CONTROL OF FLEXIBLE JOINT ROBOT MANIPULATORS BY COMPENSATING FLEXIBILITY

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Abstract. A flexible-joint robot manipulator is a complex system because it is nonlinear, multivariable, highly coupled along with joint flexibility and uncertainty. To overcome flexibility, several methods have been proposed based on flexible model. This paper presents a novel method for controlling flexible-joint robot manipulators. A novel control law is presented by compensating flexibility to form a rigid robot and then control methods for rigid robots are applied. Feedback linearization and direct adaptive fuzzy control, based on rigid model, are designed with torque control strategy. A decentralized adaptive fuzzy controller is designed because of simplicity and ease of implementation. Effectiveness of the proposed control approach is demonstrated by simulations, using a three-joint articulated flexible-joint robot, driven by permanent magnet dc motors.

1. Introduction

Electrically driven robots have various applications in different fields. Their motors provide low torque at high speed, and therefore, in order to provide high torque at low speed for performing the tasks, they are equipped with power transmission systems. However, deformation of the transmission system produces flexibility in the joints. This phenomenon is the main source of vibration in industrial robot manipulators [19]. Due to flexibility in their joints, the number of degrees of freedom are twice as the number of control actions compared with rigid robots. Moreover, there is no matching between nonlinearities and inputs [1]. Applying flexible-joint robot high-precision tasks are difficult since the link position cannot follow the actuator position directly. As a result, to improve the performance and to avoid unwanted oscillations, flexibility in joints should be compensated very well.

There are many challenges for flexible-joint robot manipulator such as nonlinearity, largeness of model, coupling, uncertainty, and joint flexibility in the modelling and control. Therefore, a great deal of research have been dedicated to develop advance controls. For instance, robust control [27], PD control [4], sliding mode control [12], adaptive control [17], fuzzy control [2], learning control [26], neural network approach [28], passivity based impedance control [9], state observer based control [20] have been proposed to deal with the control of flexible-joint robot. In all the aforementioned controllers, multivariable control systems are used to provide the torque.

The decentralized control is the most favorite control scheme in multivariable control because it has many advantages, such as flexibility in operation, failure tolerance, and simplified design and tuning [13]. The decentralized controller is used in many robotic systems in favor of computational simplicity and a low-cost hardware setup [8]. In decentralized control an actuator is commanded to drive a joint by taking feedbacks from that joint [14]. For this purpose, a robotic system is decomposed into individual single-input/single-output systems. The effect of interactions between decomposed systems are considered as disturbances and they should be compensated. Performance of the decentralized control can be improved by utilizing the robust and adaptive techniques.

Feedback linearization is a helpful approach to decouple the robotic system and facilitate the control design by cancelling the nonlinearities. Generally, a flexible-joint robot cannot be feedback linearized by static feedback [21]. However, under some assumptions, a simplified model was introduced which can be feedback linearized [18]. Moreover, it was proven that the whole class of elastic joint robots could be linearized via dynamic feedback [3]. One problem is that the model of the flexible robot is very large, computationally extensive and imprecise. Thus, model-based techniques such as feedback linearization cannot work well.

Fuzzy control is an efficient alternative for control of nonlinear uncertain systems. It is superior to conventional control due to using information from experts in linguistic rules [25]. An important feature of a fuzzy system is that it can be used as a universal approximator for any nonlinear function. This feature has given great attention to design many fuzzy controllers such as adaptive fuzzy controllers. The direct method of Lyapunov has been used in the design and analysis of adaptive fuzzy control. Based on the Lyapanov stability method, a multi-input/multi-output adaptive fuzzy terminal sliding mode controller for robotic manipulators was designed [10]. Adaptive fuzzy control is adopted by different nonlinear systems such as large-scale systems [24], single-input/single-output systems [23], stochastic strict-feedback systems [22], and multi-input/multi-output systems [11]. The design of adaptive fuzzy control in decentralized structure with guaranteed stability, robustness, satisfactory performance and ease of implementation is presented for robot manipulators with voltage control strategy [7, 6]. In this strategy control signal is the voltage of motors to control the motors of the joints of the manipulators

Most controllers for industrial manipulators have been designed based on the rigid robot assumption. Control of rigid robots has been well understood in the past decades. In this paper, by compensating flexibility effect, the robot turns to rigid robot and then control law based on rigid model are designed and applied.

