FUZZY SOFT SET THEORY AND ITS APPLICATIONS

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Abstract. In this work, we define a fuzzy soft set theory and its related properties. We then define fuzzy soft aggregation operator that allows constructing more efficient decision making method. Finally, we give an example which shows that the method can be successfully applied to many problems that contain uncertainties.

1. Introduction

Molodtsov [23] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. Molodtsov applied this theory to several directions [23, 24, 25], and then formulated the notions of soft number, soft derivative, soft integral, etc. in [26]. The soft set theory has been applied to many different fields with great success. Maji *et al.* [20] worked on theoretical study of soft sets in detail, and [19] presented an application of soft set in the decision making problem using the reduction of rough sets [29]. Chen *et al.* [6] proposed parametrization reduction of soft sets, and then Kong *et al.* [16] presented the normal parametrization reduction of soft sets.

Recently, many scholars study the properties and applications on the soft set theory. Xiao *et al.* [34] studied synthetically evaluating method for business competitive capacity and also Xiao *et al.* [35] gave a recognition for soft information based on the theory of soft sets. Pei and Miao [30] showed that the soft sets are a class of special information systems. Mushrif *et al.* [27] presented a new algorithm based on the notions of soft set theory for classification of the natural textures. Kovkov *et al.* [17] considered the optimization problems in the framework of the theory of soft sets which is directed to formalization of the concept of approximate object description. Zou and Xiao [42] presented data analysis approaches of soft sets under incomplete information. Majumdar and Samanta [21] studied the similarity measure of soft sets. Ali *et al.* [1] introduced the analysis of several operations on soft sets.

The algebraic structure of soft set theory dealing with uncertainties has also been studied in more detail. Aktaş and Çağman [2] introduced a definition of soft groups, and derived their basic properties. Park *et al.* [28] worked on the notion of soft WS-algebras, soft subalgebras and soft deductive systems. Jun [9] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. Jun

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and Park [10] presented the notion of soft ideals, idealistic soft and idealistic soft BCK/BCI-algebras. Jun *et al.* [11] applied soft set theory to commutative ideals in BCK-algebras. Jun and Park [12] make application of soft sets in Hilbert Algebras. Jun *et al.* [13, 14] presented Pseudo d-algebras and applied soft set theory to ideals in d-algebras. Sun *et al.* [33] gave the definition of soft modules. In [8], Feng *et al.* defined the notion of a soft semiring and its algebraic properties. Zou and Chen [40] worked on soft set theory and parameters reduction based on relational algebra.

Maji *et al.* [18] presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets [39]. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji [31] gave some applications of fs-sets. Som [32] defined soft relation and fuzzy soft relation on the theory of soft sets. Mukherjee and Chakraborty [22] worked on intuitionistic fuzzy soft relations. Aktaş and Çağman [2] compared soft sets with the related concepts of fuzzy sets and rough sets. Yang et al. [37] defined the operations on fuzzy soft sets which are based on three fuzzy logic operators: negation, triangular norm and triangular conorm. Zou and Xiao [42] introduced the soft set and fuzzy soft set into the incomplete environment. Xiao *et al.* [36] used forecasting accuracy as the criterion of fuzzy membership function, and purposed a combined forecasting approached based on fs-sets. Yang *et al.* [38] presented the combination of interval-valued fuzzy set and soft set. Kong *et al.* [15] defined the normal parameter reduction in the fs-sets, and showed that Roy and Maji's [31] algorithm is not convenient in general cases.

The operations of the fs-sets and soft sets defined by Maji *et al.* [18, 20] are used in all the works mentioned above. But, Chen *et al.* [6], Pei and Miao [30], Kong *et al.* [15] and Ali *et al.* [1] pointed out that these works have some weak points. Therefore, to develop the theory, Çağman and Enginoğlu [3] redefined operations of the soft sets which are more functional for improving several new results. By using these new operations, Çağman and Enginoğlu [4] presented a soft matrix theory. Çağman *et al.* [5] defined a fuzzy parameterized soft set theory and its decision making method.

