A NEW APPROACH BASED ON OPTIMIZATION OF RATIO FOR SEASONAL FUZZY TIME SERIES

U. YOLCU

Abstract. In recent years, many studies have been done on forecasting fuzzy time series. First-order fuzzy time series forecasting methods with first-order lagged variables and high-order fuzzy time series forecasting methods with consecutive lagged variables constitute the considerable part of these studies. However, these methods are not effective in forecasting fuzzy time series which contain seasonal structures. In this respect, it would be more appropriate to use methods that consider the seasonal relations in seasonal fuzzy time series forecasting. Although seasonal fuzzy time series forecasting methods exist in literature, these methods use equal interval lengths in partition of the universe of discourse. This situation incapacitates the performance of the method in forecasting time series including seasonality and trend. In this study, a new fuzzy time series forecasting method in which intervals constituting partition of the universe of discourse increase in time at a rate that obtained based on optimization was proposed. The proposed method was applied to two real time series and obtained results were compared with other methods and the superior performance of the proposed method was proved.

1. Introduction

Fuzzy set theory proposed by Zadeh [43] provided a basis for fuzzy time series concept that was first proposed by Song and Chissom [35]. Song and Chissom [36] classified fuzzy time series as time-variant and time-invariant depending whether the relation found between any t and previous $t\text{-}1$ is the same. If the relation is always the same, it is time-variant, if not it is called time-invariant. Although, solution methods have been developed for both time- variant and time-invariant fuzzy time series, most of the studies are composed of time-invariant fuzzy time series.

Fuzzy time series analysis method consists of three steps as fuzzifying the observations, identifying fuzzified relations and defuzzifying. In the fuzzifying step, interval lengths that are needed in partition of the universe of discourse are highly effective. Therefore, several studies on the determination of the length of intervals are available in the literature. In many of these studies, fixed interval lengths are determined. While Song and Chissom [35, 36, 37] and Chen [9, 10] determined the length of intervals subjectively, Huarng [24] proposed average and distribution based approaches to show that lengths of interval determined in the partition stage

Received: July 2013; Revised: October 2015; Accepted: January 2016

Key words and phrases: Seasonal fuzzy time series, Optimization, Forecasting, Feed forward neural networks.

of universal set have an effect on forecasting performance. Egrioglu et al. [18, 19] proposed approaches based on optimization while determining the interval lengths. Additionally, based on the idea that the universe of discourse are partitioned by changing interval lengths rather than fixed interval lengths, Kuo et al. [28, 29], Davari et al. [14], Park et al. [33], and Hsu et al. [23] used particle swarm optimization whereas Chen and Chung [11], Lee at al. [30], and Bas et al. [6] used genetic algorithms. In the analysis of time series with trend, Huarng and Yu [25] and Yolcu et al. [38] proposed approaches in which the increase of interval lengths are determined in time. In the fuzzification step, while Cheng et al. [13] and Alpaslan et al. [5] used C- means, Egrioglu et al. [20] used fuzzy clustering methods which do not need to partition of the universe of discourse such as Gustafson-Kessel.

In the determination of fuzzy relations, Chen [9] facilitated this step using fuzzy logic group relation tables rather than using fuzzy relation matrix which was proposed by Song and Chissom [35]. Besides, Huarng [24], Yu [40], Huarng and Yu [25], Cheng et al. [13], and Egrioglu et al. [18, 19] used fuzzy logic group relation tables which are currently used in many studies in literature for the determination of fuzzy relations. Moreover, artificial neural networks (ANN), which were first used by Huarng and Yu [26] for forecasting fuzzy time series, was used in studies conducted by Aladag et al. [1, 2, 3], Egrioglu et al. [16, 17], Yu and Huarng [42], Yolcu et al. [39] and Cagcag Yolcu [8] and also Khashei et al. [27] proposed an approach based on ANN and fuzzy regression model.

In the defuzzification step, while almost all studies in literature used centralization method, Song and Chissom [37] used artificial neural networks. Cheng et al. [12] and Aladag et al. [2] managed to increase forecasting performance using centralization and adaptive expectation methods.

In addition, in some other studies, while Egrioglu [15] and Egrioglu et al. [21] used particle swarm optimization to determine fuzzy relation matrix, Aladag et al. [4] used genetic algorithms for fuzzy lagged variable selection.

In literature, some studies include first-order fuzzy time series forecasting models whereas others include high order fuzzy time series forecasting models. While first-order fuzzy time series forecasting models include only one lagged variable, high-order fuzzy time series forecasting models include all consecutive lagged variables. It is clear that these models would not be effective in forecasting seasonal fuzzy time series which are frequently encountered in real life. Additionally, while using a high-order models in seasonal fuzzy time series whose period is p, the use of lagged variable as the number of p is needed (AR (p), $p > 1$). This includes variables which are not required in explaining relations constituting time series structure in the model. Song [34], proposed a fuzzy time series analysis method in which.

F(t-p) was used as input and *F(t)* was used as output in the analysis of fuzzy time series. Song's method was similar to the first-order seasonal autoregressive model SAR(1) but this is not efficient in explaining seasonal relations. Real time series data rarely bare this structure. There exist more complicated time series data so that analyzing of them requires models such as either SAR(2), SAR(3), etc. Besides, the time series encountered in real life have not only autoregressive (AR) structure but also moving average (MA) structure. In that case, in addition to the AR (autoregressive) and SAR (seasonal autoregressive) terms, MA (moving average) and SMA (seasonal moving average) terms should be considered in the model. Such a model contains terms of AR, SAR structure and MA, SMA structure. Namely, the inputs of model include both the terms of time series (X_t) and the terms of error (a_t) series. This situation requires that bivariate case should be employed. For this purpose, Egrioglu et al. [16] proposed a partial high order bivariate fuzzy time series forecasting approach which combines SARIMA (seasonal autoregressive integrated moving average) and fuzzy time series. Although, Egrioglu et al.'s method provides advantages in seasonal fuzzy time series analysis, it employs subjectively determined fixed intervals in partition of the universe of discourse. In forecasting seasonal fuzzy time series as well as time-variant fuzzy time series, having ever increasing interval lengths instead of fixed lengths would improve forecasting performance. In this study, a partial high-order bivariate fuzzy time series forecasting method which combines SARIMA and fuzzy time series proposed by Egrioglu et al. [16] was developed. For this purpose, a new approach was suggested based on optimization of intervals constituting partition of the universe of discourse in forecasting fuzzy time series whose observations have a tendency to increase in time

The advantages and the main features of the proposed method are as follows:

•In contrast with the other seasonal fuzzy time series found in literature, the proposed method uses changing interval lengths in partition of the universe of discourse.

