

## COUNTABLY NEAR PS-COMPACTNESS IN $L$ -TOPOLOGICAL SPACES

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ABSTRACT. In this paper, the concept of countably near PS-compactness in  $L$ -topological spaces is introduced, where  $L$  is a completely distributive lattice with an order-reversing involution. Countably near PS-compactness is defined for arbitrary  $L$ -subsets and some of its fundamental properties are studied.

### 1. Introduction

We know that since fuzzy topological spaces were introduced by Chang [5], many authors have established various kinds of fuzzy compactness [1-3,5,6,8-10,12-14]. Originally fuzzy compactness [5,6,9] was defined only for the whole fuzzy topological space rather than for arbitrary fuzzy subsets. Then Wang [12] introduced a new concept based on the fuzzy nets of Pu and Liu [11] and called it nice compactness (N-compactness), which is defined for any fuzzy subset and has more advantages. Zhao [14] generalized N-compactness theory to  $L$ -topological spaces (where  $L$  is a completely distributive lattice with an order-reversing involution). This has all the advantages of Wang's theory, and has been extensively used (e.g. [7]) to further the study of fuzzy compactness. In [1-3], we studied SR-compactness, PS-compactness and near PS-compactness in  $L$ -topological spaces, respectively. Every PS-compact set is not only SR-compact but also near PS-compact [2,3]; every SR-compact set is N-compact [1], and every near PS-compact set is Lowen's fuzzy compact [9,3]. That the converse of the above statements need not be true, is shown by Examples 5.6 and 5.7 in [3].

In this paper we use near PS-compactness as a background for the introduction of the concept of countably near PS-compactness of arbitrary  $L$ -subsets in  $L$ -topological spaces. It is an " $L$ -good extension [8,13]" of general countably PS-compactness and hereditary for pre-semiclosed subsets. Moreover, it is finitely additive and can be preserved under PS-irresolute mapping and described with the cover forms and finite intersection properties.

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## 2. Preliminaries

In this paper,  $L$  always denotes a completely distributive lattice with an order-reversing involution “ ’ ”, 0 and 1 denote the least and the greatest elements in  $L$  respectively,  $X$  denotes a nonempty crisp set and  $L^X$  denotes the set of all  $L$ -subsets on  $X$ .  $r \in L$  is called a prime element of  $L$ , if  $a \wedge b \leq r$  implies  $a \leq r$  or  $b \leq r$ , where  $a, b \in L$  [8,13]. The set of all prime elements of  $L$  which are not 1 is denoted by  $pr(L)$ .  $\alpha \in L$  is called a union-irreducible element of  $L$ , if for arbitrary  $a, b \in L$  with  $\alpha \leq a \vee b$ , either  $\alpha \leq a$  or  $\alpha \leq b$  [8,13].  $M(L)$  and  $M^*(L^X)$  denote the set of all nonzero union-irreducible elements of  $L$  and  $L^X$  respectively. Clearly,  $r \in pr(L)$  iff  $r' \in M(L)$ . For each  $\psi \subset L$ , we define  $\psi' = \{A' : A \in \psi\}$ . We will denote  $L$ -topological space by  $L$ -ts, and  $\varepsilon_r(A) = \{x \in X : A(x) \geq r\}$ .

Let  $(L^X, \delta)$  be an  $L$ -ts.  $A \in L^X$  is called a pre-semiopen set iff  $A \leq (A^-)_o$ , and  $A$  is called a pre-semiclosed set iff  $A \geq (A^o)_-$ , where  $A^o, A^-, A_o$  and  $A_-$  are the interior, closure, semi-interior and semiclosure of  $A$ , respectively.  $PSO(L^X)$  and  $PSC(L^X)$  will always denote the family of pre-semiopen sets and family of pre-semiclosed sets of an  $L$ -ts  $(L^X, \delta)$ , respectively. It is clear that every semiopen set is pre-semiopen and every preopen set is pre-semiopen in  $L$ -ts. That none of the converses need be true is shown by Example 3.3 in [4].

**Definition 2.1.** [2] Let  $(L^X, \delta)$  be an  $L$ -ts and  $x_\lambda \in M^*(L^X)$ .  $A \in PSC(L^X)$  is called a pre-semiclosed remote-neighborhood, or briefly, PSC-RN of  $x_\lambda$ , if  $x_\lambda \notin A$ . The set of all PSC-RNs of  $x_\lambda$  is denoted by  $\zeta(x_\lambda)$ .

