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ON n-ARY HYPERGROUPS AND FUZZY n-ARY HOMOMORPHISM

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Abstract. The aim of this paper is to introduce the notion of fuzzy homomorphism and fuzzy isomorphism between two n-ary hypergroups and to extend the fuzzy results of fundamental equivalence relations to n-ary hypergroups. We study some of their properties and prove the decomposition theorems for fuzzy homomorphism and fuzzy isomorphism.

1. Introduction

Hypergroup which is based on the notion of hyperoperation has been introduced by Marty in [27] and studied extensively by many mathematicians. Hypergroup theory extends some well-known group results and also introduces new topics leading thus to a wide variety of applications, as well as to a broadening of the investigation fields, see [5,7,14,32]. A recent book [7] contains a wealth of applications. There are applications to the following subjects: geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, combinatorics, codes, artificial intelligence, and probabilistic.

The notion of an n-ary group is a natural generalization of the notion of a group and has many applications in different branches. The idea of investigations of such groups seems to be going back to E. Kasner's lecture at the fifty-third annual meeting of the American Association for the Advancement of Science in 1904 [22]. But the first paper concerning the theory of n-ary groups was written (under inspiration of Emmy Noether) by W. Dörnte in 1928 (Also see [18,20 23]).

n-ary generalizations of algebraic structures is the most natural way for further development and deeper understanding of their fundamental properties. Ameri and Zahedi in [2] studied algebraic hypersystems. In [12], Davvaz studied the relation between rough sets and algebraic systems. In [15], Davvaz and Vougiouklis introduced the concept of n-ary hypergroups as a generalization of hypergroups in the sense of Marty. Also, we can consider n-ary hypergroups as a nice generalization of n-ary groups. Leoreanu-Fotea and Davvaz in [25] introduced and studied the notion of a partial n-hypergroupoid, associated with a binary relation. Some important results, concerning Rosenberg partial hypergroupoids, induced by relations, are generalized to the case of n-hypergroupoids.

After the introduction of fuzzy sets by Zadeh [34], reconsideration of the concept

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of classical mathematics began. On the other hand, because of the importance of group theory in mathematics, as well as its many areas of application, the notion of fuzzy subgroups was defined by Rosenfeld [29] and its structure was investigated, also see [9]. Further studies of fuzzy subgroups as proposed by Das were undertaken by Mashinchi and Zahedi [35], who corrected some results of Das's paper. Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and so on. This provides sufficient motivations to researchers to review various concepts and results from the realm of abstract algebra in the broader framework of fuzzy setting. In [17], the notion of (normal) fuzzy n-ary subgroup of an n-ary group is introduced and some related properties are investigated. Characterizations of fuzzy n-ary subgroups are given, also see [19]. Ajmal in [1] defined a notion of containment of an ordinary kernel of a group homomorphism in a fuzzy subgroup. Using this idea, he provided the long-awaited solution of the problem of showing a one-to-one correspondence between the family of fuzzy subgroups of a group, containing the kernel of a given homomorphism, and the family of fuzzy subgroups of the homomorphic image of the given group. It is shown that an ordinary kernel gives rise to the notion of fuzzy quotient group in a natural way. Consequently, the fundamental theorem of homomorphisms is established for fuzzy subgroups. In [26], the definitions of six kinds of fuzzy homomorphisms are given. The object of [4] is to prove an analogue of the fundamental theorem of homomorphism and the second isomorphism theorem for fuzzy homomorphisms. Jin-xuan in [21] introduced the concepts of fuzzy homomorphism and fuzzy isomorphism between two fuzzy groups by a natural way, and studied some of their properties. He proved the decomposition theorems for fuzzy homomorphism and fuzzy isomorphism, also see [30].

Fuzzy sets and hyperstructures introduced by Zadeh and Marty, respectively, are now used in the world both on the theoretical point of view and for their many applications. The relations between fuzzy sets and hyperstructures have been already considered by Corsini, Davvaz, Leoreanu, Zahedi and others, for instance see [6,8,10,11,13,16,24,33,36]. In [10], Davvaz applied the concept of fuzzy sets to the theory of algebraic hyperstructures and defined fuzzy subhypergroup (resp. H_v-subgroup) of a hypergroup (resp. H_v-group) which is a generalization of the concept of Rosenfeld's fuzzy subgroup of a group, and in [13] introduced the concepts of strong homomorphism, inclusion homomorphism and fuzzy H_v-homomorphism between two H_v-groups. In [16], Davvaz and Corsini introduced the notion of a fuzzy and anti fuzzy n-ary subhypergroup of an n-ary hypergroup and to extend the fuzzy results of fundamental equivalence relations to n-ary hypergroups. Recently, lattice structure on fuzzy congruence relations of a hypergroupoid has been given by Bakhshi et.al in [3]. Now, in this paper, we introduce the notion of fuzzy homomorphism and fuzzy isomorphism between two n-ary hypergroups and to extend the fuzzy results of fundamental equivalence relations to n-ary hypergroups. We study some of their properties and prove the decomposition theorems for fuzzy homomorphism and fuzzy isomorphism.