•In the proposed method, determination of model order is done systematically by utilizing Box-Jenkins algorithm.

•In the proposed method, in addition to AR terms, MA terms are also included by using residuals (a_t).

•Fuzzy relations are determined using feed forward artificial neural networks and the model avoids using complex matrix relation and fuzzy logic group relation tables.

•The proposed method provides an effective performance in forecasting fuzzy time series whose observations tend to increase.

This paper organized as follows:

In chapter 2, basic definitions of fuzzy time series were presented. SARIMA model, which is used to determine the input and order of fuzzy time series forecasting model, and artificial neural networks which are used to determine fuzzy time series were summarized in chapter 3. In chapter 4, the proposed method and its algorithm were summarized. Finally, in chapter 5, implementation of the proposed method was introduced and obtained results were discussed.

2. **Fuzzy Time Series**

The definition of fuzzy time series was firstly introduced by Song and Chissom [35]. In contrast to conventional time series methods, various theoretical assumptions do not need to be checked in fuzzy time series approach. The most important advantage of fuzzy time series approach is to be able to work with a very small set of data and not to require the linearity assumption. General definitions of fuzzy time series are given as follows:

Let U be the universe of discourse, where $U = \{u_1, u_2, ..., u_b\}$. A fuzzy set A_i of U is defined as $A_i = f_{A_i}(u_1)/u_1 + f_{A_i}(u_2)/u_2 + \cdots + f_{A_i}(u_b)/u_b$, where f_{A_i} is the membership function of the fuzzy set A_i; $f_{A_i} : U \to [0, 1]$. u_a is a generic element of fuzzy set A_i; $f_{A_i}(u_a)$ is the degree of belongingness of u_a to A_i; $f_{A_i}(u_a) \in [0, 1]$ and $1 \leq a \leq b$.

Definition 2.1. <Fuzzy time series> Let $Y(t)(t = \ldots, 0, 1, 2, \ldots)$, a subset of real numbers, be the universe of discourse by which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), ...$ then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2.2. Fuzzy time series relationships assume that $F(t)$ is caused only by $F(t-1)$, then the relationship can be expressed as: $F(t) = F(t-1) * R(t, t-1)$, which is the fuzzy relationship between $F(t)$ and $F(t-1)$, where * represents as an operator. Based on the type of problem, different operators as fuzzy complement, intersection (T-norm) and union (S-norm) can be used. To sum up, let $F(t-1) = A_i$ and $F(t) = A_j$. The fuzzy logical relationship between $F(t)$ and $F(t-1)$ can be denoted as $A_i \to A_j$ where A_i refers to the left-hand side and A_j refers to the right-hand side of the fuzzy logical relationship. Furthermore, these fuzzy logical relationships can be grouped to establish different fuzzy relationships.

The definition of the first order seasonal fuzzy time series model for forecasting proposed by Song [34] is given as follows:

Definition 2.3. Let $F(t)$ be a fuzzy time series. Assume there exists seasonality in $\{F(t)\}$, first order seasonal fuzzy time series forecasting model:

$$F(t - m) \to F(t) \tag{1}$$

where m denotes the period.

The high order fuzzy time series model proposed by Chen [10] is given as follows:

Definition 2.4. Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1), F(t-2), \ldots$, and $F(t-n)$, then this fuzzy logical relationship is represented by

$$F(t - n), ..., F(t - 2), F(t - 1) \to F(t) \tag{2}$$

and it is called the n-th order fuzzy time series forecasting model.

Bivariate fuzzy time series model defined by Yu and Huarng [41] is given as follows:

Definition 2.5. Let *F(t)* and *G(t)* be two fuzzy time series. Suppose that $F(t-1) = A_i$, $G(t-1) = B_k$ and $F(t) = A_j$. A bivariate fuzzy logical relationship is defined as A_i, $B_k \to A_j$ where A_i, B_k are referred to as the left hand side and A_j as the right hand side of the bivariate fuzzy logical relationship.

Therefore, first order bivariate fuzzy time series forecasting model is as follows:

$$F(t - 1), G(t - 1) \to F(t) \tag{3}$$

The definition of the Egrioglu et al.'s [16] model which is called a $(k,\ l)^{th}$ order partial bivariate fuzzy time series forecasting model is given as follows:

Definition 2.6. Let F and G be two fuzzy time series. If $F(t)$ is caused by $F(t-m_1),...,F(t-m_{k-1}),F(t-m_k)$,$G(t-n_1),...,G(t-n_{l-1}),G(t-n_l)$, where m_i $(i=1,2,..,k)$ and n_j $(j=1,2,..,l)$ are integers $1 \leq m_1 < ... < m_k$, , $1 \leq n_1 < ... < n_l$ then this FLR is represented by

$$F(t-m_1),...,F(t-m_{k-1}),F(t-m_k),G(t-n_1),...,G(t-n_{l-1}),G(t-n_l) \rightarrow F(t) \quad (4)$$

3. **Preliminary**

This chapter includes summary information on SARIMA which is used for the determination of model input, model order and artificial neural networks which are used in creating fuzzy relations.

3.1. **SARIMA Models.** If X_t is a time series with mean μ, then the model is expressed in Equation (1).

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D(X_t-\mu)=\theta(B)\Theta(B^s)a_t \quad (5)$$

Model parameters can be given as follows:

$$\phi(B)=(1-\phi_1 B-\cdots-\phi_p B^p) \quad (6)$$

$$\theta(B)=(1+\theta_1 B+\cdots+\theta_q B^q) \quad (7)$$

$$\Phi(B)=(1-\Phi_1 B^s-\cdots-\Phi_P B^{sP}) \quad (8)$$

$$\Theta(B)=(1+\Theta_1 B^s+\cdots+\Theta_Q B^{sQ}) \quad (9)$$

The seasonal autoregressive integrated moving average (SARIMA) model incorporates both non-seasonal and seasonal factors in a multiplicative model. One shorthand notation for the model is $\mathrm{SARIMA(p,d,q)(P,D,Q)_s}$ with p; non-seasonal AR (Autoregressive) order, d; non-seasonal differencing, q; non-seasonal MA (Moving average) order, P; seasonal AR order, D; seasonal differencing, Q; seasonal MA order, and s; time span of repeating seasonal pattern and also μ; the mean of X_t. Detailed information on the model which is called SARIMA can be obtained from Box and Jenkins study [7].

3.2. **Artificial Neural Networks.** Artificial neural networks (ANN) can be defined as the mathematical algorithm that is inspired by the biological neural networks. Artificial neural networks, defined as the mathematical algorithms, are the algorithms that can learn from the examples and that can generalize what is learnt. Network presentation is the graphical expression of mathematical algorithms (see [22]). Artificial neural networks are much more different from biological ones in terms of their structure and ability (see [45]). Artificial neural networks compose of a mathematical model (see [44]).