The rest of the paper is organized as follows: Section 2 presents the modelling of electrically driven robot manipulators and introduces a model for the electrical flexible-joint manipulator driven by the geared permanent magnet dc motors. Section 3 develops the feedback linearization control. Section 4 describes decentralized direct adaptive fuzzy control design. Section 5 presents stability analysis of the proposed control laws. Section 6 presents the simulation results and finally, Section 7 concludes the paper.

2. Modelling

Consider an electrical robot driven by geared permanent magnet dc motors [16]. The dynamics of robot can be expressed as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau_l \tag{1}$$

Where $q \in R_n$ is the vector of joint positions, $D(q) \in R_{n \times n}$ is the matrix of manipulator inertia, $C(q,\dot{q})\dot{q} \in R_n$ is the vector of centrifugal and Coriolis torques, $G(q) \in R_n$ is the vector of gravitational torques, and τ_l is the torque vector of robot. Note that vectors and matrices are bold for clarity.

The inserted torque on the joint to drive the manipulator is the load torque of motor, which is considered in a dynamic equation formed as:

$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m + r\tau_l = \tau_m \tag{2}$$

Where θ_m is the rotor position, τ_l is the load torque, τ_m is the motor torque, r is the gear reduction coefficient, J_m is the sum of actuator and gear inertia, and B_m is the damping coefficient.

The reduction gear relates the motor position to the joint position as:

$$q = r\theta_m \tag{3}$$

In a simplified model of flexible-joint robot [12], if the joint flexibility is modelled by a linear torsional spring, dynamic equation of motion can be expressed as:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = K(r\theta_m - q) \tag{4}$$

$$J_m\ddot{\theta}_m + B\dot{\theta}_m + rK(r\theta_m - q) = \tau_m \tag{5}$$

Thus, this system possesses $2n$ coordinates as $[q\theta_m]$. The diagonal matrix K represents the lumped flexibility provided by the joint and reduction gear. To simplify the model, both joint stiffness and gear coefficients are assumed constant. Since flexible joint robots can not directly follow the motor position for each motor we have

$$\tau_l = K(r\theta_m - q) \tag{6}$$

Equation (6) yields:

$$q = r\theta_m - K^{-1}\tau_l \tag{7}$$

Equations (3) and (7) show the difference between rigid and flexible-joint robot manipulators. Comparing these two equations, indicates that if this relationship $q = r\theta_m$ is established, the robot acts like a rigid robot. In other words, $\theta_m \to r^{-1}q$ becomes our control objective. In addition, our main goal in a robotic system is to $q \to q_d$ hence, by means of a control law for a rigid robot, we can reach this goal.

3. Feedback Linearization

According to dynamic equation of (1), in order to cancel nonlinear terms a control law is proposed as [15]:

$$\tau_l = D(q)u + C(q,\dot{q})\dot{q} + G(q) \tag{8}$$

Where u is a new control law. Substitute (8) into (1) results in a linear time invariant system formed as:

$$\ddot{q} = u \tag{9}$$

In the system obtained above, u is the input. Therefore for tracking a desired trajectory, a linear control law is suggested as:

$$u = \ddot{q}_d + K_d(\dot{q}_d - q) + K_p(q_d - q) \tag{10}$$

Where q_d is the desired trajectory, $\dot{q}_d$ is the desired joint velocity, $\ddot{q}_d$ is the desired joint acceleration. K_p, K_d are diagonal coefficient matrices.