**Definition 2.2.** [2] Let  $(L^X, \delta)$  be an  $L$ -ts,  $A \in L^X$  and  $\alpha \in M(L)$ ,  $\phi \subset PSC(L^X)$  is called an  $\alpha$ -PSC-remote neighborhood family of  $A$  (briefly  $\alpha$ -PSC-RF of  $A$ ) if, for each  $x_\alpha$  in  $A$ , there is  $P \in \phi$  such that  $P \in \zeta(x_\alpha)$ .

**Definition 2.3.** [3] Let  $(L^X, \delta)$  be an  $L$ -ts and  $A \in L^X$ .  $A$  is called near PS-compact if every  $\alpha$ -PSC-RF  $\phi$  of  $A$  has a finite subfamily  $\psi$  of  $\phi$  such that  $\psi$  is an  $\alpha$ -PSC-RF of  $A$  ( $\alpha \in M(L)$ ). Specifically, when  $A = 1_X$  is near PS-compact, we call  $(L^X, \delta)$  a near PS-compact space.

### 3. Countably Near PS-compactness and its Characterizations

**Definition 3.1.** Let  $(L^X, \delta)$  be an  $L$ -ts and  $A \in L^X$ .  $A$  is called a countably near PS-compact set if every countable  $\alpha$ -PSC-RF  $\Phi$  of  $A$  has a finite subfamily  $\Psi$  of  $\Phi$  such that  $\Psi$  is an  $\alpha$ -PSC-RF of  $A$  ( $\alpha \in M(L)$ ). Specifically, when  $A = 1_X$  is countably near PS-compact, we call  $(L^X, \delta)$  a countably near PS-compact space.

Clearly, every near PS-compact set is countably near PS-compact.

**Definition 3.2.** [2] Let  $(L^X, \delta)$  be an  $L$ -ts,  $A \in L^X$ ,  $\mu \subset PSO(L^X)$ ,  $r \in pr(L)$ .  $\mu$  is called an  $r$ -PS-cover of  $A$  if for each  $x \in \varepsilon_{r'}(A)$ , there is  $U \in \mu$  such that  $U(x) \not\leq r$ .

**Theorem 3.3.** Let  $(L^X, \delta)$  be an  $L$ -ts,  $r \in pr(L)$ .  $A \in L^X$  is a countably near PS-compact set iff every countable  $r$ -PS-cover  $\mu$  of  $A$  has a finite subfamily  $\nu$  of  $\mu$  such that  $\nu$  is an  $r$ -PS-cover of  $A$ .

*Proof.* Let  $A$  be a countably near PS-compact set,  $\mu$  be a countable  $r$ -PS-cover of  $A$  and  $r \in pr(L)$ . Put  $\Phi = \mu'$ , then  $\Phi \subset PSC(L^X)$  and for each  $x \in \varepsilon_{r'}(A)$  there is  $Q = U' \in \Phi$  such that  $U(x) \not\leq r$ , i.e.,  $r' \not\leq Q(x)$ . Now  $r \in pr(L)$ , implies that  $r' \in M(L)$  and since  $x_{r'} \not\leq Q$ , we have  $Q \in \zeta(x_{r'})$ . Hence  $\Phi$  is an  $r'$ -PSC-RF of  $A$ . Since  $A$  is countably near PS-compact, there is a finite subfamily  $\nu$  of  $\mu$  such that  $\Psi = \nu'$  is an  $r'$ -PSC-RF of  $A$ , i.e. for each  $x \in \varepsilon_{r'}(A)$ , there is  $V \in \nu$  such that  $V' \in \zeta(x_{r'})$ , i.e.  $(x_{r'}) \notin V'$  or, equivalently,  $r' \not\leq V'(x)$ ; for each  $x \in \varepsilon_{r'}(A)$ , there is  $V \in \nu$  such that  $V(x) \not\leq r$ . Thus  $\mu$  has a finite subfamily  $\nu$  which is an  $r$ -PS-cover of  $A$ .