2. Some Basic Definitions and Examples

We start by giving some known and useful definitions and notations. The definitions may be found in references [15,16]. Let H be a non-empty set and f be a mapping $f : H \times H \to P^*(H)$, where $P^*(H)$ is the set of all non-empty subsets of H. Then f is called a *binary hyperoperation* on H. We denote by H^n the cartesian product $H \times ... \times H$, where H appears n times. An element of H^n will be denoted by $(x_1, ..., x_n)$, where $x_i \in H$ for any i with $1 \leq i \leq n$. In general, a mapping $f : H^n \to P^*(H)$ is called an *n-ary hyperoperation* and n is called the *arity* of the hyperoperation f. Let f be an n-ary hyperoperation on H and $A_1, ..., A_n$ subsets of H. We define

$$f(A_1, ..., A_n) = \cup\{f(x_1, ..., x_n) \mid x_i \in A_i, \quad i = 1, ..., n\}.$$

We shall use the following abbreviated notation: The sequence $x_i, x_{i+1}, ..., x_j$ will be denoted by x_i^j. For $j < i$, x_i^j is the empty set. Thus $f(x_1, ..., x_i, y_{i+1}, ..., y_j, z_{j+1}, ..., z_n)$ will be written as $f(x_1^i, y_{i+1}^j, z_{j+1}^n)$.

A non-empty set H with an n-ary hyperoperation $f : H^n \to P^*(H)$ is called an *n-ary hypergroupoid* and denote by (H, f). An n-ary hypergroupoid (H, f) is called an *n-ary semihypergroup* if and only if the following associative axiom holds:

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) = f(x_1^{j-1}, f(x_j^{n+j-1}), x_{n+j}^{2n-1})$$

for every $i, j \in \{1, 2, ..., n\}$ and $x_1, x_2, ..., x_{2n-1} \in H$. If for all $(a_1, a_2, ..., a_n) \in H^n$, the set $f(a_1, a_2, ..., a_n)$ is a singleton, then f is called an *n-ary operation* and (H, f) is called an *n-ary groupoid* (rep.*n-ary semigroup*). An n-ary semihypergroup (H, f) in which the equation

$$b \in f(a_1^{i-1}, x_i, a_{i+1}^n). \tag{*}$$

has a solution $x_i \in H$ for every $a_1, ..., a_{i-1}, a_{i+1}, ..., a_n, b \in H$ and $1 \leq i \leq n$, is called an *n-ary hypergroup.* If f is an n-ary operation, then the equation (*) becomes:

$$b = f(a_1^{i-1}, x_i, a_{i+1}^n). \tag{**}$$

In this case (H, f) is an *n-ary group.*

Definition 2.1. [15] Let (H, f) be an n-ary hypergroup and B be a non-empty subset of H. Then B is an *n-ary sub-hypergroup* of H if the following conditions hold:

(1) B is closed under the n-ary hyperoperation f, i.e., for every $(x_1, ..., x_n) \in B^n$ we have $f(x_1, ..., x_n) \subseteq B$.
(2) Equation $b \in f(b_1^{i-1}, x_i, b_{i+1}^n)$ has a solution $x_i \in B$ for every $b_1, ..., b_{i-1}, b_{i+1}, ..., b_n$, $b \in B$ and $1 \leq i \leq n$.

Definition 2.2. [15] Let (A, f) and (B, g) be two n-ary hypergroups. A *homomorphism* from A to B is a mapping $\varphi : A \to B$ such that

$$\varphi(f(x_1, x_2, \ldots, x_n)) = g(\varphi(x_1), \varphi(x_2), \ldots, \varphi(x_n))$$

holds for all $x_1, x_2, \ldots, x_n \in A$. If φ is injective, then it is called an *embedding.* The map φ is an *isomorphism* if it is bijective and homomorphism. We say that A is *isomorphic* to B and denot by $A \cong B$ if there exists an isomorphism from A to B.

Theorem 2.3. [15] *Let (A, f) and (B, g) be two n-ary hypergroups and $\varphi : A \to B$ a homomorphism. Then*

(1) *If S is an n-ary sub-hypergroup of A, then $\varphi(S)$ is an n-ary sub-hypergroup of B,*
(2) *If K is an n-ary sub-hypergroup of B such that $\varphi^{-1}(K) \neq \emptyset$, then $\varphi^{-1}(K)$ is an n-ary sub-hypergroup of A.*

The concept of fuzzy sets was introduced by Zadeh [34] in 1965. A mapping $\mu : X \to [0,1]$, where X is an arbitrary non-empty set, is called a fuzzy subset of X. The complement of μ, denoted by μ^c, is the fuzzy subset given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$. In 1971, Rosenfeld [29] applied the concept of fuzzy sets to the theory of groups and studied fuzzy subgroups of a group. Davvaz [10] applied fuzzy sets to the theory of algebraic hyperstructures and defined the concept of fuzzy sub-hypergroup (respectively fuzzy H_v-subgroups). We shall use the following abbreviated notation: the sequence $\mu(a_i), \mu(a_{i+1}), ..., \mu(a_j)$ will be denoted by $\mu_{a_i}^{a_j}$.

Definition 2.4. [16] Let(H, f) be an n-ary hypergroup and μ a fuzzy subset of H. Then μ is said to be a *fuzzy n-ary sub-hypergroup* of H if the following axioms hold:

(1) $\min\{\mu_{x_1}^{x_n}\} \leq \bigwedge_{z \in f(x_1^n)}\{\mu(z)\}$ for all $x_1^n \in H$,
(2) for all $a_1^{i-1}, a_{i+1}^n, b \in H$ and $1 \leq i \leq n$, there exist $x_i \in H$ such that $b \in f(a_1^{i-1}, x_i, a_{i+1}^n)$ and $\min\{\mu_{a_1}^{a_{i-1}}, \mu_{a_{i+1}}^{a_n}, \mu(b)\} \leq \mu(x_i)$.

Let (H, f) be an n-ary hypergroup and $B \subseteq H$. Then it is not difficult to see that the characteristic function χ_H is a fuzzy n-ary sub-hypergroup of H if and only if B is an n-ary sub-hypergroup of H.

