(t)M_1 + x^T(t-\tau(t))M_2 + x^T(t-\sigma)M_3]\Big[(h_2-h_1)x(t) - \int_{t-h_2}^{t-\tau(t)} x(s)ds \\
& -\int_{t-\tau(t)}^{t-h_1} x(s)ds - \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}(s)dsd\theta\Big].
\end{aligned} \tag{18}
$$

From Lemma 2.3, we have the following inequalities

$$-\quad 2\zeta^T(t)N\int_{-h_2}^{0}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta \le \frac{1}{2}h_2^2\zeta^T(t)NR_1^{-1}N^T\zeta(t)+\int_{-h_2}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R_1\dot{x}(s)dsd\theta,$$

$$-\quad 2\zeta^T(t)M\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}(s)dsd\theta \le \frac{1}{2}(h_2^2-h_1^2)\zeta^T(t)MR_2^{-1}M^T\zeta(t)+\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}^T(s)R_2\dot{x}(s)dsd\theta.$$

Substituting (10)-(18) into (9), we have

$$\dot{V}(t)\le\sum_{i=1}^{r}\rho_i(\theta(t))\zeta^T(t)\Pi^i\zeta(t)<0. \tag{19}$$

where

$$\Pi^i = \Xi^i+\frac{1}{2}h_2^2NR_1^{-1}N^T+\frac{1}{2}(h_2^2-h_1^2)MR_2^{-1}M^T+\Upsilon_i^T(h_2^2Z_2+(h_2-h_1)^2Z_3+\frac{h_2^2}{2}R_1+\frac{h_2^2-h_1^2}{2}R_2)\Upsilon_i$$

$$\zeta^T(t) = \Big[x^T(t)\quad x^T(t-\tau(t))\quad x^T(t-h_1)\quad x^T(t-h_2)\quad x^T(t-\sigma)\quad \int_{t-\sigma}^{t}x^T(s)ds$$
$$\int_{t-\eta(t)}^{t}x^T(s)ds\quad \int_{t-\tau(t)}^{t}x^T(s)ds\quad \int_{t-h_2}^{t-\tau(t)}x^T(s)ds\quad \int_{t-\tau(t)}^{t-h_1}x^T(s)ds\Big],$$

and $\Upsilon^i = [0\quad B_i\quad 0\quad 0\quad A_i\quad 0\quad C_i\quad 0\quad 0\quad 0].$

Considering the first term on the right hand of (8), one can easily obtain that

$$\begin{aligned}\| x(t)+A_i\int_{t-\sigma}^{t}x(s)ds\|^2 &\le \frac{1}{\lambda_{min}(P)}\big[x(t)+A_i\int_{t-\sigma}^{t}x(s)ds\big]^TP\big[x(t)+A_i\int_{t-\sigma}^{t}x(s)ds\big]\\ &\le \frac{V(t)}{\lambda_{min}(P)}\le\frac{V(0)}{\lambda_{min}(P)},\quad t>0,\end{aligned}$$

which implies that

$$\| x(t)\|\le\| A_i\int_{t-\sigma}^{t}x(s)ds\|+\sqrt{\frac{V_0(t)}{\lambda_{min}(P)}}\quad\le\quad\| A_i\|\int_{t-\sigma}^{t}\| x(s)\| ds+\sqrt{\frac{V(0)}{\lambda_{min}(P)}}\quad t>0,$$

where $\| A_i\| = \sqrt{\sum_{l=1}^{n}\sum_{k=1}^{n}a_{ilk}^2}$.

From the well-known Gronwall inequality, one obtains

$$\| x(t)\|\le\sqrt{\frac{V(0)}{\lambda_{min}(P)}}e^{\| A_i\|\sigma}. \tag{20}$$

Note that

$$\begin{aligned}V(0) \le\ & [\phi(0)+A_i\int_{-\sigma}^{0}\phi(s)ds]^TP[\phi(0)+A_i\int_{-\sigma}^{0}\phi(s)ds]+\int_{-h_1}^{0}\phi^T(s)Q_1\phi(s)ds\\ &+\int_{-h_2}^{0}\phi^T(s)Q_2\phi(s)ds+\int_{-\tau(0)}^{0}\phi^T(s)Q_3\phi(s)ds+\int_{-\sigma}^{0}\phi^T(s)Q_4\phi(s)ds\\ &+\sigma\int_{-\sigma}^{0}\int_{\theta}^{0}\phi^T(s)Z_1\phi(s)dsd\theta+h_2\int_{-h_2}^{0}\int_{\theta}^{0}\dot{\phi}^T(s)Z_2\dot{\phi}(s)dsd\theta\end{aligned}$$

$$+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{\theta}^{0}\dot{\phi}^T(s)Z_3\dot{\phi}(s)dsd\theta+\int_{-h_2}^{0}\int_{\theta}^{0}\phi^T(s)Z_4\phi(s)dsd\theta$$
$$+\int_{-h_2}^{-h_1}\int_{\theta}^{0}\phi^T(s)Z_5\phi(s)dsd\theta+\bar{\eta}\int_{-\bar{\eta}}^{0}\int_{\theta}^{0}\phi^T(s)S\phi(s)dsd\theta$$
$$+\int_{-h_2}^{0}\int_{\theta}^{0}\int_{\lambda}^{0}\dot{\phi}^T(s)R_1\dot{\phi}(s)dsd\lambda d\theta+\int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{\lambda}^{0}\dot{\phi}^T(s)R_2\dot{\phi}(s)dsd\lambda d\theta.$$
$$\leq \{2\lambda_{max}(P)(1+\sigma\parallel A_i\parallel))+h_1\lambda_{max}(Q_1)+h_2\lambda_{max}(Q_2)+h_2\lambda_{max}(Q_3)+\sigma\lambda_{max}(Q_4)$$
$$+\sigma^3\lambda_{max}(Z_1)+h_2^3\lambda_{max}(Z_2)+(h_2-h_1)^3\lambda_{max}(Z_3)+h_2^2\lambda_{max}(Z_4)+(h_2-h_1)^2\lambda_{max}(Z_5)$$
$$+\bar{\eta}^3\lambda_{max}(S)+h_2^3\lambda_{max}(R_1)+(h_2-h_1)^3\lambda_{max}(R_2)\}\parallel\phi\parallel_{\tau*}^2<\infty. \tag{21}$$