Substituting (10) into (8) the final control law for rigid robot is

$$\tau = D(q)\big(\ddot{q}_d + K_d(\dot{q}_d - q) + K_p(q_d - q)\big) + C(q, \dot{q})\dot{q} + G(q) \tag{11}$$

In dynamic equation of flexible joint robots substituting (4) into (5) yields

$$J\ddot{\theta}_m + B\dot{\theta}_m + rD(q)\ddot{q} + rC(q, \dot{q})\dot{q} + rG(q) = \tau_m \tag{12}$$

flexibility effect in (5) can be defined as

$$\theta_m = r^{-1}q + \delta \tag{13}$$

substituting (13) into (11) and arranging it yields

$$\big(rD(q) + Jr^{-1}\big)\ddot{q} + \big(rC(q, \dot{q}) + Br^{-1}\big)\dot{q} + rG(q) + \eta = \tau_m \tag{14}$$

Where η is the flexibility effect with dynamic equation of

$$\eta = J_m\ddot{\delta} + B_m\dot{\delta} \tag{15}$$

The control law is proposed as

$$\tau_m = \tau_{m_1} + \tau_{m_2} \tag{16}$$

Where τ_{m_1} is a control law for rigid robot and τ_{m_2} is used for compensating flexibility with the purpose of establishing $q = r\theta_m$.

τ_{m_1} is proposed as feedback linearization control law similar to (11) as

$$\tau_{m_1} = (rD + Jr^{-1})[\ddot{q}_d + K_d(\dot{q}_d - \dot{q}) + K_p(q_d - q)] + (rC + Br^{-1})\dot{q} + rG \tag{17}$$

τ_{m_2} is proposed by a direct adaptive fuzzy controller for compensating flexibility as

$$\tau_{m_2} = \hat{f}^T\psi(\delta, \dot{\delta}) \tag{18}$$

The closed loop system is formed by substituting (17) and (18) into (14) as

$$(rD + Jr^{-1})[\ddot{e} + K_d\dot{e} + K_pe] = \hat{f}^T\psi(\delta, \dot{\delta}) - \eta \tag{19}$$

Where $e = q_d - q$ is the tracking error.

Provided that δ is compensated i.e $\delta \to 0$, then $\hat{f} \to \eta$. As a result the right side of equation (19) goes to zero therefore $q \to q_d$.

In order to design decentralized controller the dynamic of ith element of η in (15) is

$$\eta = J\ddot{\delta} + B\dot{\delta} \tag{20}$$

Where $\ddot{\delta}, \dot{\delta}$ are the ith element of vector $\ddot{\delta}, \dot{\delta}$ respectively. J and B are the inertia and damping of ith motor.

A decentralized fuzzy controller is designed by the use of two variables as inputs to the fuzz controller,

δ and its derivative $\dot{\delta}$, where δ is expressed as $\delta = \theta - r^{-1}q$. The motor torque, τ_{m_2}, is the output of the controller.

If three membership functions are given to each fuzzy input, the whole control space is covered by nine fuzzy rules. The linguistic fuzzy rules are proposed in the form of the Mamdani-type as

$$\text{Rule } l : \text{If } \delta \text{ is } A_l \text{ and } \dot{\delta} \text{ is } B_l \text{ Then } \tau_{m_2} \text{ is } C_l \qquad (21)$$

where Rule l denotes the lth fuzzy rule for $l = 1, \cdots, 9$. In the lth rule, Al, Bl, and Cl are fuzzy membership functions belonging to the fuzzy variables $\delta, \dot{\delta}$ and τ_{m_2}, respectively. Three membership functions P, Z and N are given to the input δ as shown in Figure 1. They are expressed as

$$\mu_N(x) = \begin{cases} x \leq -1 \\ 1 - 2(x+1)^2 & -1 \leq x \leq -0.5 \\ 2x^2 & -0.5 \leq x \leq 0 \\ 0 & x \geq 0 \end{cases},$$

$$\mu_P(x) = \begin{cases} x \leq 0 \\ 2x^2 & 0 \leq x \leq 0.5 \\ 1 - 2(x+1)^2 & 0.5 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases},$$

$$\mu_Z(x) = \exp\left(-\frac{x^2}{2\sigma^2}\right), \quad \sigma = 0.3 \qquad (22)$$