Three basic components that direct the operation of artificial neural networks are as follows:

Architectural Structure: The architecture structure of multilayer feed forward artificial neural network consists of input layer, hidden layer(s) and output layer (see Figure 1). Each layer consists of neurons. The architecture structure is determined based on deciding the number of neuron in each layer. These neurons are linked each other by weights. There is no link among the neurons in the same layer.

Learning Algorithm: Although, there are many learning algorithms used in the determination of weights in artificial neural networks, the most widely used one is Back Propagation algorithm which updates weights based on the difference between available data and the output of the network. Learning parameter which is used in back propagation algorithm and which can be taken fixedly or updated in the algorithm dynamically, plays an important role in reaching optimal results.

Activation Function: The proper selection of activation function that enables curvilinear matching between input and output units, significantly affect the performance of the network. When the activation function, generally selected as unipolar, bipolar or linear, is not linear, slope parameter should be determined. Slope parameter is another factor that plays an important role in reaching the appropriate conclusion.

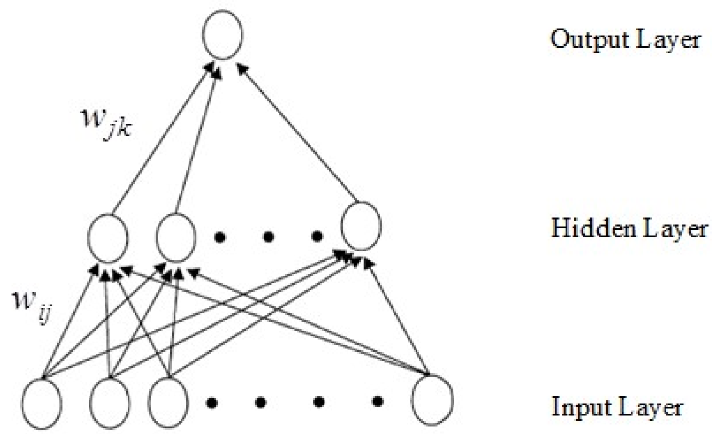

Figure 1. Architecture of Multilayer Feed Forward Neural Network

4. The Proposed Method

While seasonal fuzzy time series forecasting model, which was first proposed by Song [34], includes only one input, when high order fuzzy time series forecasting method, proposed by Chen [10], is used in the analysis of a seasonal fuzzy time series with a p period, the number of input will be the number of p. Nevertheless, both Song's [34] and Chen's [10] models will be AR-structured. However, accurate forecasting of many fuzzy time series need to consider MA terms. For this purpose, Egrioglu et al. [16] proposed a partial high order bivariate fuzzy time series forecasting model including both AR and MA terms. Although, this method provides some advantages in the analysis of seasonal fuzzy time series, it employs subjectively determined fixed intervals in partition of the universe of discourse. In forecasting seasonal fuzzy time series as well as time-variant fuzzy time series, having ever increasing interval lengths rather than fixed lengths would improve forecasting performance. In this study, a partial high-order bivariate fuzzy time series forecasting method which combines SARIMA and fuzzy time series which was proposed by Egrioglu et al. [16] was developed and a new approach in which the intervals constituting partition of the universe of discourse in fuzzification step are determined based on an optimized rate was introduced. Algorithm of new approach can be given as below.

Algorithm 4.1.

Step 1: According to Box-Jenkins method, appropriate SARIMA model is determined for the related time series and residuals (a_t) are calculated.

Let X_t and $\hat{X}_t$ represent time series and predicted time series by using appropriate $\mathrm{SARIMA(p,d,q)(P,D,Q)_s}$ model. Residuals (a_t) will be the difference between values of X_t and $\hat{X}_t$ ($a_t = X_t - \hat{X}_t$).

Step 2: Universal set (U and V) is defined for the X_t, the main factor, and for the a_t, second factor in the model and sub-intervals are detected.

Based on *min* and *max* values in the data set (X_t and a_t), $D_{\min}$ and $D_{\max}$ variables are defined. Then choose two arbitrary positive numbers which are D_1 and D_2 in order to divide the interval evenly.

$$U = [D_{\min} - D_1, D_{\max} + D_2] \tag{10}$$

Sub-intervals for a_t(second factor) are determined arbitrarily whereas sub- intervals for X_t(main factor) are determined using the formula below.

$$\begin{gathered} upper_0 = initial \\ For\ \ j \geq 1 \\ lower_j = upper_{j-1} \\ upper_j = (1+r)^j \times upper_0 \\ interval_j = [lower_j, upper_j] \end{gathered} \tag{11}$$

where r represents the ratio. Determination of ratio (r) is an optimization process. During optimization, r is limited in the range of [0.001, 0.2] considering that wide interval would destroy the fluctuation in the fuzzy time series. MATLAB function named "fminbnd" was used in the optimization of r to be used in determining the intervals. Optimization problem can be formulated as follows:

$$\begin{gathered} \min \ \ RMSE(r) \\ Subject\ to: \ \ ratio \in [0.001\ ,\ 0.2] \end{gathered} \tag{12}$$

where RMSE (Root Mean Square Error)is a function of r. On the other hand, optimization problem can also be expressed with MAPE (Mean Absolute Percentage Error) value which can be expressed as a function of the ratio:

$$\begin{gathered} \min \ \ MAPE(r) \\ Subject\ to: \ \ ratio \in [0.001\ ,\ 0.2] \end{gathered} \tag{13}$$

Step 3: Fuzzy sets are determined depending on universal set and sub-intervals.

Based on U and V the universe of discourse, $A_1, A_2, ..., A_{k_1}$ and $B_1, B_2, ..., B_{k_2}$ linguistic variables for time series and residuals are defined as follows:

$$\begin{array}{ll} A_1 = a_{11}/u_1 + a_{12}/u_2 + ... + a_{1k_1}/u_{k_1} & B_1 = b_{11}/v_1 + b_{12}/v_2 + ... + b_{1k_2}/v_{k_2} \\ A_2 = a_{11}/u_1 + a_{12}/u_2 + ... + a_{1k_1}/u_{k_1} & B_2 = b_{11}/v_1 + b_{12}/v_2 + ... + b_{1k_2}/v_{k_2} \\ \vdots & \vdots \\ A_{k_1} = a_{11}/u_1 + a_{12}/u_2 + ... + a_{1k_1}/u_{k_1} & B_{k_2} = b_{11}/v_1 + b_{12}/v_2 + ... + b_{1k_2}/v_{k_2} \end{array} \tag{14}$$

Here a_{ij}, are the membership values u_i sub-interval, $a_{ij} \in [0,1]$, $1 \leq i \leq k_1$; similarly, b_{lm}, are the membership values of v_l sub-interval and expressed as $b_{lm} \in [0,1]$, $1 \leq l \leq k_2$.