For any fuzzy subset μ of a non-empty set X and any $t \in (0,1]$, we define the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$. Then μ_t is called a *level subset* of μ and the set $\{x \in X \mid \mu(x) > 0\}$ is called the *support* of μ and is denoted by $\mu_0^>$. A fuzzy subset μ of X of the form

$$\mu(y) = \begin{cases} t \neq 0 & \text{if} \quad y = x, \\ 0 & \text{if} \quad y \neq x. \end{cases}$$

is called a *fuzzy point* with support x and value t and is denoted by x_t [28]. Then the point x is called the *support point* and t is called the *height* of x_t which are denoted by *supp*μ and *hgt*μ, respectively. For any fuzzy subset μ, we define

$$\widetilde{\mu} = \bigcup_{t \in (0,1]} \{x_t \mid \mu(x) \geq t\}.$$

Hence $\widetilde{X}$ will denote the family of all fuzzy points in X. Let $\widetilde{\psi}$ be a mapping of $\widetilde{X}$ into $\widetilde{Y}$ and μ be a fuzzy subset of X. We define $\widetilde{\psi}(\mu) = \bigvee\{\widetilde{\psi}(x_t) \mid x_t \in \widetilde{\mu}\}$. Then $\widetilde{\psi}(\mu)$ is called the image of μ for $\widetilde{\psi}$.

Theorem 2.5. [16] *Let (H, f) be an n-ary hypergroup and μ a fuzzy subset of H. Then μ is a fuzzy n-ary sub-hypergroup of H if and only if for every $t \in (0, 1]$, μ_t $(\neq \emptyset)$ is an n-ary sub-hypergroup of H.*

Example 2.6. Let $\mathbb{Z}$ be an integer numbers with an n-ary hyperoperation f as follows:

$$f : \mathbb{Z}^n \to P^*(\mathbb{Z}), f(x_1^n) = \{m_1x_1 + m_2x_2 + \ldots + m_nx_n \mid m_1, m_2, \ldots, m_n \in \mathbb{Z}\}.$$

Then $(\mathbb{Z}, f)$ is an n-ary hypergroup. If $\mu : \mathbb{Z} \to [0, 1]$ is defined by

$$\mu(x) := \begin{cases} 0.8 & \text{if} \quad x \in \langle 4 \rangle\,, \\ 0.6 & \text{if} \quad x \in \langle 2 \rangle \setminus \langle 4 \rangle\,, \\ 0.4 & \text{if} \quad x \in \mathbb{Z} \setminus \langle 2 \rangle\,. \end{cases}$$

then it is easy to see that μ is a fuzzy n-ary sub-hypergroup of $\mathbb{Z}$.

Now, in the following example, we give a generalization of Theorem 4 of [6].

Example 2.7. Let H be a non-empty set and μ be a fuzzy subset of H. We define an n-ary hyperoperation f as follows:

$$f(x_1, x_2, \ldots, x_n) = \{t \mid \bigwedge_{i=1}^{n} \mu(x_i) \leq \mu(t) \leq \bigvee_{i=1}^{n} \mu(x_i)\}.$$

Then (H, f) is an n-ary hypergroup and μ is a fuzzy n-ary sub-hypergroup of H.

It is easy to see that

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) \subseteq \{t \mid \bigwedge_{i=1}^{2n-1} \mu(x_i) \leq \mu(t) \leq \bigvee_{i=1}^{2n-1} \mu(x_i)\}.$$

Now, we will show that

$$f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1}) \supseteq \{t \mid \bigwedge_{i=1}^{2n-1} \mu(x_i) \leq \mu(t) \leq \bigvee_{i=1}^{2n-1} \mu(x_i)\}.$$

Denote by $A = \{t \mid \bigwedge_{i=1}^{2n-1} \mu(x_i) \leq \mu(t) \leq \bigvee_{i=1}^{2n-1} \mu(x_i)\}$. Suppose that t be an arbitrary element of A. Put

$$\mu(z) = \mu(x_i) \wedge \mu(x_{i+1}) \wedge \ldots \wedge \mu(x_{n+i-1}) \quad \textit{and} \quad \mu(\omega) = \mu(x_i) \vee \mu(x_{i+1}) \vee \ldots \vee \mu(x_{n+i-1}).$$

If $\mu(t) \leq \mu(z)$, then $z \in f(x_i^{n+i-1})$ and

$$\bigwedge_{j=1}^{i-1} \mu(x_j) \wedge \mu(z) \wedge \bigwedge_{j=n+i}^{2n-1} \mu(x_j) \leq \mu(z) \leq \bigvee_{j=1}^{i-1} \mu(x_j) \vee \mu(z) \vee \bigvee_{j=n+1}^{2n-1} \mu(x_j)$$

Hence $t \in f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1})$.

If $\mu(\omega) \leq \mu(t)$, similarly we obtain $t \in f(x_1^{i-1}, f(x_i^{n+i-1}), x_{n+i}^{2n-1})$.

Otherwise, we have $\mu(z) < \mu(t) < \mu(\omega)$. It means that $t \in f(x_i^{n+i-1})$ and

$$\bigwedge_{j=1}^{i-1}\mu(x_j)\wedge\mu(t)\wedge\bigwedge_{j=n+i}^{2n-1}\mu(x_j)\leq\mu(t)\leq\bigvee_{j=1}^{i-1}\mu(x_j)\vee\mu(t)\vee\bigvee_{j=n+1}^{2n-1}\mu(x_j).$$

Therefore, $t\in f(x_1^{i-1},f(x_i^{n+i-1}),x_{n+i}^{2n-1})$. Now, for every $a_1^{i-1},a_{i+1}^n,b\in H$ and $1\leq i\leq n$, we have $b\in f(a_1^{i-1},b,a_{i+1}^n)$. Therefore, (H,f) is an n-ary hypergroup. Using the Definition 2.4 we obtain μ is a fuzzy n-ary sub-hypergroup of H.