Conversely, suppose every countable  $r$ -PS-cover of  $A$  has a finite subfamily which is an  $r$ -PS-cover of  $A$ . Let  $\Phi$  be an  $\alpha$ -PSC-RF of  $A$ ,  $\mu = \Phi'$  and  $r = \alpha'$ . Since  $\alpha \in M(L)$ , hence  $r \in pr(L)$ . For each  $x \in \varepsilon_\alpha(A) = \varepsilon_{r'}(A)$ , i.e.  $x_\alpha \in A$ , there is  $P \in \Phi$  such that  $p \in \zeta x_\alpha$ , i.e.  $x_\alpha \notin P$ , or  $\alpha \not\leq P(x)$ . So  $P'(x) \not\leq \alpha'$ . Put  $U = P'$ , then  $U \in \mu$  and  $U(x) \not\leq r$ . Hence  $\mu$  is a countable  $r$ -PS-cover of  $A$ . By supposition,  $\Phi$  has a finite subfamily  $\Psi$  such that  $\Psi'$  is an  $r$ -PS-cover of  $A$ . So for each  $x \in \varepsilon_{r'}(A) = \varepsilon_\alpha(A)$ , i.e.  $x_\alpha \in A$ , there is  $Q \in \Psi$  such that  $Q'(x) \not\leq r$ . Hence,  $r' = \alpha \not\leq Q(x)$ , i.e.  $Q \in \zeta(x_\alpha)$ . This shows that  $\Psi$  is an  $\alpha$ -PSC-RF of  $A$ . Thus  $A$  is countably near PS-compact.  $\square$

**Definition 3.4.** Let  $(L^X, \delta)$  be an  $L$ -ts,  $A \in L^X$ ,  $r \in pr(L)$  and  $\Phi \subset L^X$ . If for every finite subfamily  $\Psi$  of  $\Phi$ , there is  $x \in \varepsilon_{r'}(A)$  such that  $(\bigwedge \Psi)(x) \geq r'$ , then we say that  $\Phi$  has an  $r$ -finite intersection property in  $A$ .

**Theorem 3.5.** Let  $(L^X, \delta)$  be an  $L$ -ts,  $A \in L^X$  and  $r \in pr(L)$ .  $A$  is a countably near PS-compact set iff for every countable subfamily  $\Phi \subset PSC(L^X)$  which has the  $r$ -finite intersection property in  $A$ , there is  $x \in \varepsilon_{r'}(A)$  such that  $(\bigwedge \Phi)(x) \geq r'$ .

*Proof.* Let  $A$  be a countably near PS-compact set. Suppose there is an  $r \in pr(L)$  and some countable subfamily  $\Phi \subset PSC(L^X)$  which has an  $r$ -finite intersection property in  $A$ , for each  $x \in \varepsilon_{r'}(A)$  such that  $(\bigwedge \Phi)(x) \not\geq r'$ . Then there exists  $P \in \Phi$  such that  $P(x) \not\geq r'$ , i.e.,  $P'(x) \not\leq r$ . This shows that  $\Phi'$  is a countable  $r$ -PS-cover of  $A$ . By Theorem 3.3, there is a finite subfamily  $\Psi = \{P_1, \dots, P_n\}$  of  $\Phi$  such that  $\Psi'$  is an  $r$ -PS-cover of  $A$ . Hence for each  $x \in \varepsilon_{r'}(A)$ , there is  $P_i \in \Psi$  such that  $P'_i(x) \not\leq r$ . And so

$$\left(\bigvee_{i=1}^n P'_i\right)(x) \not\leq r \text{ i.e. } (\bigwedge \Psi)(x) = \left(\bigwedge_{i=1}^n P_i\right)(x) \not\geq r',$$

which contradicts the fact that  $\Phi$  has an  $r$ -finite intersection property in  $A$ .