In this work, we redefine the fs-sets and their operations, and then define fuzzy soft aggregation operator which allows constructing more efficient decision processes. We finally give an example which shows that the method can be successfully applied to many problems containing uncertainties.

2. **Preliminaries**

In this section, we present the basic definitions of soft set theory [23] and fuzzy set theory [39] that are useful for subsequent discussions. These definitions and more detailed explanations related to the soft sets and fuzzy sets can be found in [3, 20, 23] and [7, 41], respectively.

Throughout this work, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subseteq E$.

Definition 2.1. A soft set F_A over U is a set defined by a function f_A representing a mapping

$$f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \notin A$$

Here, f_A is called approximate function of the soft set F_A, and the value $f_A(x)$ is a set called *x-element* of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary, empty, or have nonempty intersection. Thus a soft set over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}.$$

Note that the set of all soft sets over U will be denoted by $S(U)$.

Example 2.2. Let $U = \{u_l, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $A = \{x_1, x_2, x_4\} \subseteq E$, $f_A(x_1) = \{u_2, u_4\}$, $f_A(x_2) = U$ and $f_A(x_4) = \{u_1, u_3, u_5\}$, then the soft set F_A is written by

$$F_A = \{(x_1, \{u_2, u_4\}), (x_2, U), (x_4, \{u_1, u_3, u_5\})\}.$$

Definition 2.3. Let U be a universe. A fuzzy set X over U is a set defined by a function μ_X representing a mapping

$$\mu_X : U \to [0, 1]$$

μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follows:

$$X = \{(\mu_X(u)/u) : u \in U, \mu_X(x) \in [0, 1]\}.$$

Note that the set of all the fuzzy sets over U will be denoted by $F(U)$.

3. **Fuzzy Soft Sets**

In this section, we have defined fs-sets and their operations. In the soft sets, given in Section 2, the parameter sets and the approximate functions are crisp. But in the fs-sets, while the parameters sets are crisp, the approximate functions are fuzzy subsets of U. From now on, we will use Γ_A, Γ_B, Γ_C,..., etc. for fs-sets and γ_A, γ_B, γ_C,..., etc. for their fuzzy approximate functions, respectively.

Definition 3.1. An fs-set Γ_A over U is a set defined by a function γ_A representing a mapping

$$\gamma_A : E \to F(U) \text{ such that } \gamma_A(x) = \emptyset \text{ if } x \notin A.$$

Here, γ_A is called fuzzy approximate function of the fs-set Γ_A, and the value $\gamma_A(x)$ is a set called x-element of the fs-set for all $x \in E$. Thus, an fs-set Γ_A over U can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}.$$

Note that the set of all fs-sets over U will be denoted by $FS(U)$.

Example 3.2. Let $U = \{u_l, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $A = \{x_1, x_2, x_4\} \subseteq E$, $\gamma_A(x_1) = \{0.9/u_2, 0.5/u_4\}$, $\gamma_A(x_2) = U$, and $\gamma_A(x_4) = \{0.2/u_1, 0.4/u_3, 0.8/u_5\}$, then the soft set F_A is written by

$$F_A = \{(x_1, \{0.9/u_2, 0.5/u_4\}), (x_2, U), (x_4, \{0.2/u_1, 0.4/u_3, 0.8/u_5\})\}.$$

Definition 3.3. Let $\Gamma_A \in FS(U)$. If $\gamma_A(x) = \emptyset$ for all $x \in E$, then Γ_A is called an empty fs-set, denoted by Γ_Φ.

Definition 3.4. Let $\Gamma_A \in FS(U)$. If $\gamma_A(x) = U$ for all $x \in A$, then Γ_A is called A-universal fs-set, denoted by $\Gamma_{\tilde{A}}$.

If $A = E$, then the A-universal fs-set is called universal fs-set, denoted by $\Gamma_{\tilde{E}}$.

Example 3.5. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters.

If $A = \{x_2, x_3, x_4\}$, $\gamma_A(x_2) = \{0.5/u_2, 0.9/u_4\}$, $\gamma_A(x_3) = \emptyset$ and $\gamma_A(x_4) = U$, then the fs-set Γ_A is written by $\Gamma_A = \{(x_2, \{0.5/u_2, 0.9/u_4\}), (x_4, U)\}$.