The membership functions of $\dot{\delta}$ are given the same as δ. The membership functions of output τ_{m_2} in the Gaussian shapes are expressed by
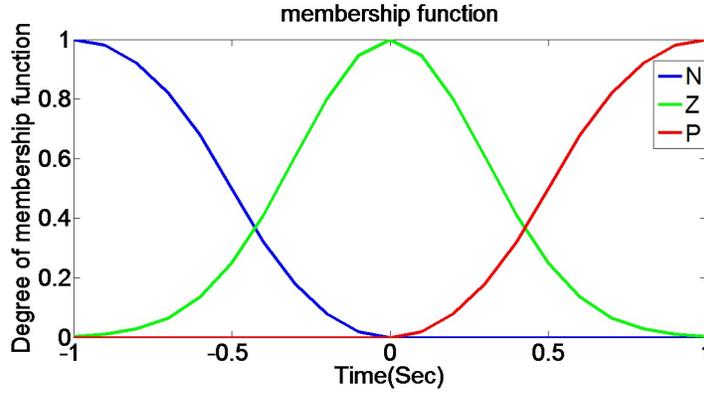

FIGURE 1. Membership Function for δ

$$\mu_{c_l} = \exp(-(\tau_{m_2} - \hat{f}_l)^2/(2\sigma^2)) \qquad (23)$$

Rule l	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
A_l	P	P	P	Z	Z	Z	N	N	N
B_l	P	Z	N	P	Z	N	P	Z	N
C_l	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9

TABLE 1. Fuzzy Rules

Where $\hat{f}_l$ is the center of Cl. Using (21)–(23), fuzzy rules are obtained as Table 1. If we use the Mamdani-type inference engine, the singleton fuzzifier and the center average defuzzifier, τ_{m_2} is calculated as [25]

$$\tau_{m_2} = \sum_{l=1}^{9} \hat{f}_l \psi_l(\delta, \dot{\delta}) = \hat{f}^T \psi(\delta, \dot{\delta}) \tag{24}$$

Where $\psi = [\psi_1 \cdots \psi_9]^T$ in which ψ_l is a positive value expressed as

$$\psi_l(\delta, \dot{\delta}) = \frac{\mu_{A_l}(\delta)\mu_{B_l}(\dot{\delta})}{\sum_{l=1}^{l=9} \mu_{A_l}(\delta)\mu_{B_l}(\dot{\delta})} \tag{25}$$

Where $\mu_l \in [0, 1]$. The parameters $\hat{f}$ in (23) are determined by the adaptive rule afterward.

Applying control law (23) to system (19) yields

$$J\left(\ddot{\delta} + \frac{B}{J}\dot{\delta}\right) = \hat{f}^T \psi(\delta, \dot{\delta}) \tag{26}$$

Where $\hat{f}$ is the estimation of f used into a fuzzy system which approximates the following function based on universal approximation theorem of fuzzy systems as

$$J\left(\frac{B}{J}\dot{\delta} - k_1\dot{\delta} - k_2\delta\right) = f^T \psi(\delta, \dot{\delta}) + \varepsilon \tag{27}$$

Where k_1 and k_2 are positive gains and ε is the approximation error. In order to obtain the adaptive law, we form the tracking system from (27) to (26) as:

$$(\ddot{\delta} + k_1\dot{\delta} + k_2\delta) = J^{-1}(\hat{f} - f)^T \psi(\delta, \dot{\delta}) - J^{-1}\varepsilon \tag{28}$$