From (20) and (21), it can be deduced that the trivial solution of system (5) is locally stable. Then the solution $x(t)=x(t,0,\phi)$ of system (5) is bounded on $[0,\infty)$. Considering (5), we know that $\dot{x}(t)$ is bounded on $[0,\infty)$, which leads to the uniform continuity of the solution $x(t)$ on $[0,\infty)$. From (19), we note that the following inequality holds:

$$\lambda_{min}(\Pi^i)\int_0^t x^T(s)x(s)ds\leq V(t)+\int_0^t\zeta^T(s)\Pi^i\zeta(s)ds\leq V(0)<\infty, t>0,$$

By Barbalat's lemma, (see Gopalsamy [6]), it holds that $\parallel x(t)\parallel\longrightarrow 0$ as $t\longrightarrow\infty$. This completes the proof. □

Now consider the following time delay system without leakage delay and distributed delay described by

$$\begin{aligned}\dot{x}(t) &= -\bar{A}_i x(t)+\bar{B}_i x(t-\tau(t))\\ x(t) &= \phi(t)\quad t\in[-h_2,0],\end{aligned}\tag{22}$$

where $\bar{A}_i=\sum_{i=1}^r\rho_i(\theta(t))A_i$ and $\bar{B}_i=\sum_{i=1}^r\rho_i(\theta(t))B_i$. The corresponding result of the above system (22) is summarized in the following Corollary.

Corollary 3.2. *For given scalars $h_2>h_1\geq 0$, and μ, the equilibrium point of system (22) is globally asymptotically stable if there exist symmetric matrices $P>0$, $Q_l>0, l=1,2,3$, $Z_k>0$, $k=2,3,4,5$, $R_1>0$, $R_2>0$, for any matrices N_1,N_2,M_1 and M_2 such that the following LMI is feasible*

$$\begin{bmatrix}\Omega^i & \frac{1}{\sqrt{2}}h_2\widetilde{N} & \sqrt{\frac{1}{2}(h_2^2-h_1^2)}\widetilde{M} & \bar{\Upsilon}^{iT}U\\ * & -R_1 & 0 & 0\\ * & * & -R_2 & 0\\ * & * & * & -U\end{bmatrix}<0, \tag{23}$$

where $\Omega^i=(\Omega^i_{lk})_{7\times 7}$ with

$$
\begin{aligned}
\Omega^i_{11} &= PA_i + A_i^T P - Z_2 + Q_1 + Q_2 + Q_3 + h_2 Z_4 + (h_1 - h_2) Z_5 + h_2 N_1 + h_2 N_1^T \\
&\quad +(h_2 - h_1) M_1 + (h_2 - h_1) M_1^T, \quad \Omega^i_{12} = PB_i + Z_2 + h_2 N_2^T + (h_2 - h_1) M_2^T, \\
\Omega^i_{13} &= \Omega^i_{14} = 0, \quad \Omega^i_{15} = -N_1, \quad \Omega^i_{16} = -N_1 - M_1, \quad \Omega^i_{17} = -M_1, \\
\Omega^i_{22} &= -Z_2 - Z_3 - Z_3^T - (1-\mu) Q_3, \quad \Omega^i_{23} = \Omega^i_{24} = Z_3, \quad \Omega^i_{25} = -N_2, \; \Omega^i_{26} = -N_2 - M_2, \\
\Omega^i_{27} &= -M_2, \quad \Omega^i_{33} = -Z_3 - Q_1, \quad \Omega^i_{34} = \ldots = \Omega^i_{37} = 0, \quad \Omega^i_{44} = -Z_3 - Q_2, \\
\Omega^i_{45} &= \ldots = \Omega^i_{47} = 0, \quad \Omega^i_{55} = -\frac{1}{h_2} Z_4, \quad \Omega^i_{56} = \Xi^i_{5,7} = 0, \quad \Omega^i_{66} = -\frac{1}{(h_2 - h_1)} (Z_4 + Z_4), \\
\Omega^i_{67} &= 0, \quad \Omega^i_{77} = -\frac{1}{(h_2 - h_1)} Z_5, \quad U = h_2^2 Z_2 + (h_2 - h_1)^2 Z_3 + \frac{h_2^2}{2} Z_4 + \frac{h_2^2 - h_1^2}{2} Z_5, \\
\widetilde{N} &= [N_1^T \; N_2^T \; 0 \; 0 \; 0 \; 0 \; 0]^T, \quad \widetilde{M} = [M_1^T \; M_2^T \; 0 \; 0 \; 0 \; 0 \; 0]^T, \quad \bar{\Upsilon}^i = [A_i^T \; B_i^T \; 0 \; 0 \; 0 \; 0 \; 0].
\end{aligned}
$$

Proof. Consider the Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{24}$$

where

$$
\begin{aligned}
V_1(t) &= x^T(t) P x(t), \\
V_2(t) &= \int_{t-h_1}^{t} x^T(s) Q_1 x(s) ds + \int_{t-h_2}^{t} x^T(s) Q_2 x(s) ds + \int_{t-\tau(t)}^{t} x^T(s) Q_3 x(s) ds, \\
V_3(t) &= h_2 \int_{-h_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta + (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \dot{x}^T(s) Z_3 \dot{x}(s) ds d\theta \\
&\quad + \int_{-h_2}^{0} \int_{t+\theta}^{t} x^T(s) Z_4 x(s) ds d\theta + \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} x^T(s) Z_5 x(s) ds d\theta \\
V_4(t) &= \int_{-h_2}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds d\lambda d\theta + \int_{-h_2}^{-h_1} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds d\lambda d\theta.
\end{aligned}
$$

□

Proof. The remaining part of the proof is immediately follows from Theorem 3.1. □

4. **Robust Stability Analysis**

In this section, the problem of delay-dependent robust stability analysis for model (5) or (6) will be investigated in the following Theorem 4.1.

