A NEW APPROACH TO STABILITY ANALYSIS OF FUZZY RELATIONAL MODEL OF DYNAMIC SYSTEMS

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Abstract. This paper investigates the stability analysis of fuzzy relational dynamic systems. A new approach is introduced and a set of sufficient conditions is derived which sustains the unique globally asymptotically stable equilibrium point in a first-order fuzzy relational dynamic system with sum-product fuzzy composition. This approach is also investigated for other types of fuzzy relational composition.

1. Introduction

Having some tools for the stability analysis of linguistic fuzzy models of dynamic systems leads to more reliable fuzzy models of plants and controllers. This is studied by several researchers and it is still an ongoing issue. The studies on the stability analysis of TSK fuzzy models of dynamic systems have been much more fruitful, as seen for example in [1], [4], and [17]. Different approaches have been taken on the stability analysis of linguistic fuzzy systems, e.g., frequency domain analysis, [12] and [20], and stability analysis based on petri-nets, [6], [15] and [7]. Several methods of the stability analysis of linguistic fuzzy models are referred to [10]. Other useful references are [5], [9], [11], [16], and [18].

A fuzzy relational model (FRM) is in fact an extended linguistic fuzzy model in which all possible rules exist and each rule is associated with a truth degree. The truth degrees are the entries of the relational matrix and this matrix is the core of the fuzzy relational model. A fuzzy relational dynamic system (FRDS) is in fact a fuzzy relational model of a dynamic system. Fuzzy relational models have good modeling capability and many papers are available on the function approximation and the model identification by FRMs, [13], [2], [3], and [14]. The operation of an FRM is based on fuzzy relational equations (FREs) introduced in 1976 by Sanchez [19]. A typical FRE is written as $\underline{b} = \underline{a} \circ R$, in which $\underline{a}$ and $\underline{b}$ are the input and output fuzzy vectors respectively. Various aspects of real world problems can be modeled by FREs. Then solving the FRE for the unknown variable solves the problem. See for example [8] for an application in image compression and reconstruction.

In this paper an FRE is the core of the model, FRM. Throughout this paper an underlined letter stands for a row vector. Figure 1 shows the model of a dynamic system based on a static operator f.

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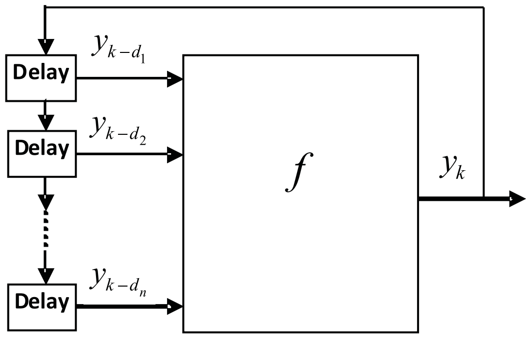

FIGURE 1. Model of a Dynamic System without any External Input

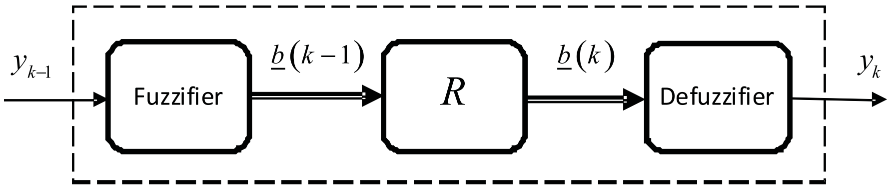

FIGURE 2. General Block Diagram of a Simple FRM

If the actual system is not too complex, only one of the past values of the output variable can be used. Then, the FRE of the FRM can be written as $\underline{b}(k) = \underline{b}\,(k-d) \circ R$ with $q \times q$ fuzzy relational matrix R, where d is a proper delay value and q is the number of the linguistic terms adopted by the designer. This model is called a *first-order FRDS* in this paper.

Details of the FRDS considered in this paper is represented in section 2. sections 3 and 4 contain the main results of this paper and include the stability analysis of the considered FRDS. A set of sufficient conditions are obtained in section 3 to ensure the existence of a unique equilibrium point and its asymptotic stability. The extension of the results is covered in section 4. Finally a brief conclusion is presented in section 5.