The state space equation in the tracking space is obtained using (28) as

$$\dot{X} = AX + Bw \tag{29}$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad X = \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix}, \quad w = J^{-1}(\hat{f} - f)^T \psi(\delta, \dot{\delta}) - J^{-1}\varepsilon \tag{30}$$

A positive definite function F is suggested as

$$F = \frac{1}{2}X^T P X + \frac{J^{-1}}{2\gamma}(\hat{f} - f)^T(\hat{f} - f) \tag{31}$$

Where the constant $\gamma > 0$, P and Q are the unique symmetric, and positive definite matrices satisfying matrix Lyapunov equation

$$A^T P + PA = -Q \tag{32}$$

Then $\dot{F}$ is calculated using (29)–(32) as

$$\dot{F} = -\frac{1}{2}X^T Q X + X^T P_2 J^{-1}\big((\hat{f}-f)^T \psi(\delta,\dot{\delta}) - \varepsilon\big) + \frac{J^{-1}}{\gamma}\dot{\hat{f}}^T(\hat{f}-f) \qquad (33)$$

Where P_2 is the second column of P. We can write (33) as

$$\dot{F} = -\frac{1}{2}X^T Q X + J^{-1}(\hat{f}-f)^T\big(X^T P_2 \psi(\delta,\dot{\delta}) + \frac{1}{\gamma}\dot{\hat{f}}\big) - X^T J^{-1} P_2 \varepsilon \qquad (34)$$

If the adaptive law is given by

$$\dot{\hat{f}} = -\gamma X^T P_2 \psi(\delta,\dot{\delta}) \qquad (35)$$

We have

$$\dot{F} = -\frac{1}{2}X^T Q X - X^T P_2 J^{-1}\varepsilon \qquad (36)$$

X reduces if $\dot{F} < 0$. Thus, satisfying $\dot{F} < 0$ results in

$$-X^T P_2 J^{-1}\varepsilon \leq 0.5 X^T Q X \qquad (37)$$

Since $\lambda_{\min}(Q)\|X\|^2 \leq X^T Q X \leq \lambda_{\max}(Q)\|X\|^2$, where $\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ are the minimum and maximum eigenvalues of Q, respectively. To satisfy $\dot{F} < 0$, it is sufficient that

$$2\left|J^{-1}\right|\|P_2\|\,|\varepsilon|\,/\lambda_{\min}(Q) \leq \|X\| \qquad (38)$$

Thus X becomes small in the area defined by (38). As a result, the X ultimately enters into the ball with the radius of $2\left|J^{-1}\right|\|P_2\|\,|\varepsilon|\,/\lambda_{\min}(Q)$.

4. Direct Adaptive Fuzzy

Control law (17) is a model-based control law. In many cases, the exact model of robot is not available and we have uncertainty in dynamic equation of robot manipulators. Direct adaptive fuzzy method can control nonlinear systems along with uncertainty with high accuracy. Hence, in this section, design of direct adaptive fuzzy control for rigid robot is presented and it is applied on flexible joint robot manipulator with compensator (18).

From dynamic equation of (14) we have

$$(rD + Jr^{-1})\ddot{q} + (rC + Br^{-1})\dot{q} + rG + \phi + \eta = \tau_m \qquad (39)$$

Where ϕ is the vector of external disturbances and η is expressed as (15). Applying control law (16) yields

$$(rD + Jr^{-1})\ddot{q} + (rC + Br^{-1})\dot{q} + rG + \phi = \tau_{m_1} + \tau_{m_2} - \eta \qquad (40)$$

Where τ_{m_2} is (24). For designing τ_{m_1} first we omit $(rD + Jr^{-1})$ because direct adaptive fuzzy controllers are implemented on systems with constant input coefficient while in torque model it is variable.