Step 4: X_t and a_t time series are fuzzified.

In this step, each observation belonging to X_t and a_t time series are matched with the fuzzy set with the highest value of intervals. Time series consisting fuzzy sets is called fuzzy time series. Fuzzy time series obtained for X_t is expressed by $F(t)$ whereas fuzzy time series obtained for $a_i$is expressed by $G(t)$.

Step 5: According to the inputs of SARIMA model, the order of model (k,l) and $m_1, ..., m_k$ and $n_1, ..., n_l$ values are determined.

Step 6: Fuzzy relations are determined.

Fuzzy relations are established by taking the lagged variables belonging to fuzzy time series $F(t-m_1), ..., F(t-m_{k-1}), F(t-m_k)$ and lagged variables belonging to fuzzy errors $G(t-n_1), ..., G(t-n_{l-1}), G(t-n_l)$as input and by taking the $F(t)$ as the target. In this step, feed forward artificial neural network is trained according to determined input and target values.

Step 7: Forecasts are obtained.

In consequence of training artificial neural networks, if network inputs are $F(t-m_1), ..., F(t-m_{k-1}), F(t-m_k)$, $G(t-n_1), ..., G(t-n_{l-1}), G(t-n_l)$ and target is $F(t)$, then $\hat{F}(t)$ obtained as the input of the network will be the fuzzy forecast (see Figure 2).

Step 8: Defuzzified forecasts are obtained.

Centralization method is used in defuzzification step. If the fuzzy forecast $\hat{F}(t)$ is obtained as A_j, then defuzzified forecast will be the midpoint of the interval having the highest membership value.

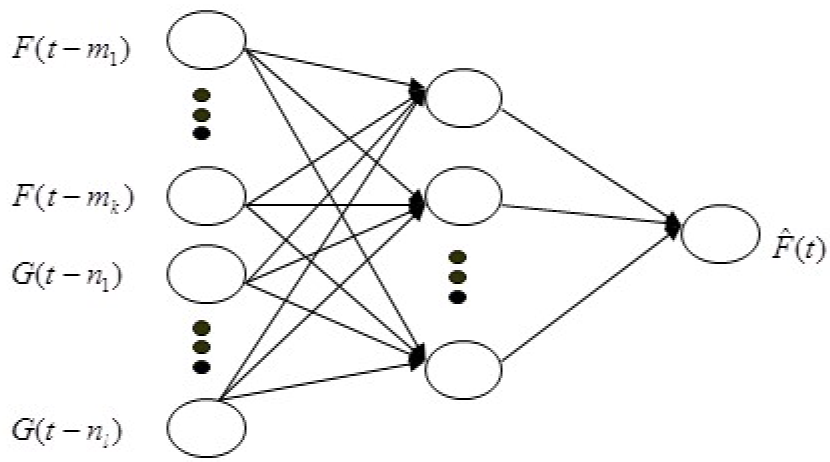

FIGURE 2. ANN Architecture

Step 9: RMSE or MAPE performance criteria are calculated.

Depending of the defuzzified forecast value, RMSE and MAPE performance criteria are calculated as fallows;

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n} (actual_t - forecast_t)^2}{n}} \tag{15}$$

$$MAPE = \frac{\sum_{t=1}^{n} \left| \frac{actual_t - forecast_t}{actual_t} \right|}{n} \tag{16}$$

5. Application

5.1. **The Number of Foreign Tourist Arriving in Turkey.** Firstly, the new method proposed in this study was applied to the time series of "the number of foreign tourist arriving in Turkey" between January 1998 and December 2008 which exhibits seasonality and a trend increasing in time. The graph is shown in Figure 3.

The last 12 observations were grouped as a test set and were used in performance evaluation. Stages of implementation can be given as a stepwise algorithm in parallel with the one introduced in the previous section.

Step 1: According to Box-Jenkins method, appropriate SARIMA model is determined as SARIMA $(1,0,2)\,(0,1,1)_{12}$ for the time series and residuals in this model (a_t) are calculated. The obtained method can be expressed as follows where f is a linear function.

$$X_t = f(X_{t-1}, X_{t-12}, X_{t-13}, a_{t-1}, a_{t-2}, a_{t-12}, a_{t-13}, a_{t-14}) \tag{17}$$

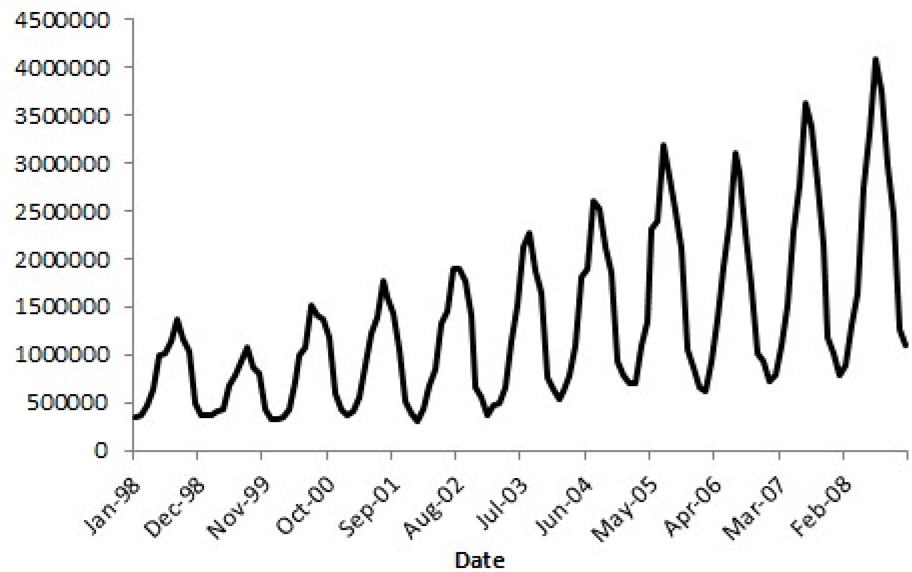

FIGURE 3. The Number of Foreign Tourist Arriving in Turkey between January 1998 and December 2008

Step 2: Universal sets for X_t, the main factor, and for a_t, second factor, in the model, are determined as;

$$U = [300000\ ,\ 4296621]\, V = [-350000\ ,\ 370000]$$

In consequence of optimization, when the ratio in (13) formula is 0.135136 for X_t (the main factor), sub-intervals can be created as follows:

$u_1 = [300000\,, 340541]$ $u_2 = [340541\,, 386560]$
$u_3 = [386560\,, 438798]$ $u_4 = [438798\,, 565406]$
$u_5 = [565406\,, 641813]$ $u_6 = [641813\,, 728545]$
$u_7 = [728545\,, 826998]$ $u_8 = [826998\,, 938755]$
$u_9 = [938755\,, 1065615]$ $u_{10} = [1065615\,, 1209618]$
$u_{11} = [1209618\,, 1373081]$ $u_{12} = [1373081\,, 1558633]$
$u_{13} = [1558633\,, 1769261]$ $u_{14} = [1769261\,, 2008351]$
$u_{15} = [2008351\,, 2279752]$ $u_{16} = [2279752\,, 2587829]$
$u_{17} = [2587829\,, 2937537]$ $u_{18} = [2937537\,, 3334504]$
$u_{19} = [3334504\,, 3785116]$ $u_{20} = [3785116\,, 4296621]$

Subintervals for second factor a_t are determined by increasing 10000 between 20000 and 100000 for each condition. If sub-intervals are taken as 80000 for a_t (second factor), subintervals are obtained as follows:

Step 3: A_j and B_i fuzzy sets are determined depending on universal set and sub-intervals. Given the subintervals in Step 2, fuzzy sets can be determined

$v_1 = [-350000\,,\,-270000]$ $v_2 = [-270000\,,\,-190000]$ $v_3 = [-190000\,,\,-110000]$
$v_4 = [-110000\,,\,-30000]$ $v_5 = [-30000\,,\,50000]$ $v_6 = [50000\,,\,130000]$
$v_7 = [130000\,,\,210000]$ $v_8 = [210000\,,\,290000]$ $v_9 = [290000\,,\,370000]$

as follows:

$$A_j = a_{j1}/u_1 + a_{j2}/u_2 + ... + a_{jk_1}/u_{k_1} a_{jk} = \begin{cases} 1 & , k_1 = j \\ 0.5 & , k_1 = j-1 \\ 0.5 & , k_1 = j+1 \\ 0 & , o.w. \end{cases} \quad , j = 1, \ldots, 20 \tag{18}$$

$$B_i = b_{i1}/v_1 + b_{i2}/v_2 + ... + b_{ik_2}/v_{k_2} b_{ik} = \begin{cases} 1 & , k_2 = i \\ 0.5 & , k_3 = i-1 \\ 0.5 & , k_2 = i+1 \\ 0 & , o.w. \end{cases} \quad , i, = 1, \ldots, 9 \tag{19}$$

Step 4: X_t and a_t time series are fuzzified.

For example, given the subintervals and fuzzy sets in Step 2 and 3, as $X(t = 01/2008) = 782786$ observation belonging to January corresponds u_7 intervals, fuzzy observation of this period will be A_7, which has got the highest membership value. That $F(t = 01/2008) = A_7$. Similarly, as $a(t = 01/2008) = 2843.03$ error corresponds v_5 interval, fuzzy observation will beB_5, which has got the highest membership value. That is $G(t = 01/2008) = B_5$.

Step 5: The order of model (k, l) and $m_1, ..., m_k$ and $n_1, ..., n_l$ values are determined according to inputs of SARIMA model.

Considering that appropriate model obtained in the first step is SARIMA$(1, 0, 2)(0, 1, 1)_{12}$, partial high order bivariate forecasting model where $k = 3$ and $l = 5$ is as follows:

$$F(t-1), F(t-12), F(t-13), G(t-1), G(t-2)G(t-12), G(t-13), G(t-14) \to F(t) \tag{20}$$

Here, $m_1 = 1$, $m_2 = 12$, $m_3 = 13$, $n_1 = 1$, $n_2 = 2$, $n_3 = 12$, $n_4 = 13$, $n_5 = 14$ and $F(t)$ is fuzzy X_t and $G(t)$ is fuzzy a_t.

Step 6: Fuzzy relations are determined.

In the determination of fuzzy relation, inputs of a feed forward artificial neural network is $F(t-1), F(t-12), F(t-13), G(t-1), G(t-2)G(t-12), G(t-13), G(t-14)$ based on the partial high order bivariate forecasting model which is $k = 3$ and $l = 5$, the output is $\hat{F}(t)$. In this case, input layer of feed forward artificial neural network consists of $k+l = 8$, whereas output layer consists of 1 unit. The number of units in the hidden layer is limited at 1/8 range. Additionally, Levenberg-Marquardt learning algorithm [31, 32] was employed as a learning algorithm, and logistic activation functions presented in Equation (21) was employed in each layer of feed forward artificial neural network.

$$f(x) = \frac{1}{1 + \exp(-\lambda\, x)} \tag{21}$$

Step 7: Forecasts are obtained.

As well as the parameters obtained from training of feed forward artificial neural network which is used for determination of fuzzy relations with learning algorithm and the outputs obtained from giving observations in test set as inputs constitute fuzzy forecast and is expressed as $\hat{F}(t)$.

Step 8: Defuzzified forecasts are obtained.

Centralization method is used in defuzzification step. If the fuzzy forecast $\hat{F}(t)$ is obtained as A_{20}, then defuzzified forecast will be the midpoint (4040868.50) of the interval $u_{20} = [3785116\,,\,4296621]$ having the highest membership value inA_{20} fuzzy set.

Step 9: RMSE or MAPE performance criteria are calculated.

Depending of the defuzzified forecasts, RMSE and MAPE performance criteria are calculated for the determined test set.

In the analysis having these characters, in the optimization of the ratio, two analyses were made separately with the aim of minimizing both RMSE and MAPE values. Along with the results for the best conditions obtained from the analysis of the proposed method and those found in literature are presented in Table 1 and Table 2 for RMSE and MAPE, respectively.

Methods	**Time series**	**Residuals**	**Hidden layer neurons number**	**RMSE**
[9]	la=550000	-	-	646057
[33]	la=550000	-	-	532778
[10] / second order	la=500000	-	-	569492
[16]	la=150000	la=90000	3	228129
The proposed method	r=0.076570	la=100000	3	**205426**

la: length of interval r: ratio

TABLE 1. The Best RMSE Values of the Methods

Methods	**Time series**	**Residuals**	**Hidden layer neurons number**	**MAPE**
[9]	la=250000	-	-	0.2544
[33]	la=550000	-	-	0.2297
[10] / second order	la=500000	-	-	0.2954
[16]	la=300000	la=90000	5	0.0809
The proposed method	r=0.135136	la=80000	2	**0.0536**

la: length of interval r: ratio

TABLE 2. The Best MAPE Values of the Methods

When Table 1 is analyzed, it is seen that optimal ratio is 0.07657 and RMSE value is 205426 as a result of the analysis aiming the minimum RMSE value. Again, when Table 2 is analyzed, it is seen that optimal ratio is 0.135136 and MAPE value is 0.0536 as a result of the analysis aiming the minimum MAPE value. As can be seen from the both two tables, the method which was proposed in this study and which was used for the analysis of fuzzy time series exhibiting seasonality and a tendency increasing in time is superior to other methods in terms of forecasting performance. Moreover, along with the forecasts obtained from the analysis of the proposed method and actual values belonging to test data are presented in Figure 4 and Figure 5 for RMSE and MAPE, respectively. When Figure 4 and Figure 5 are analyzed, once again, it is seen that proposed method has got superior forecasting performance.