Theorem 4.1. *For given scalars $h_2 > h_1 \geq 0$, σ, $\bar{\eta}$ and μ, the equilibrium point of system (5) or (6) is globally robustly asymptotically stable if there exist symmetric matrices $P > 0$, $Q_l > 0, l = 1, 2, 3, 4$, $Z_k > 0$, $k = 1, 2, 3, 4, 5$, $R_1 > 0$, $R_2 > 0$, $S > 0$, for any matrices $N_1, N_2, N_3, M_1, M_2, M_3$ and positive scalar ϵ_i such that the following LMI is feasible*

$$\Psi^i = \begin{bmatrix} \hat{\Pi}^i & \bar{P}H_i & \Theta_1^{iT}P & 0 & \epsilon_i\bar{\Theta}_1^{iT} & \Theta_2^{iT}P & 0 & \epsilon_i\bar{\Theta}_2^{iT} & \Theta_3^{iT}U & 0 & \epsilon_i\bar{\Theta}_3^{iT} \\ * & -\epsilon_i I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P & PH_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_i I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P & PH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon_i I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon_i I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -U & UH_i & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon_i I \end{bmatrix} < 0, \tag{25}$$

where

$$\hat{\Pi}^i = \begin{bmatrix} \hat{\Xi}^i & \frac{1}{\sqrt{2}}h_2N & \sqrt{\frac{1}{2}(h_2^2-h_1^2)}M \\ * & -R_1 & 0 \\ * & * & -R_2 \end{bmatrix},$$

and $\hat{\Xi}^i = (\hat{\Xi}^i_{lk})_{10\times 10}$ *with*

$$\begin{aligned}
\hat{\Xi}^i_{11} &= PA_i + A_i^TP + \sigma^2Z_1 - Z_2 + Q_1 + Q_2 + Q_3 + Q_4 + \bar{\eta}^2S + h_2Z_4 + (h_1-h_2)Z_5 \\
&\quad + h_2N_1 + h_2N_1^T + (h_2-h_1)M_1 + (h_2-h_1)M_1^T + \epsilon_iE_{1i}^TE_{1i}, \hat{\Xi}^i_{12} = PB_i + Z_2 + h_2N_2^T \\
&\quad + (h_2-h_1)M_2^T, \quad \hat{\Xi}^i_{13} = \hat{\Xi}^i_{14} = 0, \quad \hat{\Xi}^i_{15} = h_2N_3^T + (h_2-h_1)M_3^T, \quad \hat{\Xi}^i_{16} = 0, \quad \hat{\Xi}^i_{17} = PC_i \\
\hat{\Xi}^i_{18} &= -N_1, \quad \hat{\Xi}^i_{19} = -N_1 - M_1, \quad \hat{\Xi}^i_{1,10} = -M_1, \quad \hat{\Xi}^i_{22} = -Z_2 - Z_3 - Z_3^T - (1-\mu)Q_3 + \epsilon_iE_{2i}^TE_{2i}, \\
\hat{\Xi}^i_{23} &= \hat{\Xi}^i_{24} = Z_3, \quad \hat{\Xi}^i_{25} = 0, \quad \hat{\Xi}^i_{26} = 0, \quad \hat{\Xi}^i_{27} = 0, \ \hat{\Xi}^i_{28} = -N_2, \ \hat{\Xi}^i_{29} = -N_2 - M_2, \\
\hat{\Xi}^i_{2,10} &= -M_2, \quad \hat{\Xi}^i_{33} = -Z_3 - Q_1, \quad \hat{\Xi}^i_{34} = \ldots = \hat{\Xi}^i_{3,10} = 0, \quad \hat{\Xi}^i_{44} = -Z_3 - Q_2, \\
\hat{\Xi}^i_{45} &= \ldots = \hat{\Xi}^i_{4,10} = 0, \quad \hat{\Xi}^i_{55} = -Q_4, \quad \hat{\Xi}^i_{56} = \hat{\Xi}^i_{57} = 0, \ \hat{\Xi}^i_{58} = -N_3, \ \hat{\Xi}^i_{59} = -N_3 - M_3, \\
\hat{\Xi}^i_{5,10} &= -M_3, \quad \hat{\Xi}^i_{66} = -Z_1, \quad \hat{\Xi}^i_{67} = 0, \quad \hat{\Xi}^i_{68} = \hat{\Xi}^i_{69} = \Xi^i_{6,10} = 0, \quad \hat{\Xi}^i_{77} = -S + \epsilon_iE_{3i}^TE_{3i}, \\
\hat{\Xi}^i_{78} &= \Xi^i_{79} = \hat{\Xi}^i_{7,10} = 0, \quad \hat{\Xi}^i_{88} = -\frac{1}{h_2}Z_4, \quad \hat{\Xi}^i_{89} = \hat{\Xi}^i_{8,10} = 0, \quad \hat{\Xi}^i_{99} = -\frac{1}{(h_2-h_1)}(Z_4 + Z_4), \\
\hat{\Xi}^i_{9,10} &= 0, \quad \hat{\Xi}^i_{10,10} = -\frac{1}{(h_2-h_1)}Z_5.
\end{aligned}$$

Proof. Replacing A_i, B_i and C_i respectively by $A_i + H_iF_i(t)E_{1i}$, $B_i + H_iF_i(t)E_{2i}$ and $C_i + H_iF_i(t)E_{3i}$
in (19), the corresponding formula of (19) can be written as

$$\begin{aligned}
\dot{V}(t) &\le \zeta^T(t)\Big[\bar{\Phi}^i + 2[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ A_i(t)\ \ 0\ \ 0\ \ 0\ \ 0]^TP[A_i(t)\ \ B_i(t)\ \ 0\ \ 0\ \ 0\ \ 0\ \ C_i(t)\ \ 0\ \ 0\ \ 0] \\
&\quad + [0\ \ B_i(t)\ \ 0\ \ 0\ \ A_i(t)\ \ 0\ \ C_i(t)\ \ 0\ \ 0\ \ 0]^TU[0\ \ B_i(t)\ \ 0\ \ 0\ \ A_i(t)\ \ 0\ \ C_i(t)\ \ 0\ \ 0\ \ 0]\Big]\zeta(t) < 0,
\end{aligned}$$

Using Lemma 2.1 (i), we have

$$\begin{aligned}\dot{V}(t) \quad \le \quad & \zeta^T(t)\Big[\bar{\Phi}^i + \Big[[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ A_i\ \ 0\ \ 0\ \ 0\ \ 0] + H_iF_i(t)[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_{1i}\ \ 0\ \ 0\ \ 0\ \ 0]\Big]^T \\ & \times P\Big[[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ A_i\ \ 0\ \ 0\ \ 0\ \ 0] + H_iF_i(t)[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_{1i}\ \ 0\ \ 0\ \ 0\ \ 0]\Big] + \Big[[A_i\ \ B_i\ \ 0\ \ 0 \\ & 0\ \ 0\ \ C_i\ \ 0\ \ 0\ \ 0] + H_iF_i(t)[E_{1i}\ \ E_{2i}\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_{3i}\ \ 0\ \ 0\ \ 0]\Big]^T P\Big[[A_i\ \ B_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ C_i] \\ & +H_iF_i(t)[E_{1i}\ \ E_{2i}\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_{3i}\ \ 0\ \ 0\ \ 0]\Big] + \Big[[0\ \ B_i\ \ 0\ \ 0\ \ A_i\ \ 0\ \ C_i\ \ 0\ \ 0\ \ 0] \\ & +H_iF_i(t)[0\ \ E_{2i}\ \ 0\ \ 0\ \ E_{1i}\ \ 0\ \ E_{3i}\ \ 0\ \ 0\ \ 0]\Big]^T U\Big[[0\ \ B_i\ \ 0\ \ 0\ \ A_i\ \ 0\ \ C_i\ \ 0\ \ 0\ \ 0] \\ & +H_iF_i(t)[0\ \ E_{2i}\ \ 0\ \ 0\ \ E_{1i}\ \ 0\ \ E_{3i}\ \ 0\ \ 0\ \ 0]\Big]\zeta(t) < 0,\end{aligned}$$