3. Fuzzy n-ary Homomorphism

Definition 3.1. Let μ be a fuzzy n-ary sub-hypergroup of H. We define the following n-ary hyperoperation on $\widetilde{\mu}$,

$$\widetilde{f}:\widetilde{\mu}\times\widetilde{\mu}\times\ldots\times\widetilde{\mu}\to P^*(\widetilde{\mu})$$

$$\widetilde{f}((x_1)_{t_1},(x_2)_{t_2},\ldots,(x_n)_{t_n})=\{z_{t_1\wedge t_2\wedge\ldots\wedge t_n}\mid z\in f(x_1^n)\},$$

where $t_1\wedge t_2\wedge\ldots\wedge t_n=\min\{t_1,t_2,\ldots t_n\}$.

Suppose that $(x_1)_{t_1},(x_2)_{t_2},\ldots,(x_n)_{t_n}\in\widetilde{\mu}$, then $\mu(x_1)\geq t_1,\mu(x_2)\geq t_2,\ldots,\mu(x_n)\geq t_n$, and so $\min\{\mu_{x_1}^{x_n}\}\geq t_1\wedge t_2\wedge\ldots\wedge t_n$, which implies that

$$\bigwedge_{z\in f(x_1^n)}\mu(z)\geq\min\{\mu_{x_1}^{x_n}\}\geq t_1\wedge t_2\wedge\ldots\wedge t_n.$$

Therefore for every $z\in f(x_1^n)$, we have $z_{t_1\wedge t_2\wedge\ldots\wedge t_n}\in\widetilde{\mu}$.

Lemma 3.2. *$(\widetilde{\mu},\widetilde{f})$ is an n-ary hypergroup.*

Proof. For every $(x_1)_{t_1},(x_2)_{t_2},\ldots,(x_{2n-1})_{t_{2n-1}}\in\widetilde{\mu}$ we have

$$\begin{aligned}&\widetilde{f}((x_1)_{t_1},\ldots,(x_{i-1})_{t_{i-1}},\widetilde{f}((x_i)_{t_i},\ldots,(x_{n+i-1})_{t_{n+i-1}}),(x_{n+i})_{t_{n+i}},\ldots,(x_{2n-1})_{t_{2n-1}})\\&=\{z_{t_1\wedge t_2\wedge\ldots\wedge t_{2n-1}}\mid z\in f(x_1,x_2,\ldots,x_{i-1},f(x_i,\ldots,x_{n+i-1},x_{n+i},\ldots,x_{2n-1})\}\end{aligned}\quad(1)$$

On the other hand

$$\begin{aligned}&\widetilde{f}((x_1)_{t_1},\ldots,(x_{j-1})_{t_{j-1}},\widetilde{f}((x_j)_{t_j},\ldots,(x_{n+j-1})_{t_{n+j-1}}),(x_{n+j})_{t_{n+j}},\ldots,(x_{2n-1})_{t_{2n-1}})\\&=\{z_{t_1\wedge t_2\wedge\ldots\wedge t_{2n-1}}\mid z\in f(x_1,x_2,\ldots,x_{j-1},f(x_j,\ldots,x_{n+j-1},x_{n+j},\ldots,x_{2n-1}))\}\end{aligned}\quad(2)$$

Since (H,f) is associative, from (1) and (2) we get $(\widetilde{\mu},\widetilde{f})$ is associative, therefore $(\widetilde{\mu},\widetilde{f})$ is an n-ary semihypergroup. Let$(a_1)_{t_1},\ldots,(a_{i-1})_{t_{i-1}},(a_{i+1})_{t_{i+1}},\ldots,(a_n)_{t_n},b_s\in\widetilde{\mu}$. Since μ is a fuzzy n-ary hypergroup, there exists $x_i\in H$ such that $b\in f(a_1^{i-1},x_i,a_{i+1}^n)$ and $\mu(a_1)\wedge\ldots\wedge\mu(a_{i-1})\wedge\mu(a_{i+1})\wedge\ldots\wedge\mu(a_n)\wedge\mu(b)\leq\mu(x_i)$, Hence $t_1\wedge\ldots\wedge t_{i-1}\wedge t_{i+1}\wedge\ldots\wedge t_n\wedge s\leq\mu(x_i)$. Put $t=t_1\wedge\ldots\wedge t_{i-1}\wedge t_{i+1}\wedge\ldots\wedge t_n\wedge s$, we obtain $(x_i)_t\in\widetilde{\mu}$ and $b_t\in\widetilde{f}((a_1)_{t_1},\ldots,(a_{i-1})_{t_{i-1}},(x_i)_t,(a_{i+1})_{t_{i+1}},\ldots,(a_n)_{t_n})$. □

Proposition 3.3. *Let μ be a fuzzy n-ary sub-hypergroup of H, then the set $\mu_0^>$ is an n-ary sub-hypergroup of H.*

Proof. This proposition is an immediate concequence of Theorem 2.5 and Lemma 3.2 in [3]. □