According to (40) we can write

$$\ddot{q} - \ddot{q} + (rD + Jr^{-1})\ddot{q} + (rC + Br^{-1})\dot{q} + rG + \phi = \tau_{m_1} \qquad (41)$$

Where $\ddot{q}$ is added and then subtracted. Hence, we have

$$\ddot{q} + a\dot{q} + \mu = \tau_{m_1} \qquad (42)$$

Where a is a diagonal matrix and μ is the vector of lumped uncertainty [5] expressed by

$$a = (rC + Br^{-1}) \tag{43}$$

$$\mu = \left(rG + \left((rD + Jr^{-1}) - I \right) \ddot{q} + \phi \right) \tag{44}$$

According to (42), the dynamics of the ith joint in the scalar form is represented as

$$\ddot{q} + a\dot{q} + \mu = \tau_{m_1} \tag{45}$$

Where the presented $\ddot{q}, \dot{q}$ and μ are the ith elements of vectors respectively. The coefficient a is the ith elements of the matrix. The design procedure is similar to previous section. The decentralized controller has two inputs of e and $\dot{e}$ and an output τ_{m_1} which is torque-force. Three membership functions of P, Z and N the same as in Figure 1 are given to every input. The Gaussian membership functions are in the form of (23) with the centers of $\hat{y}$. As a result of using fuzzy rules in the structure (21) and Table 1, the Mamdani-type inference engine, the singleton fuzzifier, and the center average defuzzifier, τ_{m_1} is calculated as

$$\tau_{m_1} = \hat{y}^T \psi(e, \dot{e}) \tag{46}$$

Where $e = q_d - q$ is the tracking error. Applying control law (46) to system (45) yields

$$\ddot{q} + a\dot{q} + \mu = \hat{y}^T \psi(e, \dot{e}) \tag{47}$$

Where $\hat{y}$ is the estimation of y used into a fuzzy system, $y^T \psi(e, \dot{e})$ that approximates the following function based on the universal approximation theorem of fuzzy systems as

$$\left(\ddot{q}_d + k_1(\dot{q}_d - \dot{q}) + k_2(q_d - q) + a\dot{q} + \mu \right) = y^T \psi(e, \dot{e}) + \varepsilon \tag{48}$$

Where ε is the approximation error, coefficients $k_1 > 0$ and $k_2 > 0$.

In order to obtain the adaptive law, we form the tracking system from (47) and (48) as

$$\ddot{q}_d - \ddot{q} + k_1(\dot{q}_d - q) + k_2(q_d - q) = (y - \hat{y})^T \psi(e, \dot{e}) + \varepsilon \tag{49}$$

The state space equation in the tracking space is obtained using (49) as

$$\dot{E} = AE + Bw \tag{50}$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}, \quad w = (y - \hat{y})^T \psi(e, \dot{e}) + \varepsilon \tag{51}$$

The positive definite function V is suggested as

$$V = \frac{1}{2} E^T P E + \frac{1}{2\alpha}(y - \hat{y})^T(y - \hat{y}) \tag{52}$$

Where the constant $\alpha > 0$, P and Q are the unique symmetric, and positive definite matrices satisfying matrix Lyapunov equation (32).

$\dot{V}$ is calculated using (50)–(52) and (32) as

$$\dot{V} = -\frac{1}{2} E^T Q E + \left(E^T P_2(y^T - \hat{y}^T)\psi(e, \dot{e}) - \frac{1}{\alpha}(y^T - \hat{y}^T)\dot{\hat{y}} \right) \tag{53}$$

Where $P2$ is the second column of P. Equation (53) can be rewritten as

$$\dot{V} = -\frac{1}{2}E^T Q E + (y^T - \hat{y}^T)\left(E^T P_2 \psi(e, \dot{e}) - \frac{1}{\alpha}\dot{\hat{y}}\right) + E^T P_2 \varepsilon \qquad (54)$$