where

$\bar{\Phi}^i = \Phi^i + \frac{1}{2}h_2^2 NR_1^{-1}N^T + \frac{1}{2}(h_2^2 - h_1^2)MR_2^{-1}M^T$ and $\Phi^i \ne \Xi^i$ $\Big((l,k) = ((1,1),(1,2),(1,6),(1,7),(2,6),(6,7)\Big)$

$$\begin{aligned}\Phi_{11}^i \quad = \quad & PA_i(t) + A_i^T(t)P + \sigma^2 Z_1 - Z_2 + Q_1 + Q_2 + Q_3 + Q_4 + \bar{\eta}^2 S + h_2 Z_4 + (h_1 - h_2)Z_5 \\ & +h_2N_1 + h_2N_1^T + (h_2 - h_1)M_1 + (h_2 - h_1)M_1^T,\ \Phi_{12}^i = PB_i + Z_2 + h_2N_2^T + (h_2 - h_1)M_2^T, \\ \Phi_{16}^i \quad = \quad & 0,\quad \Phi_{17}^i = PC_i(t)\quad \Phi_{26}^i = 0,\quad \Phi_{67}^i = 0.\end{aligned}$$

According to Lemma 2.1 ((ii) and (iii)), we have

$$\begin{aligned}\dot{V}(t) \quad \le \quad & \zeta^T(t)\Big[\hat{\Pi}^i + \epsilon_i^{-1}\bar{P}H_i(\bar{P}H_i)^T + \Theta_1^{iT}(P^{-1} - \epsilon_i^{-1}H_iH_i^T)^{-1}\Theta_1^i + \epsilon_i\bar{\Theta}_1^{iT}\bar{\Theta}_1^i \\ & +\Theta_2^{iT}(P^{-1} - \epsilon_i^{-1}H_iH_i^T)^{-1}\Theta_2^i + \epsilon_i\bar{\Theta}_2^{iT}\bar{\Theta}_2^i + \Theta_3^{iT}(U^{-1} - \epsilon_i^{-1}HH^T)^{-1}\Theta_3^i + \epsilon_i\bar{\Theta}_3^{iT}\bar{\Theta}_3^i\Big]\zeta(t) < 0,\end{aligned} \tag{26}$$

where

$$\begin{aligned}\Theta_1^i \quad = \quad & [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ A_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],\quad \bar{\Theta}_1^i = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_{1i}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0], \\ \Theta_2^i \quad = \quad & [A_i\ \ B_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ C_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],\quad \bar{\Theta}_2^i = [E_{1i}\ \ E_{2i}\ \ 0\ \ 0\ \ 0\ \ 0\ \ E_{3i}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0], \\ \Theta_3^i \quad = \quad & [0\ \ B_i\ \ 0\ \ 0\ \ A_i\ \ 0\ \ C_i\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],\quad \bar{\Theta}_3^i = [0\ \ E_{2i}\ \ 0\ \ 0\ \ E_{1i}\ \ 0\ \ E_{3i}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0], \\ \bar{P} \quad = \quad & [P\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T.\end{aligned}$$

By Schur complement, we have

$$\dot{V}(t) \le \sum_{i=1}^{r} \rho_i(\theta(t))\zeta^T(t)\Psi^i\zeta(t) < 0, \tag{27}$$

Ψ^i is defined in (25). One may easily obtain that

$$\begin{aligned}\| x(t) + A_i(t)\int_{t-\sigma}^t x(s)ds \|^2 \quad \le \quad & \frac{1}{\lambda_{min}(P)}[x(t) + A_i(t)\int_{t-\sigma}^t x(s)ds]^T P[x(t) + A_i(t)\int_{t-\sigma}^t x(s)ds] \\ \le \quad & \frac{V(t)}{\lambda_{min}(P)} \le \frac{V(0)}{\lambda_{min}(P)},\quad t > 0\end{aligned}$$

which implies that

$$\| x(t) \| \le \| A_i(t)\int_{t-\sigma}^t x(s)ds \| + \sqrt{\frac{V_0(t)}{\lambda_{min}(P)}} \quad \le \quad \| A_i(t) \| \int_{t-\sigma}^t \| x(s) \| ds + \sqrt{\frac{V(0)}{\lambda_{min}(P)}} \quad t > 0,$$

where $\| A_i(t) \| = \sqrt{\sum_{l=1}^{n} \sum_{k=1}^{n} a_{ilk}^2}$. From the well-known Gronwall inequality, one obtains

$$\| x(t) \| \leq \sqrt{\frac{V(0)}{\lambda_{min}(P)}} e^{\| A_i(t) \| \sigma}. \tag{28}$$