Conversely, let  $\mu$  be a countable  $r$ -PS-cover of  $A$  and  $r \in pr(L)$ . If none of the finite subfamily  $\nu$  of  $\mu$  is an  $r$ -PS-cover of  $A$ , then there exists  $x \in \varepsilon_{r'}(A)$  such that  $B(x) \leq r$  for each  $B \in \nu$  and so  $(\bigvee \nu)(x) \leq r$  or, equivalently,  $(\bigwedge \nu')(x) \geq r'$ . This shows that the subfamily  $\mu' \subset PSC(L^X)$  has an  $r$ -finite intersection property in  $A$ . Hence there is  $x \in \varepsilon_{r'}(A)$  such that  $(\bigwedge \mu')(x) \geq r'$ , i.e.  $(\bigvee \mu)(x) \leq r$ . This implies that  $\mu$  is not a countable  $r$ -PS-cover of  $A$ , a contradiction. By Theorem 3.3,  $A$  is countably near PS-compact.  $\square$

**Lemma 3.6.** [2] *Let  $(X, \delta)$  be a crisp topological space and  $A \subset X$ . If  $A$  is pre-semiopen in  $(X, \delta)$  then  $\chi_A$  is pre-semiopen in the  $L$ -ts  $(L^X, \omega_L(\delta))$ , where  $(L^X, \omega_L(\delta))$  is  $L$ -ts topologically generated by a crisp topological space  $(X, \delta)$ .*

**Definition 3.7.** Let  $(L^X, \delta)$  be an  $L$ -ts,  $A \in L^X$  and  $r \in pr(L)$ . Put  $l_r(A) = \{x \in X : A(x) \not\leq r\}$ , and  $l_r(\delta) = \{l_r(A) : A \in \delta\}$ , then  $l_r(\delta)$  is a crisp topology on  $X$ , called the  $r$ -cut topology of  $\delta$  [13]. Also,  $l_r(PSO(L^X)) = \{l_r(A) : A \in PSO(L^X)\}$  is the family of pre-semiopen sets in  $r$ -cut topological space  $(X, l_r(\delta))$ .

We say that a topological space  $(X, \delta)$  is countably PS-compact iff every countable PS-cover of  $X$  has a finite subcover.

**Theorem 3.8.** *Let  $(L^X, \delta)$  be an  $L$ -ts. Then  $A \in L^X$  is countably near PS-compact iff the subset  $\varepsilon_{r'}(A)$  of  $(X, l_r(\delta))$  is countably PS-compact for each  $r \in pr(L)$ .*

*Proof.* Necessity. For any  $r \in pr(L)$ , suppose that  $\mu$  is a countable PS-cover of  $\varepsilon_{r'}(A)$ . Then there exists a countable family  $\psi \in PSO(L^X)$  such that  $\mu = l_r(\psi) = \{l_r(U) : U \in \psi\}$ . For each  $x \in \varepsilon_{r'}(A)$ , there is  $U \in \psi$  such that  $x \in l_r(U)$ , i.e.  $U(x) \not\leq r$ . Hence  $\psi$  is a countable  $r$ -PS-cover of  $A$ . Since  $A$  is countably near PS-compact, from Theorem 3.3 there is a finite subfamily  $\nu$  of  $\psi$  such that  $\nu$  is an  $r$ -PS-cover of  $A$ . We now prove that a finite subfamily  $l_r(\nu)$  is a PS-cover of  $\varepsilon_{r'}(A)$ . In fact, for each  $x \in \varepsilon_{r'}(A)$ , by  $\nu$  is an  $r$ -PS-cover of  $A$  there is  $U \in \nu$  such that  $U(x) \not\leq r$ , i.e.  $x \in l_r(U) \in l_r(\nu)$ . This shows that  $l_r(\nu)$  is indeed a PS-cover of  $\varepsilon_{r'}(A)$ , and thus  $\varepsilon_{r'}(A)$  is countably PS-compact.

Sufficiency. Suppose that  $\mu$  is a countable  $r$ -PS-cover of  $A$  ( $r \in pr(L)$ ). Then for each  $x \in \varepsilon_{r'}(A)$ , there exists  $U \in \mu$  such that  $U(x) \not\leq r$ , i.e.  $x \in l_r(U)$ . Hence  $l_r(\mu) = \{l_r(U) : U \in \mu\}$  is a countable PS-cover of  $\varepsilon_{r'}(A)$ . Since  $\varepsilon_{r'}(A)$  is countably PS-compact, there is a finite subfamily  $\nu$  of  $\mu$  such that  $l_r(\nu)$  is a PS-cover of  $\varepsilon_{r'}(A)$ . It is not difficult to see that  $\nu$  is an  $r$ -PS-cover of  $A$ . By Theorem 3.3,  $A$  is countably near PS-compact.  $\square$