2. **Model Configuration and Preliminaries**

The first order FRDS is considered in this paper. Figure 2 shows the three main parts of a typical FRM, i.e., a fuzzifier, a defuzzifier, and a relational matrix with the relative fuzzy composition operator.

The specific type of each part of the model is selected by the model designer. In this paper the part types are selected as follows:

• The fuzzifier is made up of q triangular membership functions with the property of sum-normality, i.e., the sum of the membership function values for every non-fuzzy value being 1. Such a fuzzifier is called *standard fuzzifier* henceforth. An example of a standard fuzzifier with five membership functions is shown in Figure 3.

• The defuzzifier is of *weighted average* type as follows:

$$y = \sum_{i=1}^{q} b_i c_i \Big/ \sum_{i=1}^{q} b_i \tag{1}$$

where b_i's are the elements of the fuzzy vector $\underline{b}$ and c_i's are the centers of the defuzzifier.

• The relational matrix is a $q \times q$ matrix $R = [r_{ij}]$, where $r_{ij} \in [0,1]$.

• For the t-conorm s and t-norm t, the FRE is $\underline{b}(k) = \underline{b}\,(k-1) \circ R$, where "$\circ$" represents an s-t fuzzy relational composition as

$$b_j(k) = s(t(b_1(k-1), r_{1j}), \cdots, t(b_q(k-1), r_{qj})).$$

Throughout the paper, $\underline{a} = (a_1, \cdots, a_p)$ with $a_i \in [0,1]$ for all $i = 1, \ldots, p$ and $\underline{b} = (b_1, \cdots, b_q)$ with $b_j \in [0,1]$ for all $j = 1, \ldots, q$, unless otherwise expressed.

Remark 2.1. Regarding the standard fuzzifier note that the centers of the membership functions does not have to be spaced equally in the non-fuzzy universe of discourse.

3. Convergence to the Unique Equilibrium Point of a Sum-Prod FRDS

Throughout this section, the t-norm t and the t-conorm s are both selected as algebraic type as follows:

$$t(a,b) = ab, \quad s(a,b) = \min\{(a+b), 1\}$$

A fuzzy relational composition composed of such t-conorm and t-norm is called *sum-prod composition*, and an FRDS with sum-prod composition, is called *sum-prod FRDS*.

A set of sufficient conditions for asymptotic convergence of the output to the center of a linguistic term is obtained and then the possibility of extending this result to other kinds of fuzzy composition is investigated.

Remark 3.1. Selecting the part types as in section 2 simplifies analysis of FRM while preserving the good modeling capability of the model. To see the capability of an FRM with such configuration see [3].

Let us introduce two useful concepts.

Definition 3.2. Let Γ_f be the hyperplane of all q-tuples $(x_1, ..., x_q)$ such that $\sum_{j=1}^{q} x_j = 1$, where $x_i \in [0,1]$, for $i = 1, ..., q$. An FRDS described by $\underline{b}(k+1) = \underline{b}(k) \circ R$ has the property of *intraplanarity* on Γ_f, if $\underline{b}(k+1) \in \Gamma_f$ for every $\underline{b}(k) \in \Gamma_f$. In other words, the FRDS is called *intraplanar* if Γ_f is invariant under the sum-prod fuzzy composition.

Definition 3.3. A $p \times q$ matrix $R = [r_{ij}]$ is called *unit-row matrix* if

$$\sum_{j=1}^{q} r_{ij} = 1 \qquad \forall i \in \{1, \dots, p\}\,.$$

Lemma 3.4. *Let $R = [r_{ij}]$ be a $q \times q$ relational matrix and "$\circ R$" be sum-prod fuzzy composition with R, as in $\underline{b} = \underline{a} \circ R$. Then the hyperplane Γ_f is invariant under "$\circ R$" if and only if R is a unit-row matrix.*