If $B = \{x_1, x_3\}$, and $\gamma_B(x_1) = \emptyset$, $\gamma_B(x_3) = \emptyset$, then the fs-set Γ_B is an empty fs-set, i.e., $\Gamma_B = \Gamma_\Phi$.

If $C = \{x_1, x_2\}$, $\gamma_C(x_1) = U$, and $\gamma_C(x_2) = U$, then the fs-set Γ_C is a C-universal fs-set, i.e., $\Gamma_C = \Gamma_{\tilde{C}}$.

If $D = E$, and $\gamma_D(x_i) = U$ for all $x_i \in E$, where $i = 1, 2, 3, 4$, then the fs-set Γ_D is a universal fs-set, i.e., $\Gamma_D = \Gamma_{\tilde{E}}$.

Definition 3.6. Let $\Gamma_A, \Gamma_B \in FS(U)$. Then, Γ_A is an fs-subset of Γ_B, denoted by $\Gamma_A \tilde{\subseteq} \Gamma_B$, if $\gamma_A(x) \subseteq \gamma_B(x)$ for all $x \in E$.

Remark 3.7. $\Gamma_A \tilde{\subseteq} \Gamma_B$ does not imply that every element of Γ_A is an element of Γ_B as in the definition of the classical subset.

For example, assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set of objects and $E = \{x_1, x_2, x_3\}$ is a set of all the parameters. If $A = \{x_1\}$, $B = \{x_1, x_3\}$, $\Gamma_A = \{(x_1, \{0.2/u_2\})\}$ and $\Gamma_B = \{(x_1, \{0.9/u_2, 0.3/u_3, 0.5/u_4\}),$
$(x_3, \{0.2/u_1, 0.7/u_5\})\}$, then for all $x \in E$, $\gamma_A(x) \subseteq \gamma_B(x)$ is valid. Hence, $\Gamma_A \tilde{\subseteq} \Gamma_B$. It is clear that $(x_1, \{0.2/u_2\}) \in \Gamma_A$, but $(x_1, \{0.2/u_2\}) \notin \Gamma_B$.

Proposition 3.8. *Let $\Gamma_A, \Gamma_B \in FS(U)$. Then,*

(1) $\Gamma_A \tilde{\subseteq} \Gamma_{\tilde{E}}$
(2) $\Gamma_\Phi \tilde{\subseteq} \Gamma_A$
(3) $\Gamma_A \tilde{\subseteq} \Gamma_A$
(4) $\Gamma_A \tilde{\subseteq} \Gamma_B$ *and* $\Gamma_B \tilde{\subseteq} \Gamma_C \Rightarrow \Gamma_A \tilde{\subseteq} \Gamma_C$

Proof. They can be proved easily by using the fuzzy approximate function of the fs-sets. □

Definition 3.9. Let $\Gamma_A, \Gamma_B \in FS(U)$. Then, Γ_A and Γ_B are fs-equal, written as $\Gamma_A = \Gamma_B$, if and only if $\gamma_A(x) = \gamma_B(x)$ for all $x \in E$.

Proposition 3.10. *Let $\Gamma_A, \Gamma_B, \Gamma_C \in FS(U)$. Then,*

(1) $\Gamma_A = \Gamma_B$ *and* $\Gamma_B = \Gamma_C \Rightarrow \Gamma_A = \Gamma_C$
(2) $\Gamma_A \tilde{\subseteq} \Gamma_B$ *and* $\Gamma_B \tilde{\subseteq} \Gamma_A \Leftrightarrow \Gamma_A = \Gamma_B$

Proof. The proofs are trivial. □

Definition 3.11. Let $\Gamma_A \in FS(U)$. Then, the complement $\Gamma_A^{\tilde{c}}$ of Γ_A is an fs-set such that

$$\gamma_{A^{\tilde{c}}}(x) = \gamma_A^c(x), \text{ for all } x \in E,$$

where $\gamma_A^c(x)$ is complement of the set $\gamma_A(x)$.