Definition 3.4. Let μ_1 and μ_2 be fuzzy n-ary sub-hypergroups of H_1 and H_2, respectively. Let $\widetilde{\varphi}$ be a mapping from $\widetilde{\mu_1} \to \widetilde{\mu_2}$ such that $supp\widetilde{\varphi}(x_t) = supp\widetilde{\varphi}(x_s)$, for all $x_t, x_s \in \widetilde{\mu_1}$. Then $\widetilde{\varphi}$ is called a *fuzzy n-ary homomorphism* if

$$\widetilde{\varphi}(\widetilde{f}((x_1)_{t_1}, (x_2)_{t_2}, \ldots, (x_n)_{t_n})) = \widetilde{g}(\widetilde{\varphi}((x_1)_{t_1}), \widetilde{\varphi}((x_2)_{t_2}), \ldots, \widetilde{\varphi}((x_n)_{t_n}))$$

for all $(x_1)_{t_1}, (x_2)_{t_2}, \ldots, (x_n)_{t_n} \in \widetilde{\mu_1}$. If $\widetilde{\varphi}$ is injective, then it is called an *embedding*. A mapping $\widetilde{\varphi} : \widetilde{\mu_1} \to \widetilde{\mu_2}$ is called a *fuzzy n-ary isomorphism* if it is bijective and fuzzy n-ary homomorphism. Two fuzzy n-ary sub-hypergroups μ_1 and μ_2 are called *fuzzy n-ary isomorphic*, denoted by $\mu_1 \cong \mu_2$ if there exists a fuzzy n-ary isomorphism from $\widetilde{\mu_1}$ to $\widetilde{\mu_2}$.

Example 3.5. Let $H = \{x, y, z\}$ be a set and f and g be two 3-ary hyperoperations on H as follow:

$$\begin{array}{lll}
f(x,x,x) = \{x,z\} & f(y,x,x) = H & f(z,x,x) = \{x,z\} \\
f(x,x,y) = H & f(y,x,y) = H & f(z,x,y) = H \\
f(x,x,z) = \{x,z\} & f(y,x,z) = H & f(z,x,z) = \{x,z\} \\
f(x,y,x) = H & f(y,y,x) = H & f(z,y,x) = H \\
f(x,y,y) = H & f(y,y,y) = \{y,z\} & f(z,y,y) = \{y,z\} \\
f(x,y,z) = H & f(y,y,z) = \{y,z\} & f(z,y,z) = \{y,z\} \\
f(x,z,x) = \{x,z\} & f(y,z,x) = H & f(z,z,x) = \{x,z\} \\
f(x,z,y) = H & f(y,z,y) = \{y,z\} & f(z,z,y) = \{y,z\} \\
f(x,z,z) = \{x,z\} & f(y,z,z) = \{y,z\} & f(z,z,z) = z.
\end{array}$$

and

$$\begin{array}{lll}
g(x,x,x) = H & g(y,x,x) = H & g(z,x,x) = H \\
g(x,x,y) = H & g(y,x,y) = H & g(z,x,y) = H \\
g(x,x,z) = H & g(y,x,z) = H & g(z,x,z) = H \\
g(x,y,x) = H & g(y,y,x) = H & g(z,y,x) = H \\
g(x,y,y) = H & g(y,y,y) = \{y,z\} & g(z,y,y) = \{y,z\} \\
g(x,y,z) = H & g(y,y,z) = \{y,z\} & g(z,y,z) = \{y,z\} \\
g(x,z,x) = H & g(y,z,x) = H & g(z,z,x) = H \\
g(x,z,y) = H & g(y,z,y) = \{y,z\} & g(z,z,y) = \{y,z\} \\
g(x,z,z) = H & g(y,z,z) = \{y,z\} & g(z,z,z) = z.
\end{array}$$

It is easy to see that (H, f) and (H, g) are 3-ary hypergroups. Let $\mu_1 : H \to [0, 1]$ and $\mu_2 : H \to [0, 1]$ be defined by

$$\mu_1(z) = 1, \quad \mu_1(x) = 0.5, \quad \mu_1(y) = 0 \quad and \quad \mu_2(z) = 1, \quad \mu_2(x) = 0, \quad \mu_2(y) = 0.5.$$

Then μ_1 and μ_2 are fuzzy 3-ary sub-hypergroups of (H, f) and (H, g), respectively. Also, we have $(\mu_1)_0^{>} = \{x, z\}$ and $(\mu_2)_0^{>} = \{y, z\}$. Let us define a fuzzy function $\widetilde{\varphi} : \widetilde{\mu_1} \to \widetilde{\mu_2}$ as follows: $\widetilde{\varphi}(x_t) = y_t$ and $\widetilde{\varphi}(z_t) = z_t$. By routine calculation, we can check that $\widetilde{\varphi}$ is a fuzzy 3-ary homomorphism.

Theorem 3.6. *Let μ_1 and μ_2 be fuzzy n-ary sub-hypergroups of H_1 and H_2, respectively, and let $\widetilde{\varphi}:\widetilde{\mu_1}\to\widetilde{\mu_2}$ be a fuzzy n-ary homomorphism. Then we have*

(i) $hgt\widetilde{\varphi}(x_t)=hgt\widetilde{\varphi}(y_t)$,
(ii) $hgt\widetilde{\varphi}(x_t)\leq hgt\widetilde{\varphi}(x_s)$, *whenever* $t\leq s$.