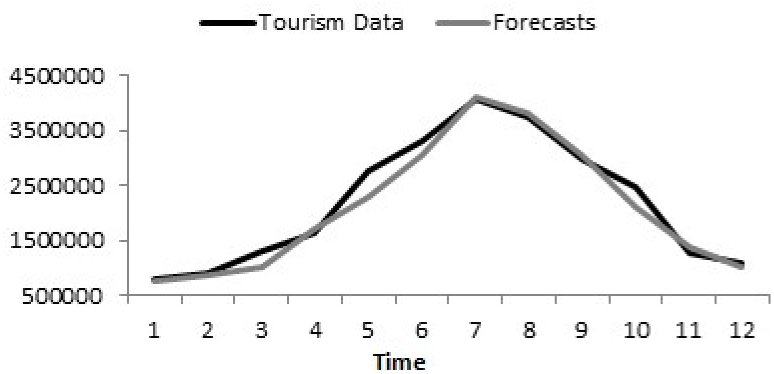

FIGURE 4. Forecasts of Tourism Data for Optimization of RMSE

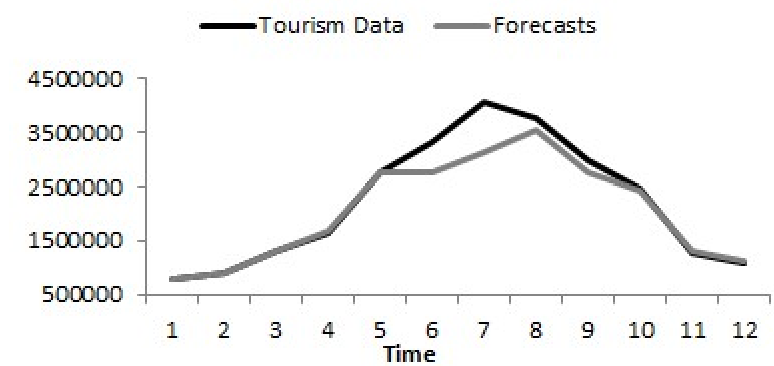

FIGURE 5. Forecasts of Tourism Data for Optimization of MAPE

5.2. **International Airline Passengers: Monthly Totals.** Secondly, the new method proposed was applied to the time series of "international airline passengers: monthly totals (series G)" between January 1949 and December 1960 which exhibits seasonality and a trend increasing in time and well known in the literature. The graph of Series G is shown in Figure 6.

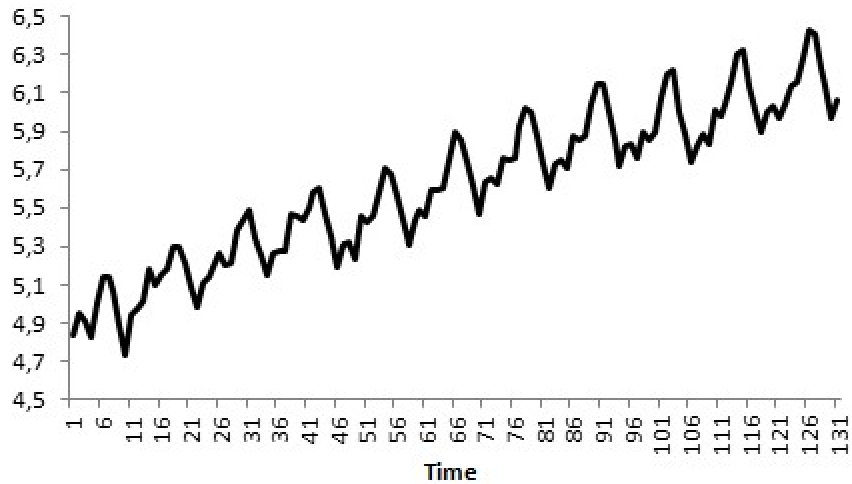

FIGURE 6. Series G

The last 13 observations were grouped as a test set and were used in performance evaluation. Because stages of previous implementation were given as a stepwise algorithm in parallel with the one introduced in the section four, stages of this implementation were not given as a stepwise algorithm.

According to Box-Jenkins method, appropriate SARIMA model is determined as SARIMA $(0,1,1)\,(0,1,1)_{12}$ for the time series and residuals in this model (a_t) are calculated. The obtained method can be expressed as follows where f is a linear function.

$$X_t = f(X_{t-1}, X_{t-12}, X_{t-13}, a_{t-1}, a_{t-12}, a_{t-13}) \tag{22}$$

Therefore, in the second phase, both k and l, which are the order of partial high order fuzzy time series forecasting model, are 3. Based on the lagged variables of the determined SARIMA model, the parameters of the model are $\mathrm{m}_1 = 1$, $\mathrm{m}_2 = 12$, $\mathrm{m}_3 = 13$, $\mathrm{n}_1 = 1$, $\mathrm{n}_2 = 12$, $\mathrm{n}_3 = 13$. The expression of the $(3,3)^{th}$ ordered partial bivariate fuzzy time series model can be written as follows:

$$F(t-1), F(t-12), F(t-13), G(t-1), G(t-12), G(t-13) \rightarrow F(t) \tag{23}$$

Subintervals for second factor a_t are taken as 0.01 and 0.02. In the analysis having these characters, in the optimization of the ratio, again, two analyses were made separately with the aim of minimizing both RMSE and MAPE values. Along with the results for the best conditions obtained from the analysis of the proposed method and those found in literature are presented in Table 3 and Table 4 for RMSE and MAPE, respectively.