Proof. First the sufficient condition is proved. Since R is a unit-row matrix and $\underline{a} \in \Gamma_f$, $\underline{a}R = (\sum_{i=1}^{q} a_i r_{i1}, \cdots, \sum_{i=1}^{q} a_i r_{iq})$ is a unit-row matrix, because

$$\sum_{j=1}^{q}\sum_{i=1}^{q} a_i r_{ij} = \sum_{i=1}^{q}\sum_{j=1}^{q} a_i r_{ij} = \sum_{i=1}^{q} a_i \sum_{j=1}^{q} r_{ij} = 1. \tag{2}$$

Hence $\sum_{i=1}^{q} a_i r_{ij} \leq 1$, for every $j = 1, \dots, q$. This shows that

$$b_j = \min\{\sum_{i=1}^{q} a_i r_{ij}, 1\} = \sum_{i=1}^{q} a_i r_{ij},$$

and by (2) the result follows:

For the necessary condition, let $e_i \in \Gamma_f$ be such that its ith element equals 1, for every $i = 1, \dots, q$. The ith row of R is in Γ_f since it is equal to $e_i \circ R \in \Gamma_f$. This completes the proof. □

Remark 3.5. For the sum-prod fuzzy composition $\underline{b} = \underline{a} \circ R$, as driven in the proof of Lemma 3.4, $b_j = \min\{\sum_{i=1}^{p} a_i r_{ij}, 1\} = \sum_{i=1}^{p} a_i r_{ij}$, if R is a unit-row relational matrix.

Corollary 3.6. *A sum-prod FRDS with standard fuzzifier and FRE $\underline{b}(k+1) = \underline{b}(k) \circ R$, has the property of fuzzy intraplanarity on the hyperplane Γ_f if and only if R is a unit-row matrix.*

Corollary 3.7. *In a sum-prod FRDS with standard fuzzifier, weighted average defuzzifier, and FRE $\underline{b}(k+1) = \underline{b}(k) \circ R$, if the relational matrix is a unit-row matrix then the defuzzifier is simplified as $y = \sum_{j=1}^{q} b_j c_j$.*

Theorem 3.8. *In a sum-prod FRDS with standard fuzzifier, weighted average defuzzifier, FRE $\underline{b}(k+1) = \underline{b}(k) \circ R$, and unit-row relational matrix, the output (of the defuzzifier) converges asymptotically globally to c_l if:*

1. $r_{ll} = 1$,
2. $r_{il} \neq 0 \quad \forall i = 1, \dots, q$.

Proof. Since $r_{ll} = 1$,

$$b_l(k+1) = \sum_{i=1}^{q} b_i(k) r_{il} = b_l(k) + \sum_{i=1, i\neq l}^{q} b_i(k) r_{il}.$$

Therefore $\{b_l(k)\}$ is an increasing and bounded sequence and hence it is convergent. Also

$$b_l(k+1) - b_l(k) = \sum_{i \neq l} b_i(k) r_{il} > \sum_{i \neq l} b_i(k) s,$$

where $s = \min_i r_{il}$. Hence

$$0 = \lim_{k \to \infty} (b_l(k+1) - b_l(k)) = \sum_{i \neq l} \lim_{k \to \infty} b_i(k) r_{il} > s \sum_{i \neq l} \lim_{k \to \infty} b_i(k).$$

Clearly $s \sum_{i \neq l} \lim_{k \to \infty} b_i(k) \geq 0$ and so

$$0 \geq s \sum_{i \neq l} \lim_{k \to \infty} b_i(k) \geq 0.$$

Since $s \neq 0$, $\sum_{i \neq l} \lim_{k \to \infty} b_i(k) = 0$. This shows that $\lim_{k \to \infty} b_i(k) = 0$, for every $i \neq l$ which leads to $\lim_{k \to \infty} b_l(k) = 1$ by Lemma 3.4. This completes the proof. $\square$

Example 3.9. Consider a FRDS described by $\underline{b}(k+1) = \underline{b}(k) \circ R$, with sum-prod fuzzy composition and standard fuzzifier. The relational matrix is considered as follows:

$$R = \begin{pmatrix} 0.7 & 0.2 & 0.1 & 0 & 0 \\ 0 & 0.7 & 0.2 & 0.1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0.1 & 0.2 & 0.7 & 0 \\ 0 & 0 & 0.1 & 0.2 & 0.7 \end{pmatrix}$$