Proof. i) For every $x_t,y_t\in\widetilde{\mu_1}$. there exists $z\in H_1$ such that $y\in f(x,x,\dots,x,z)$ and $\min\{\mu_1(x),\mu_1(y)\}\leq\mu_1(z)$. Since $\mu_1(x)\geq t,\mu_1(y)\geq t$, we have $\mu(z)\geq t$ which implies $z_t\in\widetilde{\mu_1}$. From $y_t\in\widetilde{f}(x_t,x_t,\dots,x_t,z_t)$, we get $\widetilde{\varphi}(y_t)\in\widetilde{\varphi}(\widetilde{f}(x_t,x_t,\dots,x_t,z_t))$ or $\widetilde{\varphi}(y_t)\in\widetilde{g}(\widetilde{\varphi}(x_t),\widetilde{\varphi}(x_t),\dots,\widetilde{\varphi}(x_t),\widetilde{\varphi}(z_t))$ and so

$hgt\widetilde{\varphi}(y_t)=\min\{hgt\widetilde{\varphi}(x_t),hgt\widetilde{\varphi}(x_t),\dots,hgt\widetilde{\varphi}(x_t),hgt\widetilde{\varphi}(z_t)\}\leq hgt\widetilde{\varphi}(x_t)$ Similarly, we obtain $hgt\widetilde{\varphi}(x_t)\leq hgt\widetilde{\varphi}(y_t)$. Therefore $hgt\widetilde{\varphi}(x_t)=hgt\widetilde{\varphi}(y_t)$.

ii) Suppose $t\leq s$. If $x_t\in\widetilde{\mu_1}$, there exists $y\in H_1$ such that $x\in f(x,x,\dots,x,y)$ and $\mu_1(x)\leq\mu_1(y)$, so $y_t\in\widetilde{\mu_1}$. From $x_t\in\widetilde{f}(x_t,x_t,\dots,x_t,y_t)$ we have $\widetilde{\varphi}(x_t)\in\widetilde{\varphi}(\widetilde{f}(x_t,x_t,\dots,x_t,y_t))$ which implies that $\widetilde{\varphi}(x_t)\in\widetilde{\varphi}(\widetilde{f}(x_s,x_s,\dots,x_s,y_t))$ or $\widetilde{\varphi}(x_t)\in\widetilde{g}(\widetilde{\varphi}(x_s),\widetilde{\varphi}(x_s),\dots,\widetilde{\varphi}(x_s),\widetilde{\varphi}(y_t))$. Therefore $hgt\widetilde{\varphi}(x_t)=\min\{hgt\widetilde{\varphi}(x_s),hgt\widetilde{\varphi}(y_t)\}\leq hgt\widetilde{\varphi}(x_s)$. □

Theorem 3.7. *Let μ_1 and μ_2 be a fuzzy n-ary sub-hypergroup of H_1 and H_2, respectively. A mapping $\widetilde{\varphi}:\widetilde{\mu_1}\to\widetilde{\mu_2}$ is a fuzzy n-ary homomorphism if and only if there exists an n-ary homomorphism $\varphi:(\mu_1)_0^{>}\to(\mu_2)_0^{>}$ and an increasing function $\alpha:(0,1]\to(0,1]$ such that*

$$\widetilde{\varphi}(x_t)=[\varphi(x)]_{\alpha(t)}\quad \textit{for } x_t\in\widetilde{\mu_1}.$$

Proof. Assume that $\widetilde{\varphi}:\widetilde{\mu_1}\to\widetilde{\mu_2}$ is a fuzzy n-ary homomorphism. Similar to the proof of Theorem 3.6 in [12], we define a mapping $\varphi:(\mu_1)_0^{>}\to(\mu_2)_0^{>}$ and a function $\alpha:(0,1]\to(0,1]$ as follows:

$$\begin{aligned}&\varphi(x)=supp\widetilde{\varphi}(x_{\mu_1(x)}),\text{ for all } x\in(\mu_1)_0^{>},\\&\alpha(t)=hgt\widetilde{\varphi}(x_t),\text{ for all } t\in(0,1].\end{aligned}$$

Since $\widetilde{\varphi}$ is a fuzzy n-ary homomorphism, then $supp\widetilde{\varphi}(x_t)=supp\widetilde{\varphi}(x_{\mu_1(x)})$ and so $supp\widetilde{\varphi}(x_t)=\varphi(x)$, which implies that $\widetilde{\varphi}(x_t)=[\varphi(x)]_{\alpha(t)}$. By definition of φ and Theorem 3.6, it is easy to see that α is increasing. Therefore it remains to show that φ is an n-ary homomorphism. For every $a_1^n\in(\mu_1)_0^{>}$, we put $\mu_1(a_1)=t_1,\mu_2(a_2)=t_2,\dots,\mu_n(a_n)=t_n$. Then

$$\begin{aligned}[\varphi(f(a_1,a_2,\dots,a_n))]_{\alpha(t_1\wedge t_2\wedge\ldots\wedge t_n)}&=\bigcup_{z\in f(a_1^n)}[\varphi(z)]_{\alpha(t_1\wedge t_2\wedge\ldots\wedge t_n)}\\&=\bigcup_{z\in f(a_1^n)}\widetilde{\varphi}(z_{t_1\wedge t_2\wedge\ldots\wedge t_n})\\&=\widetilde{\varphi}(\bigcup_{z\in f(a_1^n)}z_{t_1\wedge t_2\wedge\ldots\wedge t_n}\\&=\widetilde{\varphi}(\widetilde{f}((a_1)_{t_1},(a_2)_{t_2},\dots,(a_n)_{t_n}))\\&=\widetilde{g}(\widetilde{\varphi}(a_1)_{t_1},\widetilde{\varphi}(a_2)_{t_2},\dots,\widetilde{\varphi}(a_n)_{t_n})\\&=\widetilde{g}([\varphi(a_1)]_{\alpha(t_1)},[\varphi(a_2)]_{\alpha(t_2)},\dots,[\varphi(a_n)]_{\alpha(t_n)})\\&=\bigcup_{w\in g(\varphi(a_1),\varphi(a_2),\dots,\varphi(a_n))}w_{\alpha(t_1)\wedge\alpha(t_2)\wedge\ldots\wedge\alpha(t_n)}\\&=[g(\varphi(a_1),\varphi(a_2),\dots,\varphi(a_n))]_{\alpha(t_1)\wedge\alpha(t_2)\wedge\ldots\wedge\alpha(t_n)}.\end{aligned}$$

Therefore $\varphi(f(a_1, a_2, \ldots, a_n)) = g(\varphi(a_1), \varphi(a_2), \ldots, \varphi(a_n))$, i,e, φ is an n-ary homomorphism.