Methods	Time series	Residuals	Hidden layer neurons number	RMSE
[9]	la=0.2000	-	-	0.1170
[33]	la=0.1500	-	-	0.1158
[10] / second order	la=0.2500	-	-	0.1535
[16]	la=0.1000	la=0.0200	4	0.0669
The proposed method	r=0.012222	la=0.0200	6	**0.0498**

la: length of interval r: ratio

Table 3. The Best RMSE Values of the Methods for Series G

Methods	Time series	Residuals	Hidden layer neurons number	MAPE
[9]	la=0.2000	-	-	0.0153
[33]	la=0.1500	-	-	0.0147
[10] / fifth order	la=0.2500	-	-	0.0199
[16]	la=0.1000	la=0.0200	4	0.0094
The proposed method	r=0.010909	la=0.0200	6	**0.0066**

la: length of interval r: ratio

Table 4. The Best MAPE Values of the Methods for Series G

Moreover, along with the forecasts obtained from the analysis of the proposed method and actual values belonging to test data of Series G are presented in Figure 7 and Figure 8 for RMSE and MAPE, respectively. When Figure 7 and Figure 8 are analyzed, once more again, superior forecasting performance of proposed method is seen.

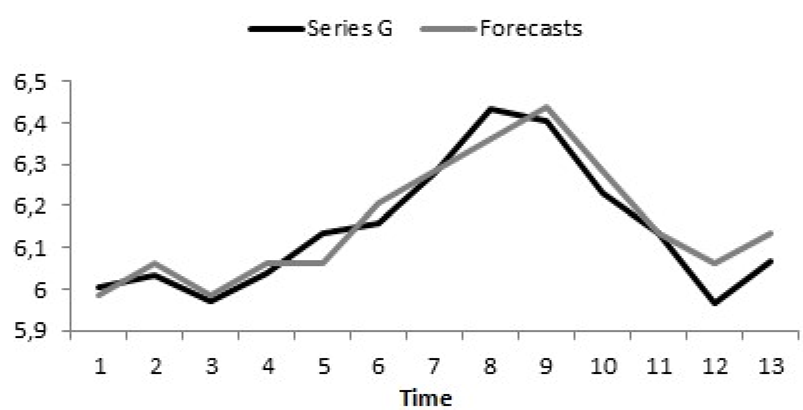

Figure 7. Forecasts of Series G for Optimization of RMSE

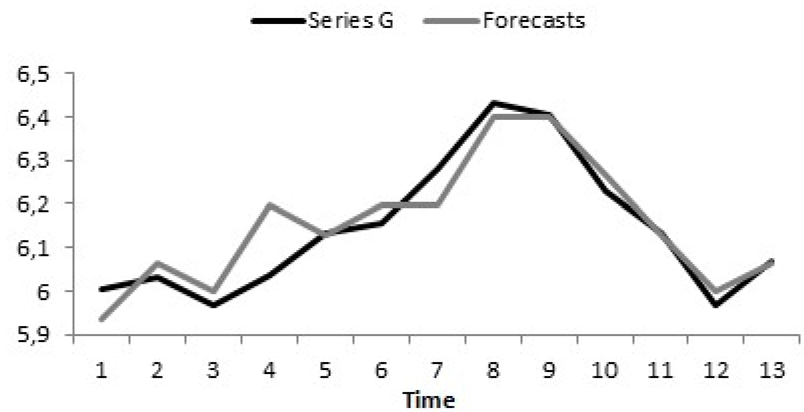

FIGURE 8. Forecasts of Series G for Optimization of MAPE

6. **Conclusion**

In the literature, there have been many studies using fuzzy time series for the purpose of forecasting. Although, each of these studies, which determine subintervals constituting partition of the universe of discourse in fuzzification step fixedly and subjectively, has some advantages. They are not efficient in forecasting time series exhibiting seasonality and tendency increasing in time. In this study, a new fuzzy time series forecasting method which exhibits seasonality and in which intervals constituting partition of the universe of discourse increase in time at a rate that was obtained based on optimization was proposed. The proposed method was applied to "the number of tourist arriving in Turkey" and "Series G" time series, and the superior performance of the proposed method was proved.

REFERENCES

[1] C. H. Aladag, M. A. Basaran, E. Egrioglu, U. Yolcu and V. R. Uslu, *Forecasting in high order fuzzy time series by using neural networks to define fuzzy relations*, Expert Systems with Applications, **36** (2009), 4228-4231.

[2] C. H. Aladag, U. Yolcu and E. Egrioglu, *A high order fuzzy time series forecasting model based on adaptive expectation and artificial neural networks*, Mathematics and Computers in Simulation, **81** (2010), 875-882.

[3] C. H. Aladag, E. Egrioglu, U. Yolcu and V. R. Uslu, *A high order seasonal fuzzy time series model and application to international tourism demand of Turkey*, Journal of Intelligent and Fuzzy Systems, **26** (2014), 295-302.

[4] C. H. Aladag, U. Yolcu, E. Egrioglu and E. Bas, *Fuzzy lagged variable selection in fuzzy time series with genetic algorithms*, Applied Soft Computing, **22** (2014), 465-473.

[5] F. Alpaslan, O. Cagcag, C. H. Aladag, U. Yolcu and E. Egrioglu, *A novel seasonal fuzzy time series method*, Hacettepe Journal of Mathematics and Statistics, **41** (2012), 375-385.

[6] E. Bas, V. R. Uslu, U. Yolcu and E. Egrioglu, *A modified genetic algorithm for forecasting fuzzy time series*, Applied Intelligence, **41** (2014), 453-463.

[7] G. E. P. Box and G. M. Jenkins, *Time series analysis: Forecasting and control.* CA: Holdan-Day, San Francisco, 1976.

[8] O. Cagcag Yolcu, *A Hybrid Fuzzy Time Series Approach Based on Fuzzy Clustering and Artificial Neural Network with Single Multiplicative Neuron Model*, Mathematical Problems in Engineering, Article ID 560472, **2013** (2013), 9 pages.

[9] S. M. Chen, *Forecasting enrollments based on fuzzy time-series*, Fuzzy Sets and Systems, **81** (1996), 311-31.

[10] S. M. Chen, *Forecasting enrolments based on high order fuzzy time series*, Cybernetics and Systems, **33** (2002), 1-16.

[11] S. M. Chen and N. Y. Chung, *Forecasting enrolments using high order fuzzy time series and genetic algorithms*, International Journal of Intelligent Systems, **21** (2006), 485-501.

[12] C. H. Cheng, T. L. Chen, H. J. Teoh and C. H. Chiang, *Fuzzy time-series based on adaptive expectation model for TAIEX forecasting*, Expert Systems with Applications, **34** (2008), 1126-1132.

[13] C. H. Cheng, G. W. Cheng and J. W. Wang, *Multi-attribute fuzzy time series method based on fuzzy clustering*, Expert Systems with Applications, **34** (2008), 1235-1242.

[14] S. Davari, M. H. F. Zarandi and I. B. Turksen, *An Improved fuzzy time series forecasting model based on particle swarm intervalization*, The 28^{th} North American Fuzzy Information Processing Society Annual Conferences (NAFIPS 2009), Cincinnati, Ohio, USA, June 14-17, 2009.