Proposition 3.12. *Let* $\Gamma_A \in FS(U)$. *Then,*
(1) $(\Gamma_A^{\tilde{c}})^{\tilde{c}} = \Gamma_A$
(2) $\Gamma_\Phi^{\tilde{c}} = \Gamma_{\tilde{E}}$

Proof. By using the fuzzy approximate functions of the fs-sets, the proofs are straightforward. □

Definition 3.13. Let $\Gamma_A, \Gamma_B \in FS(U)$. Then, the union of Γ_A and Γ_B, denoted by $\Gamma_A \tilde{\cup} \Gamma_B$, is defined by its fuzzy approximate function

$$\gamma_{A\tilde{\cup}B}(x) = \gamma_A(x) \cup \gamma_B(x) \quad \text{for all } x \in E.$$

Proposition 3.14. *Let* $\Gamma_A, \Gamma_B, \Gamma_C \in FS(U)$. *Then,*
(1) $\Gamma_A \tilde{\cup} \Gamma_A = \Gamma_A$
(2) $\Gamma_A \tilde{\cup} \Gamma_\Phi = \Gamma_A$
(3) $\Gamma_A \tilde{\cup} \Gamma_{\tilde{E}} = \Gamma_{\tilde{E}}$
(4) $\Gamma_A \tilde{\cup} \Gamma_B = \Gamma_B \tilde{\cup} \Gamma_A$
(5) $(\Gamma_A \tilde{\cup} \Gamma_B) \tilde{\cup} \Gamma_C = \Gamma_A \tilde{\cup} (\Gamma_B \tilde{\cup} \Gamma_C)$

Proof. The proofs can be easily obtained from Definition 3.13. □

Definition 3.15. Let $\Gamma_A, \Gamma_B \in FS(U)$. Then, the intersection of Γ_A and Γ_B, denoted by $\Gamma_A \tilde{\cap} \Gamma_B$, is defined by its fuzzy approximate function

$$\gamma_{A\tilde{\cap}B}(x) = \gamma_A(x) \cap \gamma_B(x) \quad \text{for all } x \in E.$$

Proposition 3.16. *Let* $\Gamma_A, \Gamma_B, \Gamma_C \in FS(U)$. *Then,*
(1) $\Gamma_A \tilde{\cap} \Gamma_A = \Gamma_A$
(2) $\Gamma_A \tilde{\cap} \Gamma_\Phi = \Gamma_\Phi$
(3) $\Gamma_A \tilde{\cap} \Gamma_{\tilde{E}} = \Gamma_A$
(4) $\Gamma_A \tilde{\cap} \Gamma_B = \Gamma_B \tilde{\cap} \Gamma_A$
(5) $(\Gamma_A \tilde{\cap} \Gamma_B) \tilde{\cap} \Gamma_C = \Gamma_A \tilde{\cap} (\Gamma_B \tilde{\cap} \Gamma_C)$

Proof. The proofs can be easily obtained from Definition 3.15. □

Remark 3.17. Let $\Gamma_A \in FS(U)$. If $\Gamma_A \neq \Gamma_\Phi$ and $\Gamma_A \neq \Gamma_{\tilde{E}}$, then $\Gamma_A \tilde{\cup} \Gamma_A^{\tilde{c}} \neq \Gamma_{\tilde{E}}$ and $\Gamma_A \tilde{\cap} \Gamma_A^{\tilde{c}} \neq \Gamma_\Phi$.

Proposition 3.18. *Let* $\Gamma_A, \Gamma_B \in FS(U)$. *Then, De Morgan's laws are valid as follows:*
(1) $(\Gamma_A \tilde{\cup} \Gamma_B)^{\tilde{c}} = \Gamma_A^{\tilde{c}} \tilde{\cap} \Gamma_B^{\tilde{c}}$

(2) $(\Gamma_A \tilde{\cap} \Gamma_B)^{\tilde{c}} = \Gamma_A^{\tilde{c}} \tilde{\cup} \Gamma_B^{\tilde{c}}$.