Note that

$$\begin{aligned} V(0) \leq\ & [\phi(0) + A_i(0) \int_{-\sigma}^{0} \phi(s)ds]^T P[\phi(0) + A_i(0) \int_{-\sigma}^{0} \phi(s)ds] + \int_{-h_1}^{0} \phi^T(s)Q_1\phi(s)ds \\ & + \int_{-h_2}^{0} \phi^T(s)Q_2\phi(s)ds + \int_{-\tau(0))}^{0} \phi^T(s)Q_3\phi(s)ds + \int_{-\sigma}^{0} \phi^T(s)Q_4\phi(s)ds \\ & \sigma \int_{-\sigma}^{0} \int_{\theta}^{0} \phi^T(s)Z_1\phi(s)dsd\theta + h_2 \int_{-h_2}^{0} \int_{\theta}^{0} \dot{\phi}^T(s)Z_2\dot{\phi}(s)dsd\theta \\ & +(h_2 - h_1) \int_{-h_2}^{-h_1} \int_{\theta}^{0} \dot{\phi}^T(s)Z_3\dot{\phi}(s)dsd\theta + \int_{-h_2}^{0} \int_{\theta}^{0} \phi^T(s)Z_4\phi(s)dsd\theta \\ & + \int_{-h_2}^{-h_1} \int_{\theta}^{0} \phi^T(s)Z_5\phi(s)dsd\theta + \bar{\eta} \int_{-\bar{\eta}}^{0} \int_{\theta}^{0} \phi^T(s)S\phi(s)dsd\theta \\ & + \int_{-h_2}^{0} \int_{\theta}^{0} \int_{\lambda}^{0} \dot{\phi}^T(s)R_1\dot{\phi}(s)dsd\lambda d\theta + \int_{-h_2}^{-h_1} \int_{\theta}^{0} \int_{\lambda}^{0} \dot{\phi}^T(s)R_2\dot{\phi}(s)dsd\lambda d\theta \\ \leq\ & \{2\lambda_{max}(P)(1 + \sigma \| A_i(t) \|)) + h_1\lambda_{max}(Q_1) + h_2\lambda_{max}(Q_2) + h_2\lambda_{max}(Q_3) + \sigma\lambda_{max}(Q_4) \\ & +\sigma^3\lambda_{max}(Z_1) + h_2^3\lambda_{max}(Z_2) + (h_2 - h_1)^3\lambda_{max}(Z_3) + h_2^2\lambda_{max}(Z_4) + (h_2 - h_1)^2\lambda_{max}(Z_5) \\ & +\bar{\eta}^3\lambda_{max}(S) + h_2^3\lambda_{max}(R_1) + (h_2 - h_1)^3\lambda_{max}(R_2)\} \| \phi \|_{\tau^*}^2 < \infty. \end{aligned} \tag{29}$$

From (28) and (29), it can be deduced that the trivial solution of system (5) is locally stable. Then the solution $x(t) = x(t, 0, \phi)$ of system (5) is bounded on $[0, \infty)$. Considering (5), we know that $\dot{x}(t)$ is bounded on $[0, \infty)$, which leads to the uniform continuity of the solution $x(t)$ on $[0, \infty)$. From (27), we note that the following inequality holds:

$$\lambda_{min}(\Psi^i) \int_0^t x^T(s)x(s)ds \leq V(t) + \int_0^t \zeta^T(s)\Psi^i\zeta(s)ds \leq V(0) < \infty, t > 0.$$

By Barbalat's lemma, (see Gopalsamy [6]), it follow that $\| x(t) \| \longrightarrow 0$ as $t \longrightarrow \infty$. This completes the proof. □

Remark 4.2. Yoneyama [33], studied robust stability and stabilizing controller design of fuzzy systems with discrete and distributed delays. However, this paper dealt with the robust stability criteria for T-S fuzzy systems with distributed delays and time delay in the leakage term. To the best of our knowledge, robust stability criteria for T-S fuzzy systems time delay in the leakage term approach have not been fully discussed in the literature. We attained this goal successfully and derived new sufficient conditions for the considered fuzzy system with time delay in the leakage term.

Remark 4.3. Now, we will discuss the robust stability for the following uncertain fuzzy system

$$\begin{aligned} \dot{x}(t) &= A_i(t)x(t-\sigma) + B_i(t)x(t-\tau(t)) \\ x(t) &= \phi(t), \quad t \in [-\bar{h}, 0] \quad \bar{h} = max\{\sigma, h_2\}. \end{aligned} \tag{30}$$

Using a simple transformation, model (30) has an equivalent form as follows:

$$\frac{d}{dt}\Big[x(t) + A_i(t)\int_{t-\sigma}^{t} x(s)ds\Big] = A_i(t)x(t) + B_i(t)x(t-\tau(t)). \tag{31}$$

For system (30) or (31), we have the following result.

Corollary 4.4. *For given scalars $h_2 > h_1 \geq 0$, σ and μ, the equilibrium point of system (30) or (31) is globally robustly asymptotically stable if there exist symmetric matrices $P > 0$, $Q_l > 0, l = 1, 2, 3, 4$, $Z_k > 0$, $k = 1, 2, 3, 4, 5$, $R_1 > 0$, $R_2 > 0$, for any matrices $N_1, N_2, N_3, M_1, M_2, M_3$ and positive scalar ϵ_i such that the following LMI is feasible*

$$\begin{bmatrix} \hat{\Lambda}^i & \bar{P}H_i & \Gamma_1^{iT}P & 0 & \epsilon_i\bar{\Gamma}_1^{iT} & \Gamma_2^{iT}P & 0 & \epsilon_i\bar{\Gamma}_2^{iT} & \Gamma_3^{iT}U & 0 & \epsilon_i\bar{\Gamma}_3^{iT} \\ * & -\epsilon_i I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P & PH_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon_i I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P & PH_i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon_i I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon_i I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -U & UH_i & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon_i I \end{bmatrix} < 0, \tag{32}$$

where

$$\hat{\Lambda}^i = \begin{bmatrix} \Lambda^i & \frac{1}{\sqrt{2}}h_2 N & \sqrt{\frac{1}{2}(h_2^2 - h_1^2)}M \\ * & -R_1 & 0 \\ * & * & -R_2 \\ * & * & * \end{bmatrix},$$