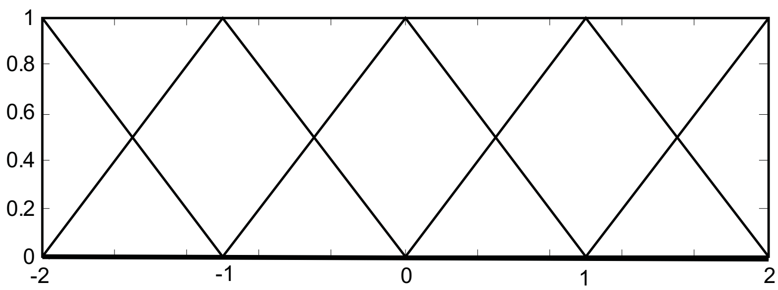

FIGURE 3. The Membership Functions of the Example on the Non-fuzzy Universe of Discourse $[-2, 2]$

Five linguistic values are considered to describe the fuzzy variable, see Figure 3. The set of the centers of membership functions is $\{-2, -1, 0, 1, 2\}$.

Both conditions of Theorem 3.8 hold for the third linguistic term. Hence the actual output of the dynamic system is bounded to converge to the center of the third membership function, i.e., 0 here, from every arbitrary initial point in the non-fuzzy universe of discourse.

Figure 4 shows the trajectory of the model output initialized from $y(0) = -2$. A simple exponential function $e^{-\alpha t}$ is fitted to the output trajectory with $\alpha = -2.8$, to

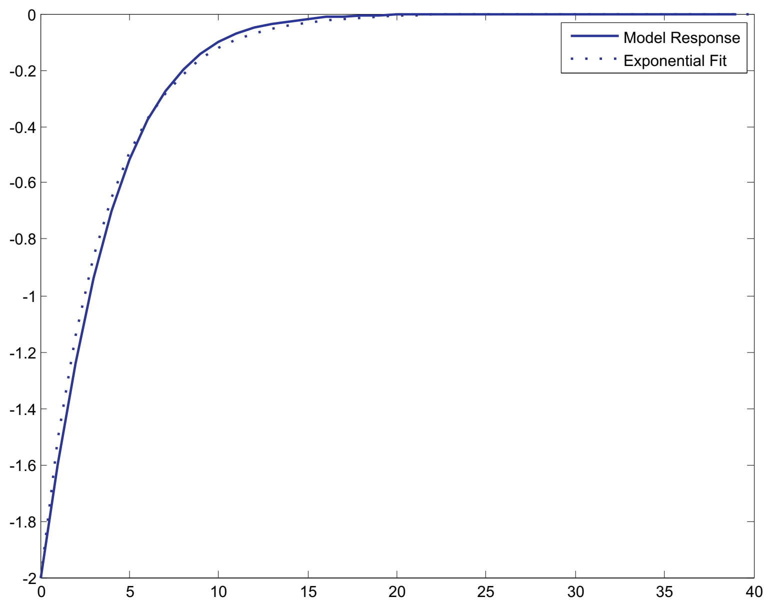

Figure 4. Trajectory of the Non-fuzzy Output of the System and Its Exponential Fit

k	$\underline{b}(k)$				
0	1	0	0	0	0
1	0.7	0.2	0.1	0	0
2	0.49	0.28	0.21	0.02	0
3	0.343	0.296	0.319	0.042	0
4	0.2401	0.2800	0.4209	0.0590	0
⋮	⋮	⋮	⋮	⋮	⋮
50	0	0	1	0	0

Table 1. Evolution of the Output Fuzzy Vector from $[1, 0, 0, 0, 0]$ to $[0, 0, 1, 0, 0]$

show the similarity of the trajectory of this specific example to that of a first-order non-fuzzy dynamic system.

Note that in Table 1, the sum of the elements of each row, i.e., the fuzzy vector, is 1, as expected. This reminds that having a unit-row relational matrix in the FRDS $\underline{b}(k+1) = \underline{b}(k) \circ R$, makes Γ_f invariant under the operation "$\circ R$".