**Theorem 3.9.** *Let  $(L^X, \omega_L(\delta))$  be an  $L$ -ts topologically generated by a crisp topological space  $(X, \delta)$ . Then  $(L^X, \omega_L(\delta))$  is countably near PS-compact iff  $(X, \delta)$  is countably PS-compact.*

*Proof.* Necessity. Suppose that  $\mu$  is a countable PS-cover of  $(X, \delta)$ . Then, by Lemma 3.6,  $\chi_\mu = \{\chi_E : E \in \mu\}$  is a family of semi-pre-open sets of  $(L^X, \omega_L(\delta))$ . For each  $r \in pr(L)$  we will prove that  $\chi_\mu$  is a countable  $r$ -PS-cover of  $1_X$ . In fact, for each  $x \in \varepsilon_{r'}(1_X) = X$ , there exists  $E \in \mu$  such that  $x \in E$ , and so  $\chi_E(x) = 1 \not\leq r$ . Hence,  $\chi_\mu$  is a countable  $r$ -PS-cover of  $1_X$ . Since  $(L^X, \omega_L(\delta))$  is countably near PS-compact, there is a finite subcover  $\nu$  of  $\mu$  such that  $\chi_\nu$  is an  $r$ -PS-cover of  $1_X$ , i.e. for each  $x \in X$ , there exists  $E \in \nu$  such that  $\chi_E(x) \not\leq r$ , and so  $x \in E$ . This shows that  $\nu$  is a PS-cover of  $X$ , and hence  $(X, \delta)$  countably PS-compact.

Sufficiency. For any  $r \in pr(L)$ , suppose that  $\mu$  is a countable  $r$ -PS-cover of  $1_X$ . Then for each  $x \in \varepsilon_{r'}(1_X) = X$ , there exists  $U \in \mu$  such that  $U(x) \not\leq r$ , i.e.  $x \in l_r(U)$ , where  $l_r(U)$  is pre-semiopen set in  $(X, \delta)$ . Hence,  $l_r(\mu) = \{l_r(U) : U \in \mu\}$  is a countable PS-cover of  $(X, \delta)$ . Since  $(X, \delta)$  is countably PS-compact, there is a finite subcover  $\nu$  of  $\mu$  such that  $l_r(\nu)$  is a cover of  $(X, \delta)$ . Thus, for each

$x \in X = \varepsilon_{r'}(1_X)$ , there exists  $U \in \nu$  such that  $x \in l_r(U) \in l_r(\nu)$ , i.e.  $U(x) \not\leq r$ . This shows that  $\nu$  is an  $r$ -PS-cover of  $1_X$ , and hence  $(L^X, \omega_L(\delta))$  is countably near PS-compact.  $\square$

#### 4. Some Other Properties

**Theorem 4.1.** *Let  $A$  be a countably near PS-compact set in  $L$ -ts  $(L^X, \delta)$ . Then for each  $B \in PSC(L^X)$ ,  $A \wedge B$  is countably near PS-compact.*

*Proof.* Let  $\Phi$  be a countable  $\alpha$ -PSC-RF of  $A \wedge B$  ( $\alpha \in M(L)$ ) and put  $\Phi_1 = \Phi \cup \{B\}$ . Then  $\Phi$  is a countable  $\alpha$ -PSC-RF of  $A$ . In fact, for each  $x_\alpha \in A$ , if  $x_\alpha \in B$  then  $x_\alpha \in A \wedge B$ . Hence, there is  $P \in \Phi \subset \Phi_1$  such that  $P \in \zeta(x_\alpha)$ . If  $x_\alpha \notin B$ , then  $B \in \Phi_1$  and  $B \in \zeta(x_\alpha)$ . Thus,  $\Phi$  is indeed a countable  $\alpha$ -PSC-RF of  $A$ . Since  $A$  is a countably near PS-compact set, there exists a finite subfamily  $\Psi_1$  of  $\Phi_1$  such that  $\Psi_1$  is an  $\alpha$ -PSC-RF of  $A$ . Let  $\Psi = \Psi_1 - \{B\}$ , then  $\Psi$  is a finite subfamily of  $\Phi$ , and  $\Psi$  is an  $\alpha$ -PSC-RF of  $A \wedge B$ . In fact,  $x_\alpha \in A \wedge B$  implies  $x_\alpha \in A$  and hence from the definition of  $\Psi_1$ , there exists  $P \in \Psi_1$  with  $P \in \zeta(x_\alpha)$ . However,  $x_\alpha \in B$  so  $P \neq B$ , and thus  $P \in \Psi_1 - \{B\} = \Psi$ . Hence,  $A \wedge B$  is countably near PS-compact.  $\square$