[15] E. Egrioglu, *PSO-based high order time invariant fuzzy time series method: Application to stock exchange data*, Economic Modelling, **38** (2014), 633-639.

[16] E. Egrioglu, C. H. Aladag, U. Yolcu, M. A. Basaran and V. R. Uslu, *A new hybrid approach based on SARIMA and partial high order bivariate fuzzy time series forecasting model*, Expert Systems with Applications, **36** (2009), 7424-7434.

[17] E. Egrioglu, C. H. Aladag, U. Yolcu, V. R. Uslu and M. A. Basaran, *A new approach based on artificial neural networks for high order multivariate fuzzy time series*, Expert Systems with Applications, **36** (2009), 10589-10594.

[18] E. Egrioglu, C. H. Aladag, U. Yolcu, V. R. Uslu and M. A. Basaran, *Finding an optimal interval length in high order fuzzy time series*, Expert Systems with Applications, **37** (2010), 5052-5055.

[19] E. Egrioglu, C. H. Aladag, M. A. Basaran, V. R. Uslu and U. Yolcu, *A New Approach Based on the Optimization of the Length of Intervals in Fuzzy Time Series*, Journal of Intelligent and Fuzzy Systems, **22** (2011), 15-19.

[20] E. Egrioglu, C. H. Aladag, U. Yolcu, V. R. Uslu and N. A. Erilli, *Fuzzy Time Series Forecasting Method Based on Gustafson-Kessel Fuzzy Clustering*, Expert Systems with Applications, **38** (2011), 10355-10357.

[21] E. Egrioglu, U. Yolcu, C. H. Aladag and C. Kocak, *An ARMA Type Fuzzy Time Series Forecasting Method Based on Particle Swarm Optimization*, Mathematical Problems in Engineering, Article ID 935815, **2013** (2013), 12 pages.

[22] S. Gunay, E. Egrioglu and C. H. Aladag, *Introduction to univariate time series analysis.* Hacettepe University Press, Ankara Turkey, 2007.

[23] L. Y. Hsu, S. J. Horng, T. W. Kao, Y. H. Chen, R. S. Run, R. J. Chen, J. L. Lai and I. H. Kuo, *Temperature prediction and TAIFEX forecasting based on fuzzy relationships and MTPSO techniques*, Expert Systems with Application, **37** (2010), 2756-2770.

[24] K. Huarng, *Effective length of intervals to improve forecasting in fuzzy time-series*, Fuzzy Sets and Systems, **123** (2001a), 387-394.

[25] K. Huarng and H. K. Yu, *Ratio-based lengths of intervals to improve fuzzy time series forecasting*, IEEE Trans. Syst. Man Cybern. B, Cybern., **36** (2006), 328-340.

[26] K. Huarng and H. K. Yu, *The application of neural networks to forecast fuzzy time series*, Physica A, **363** (2006), 481-491.

[27] M. Khashei, S. R. Hejazi and M. Bijari, *A new hybrid artificial neural networks and fuzzy regression model for time series forecasting*, Fuzzy Sets and Systems, **159(7)** (2008), 769-786.

[28] I. H. Kuo, S. J. Horng, T. W. Kao, T. L. Lin, C. L. Lee and Y. Pan, *An improved method for forecasting enrollments based on fuzzy time series and particle swarm optimization*, Expert Systems with Application, **36** (2009), 6108-6117.

[29] I. H. Kuo, S. J. Horng, Y. H. Chen, R. S. Run, T. W. Kao, R. J. Chen, J. L. Lai and T. L. Lin, *Forecasting TAIFEX based on fuzzy time series and particle swarm optimization*, Expert Systems with Application, **37** (2010), 1494-1502.

[30] L. W. Lee, L. H. Wang and S. M. Chen, *Temperature prediction and TAIFEX forecasting based on fuzzy logical relationships and genetic algorithms*, Expert Systems with Applications, **33** (2007), 539-550.

[31] K. Levenberg, *A Method for the Solution of Certain Non-Linear Problems in Least Squares*, The Quarterly of Applied Mathematics, **2** (1944), 164-168.

[32] D. W. Marquardt, *An algorithm for least-squares estimation of nonlinear parameters*, Journal of the Society for Industrial and Applied Mathematics, **11** (1963), 431-441.

[33] J. I. Park, D. J. Lee, C. K. Song and M. G. Chun, *TAIFEX and KOSPI 200 forecasting based on two factors high order fuzzy time series and particle swarm optimization*, Expert Systems with Application, **37** (2010), 959-967.

[34] Q. Song, *Seasonal forecasting in fuzzy time series*, Fuzzy Sets and Systems, **107** (1999), 235-236.

[35] Q. Song and B. S. Chissom, *Fuzzy time series and its models*, Fuzzy Sets and Systems, **54** (1993), 269-277.

[36] Q. Song and B. S. Chissom, *Forecasting enrollments with fuzzy time series- Part I*, Fuzzy Sets and Systems, **54** (1993), 1-10.

[37] Q. Song and B. S. Chissom, *Forecasting enrollments with fuzzy time series- Part II*, Fuzzy Sets and Systems, **62** (1994), 1-8.

[38] U. Yolcu, E. Egrioglu, V. R. Uslu, M. A. Basaran and C. H. Aladag, *A new approach for determining the length of intervals for fuzzy time series*, Applied Soft Computing, **9(2)** (2009), 647-651.

[39] U. Yolcu, C. H. Aladag, E. Egrioglu and V. R. Uslu, *Time series forecasting with a novel fuzzy time series approach: an example for Istanbul stock market*, Journal of Statistical Computation and Simulation, **83(4)** (2013), 597-610.

[40] H. K. Yu, *Weighted fuzzy time series models for TAIEX forecasting*, Physica A, **349** (2005), 609- 624.

[41] H. K. Yu and K. Huarng, *A bivariate fuzzy time series model to forecast TAIEX*, Expert Systems with Applications, **34** (2008), 2945-2952.

[42] H. K. Yu and K. Huarng, *A neural network- based fuzzy time series model to improve forecasting*, Expert Systems with Application, **37** (2010), 3366-3372.

[43] L. A. Zadeh, *Fuzzy Sets*, Inform and Control, **8** (1965), 338-353.

[44] G. P., Zhang, B. E., Patuwo and Y. M. Hu, *Forecasting with artificial neural networks: The state of the art*, International Journal of Forecasting, **14** (1998), 35–62.

[45] J. M. Zurada, *Introduction of artificial neural systems.* St. Paul: West Publishing, (1992), 26-27.

Ufuk Yolcu, Department of Statistics, Faculty of Science, Ankara University, 06100 Ankara, Turkey

E-mail address: uyolcu@ankara.edu.tr; varyansx@hotmail.com