and $\Lambda^i = (\Lambda^i_{lk})_{9\times 9}$ with

$$\begin{aligned} \Lambda^i_{11} &= PA_i + A_i^T P + \sigma^2 Z_1 - Z_2 + Q_1 + Q_2 + Q_3 + Q_4 + \bar{\eta}^2 S + h_2 Z_4 + (h_1 - h_2)Z_5 \\ &\quad + h_2 N_1 + h_2 N_1^T + (h_2 - h_1)M_1 + (h_2 - h_1)M_1^T + \epsilon_i E_{1i}^T E_{1i}, \ \Lambda^i_{12} = PB_i + Z_2 + h_2 N_2^T \\ &\quad + (h_2 - h_1)M_2^T, \quad \Lambda^i_{13} = \Lambda^i_{14} = 0, \quad \Lambda^i_{15} = h_2 N_3^T + (h_2 - h_1)M_3^T, \quad \Lambda^i_{16} = 0, \\ \Lambda^i_{17} &= -N_1, \quad \Lambda^i_{18} = -N_1 - M_1, \quad \Lambda^i_{19} = -M_1, \quad \Lambda^i_{22} = -Z_2 - Z_3 - Z_3^T - (1-\mu)Q_3 + \epsilon_i E_{2i}^T E_{2i}, \\ \Lambda^i_{23} &= \Lambda^i_{24} = Z_3, \quad \Lambda^i_{25} = 0, \quad \Lambda^i_{26} = 0, \ \Lambda^i_{27} = -N_2, \ \Lambda^i_{28} = -N_2 - M_2, \\ \Lambda^i_{29} &= -M_2, \quad \Lambda^i_{33} = -Z_3 - Q_1, \quad \Lambda^i_{34} = \ldots = \Lambda^i_{39} = 0, \quad \Lambda^i_{44} = -Z_3 - Q_2, \\ \Lambda^i_{45} &= \ldots = \Lambda^i_{49} = 0, \quad \Lambda^i_{55} = -Q_4, \quad \Lambda^i_{56} = 0, \ \Lambda^i_{57} = -N_3, \ \Lambda^i_{58} = -N_3 - M_3, \\ \Lambda^i_{59} &= -M_3, \quad \Lambda^i_{66} = -Z_1, \quad \Lambda^i_{67} = \Lambda^i_{68} = \Lambda^i_{69} = 0, \quad \Lambda^i_{77} = -\frac{1}{h_2}Z_4, \quad \Lambda^i_{78} = \Lambda^i_{79} = 0, \\ \Lambda^i_{88} &= -\frac{1}{(h_2 - h_1)}(Z_4 + Z_4), \quad \Lambda^i_{89} = 0, \quad \Lambda^i_{99} = -\frac{1}{(h_2 - h_1)}Z_5. \end{aligned}$$

$$\begin{aligned}
\Gamma_1^i &= [0\ 0\ 0\ 0\ 0\ A_i\ 0\ 0\ 0\ 0\ 0], \quad \bar{\Gamma}_1^i = [0\ 0\ 0\ 0\ 0\ E_{1i}\ 0\ 0\ 0\ 0\ 0],\\
\Gamma_2^i &= [A_i\ B_i\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0], \quad \bar{\Gamma}_2^i = [E_{1i}\ E_{2i}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],\\
\Gamma_3^i &= [0\ B_i\ 0\ 0\ A_i\ 0\ 0\ 0\ 0\ 0\ 0], \quad \bar{\Gamma}_3^i = [0\ E_{2i}\ 0\ 0\ E_{1i}\ 0\ 0\ 0\ 0\ 0\ 0],\\
\bar{P} &= [P\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T.
\end{aligned}$$

Remark 4.5. Now, we will discuss the robust stability for the following uncertain fuzzy system

$$\begin{aligned}
\dot{x}(t) &= A_i(t)x(t) + B_i(t)x(t-\tau(t))\\
x(t) &= \phi(t), \quad t \in [-h_2, 0].
\end{aligned} \tag{33}$$

For system (33), we have the following result.

Corollary 4.6. *For given scalars $h_2 > h_1 \geq 0$, and μ, the equilibrium point of system (33) is globally asymptotically stable if there exist symmetric matrices $P > 0$, $Q_l > 0, l = 1,2,3,$ $Z_k > 0,$ $k = 2,3,4,5,$ $R_1 > 0,$ $R_2 > 0,$ for any matrices N_1, N_2, M_1, M_2 and positive scalar ϵ_i such that the following LMI is feasible*

$$\begin{bmatrix}
\bar{\Omega}^i & \frac{1}{\sqrt{2}}h_2\widetilde{N} & \sqrt{\frac{1}{2}(h_2^2-h_1^2)}\ \widetilde{M} & \hat{P}H_i & \Gamma_1^{iT}U & 0 & \epsilon_i\Gamma_2^{iT}\\
* & -R_1 & 0 & 0 & 0 & 0 & 0\\
* & * & -R_2 & 0 & 0 & 0 & 0\\
* & * & * & -\epsilon_i I & 0 & 0 & 0\\
* & * & * & * & -U & UH & 0\\
* & * & * & * & * & -\epsilon_i I & 0\\
* & * & * & * & * & 0 & -\epsilon_i I
\end{bmatrix} < 0, \tag{34}$$

where $\bar{\Omega}^i = \Omega^i + diag\{\ \epsilon_i E_{1i}^T E_{1i},\ \epsilon_i E_{2i}^T E_{2i},\ 0,\ 0,\ 0,\ 0,\ 0\}$, $\hat{P} = [P\ 0\ 0\ 0\ 0\ 0\ 0]^T$, $\Gamma_1^i = [A_i\ B_i\ 0\ 0\ 0\ 0\ 0]$,
$\Gamma_2^i = [E_{1i}\ E_{2i}\ 0\ 0\ 0\ 0\ 0]$ *and* $\Omega^i, \widetilde{N}, \widetilde{M}$ *are define in Corollary 3.2.*

Remark 4.7. Now, we will discuss the robust stability for the following uncertain system

$$\begin{aligned}
\dot{x}(t) &= A(t)x(t-\sigma) + B(t)x(t-\tau(t)) + C(t)\int_{t-\eta(t)}^{t} x(s)ds\\
x(t) &= \phi(t), \quad t \in [-\tau^*, 0].
\end{aligned} \tag{35}$$

Using a simple transformation, model (1) has an equivalent described by

$$\frac{d}{dt}\Big[x(t) + A(t)\int_{t-\sigma}^{t} x(s)ds\Big] = A(t)x(t) + B(t)x(t-\tau(t) + C(t)\int_{t-\eta(t)}^{t} x(s)ds. \tag{36}$$

For system (35) or (36), we have the following result.