Proof. The proofs can be obtained by using the respective approximate functions. For all $x \in E$,

$$\begin{aligned}(1): \ \gamma_{(A\tilde{\cup}B)^{\tilde{c}}}(x) &= \gamma^c_{A\tilde{\cup}B}(x)\\ &= (\gamma_A(x) \cup \gamma_B(x))^c\\ &= (\gamma_A(x))^c \cap (\gamma_B(x))^c\\ &= \gamma^c_A(x) \cap \gamma^c_B(x)\\ &= \gamma_{A^{\tilde{c}}}(x) \cap \gamma_{B^{\tilde{c}}}(x)\\ &= \gamma_{A^{\tilde{c}}\tilde{\cap}B^{\tilde{c}}}(x)\end{aligned}$$ □

The proof of (2) is similar.

Proposition 3.19. *Let* $\Gamma_A, \Gamma_B, \Gamma_C \in FS(U)$. *Then,*

(1) $\Gamma_A \tilde{\cup} (\Gamma_B \tilde{\cap} \Gamma_C) = (\Gamma_A \tilde{\cup} \Gamma_B) \tilde{\cap} (\Gamma_A \tilde{\cup} \Gamma_C)$
(2) $\Gamma_A \tilde{\cap} (\Gamma_B \tilde{\cup} \Gamma_C) = (\Gamma_A \tilde{\cap} \Gamma_B) \tilde{\cup} (\Gamma_A \tilde{\cap} \Gamma_C)$

Proof. For all $x \in E$,

$$\begin{aligned}(1): \ \gamma_{A\tilde{\cup}(B\tilde{\cap}C)}(x) &= \gamma_A(x) \cup \gamma_{B\tilde{\cap}C}(x)\\ &= \gamma_A(x) \cup (\gamma_B(x) \cap \gamma_C(x))\\ &= (\gamma_A(x) \cup \gamma_B(x)) \cap (\gamma_A(x) \cup \gamma_C(x))\ .\\ &= \gamma_{A\tilde{\cup}B}(x) \cap \gamma_{A\tilde{\cup}C}(x)\\ &= \gamma_{(A\tilde{\cup}B)\tilde{\cap}(A\tilde{\cup}C)}(x).\end{aligned}$$ □

Likewise, the proof of (2) can be made in a similar way.

We note that the binary fs-sets operations, $\tilde{\cap}$ and $\tilde{\cup}$ used in the subscripts of fuzzy approximate functions, are not classical set operations. They indicate that $\gamma_{A\tilde{\cap}B}$ and $\gamma_{A\tilde{\cup}B}$ are the fuzzy approximate function of $\Gamma_A \tilde{\cap} \Gamma_B$ and $\Gamma_A \tilde{\cup} \Gamma_B$, respectively.

4. fs-aggregation

In this section, we define an fs-aggregation operator that produces an aggregate fuzzy set from an fs-set and its cardinal set. The approximate functions of an fs-set are fuzzy. An fs-aggregation operator on the fuzzy sets is an operation by which several approximate functions of an fs-set are combined to produce a single fuzzy set which is the aggregate fuzzy set of the fs-set. Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set.

Definition 4.1. Let $\Gamma_A \in FS(U)$. Assume that $U = \{u_1, u_2, ..., u_m\}$, $E = \{x_1, x_2, ..., x_n\}$ and $A \subseteq E$, then the Γ_A can be presented by the following table,

Γ_A	x_1	x_2	...	x_n
u_1	$\mu_{\gamma_{A(x_1)}}(u_1)$	$\mu_{\gamma_{A(x_2)}}(u_1)$	...	$\mu_{\gamma_{A(x_n)}}(u_1)$
u_2	$\mu_{\gamma_{A(x_1)}}(u_2)$	$\mu_{\gamma_{A(x_2)}}(u_2)$	...	$\mu_{\gamma_{A(x_n)}}(u_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\mu_{\gamma_{A(x_1)}}(u_m)$	$\mu_{\gamma_{A(x_2)}}(u_m)$	...	$\mu_{\gamma_{A(x_n)}}(u_m)$

where $\mu_{\gamma_{A(x)}}$ is the membership function of γ_A.