Conversely, we consider a mapping $\varphi : (\mu_1)_0^{>} \to (\mu_2)_0^{>}$ such that $\widetilde{\varphi}(x_t) = [\varphi(x)]_{\alpha(t)}$. It is enough to show that $\widetilde{\varphi}$ is a fuzzy n-ary homomorphism. For every $(a_1)_{t_1}, (a_2)_{t_2}, \ldots, (a_n)_{t_n} \in \widetilde{\mu_1}$ $(t = t_1 \wedge t_2 \wedge \ldots \wedge t_n)$, we have

$$\begin{aligned}
\widetilde{\varphi}(\widetilde{f}((a_1)_{t_1}, (a_2)_{t_2}, \ldots, (a_n)_{t_n})) &= \bigcup_{z \in f(a_1^n)} \widetilde{\varphi}(z_t) \\
&= \bigcup_{z \in f(a_1^n)} [\varphi(z)]_{\alpha(t)} \\
&= [\varphi(f(a_1^n)]_{\alpha(t)} = [g(\varphi(a_1), \varphi(a_2), \ldots, \varphi(a_n)]_{\alpha(t)} \\
&= [g(\varphi(a_1), \varphi(a_2), \ldots, \varphi(a_n)]_{\alpha(t_1) \wedge \alpha(t_2) \wedge \ldots \alpha(t_n)} \\
&= \widetilde{g}(\varphi(a_1)_{\alpha(t_1)}, \varphi(a_2)_{\alpha(t_2)}, \ldots, \varphi(a_n)_{\alpha(t_1)})] \\
&= \widetilde{g}(\varphi(a_1)_{t_1}, \varphi(a_2)_{t_2}, \ldots, \varphi(a_n)_{t_n}).
\end{aligned}$$

□

Let (H_1, f) and (H_2, g) be two n-ary hypergroups and let φ be a homomorphism H_1 to H_2. We can define a mapping $\widetilde{\varphi} : H_1 \to H_2$ as follows: $\widetilde{\varphi}(x_t) = [\varphi(x)]_t$ Obviously $\widetilde{\varphi}$ is a fuzzy n-ary homomorphism from H_1 to H_2 where $\alpha(t) = t$, for all $t \in (0, 1]$. Therefore the concept of fuzzy n-ary homomorphism between two n-ary hypergroups can be seen as an extension of the concept of homomorphism between two n-ary hypergroups.

Let $\varphi : X \to Y$ and let $\alpha : (0, 1] \to (0, 1]$ be an increasing mapping. We define the mapping $\varphi_\alpha : \widetilde{X} \to \widetilde{Y}$ by $\varphi_\alpha(x_t) = [\varphi(x)]_{\alpha(t)}$. For every fuzzy subset μ of X we have

$$\varphi_\alpha(\mu)(y) = \bigvee_{x \in \varphi^{-1}(y)} \alpha(\mu(x)).$$

Theorem 3.8. *Let α be a bijective and $\varphi : H_1 \to H_2$ be a surjective n-ary homomorphism and let μ be a fuzzy n-ary sub-hypergruop of H_1. Then $\varphi_\alpha(\mu)$ is a fuzzy n-ary sub-hypergruop of H_2.*

Proof. Let μ be a fuzzy n-ary sub-hypergroup of H_1. By Theorem 2.5, for every t, $0 \leq t \leq 1$, the level subset $\mu_t (\neq \emptyset)$ is an n-ary sub-hypergroup of H_1 and by Theorem 2.3 (1), $\varphi(\mu_{\alpha^{-1}(t)})$ is an n-ary sub-hypergroup of H_2. Now it is enough to show that

$$\varphi(\mu_{\alpha^{-1}(t)}) = (\varphi_\alpha(\mu))_t.$$

For every $y \in (\varphi_\alpha(\mu))_t$, we have $\varphi_\alpha(\mu)(y) \geq t$ which implies that $\bigvee_{x \in \varphi^{-1}(y)} \{\alpha(\mu(x))\} \geq t$. Therefore there exists $x_0 \in \varphi^{-1}(y)$ such that $\alpha(\mu)(x_0) \geq t$ which implies that $\mu(x_0) \geq \alpha^{-1}(t)$ or $x_0 \in \mu_{\alpha^{-1}(t)}$ and so $\varphi(x_0) \in \varphi(\mu_{\alpha^{-1}(t)})$ implies that $y \in \varphi(\mu_{\alpha^{-1}(t)})$. Now, for every $y \in \varphi(\mu_{\alpha^{-1}(t)})$ there exists $x \in \mu_{\alpha^{-1}(t)}$ such that $y = \varphi(x)$. Since $x \in \mu_{\alpha^{-1}(t)}$ we have $\mu(x) \geq \alpha^{-1}(t)$ or $\alpha(\mu(x)) \geq t$ and so $\bigvee_{x \in \varphi^{-1}(y)} \{\alpha(\mu(x))\} \geq t$ which implies that $\varphi_\alpha(\mu)(y) \geq t$, therefore $y \in (\varphi_\alpha(\mu))_t$. □