Corollary 4.8. *For given scalars $h_2 > h_1 \geq 0$, σ, $\bar{\eta}$ and μ, the equilibrium point of system (35) or (36) is globally robustly asymptotically stable if there exist symmetric matrices $P > 0$, $Q_l > 0, l = 1, 2, 3, 4$, $Z_k > 0$, $k = 1, 2, 3, 4, 5$, $R_1 > 0$, $R_2 > 0$, $S > 0$, for any matrices $N_1, N_2, N_3, M_1, M_2, M_3$ and positive scalar ϵ such that the following LMI is feasible*

$$\Psi = \begin{bmatrix} \hat{\Pi} & \bar{P}H & \Theta_1^T P & 0 & \epsilon\bar{\Theta}_1^T & \Theta_2^T P & 0 & \epsilon\bar{\Theta}_2^T & \Theta_3^T U & 0 & \epsilon\bar{\Theta}_3^T \\ * & -\epsilon I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P & PH & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\epsilon I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\epsilon I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P & PH & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\epsilon I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\epsilon I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -U & UH & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon I \end{bmatrix} < 0. \tag{37}$$

where

$$\hat{\Pi} = \begin{bmatrix} \hat{\Xi} & \frac{1}{\sqrt{2}}h_2 N & \sqrt{\frac{1}{2}(h_2^2 - h_1^2)}M \\ * & -R_1 & 0 \\ * & * & -R_2 \end{bmatrix},$$

and $\hat{\Xi} = (\hat{\Xi}_{lk})_{10\times 10}$ *with*

$$\begin{aligned}
\hat{\Xi}_{11} &= PA + A^T P + \sigma^2 Z_1 - Z_2 + Q_1 + Q_2 + Q_3 + Q_4 + \bar{\eta}^2 S + h_2 Z_4 + (h_1 - h_2)Z_5 \\
&\quad + h_2 N_1 + h_2 N_1^T + (h_2 - h_1)M_1 + (h_2 - h_1)M_1^T + \epsilon E_1^T E_1, \; \hat{\Xi}_{12} = PB + Z_2 + h_2 N_2^T \\
&\quad + (h_2 - h_1)M_2^T, \quad \hat{\Xi}_{13} = \hat{\Xi}_{14}^i = 0, \quad \hat{\Xi}_{15} = h_2 N_3^T + (h_2 - h_1)M_3^T, \quad \hat{\Xi}_{16} = 0, \quad \hat{\Xi}_{17} = PC_i \\
\hat{\Xi}_{18} &= -N_1, \quad \hat{\Xi}_{19} = -N_1 - M_1, \quad \hat{\Xi}_{1,10} = -M_1, \quad \hat{\Xi}_{22} = -Z_2 - Z_3 - Z_3^T - (1-\mu)Q_3 + \epsilon E_2^T E_2, \\
\hat{\Xi}_{23} &= \hat{\Xi}_{24}^i = Z_3, \quad \hat{\Xi}_{25} = 0, \quad \hat{\Xi}_{26} = 0, \quad \hat{\Xi}_{27} = 0, \; \hat{\Xi}_{28} = -N_2, \; \hat{\Xi}_{29} = -N_2 - M_2, \\
\hat{\Xi}_{2,10} &= -M_2, \quad \hat{\Xi}_{33} = -Z_3 - Q_1, \quad \hat{\Xi}_{34} = \ldots = \hat{\Xi}_{3,10} = 0, \quad \hat{\Xi}_{44} = -Z_3 - Q_2, \\
\hat{\Xi}_{45} &= \ldots = \hat{\Xi}_{4,10} = 0, \quad \hat{\Xi}_{55} = -Q_4, \quad \hat{\Xi}_{56} = \hat{\Xi}_{57} = 0, \quad \hat{\Xi}_{58} = -N_3, \; \hat{\Xi}_{59} = -N_3 - M_3, \\
\hat{\Xi}_{5,10} &= -M_3, \quad \hat{\Xi}_{66} = -Z_1, \quad \hat{\Xi}_{67} = 0, \quad \hat{\Xi}_{68} = \hat{\Xi}_{69} = \hat{\Xi}_{6,10} = 0, \quad \hat{\Xi}_{77} = -S + \epsilon E_3^T E_3, \\
\hat{\Xi}_{78} &= \hat{\Xi}_{79} = \hat{\Xi}_{7,10} = 0, \quad \hat{\Xi}_{88} = -\frac{1}{h_2}Z_4, \quad \hat{\Xi}_{89} = \hat{\Xi}_{8,10}^i = 0, \quad \hat{\Xi}_{99} = -\frac{1}{(h_2 - h_1)}(Z_4 + Z_4), \\
\hat{\Xi}_{9,10} &= 0, \quad \hat{\Xi}_{10,10} = -\frac{1}{(h_2 - h_1)}Z_5, \\
\Theta_1 &= [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; A \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0], \quad \bar{\Theta}_1 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; E_1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0], \\
\Theta_2 &= [A \;\; B \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; C \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0], \quad \bar{\Theta}_2 = [E_1 \;\; E_2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; E_3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0], \\
\Theta_3 &= [0 \;\; B \;\; 0 \;\; 0 \;\; A \;\; 0 \;\; C \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0], \quad \bar{\Theta}_3 = [0 \;\; E_2^T \;\; 0 \;\; 0 \;\; E_1 \;\; 0 \;\; E_3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0], \\
\bar{P} &= [P \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T.
\end{aligned}$$

Remark 4.9. Recently few authors have discussed the triple integral terms added in the Lyapunov-Krasovkii functional, see for example [26, 25]. The free weighting matrix method has also been applied to reduce less conservative stability conditions. In [18], the authors used the few free weighting matrices and found some conservative stability results than the published papers in the literature. However, there still exists room for further improvement than the results discussed in [18]. Motivating this reason, we introduce some triple integral terms for interval time-varying delays in the Lyapunov-Krasovkii functional. This plays an important role in the further reduction of conservatism and we find an upper bound better than the result reported in [18].

5. Numerical Examples

In this section, we will give three examples showing the effectiveness of established theoretical results.

Example 5.1. Consider the system (5) with the following matrices

$$A_1 = \begin{bmatrix} -1.5 & 0 \\ 0 & -1.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.2 & 0 \\ 0 & -1.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.5 & 0.8 \\ 0.3 & 0.9 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.4 & 0.6 \\ 0.2 & -0.8 \end{bmatrix}, \quad H_1 = H_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$E_{11} = E_{21} = E_{31} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad E_{12} = E_{22} = E_{32} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

Using Matlab LMI toolbox and Theorem 4.1, we obtain the following results listed in Table 1. Table 1 describes allowable upper bounds h_2 for different h_1, μ, σ and fixed $\bar{\eta} = 0.3$.

μ	0	0.5	0.7	1.5
$h_1 = 0, \sigma = 0.1$	0.4021	0.3381	0.3269	0.3269
$h_1 = 0, \sigma = 0.2$	0.3171	0.2470	0.2470	0.2470
$h_1 = 0, \sigma = 0.3$	0.0840	0.0440	0.0440	0.0440
$h_1 = 0, \sigma = 0.4$	infeasible	infeasible	infeasible	infeasible
$h_1 = 0.4, \sigma = 0.1$	0.4127	0.4099	0.4099	0.4099
$h_1 = 0.4, \sigma = 0.2$	infeasible	infeasible	infeasible	infeasible

Table 1. Maximum Allowable Upper Bounds of h_2 for Different μ, h_1 and Fixed $\bar{\eta} = 0.3$ and $\sigma = 0.1$

The above results show that system (5) or (6) is globally robustly asymptotically stable.