If $a_{ij} = \mu_{\gamma_{A(x_j)}}(u_i)$, for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$, then the fs-set Γ_A is uniquely characterized by a matrix,

$$[a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

is called an $m \times n$ fs-matrix of the fs-set Γ_A over U.

Definition 4.2. Let $\Gamma_A \in FS(U)$. Then, the cardinal set of Γ_A, denoted by $c\Gamma_A$ and defined by

$$c\Gamma_A = \{\mu_{c\Gamma_A}(x)/x : x \in E\},$$

is a fuzzy set over E. The membership function $\mu_{c\Gamma_A}$ of $c\Gamma_A$ is defined by

$$\mu_{c\Gamma_A} : E \to [0, 1], \quad \mu_{c\Gamma_A}(x) = \frac{|\gamma_A(x)|}{|U|}$$

where $|U|$ is the cardinality of universe U, and $|\gamma_A(x)|$ is the scalar cardinality of fuzzy set $\gamma_A(x)$.

Note that the set of all cardinal sets of the fs-sets over U will be denoted by $cFS(U)$. It is clear that $cFS(U) \subseteq F(E)$.

Definition 4.3. Let $\Gamma_A \in FS(U)$ and $c\Gamma_A \in cFS(U)$. Assume that $E = \{x_1, x_2, ..., x_n\}$ and $A \subseteq E$, then $c\Gamma_A$ can be presented by the following table

E	x_1	x_2	...	x_n
$\mu_{c\Gamma_A}$	$\mu_{c\Gamma_A}(x_1)$	$\mu_{c\Gamma_A}(x_2)$	...	$\mu_{c\Gamma_A}(x_n)$

If $a_{1j} = \mu_{c\Gamma_A}(x_j)$ for $j = 1, 2, ..., n$, then the cardinal set $c\Gamma_A$ is uniquely characterized by a matrix,

$$[a_{1j}]_{1\times n} = [a_{11}\ a_{12}\ ...a_{1n}]$$

which is called the cardinal matrix of the cardinal set $c\Gamma_A$ over E.

Definition 4.4. Let $\Gamma_A \in FS(U)$ and $c\Gamma_A \in cFS(U)$. Then fs-aggregation operator, denoted by FS_{agg}, is defined by

$$FS_{agg} : cFS(U) \times FS(U) \to F(U), \quad FS_{agg}(c\Gamma_A, \Gamma_A) = \Gamma^*_A$$

where

$$\Gamma^*_A = \{\mu_{\Gamma^*_A}(u)/u : u \in U\}$$

is a fuzzy set over U. Γ^*_A is called the aggregate fuzzy set of the fs-set Γ_A. The membership function $\mu_{\Gamma^*_A}$ of Γ^*_A is defined as follows:

$$\mu_{\Gamma^*_A} : U \to [0, 1], \quad \mu_{\Gamma^*_A}(u) = \frac{1}{|E|}\sum_{x\in E} \mu_{c\Gamma_A}(x)\mu_{\gamma_{A(x)}}(u),$$

where $|E|$ is the cardinality of E.

Definition 4.5. Let $\Gamma_A \in FS(U)$ and Γ_A^* be its aggregate fuzzy set. Assume that $U = \{u_1, u_2, ..., u_m\}$, then the Γ_A^* can be presented by the following table

Γ_A	$\mu_{\Gamma_A^*}$
u_1	$\mu_{\Gamma_A^*}(u_1)$
u_2	$\mu_{\Gamma_A^*}(u_2)$
$\vdots$	$\vdots$
u_m	$\mu_{\Gamma_A^*}(u_m)$

If $a_{i1} = \mu_{\Gamma_A^*}(u_i)$ for $i = 1, 2, ..., m$, then Γ_A^* is uniquely characterized by the matrix,

$$[a_{i1}]_{m\times 1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$

which is called the aggregate matrix of Γ_A^* over U.