**Corollary 4.2.** *Let  $(L^X, \delta)$  be a countably near PS-compact space and  $B \in PSC(L^X)$ . Then  $B$  is countably near PS-compact.*

**Theorem 4.3.** *Let  $A$  and  $B$  be two countably near PS-compact sets in  $L$ -ts  $(L^X, \delta)$ . Then  $A \vee B$  is countably near PS-compact.*

*Proof.* Let  $\Phi$  be a countable  $\alpha$ -PSC-RF of  $A \vee B$  ( $\alpha \in M(L)$ ). Then  $\Phi$  is not only a countable  $\alpha$ -PSC-RF of  $A$ , but also a countable  $\alpha$ -PSC-RF of  $B$ . Since  $A$  is countably near PS-compact, there is a finite subfamily  $\Psi_1$  of  $\Phi$  such that  $\Psi_1$  is an  $\alpha$ -PSC-RF of  $A$ . Similarly, since  $B$  is countably near PS-compact, there is a finite subfamily  $\Psi_2$  of  $\Phi$  such that  $\Psi_2$  is an  $\alpha$ -PSC-RF of  $B$ . Put  $\Psi = \Psi_1 \vee \Psi_2$ . Then  $\Psi$  is a finite subfamily of  $\Phi$  and  $\Psi$  is an  $\alpha$ -PSC-RF of  $A \vee B$ . Thus,  $A \vee B$  is countably near PS-compact.  $\square$

**Definition 4.4.** [2] Let  $(L^X, \delta)$  and  $(L^Y, \tau)$  be two  $L$ -ts's and  $f : (L^X, \delta) \rightarrow (L^Y, \tau)$  an  $L$ -mapping.  $f$  is called a PS-irresolute mapping if  $f^{-1}(B) \in PSO(L^X)$  for each  $B \in PSO(L^Y)$ .

**Theorem 4.5.** *Let  $f : (L^X, \delta) \rightarrow (L^Y, \tau)$  be an PS-irresolute mapping and  $A$  be a countably near PS-compact set in  $(L^X, \delta)$ . Then  $f(A)$  is countably near PS-compact in  $(L^Y, \tau)$ .*

*Proof.* Let  $\Phi$  be a countable  $\alpha$ -PSC-RF of  $f(A)$  ( $\alpha \in M(L)$ ). To begin with, we show that  $f^{-1}(\Phi) = \{f^{-1}(P) : P \in \Phi\}$  is a countable  $\alpha$ -PSC-RF of  $A$ . Since  $f$  is a PS-irresolute mapping,  $f^{-1}(\Phi) \subset PSC(L^X)$ . Let  $x_\alpha \in A$ ; then  $f(x_\alpha) = (f(x))_\alpha \in f(A)$  and since  $\phi$  is a countable  $\alpha$ -PSC-RF of  $f(A)$ , there is  $P \in \Phi$  with  $P \in \zeta((f(x))_\alpha)$ , i.e.,  $(f(x))_\alpha \notin P$  or, equivalently,  $P(f(x)) \not\leq \alpha$ . By the definition of inverse mapping,  $f^{-1}(P)(x) = P(f(x)) \not\leq \alpha$ , hence  $x_\alpha \notin f^{-1}(P)$ , i.e.,  $f^{-1}(P) \in \zeta(x_\alpha)$ . Therefore  $f^{-1}(\Phi)$  is a countable  $\alpha$ -PSC-RF of  $A$ . Since  $A$  is

countably near PS-compact, there is a finite subfamily  $\Psi$  of  $\Phi$  such that  $f^{-1}(\Psi)$  is an  $\alpha$ -PSC-RF of  $A$ . It is easy to show that  $\Psi$  is an  $\alpha$ -PSC-RF of  $f(A)$ . Thus  $f(A)$  is countably near PS-compact.  $\square$

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