4. Studying the Results for Other Types of Fuzzy Composition

In the previous section, Theorem 3.8 was represented for sum-prod FRDS. Here the possibility of extending Theorem 3.8 for other types of fuzzy composition is investigated. Throughout this section, s-prod FRDS is considered, where the fuzzy composition comprises an arbitrary t-conorm and the algebraic t-norm. Furthermore $S_{j=1}^{q} b_j$, operating the t-conorm on all of the elements of b_j for $j = 1, \ldots, q$, can be defined due to the associativity of the t-conorm operator.

Definition 4.1. Let S_f be the hyperplane of all q-tuples $(x_1, ..., x_q)$ such that $\underset{j=1}{\overset{q}{S}} x_j = 1$, where $x_j \in [0, 1]$, for $j = 1, ..., q$. An FRDS described by $\underline{b}(k+1) = \underline{b}(k) \circ R$ has the property of *intrasurficiality* on S_f, if $\underline{b}(k+1) \in S_f$ for every $\underline{b}(k) \in S_f$. In other words, the FRDS is called *intrasurficial* if S_f is invariant under $\circ R$.

Definition 4.2. Having a set $A \subset \mathbb{R}$, the t-conorm s has the property of *A-quasi-homogeneity*, when $s(\alpha b_1, \alpha b_2) = \alpha s(b_1, b_2)$, for every $\alpha \in A$.

Lemma 4.3. *Let s be an A-quasi-homogeneous t-conorm and $\alpha \in A$. Then*

$$s(\alpha b_1, \ldots, \alpha b_q) = \alpha s(b_1, \ldots, b_q),$$

for every $q = 2, 3, \ldots$.

Proof. It is enough to prove the lemma for $q = 3$ due to the associativity of the t-conorm. Let $b_1, b_2, b_3 \in [0, 1]$. Then

$$\begin{aligned} s(\alpha b_1, \alpha b_2, \alpha b_3) &= s(\alpha b_1, s(\alpha b_2, \alpha b_3)) = s(\alpha b_1, \alpha s(b_2, b_3)) \\ &= \alpha s(b_1, s(b_2, b_3)) = \alpha s(b_1, b_2, b_3). \end{aligned}$$

□

Definition 4.4. The t-conorm s is *strictly monotone* if $a < b$ yields $s(a, c) < s(b, c)$, for every $c \in [0, 1)$.

Lemma 4.5. *For a strictly monotone t-conorm* S, *if* $\underset{j=1}{\overset{q}{S}} b_j = 1$ *then there exists a* $j \in \{1, \cdots, q\}$ *such that* $b_j = 1$.

Proof. It suffices to prove that if $b_j < 1$, for every $j \in \{1, \cdots, q\}$, then $\underset{j=1}{\overset{q}{S}} b_j < 1$. This can be proved by induction. Since s is strictly monotone, $s(b_1, b_2) < s(b_1, 1) = 1$. Now, suppose that $s(b_1, \ldots, b_{m-1}) < 1$, then

$$s(b_1, b_2, \ldots, b_{m-1}, b_m) = s(s(b_1, \ldots, b_{m-1}), b_m) < s(1, b_m) = 1.$$

Therefore $\underset{j=1}{\overset{q}{S}} b_j < 1$, and the proof is complete. □

Definition 4.6. Let s be a t-conorm. A $p \times q$ matrix $R = [r_{ij}]$ is called *s-unit-row matrix* if

$$\mathop{S}_{j=1}^{q} r_{ij} = 1, \forall i \in \{1, \ldots, p\}\,.$$

Lemma 4.7. *Let $R = [r_{ij}]$ be a $q \times q$ relational matrix and "$\circ R$" be s-prod fuzzy composition with R, in which s is $[0,1]$-quasi-homogeneous. Then the hyperplane S_f is invariant under "$\circ R$" if and only if R is an s-unit-row matrix.*