Remark 5.2. Example 5.1, system (5) or (6) is globally asymptotically stable when $\sigma = 0.1$, it is shown in Figure 1 and Figure 2. However, using Matlab LMI toolbox, if we take the leakage delay as $\sigma \geq 0.4$ the LMI (25) is not feasible from Table 1, in that case system (5) or (6) becomes unstable it is shown in Figure 3 and Figure 4.

Example 5.3. [18] Consider system (22) with the following matrices

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ -0.2 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}$$

Using Matlab LMI toolbox and Corollary 3.2, maximum allowable upper bound h_2 that guarantees the global asymptotic stability of the system (22) and listed in Table 2. Table 2 yields less conservative results than the results discussed in [8, 3, 18].

μ	0	0.1	0.5	Unkown	
[8]	1.25	-	-	-	-
[3]	3.15	-	-	-	
[18]	3.30	2.65	1.50	0.79	
Corollary 3.2	3.4040	2.7374	1.5385	0.8600	

TABLE 2. Maximum Allowable Upper Bounds of h_2 and $h_1 = 0$

Example 5.4. [18] Consider system (33) with the following matrices

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0.5 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -1.6 & 0 \\ 0 & -1 \end{bmatrix},$$

$$E_{11} = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix}, \quad E_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad E_{12} = \begin{bmatrix} 1.6 & 0 \\ 0 & -0.05 \end{bmatrix}, \quad E_{22} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}$$

$$H_1 = H_2 = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix}.$$

Using Matlab LMI toolbox and Corollary 4.6, maximum allowable upper bound h_2 that guarantees the global asymptotic stability of the system (33) and listed in Table 3. Table 3 yields less conservative results than the papers [18, 15, 16, 17].

μ	0	0.01	0.1	0.5	Unknown
[15]	0.950	0.944	0.892	0.637	-
[16]	1.158	1.155	1.113	0.929	0.443
[17]	1.168	1.163	1.122	0.934	0.499
[18]	1.353	1.348	1.303	1.147	1.081
Corollary	1.404	1.368	1.314	1.165	1.099

TABLE 3. Maximum Allowable Upper Bounds of h_2 and $h_1 = 0$

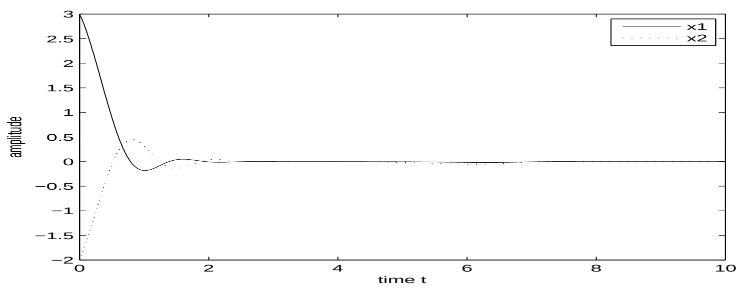

FIGURE 1. State Trajectories of Example 1 with $\sigma = 0.1$ and $i = 1$

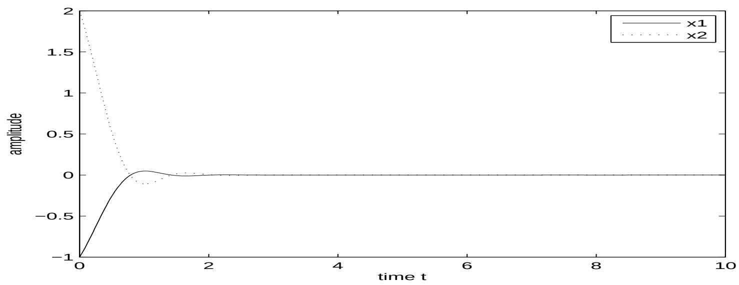

FIGURE 2. State Trajectories of Example 1 with $\sigma = 0.1$ and $i = 2$

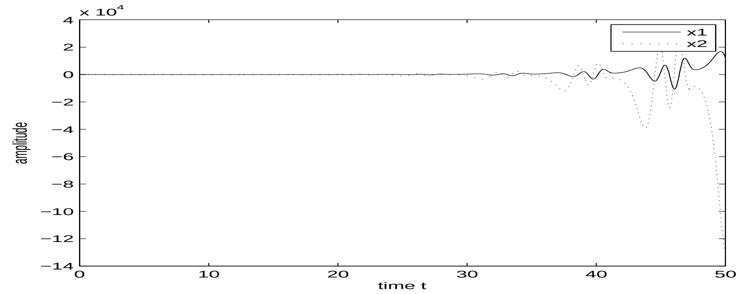

FIGURE 3. State Trajectories of Example 1 with $\sigma = 0.4$ and $i = 1$

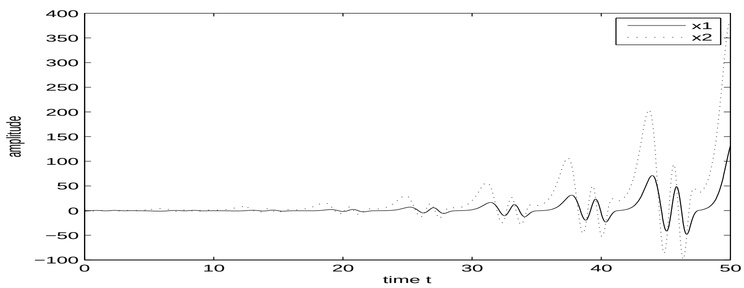

FIGURE 4. State Trajectories of Example 1 with $\sigma = 0.4$ and $i = 2$

6. **Conclusion**

The problem of robust criteria for T-S fuzzy systems with distributed delays and time delay in the leakage term has been addressed in this paper. Based on a model transformation, new Lyapunov-Krasovskii functional, some inequality techniques, new stability criteria are obtained in terms of linear matrix inequalities (LMIs). To the best of author's knowledge, there is almost no result on T-S fuzzy systems with distributed delays and time delay in the leakage term of the existing literature. The effectiveness and less conservatism of the proposed results have been demonstrated by numerical examples.

Acknowledgements. The authors are very much thankful to referees for their valuable comments and suggestions for improving this manuscript.

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