4. **On Fundamental n-ary Groups**

Let (H, f) be an n-semihypergroup. We denote

$f_{(1)} = \{f(a_1^n) \mid a_i \in H,\ \forall i \in \{1, 2, \ldots, n\}\}$,
$f_{(2)} = \{f(f(x_1^n), a_2^n) \mid x_i \in H,\ a_j \in H,\ \forall i \in \{1, \ldots, n\},\ \ \forall j \in \{2, \ldots, n\}\}$,
$f_{(3)} = \{f(f(f(z_1^n), x_2^n), a_2^n) \mid z_i \in H,\ x_j \in H,\ a_j \in H,\ \ \forall i \in \{1, 2, \ldots, n\},\ \forall j \in \{2, \ldots, n\}\}$,

and so on. Denote $U = \bigvee_{k\in\mathbb{N}^*} f_{(k)}$. Now, we can define the relation β, which is an important binary relation on an n-ary hypergroup (H, f), see [14]. We have $\beta = \bigvee_{k\geq 1} \beta_k$ and for x, y of H, define:

$$x\beta_k y \ \Leftrightarrow \exists u \in f_{(k)}, \text{such that } \ \{x, y\} \subseteq u.$$

Clearly, β is reflexive and symmetric. Let β^* be the transitive closure of β. Then β^* is the smallest equivalence relation such that the quotient $(H/\beta^*, f|_{\beta^*})$ is an n-ary semigroup, where H/β^* is the quotient set and $f|_{\beta^*}(\beta^*(a_1), \ldots, \beta^*(a_n)) = \beta^*(a)$, for any $a \in f(a_1^n)$. Leoreanu-Fotea and Davvaz in [25] proved that if (H, f) is an n-ary hypergroup, then β is transitive. β^* is called *fundamental equivalence relation.* The equivalence relation β^* first was introduced on hypergroups by Koskas [23] and studied mainly by Corsini [5] concerning hypergroups, Vougiouklis [33] concerning hyperrings, Davvaz [14].

Definition 4.1. Let β be an n-ary hypergroup and μ be a fuzzy subset of H and $\alpha : (0, 1] \to (0, 1]$ be an increasing mapping. The fuzzy subset $(\mu_{\beta^*})_\alpha$ on H/β^* defined as follows:

$$(\mu_{\beta^*})_\alpha : H/\beta^* \to [0, 1], \qquad (\mu_{\beta^*})_\alpha(\beta^*(x)) = \bigvee_{a\in\beta^*(x)} \{\alpha(\mu(a))\}$$

In particular, if α is an identity mapping, we denote $(\mu_{\beta^*})_\alpha = \mu_{\beta^*}$ [16].

Theorem 4.2. [16] Let (H, f) be an n-ary hypergroup and μ be a fuzzy n-ary sub-hypergroup of H. Then $\mu_{\beta^*_\alpha}$ is a fuzzy n-ary sub-group of H/β^*.

Theorem 4.3. Let (H, f) be an n-ary hypergroup and μ be a fuzzy n-ary sub-hypergroup of H and let $\alpha : [0, 1] \to [0, 1]$ be a mapping such that $\alpha(0) = 0$ and let α be strictly increasing on $(0, 1]$. Then

$$(\mu_{\beta^*})_0^> = ((\mu_{\beta^*})_\alpha)_0^>.$$

Proof. Let $\beta^*(x) \in (\mu_{\beta^*})_0^>$. Then $\mu_{\beta^*}(\beta^*(x)) > 0$ and so $\bigvee_{a\in\beta^*(x)}\{\mu(a)\} > 0$. Since α is strictly incresing, we obtain $\bigvee_{a\in\beta^*(x)}\{\alpha(\mu(a))\} > 0$ which implies that $(\mu_{\beta^*})_\alpha(\beta^*(x)) > 0$. Hence $\beta^*(x) \in ((\mu_{\beta^*})_\alpha)_0^>$. Therefore $(\mu_{\beta^*})_0^> \subseteq ((\mu_{\beta^*})_\alpha)_0^>$.

Conversely, assume that $\beta^*(x) \in ((\mu_{\beta^*})_\alpha)_0^>$. Then $\bigvee_{a\in\beta^*(x)}\{\alpha(\mu(a))\} > 0$ which implies that there exists $a_0 \in \beta^*(x)$ such that $\alpha(\mu(a_0)) > 0$ impliying $\mu(a_0) > 0$ and so $\bigvee_{a\in\beta^*(x)}\{\mu(a)\} > 0$. Hence $\beta^*(x) \in (\mu_{\beta^*})_0^>$. Therefore $((\mu_{\beta^*})_\alpha)_0^> \subseteq (\mu_{\beta^*})_0^>$. $\square$

Theorem 4.4. Let μ_1 and μ_2 be fuzzy n-ary sub-hypergroups of H_1 and H_2 respectively and let $\varphi_\alpha : \widetilde{\mu_1} \to \widetilde{\mu_2}$ be a fuzzy n-ary homomorphism where α is the same mapping as in Theorem 4.3 and we consider the mapping $\varphi : (\mu_1)_0^> \to (\mu_2)_0^>$. Then

$$\varphi((\mu_1)_0^>) = (\varphi_\alpha(\mu_1))_0^>$$

Proof. The proof is straightforward and omitted. □

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