Theorem 4.6. *Let $\Gamma_A \in FS(U)$ and $A \subseteq E$. If M_{Γ_A}, $M_{c\Gamma_A}$ and $M_{\Gamma_A^*}$ are representation matrices of Γ_A, $c\Gamma_A$ and Γ_A^*, respectively, then*

$$|E| \times M_{\Gamma_A^*} = M_{\Gamma_A} \times M_{c\Gamma_A}^T$$

where $M_{c\Gamma_A}^T$ is the transposition of $M_{c\Gamma_A}$ and $|E|$ is the cardinality of E.

Proof. It is sufficient to consider $[a_{i1}]_{m\times 1} = [a_{ij}]_{m\times n} \times [a_{1j}]_{1\times n}^T$. □

Theorem 4.6 is applicable to computing the aggregate fuzzy set of an fs-set.

5. Application

Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best alternative from this set. Therefore, we can make a decision by the following algorithm.

Step 1: Construct an fs-set Γ_A over U,
Step 2: Find the cardinal set $c\Gamma_A$ of Γ_A,
Step 3: Find the aggregate fuzzy set Γ_A^* of Γ_A,
Step 4: Find the best alternative from this set that has the largest membership grade by $\max \mu_{\Gamma_A^*}(u)$.

Example 5.1. Suppose a company wants to fill a position. There are eight candidates who form the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. The hiring committee consider a set of parameters, $E = \{x_1, x_2, x_3, x_4, x_5\}$. For $i = 1, 2, 3, 4, 5$, the parameters x_i stand for "experience", "computer knowledge", "young age", "good speaking" and "friendly", respectively.

After a serious discussion each candidate is evaluated from the goals and constraint point of view of according to a chosen subset $A = \{x_2, x_3, x_4\}$ of E. Finally, the committee applies the following steps:

Step 1: The committee constructs an fs-set Γ_A over U,

$$\Gamma_A = \Big\{(x_2, \{0.3/u_2, 0.5/u_3, 0.1/u_4, 0.8/u_5, 0.7/u_7\}), (x_3, \{0.4/u_1, 0.4/u_2, 0.9/u_3, 0.3/u_4\}), (x_4, \{0.2/u_1, 0.5/u_2, 0.1/u_5, 0.7/u_7, 1/u_8\})\Big\}$$

Step 2: The cardinal is computed,

$$c\Gamma_A = \{0.3/x_2, 0.25/x_3, 0.2/x_4\}$$

Step 3: The aggregate fuzzy set is found by using Theorem 4.6,

$$M_{\Gamma_A^*} = \frac{1}{5}\begin{bmatrix} 0 & 0 & 0.4 & 0.2 & 0 \\ 0 & 0.3 & 0.4 & 0.5 & 0 \\ 0 & 0.5 & 0.9 & 0 & 0 \\ 0 & 0.1 & 0.3 & 0 & 0 \\ 0 & 0.8 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0.3 \\ 0.25 \\ 0.2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.028 \\ 0.058 \\ 0.075 \\ 0.021 \\ 0.052 \\ 0 \\ 0.070 \\ 0.040 \end{bmatrix}$$

that means,

$$\Gamma_A^* = \{0.028/u_1, 0.058/u_2, 0.075/u_3, 0.021/u_4, 0.052/u_5, 0/u_6, 0.070/u_7, 0.040/u_8\}$$

Step 4: Finally, the largest membership grade is chosen by

$$\max \mu_{\Gamma_A^*}(u) = 0.075$$

which means that the candidate u_3 has the largest membership grade, hence he may be selected for the job.

6. **Conclusion**

A soft set is a mapping from parameter to the crisp subset of universe. However, the situation may be more complicated in real world because of the fuzzy characters of the parameters. In fs-sets, the soft set theory is extended to a fuzzy one, the fuzzy membership is used to describe parameter approximate elements of fuzzy soft set. To develop the theory, in this work, we first defined fs-sets and their operations. We then presented the decision making method for the fs-set theory. Finally, we provided an example demonstrating the successfully application of this method. It may be applied to many fields with problems that contain uncertainty, and it would be beneficial to extend the proposed method to subsequent studies.

However, the approach should be more comprehensive in the future to solve the related problems.

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