Proof. Let $\underline{b} = \underline{a} \circ R$. First the sufficient condition is proved. Since R is an s-unit-row matrix and $\underline{a} \in S_f$, then $\underline{a} \circ R = (\mathop{S}_{i=1}^{q} a_i r_{i1}, \cdots, \mathop{S}_{i=1}^{q} a_i r_{iq})$ is an s-unit-row matrix, because:

$$\mathop{S}_{j=1}^{q} \mathop{S}_{i=1}^{q} a_i r_{ij} = \mathop{S}_{i=1}^{q} \mathop{S}_{j=1}^{q} a_i r_{ij} = \mathop{S}_{i=1}^{q} a_i \mathop{S}_{j=1}^{q} r_{ij} = 1.$$

For the necessary condition, let $e_i \in S_f$ be such that its ith element equals 1 and the other elements are equal to 0, for every $i = 1, \ldots, q$. We have $\mathop{S}_{j=1}^{q} r_{ij} = S(e_i \circ R) = 1$, since $e_i \circ R \in S_f$, for i, $1 \leqslant i \leqslant q$. This completes the proof. □

Lemma 4.8. *For every strictly monotone t-conorm s, $s\,(a, b) > a$ if $a \neq 1$ and $b \neq 0$.*

Proof. For a strictly monotone t-conorm s, $b > c$ implies $s\,(a, b) > s\,(a, c)$. Since $b > 0$, putting $c = 0$ in the property, the result is obtained. □

Corollary 4.9. *When the operation of a strictly monotone t-conorm results in zero all its arguments are zero.*

Theorem 4.10. *Let s be an $[0,1]$-quasi-homogeneous strictly monotone t-conorm. In a s-prod FRDS with standard fuzzifier, weighted average defuzzifier, FRE $\underline{b}(k+1) = \underline{b}(k) \circ R$, where R is an s-unit-row relational matrix, the output (of the defuzzifier) converges asymptotically globally to c_l if:*

1. $r_{ll} = 1$,
2. $r_{il} \neq 0 \quad \forall i$.

Proof. Since $\underline{b}(k+1) = \underline{b}(k) \circ R$, $b_j(k+1) = \mathop{S}_{i} b_i(k) r_{ij}$ and hence

$$\begin{aligned} b_l(k+1) = \mathop{S}_{i} b_i(k) r_{il} = s\left(b_l(k) r_{ll}, \mathop{S}_{i \neq l} b_i(k) r_{il}\right) &= s(b_l(k), \mathop{S}_{i \neq l} b_i(k) r_{il}) \\ &\geqslant s(b_l(k), 0) = b_l(k). \end{aligned}$$

Therefore $\{b_l(k)\}$ is an increasing and bounded sequence and hence it is convergent. Put $\lim_{k \to \infty} b_l(k) = \alpha$. If $\alpha = 1$, then the proof is complete; if not, since

$$\alpha = s(\alpha, \lim_{k \to \infty} \mathop{S}_{i \neq l} b_i(k) r_{il}),$$

then

$$0 = \lim_{k \to \infty} \mathop{S}_{i \neq l} b_i(k) r_{il} = \mathop{S}_{i \neq l} (\lim_{k \to \infty} b_i(k) r_{il}) = \mathop{S}_{i \neq l} (r_{il} \lim_{k \to \infty} b_i(k))$$

and by Corollary 4.9, $r_{il} \lim_{k\to\infty} b_i(k) = 0$, for every $i \neq l$. By hypothesis, we have $\lim_{k\to\infty} b_i(k) = 0$, for every $i \neq l$. Hence, $\lim_{k\to\infty} b_l(k) = 1$, which is a contradiction. This completes the proof. □

5. Conclusion

A set of sufficient conditions are obtained for concluding the global asymptotic stability of simple fuzzy relational models (FRMs) with sum-prod fuzzy composition and standard fuzzifier which has sum-normal triangular membership functions. These conditions can be used as an auxiliary tool for identification of FRDS to obtain a more reliable FRM. It is also suggested to use these conditions to synthesize more reliable fuzzy relational controllers which is not dealt with in this work. The achievements of this paper can be extended in several ways, i.e., to obtain more precise sufficient conditions for the stability test, to provide sufficient conditions to ensure the stability of higher order FRMs, or to extend the results to other types of fuzzy compositions if possible.

References

[1] J. M. Andjar and A. J. Barragn, *A methodology to design stable nonlinear fuzzy control systems*, Fuzzy Sets and Systems, **154** (2005), 157-181.

[2] A. Aghili Ashtiani and M. B. Menhaj, *Introducing a new pair of differentiable fuzzy norms and its application to fuzzy relational function approximation*, Proc. 10th Joint Conf. Information Sciences, Salt Lake City, USA: World Scientific Publishing Co. ISBN: 978-981-270-967-7, (2007), 1329-1336.

[3] A. Aghili Ashtiani and M. B. Menhaj, *Numerical solution of fuzzy relational equations based on smooth fuzzy norms*, Soft Computing, **14(6)** (2010), 545-557.

[4] X. Ban, X. Z. Gao, X. Huang and A. V. Vasilakos, *Stability analysis of the simplest Takagi–Sugeno fuzzy control system using circle criterion*, Information Sciences, **177(20)** (2007), 4387-4409.

[5] J. Chen and L. Chen, *Study on stability of fuzzy closed-loop control systems*, Fuzzy Sets and Systems, **57(2)** (1993), 159-168.

[6] T. Furuhashi, H. Kakami, J. Peters and W. Pedrycz, *A stability analysis of fuzzy control system using a generalized fuzzy petri net model*, IEEE World Congress on Computational Intelligence, (1998), 95-100.

[7] T. Hasegawa and T. Furuhashi, *Stability analysis of fuzzy control systems simplified as a discrete system*, Int. J. Control and Cybernetics, **27(4)** (1998), 565-577.

[8] K. Hirota, H. Nobuhara, K. Kawamoto and S. I. Yoshida *On a lossy image compression/reconstruction method based on fuzzy relational equation*, Iranian Journal of Fuzzy Systems, **1(2)** (2004), 33-42.

[9] J. B. Kiszka, M. M. Gupta and P. N. Nikiforuk, *Energetic stability of fuzzy dynamic systems*, IEEE Transactions on Systems, Man and Cybernetics, **15** (1985), 783-792.

[10] A. Kandel, Y. Luo and Y. Q. Zhang, *Stability analysis of fuzzy control systems*, Fuzzy Sets and Systems, **105** (1999), 33-48.

[11] C. Kolodziej and R. Priemer, *Stability analysis of fuzzy systems*, Journal of the Franklin Institute, **336** (1999), 851-873.

[12] T. Leephakpreeda and C. Batur, *Stability analysis of a fuzzy control system*, Thammasat Int. J. Science and Technology, **2(l)** (1997), 1-5.

[13] W. Pedrycz, *An identification algorithm in fuzzy relational systems*, Fuzzy Sets and Systems, **13** (1984), 153-167.

[14] J. N. Ridley, I. S. Shaw and J. J. Kruger, *Probabilistic fuzzy model for dynamic systems*, Electronics Letters, **24** (1988), 890-892.

[15] A. A. Suratgar and S. K. Y. Nikravesh, *A new method for linguistic modeling with stability analysis and applications*, Intelligent automation and soft computing, **15(3)** (2009), 329-342.
[16] A. A. Suratgar and S. K. Y. Nikravesh, *Potential energy based stability analysis of fuzzy linguistic systems*, Iranian Journal of Fuzzy Systems, **2(1)** (2005), 67-74.
[17] A. A. Suratgar and S. K. Y. Nikravesh, *Necessary and sufficient conditions for asymptotic stability of a class of applied nonlinear dynamical systems*, Proc. 10th IEEE Int. Conf. Electronics, Circuits and Systems, **3** (ICECS 2003), 1062-1065.
[18] A. A. Suratgar and S. K. Y. Nikravesh, *A new sufficient condition for stability of fuzzy systems*, Proc. of Iranian Conf. Electrical Engineering, (2002), 441-445.
[19] E. Sanchez, *Resolution of composite fuzzy relation equations*, Information and Control, **30(1)** (1976), 38-48.
[20] T. Šijak, S. Tešnjak and O. Kuljaca, *Stability analysis of fuzzy control system using describing function method*, Proc. 9th Mediterranean Conf. Control and Automation, 2001.

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