If the adaptive law is given by

$$\dot{\hat{y}} = \alpha E^T P_2 \psi(e, \dot{e}) \qquad (55)$$

As a result of applying adaptive rule (55)

$$\dot{V} = -\frac{1}{2}E^T Q E + E^T P_2 \varepsilon \qquad (56)$$

The tracking error reduces if $\dot{V} < 0$. Hence

$$E^T P_2 \varepsilon \leq 0.5 E^T Q E \qquad (57)$$

As mentioned in the previous section by considering the relation

$$\lambda_{\min}(Q)\|E\|^2 \leq E^T Q E \leq \lambda_{\max}(Q)\|E\|^2 \qquad (58)$$

Hence to satisfy $\dot{V} < 0$ we have

$$2\|P_2\|\,|\varepsilon|/\lambda_{\min}(Q) \leq \|E\| \qquad (59)$$

Equation (59) shows that the tracking error is bounded. In other words, it reduces until it enters to a ball with the radius of $2\|P_2\|\,|\varepsilon|/\lambda_{\min}(Q)$.

5. Stability Analysis

In order to verify the proposed control laws in section 4 and 3, stability analysis is presented. An assumption is required for this analysis.

Assumption: the desired trajectory must be smooth so that its derivative up to necessary is available and uniformly bounded.

According to the section 3, $\dot{F} < 0$ results in

$2|J^{-1}|\|P_2\|\,|\varepsilon|/\lambda_{\min}(Q) \leq \|X\|$. Since F is a positive definite function and $\dot{F} < 0$, in the area defined by (38)

$$2|J^{-1}|\|P_2\|\,|\varepsilon|/\lambda_{\min}(Q) \leq \|X\| \leq \|X(0)\| \qquad (60)$$

Where $X(0)$ is the initial value of X. Thus X is bounded where $X^T = [x_1\ x_2]$, $x_1 = \delta = r\theta - q$, $x_2 = r\dot{\theta} - \dot{q}$. Hence the right side of equation (19) is bounded. This system is a linear second order differential equation with two bounded inputs. Coefficients K_p and K_d are positive, therefore, according to Routh criteria, $e, \dot{e}, \ddot{e}$ are bounded. Since $q = q_d - e$, $\dot{q} = \dot{q}_d - \dot{e}$ and $q_d, \dot{q}_d$ are bounded in the assumption, thus boundedness of $q, \dot{q}$ are proved. On the other hand, given that X is bounded and goes to zero thus $\delta \to 0 \Rightarrow \theta \to r^{-1}q$ and $\dot{\delta} \to 0 \Rightarrow \dot{\theta} \to r^{-1}\dot{q}$ since $q, \dot{q}$ are bounded, $\theta, \dot{\theta}$ are bounded too.

To verify control law (46), $\dot{V} < 0$ implies that

$$2\|P_2\|\,|\varepsilon|/\lambda_{\min}(Q) \leq \|E\| \leq E(0) \qquad (61)$$

Therefore, $E^T = [q_d - q\ \dot{q}_d - \dot{q}]^T$ is bounded. The desired trajectory q_d and its derivative $\dot{q}_d$ are assumed bounded in Assumption, thereby boundedness of E obtains the boundedness of q and $\dot{q}$.

6. Simulation Results

Simulation 1: The proposed nonlinear control laws (17) and (18) are applied to control an electrical flexible-joint articulated robot manipulator as shown in Figure 2. The Denavit-Hartenberg (DH) parameters of articulated robot are given in Table 2, where the parameters θ_i, d_i, a_i and α_i are link offset, link length and link twist, respectively. The dynamic parameters of the manipulators are given in table 2, that is for the ith link, m_i is the mass, $r_{ci} = \begin{bmatrix} x_{ci} & y_{ci} & z_{ci} \end{bmatrix}^T$ is the center of mass expressed in the ith frame, and I_i is the inertia tensor expressed in the center of mass frame defined as:
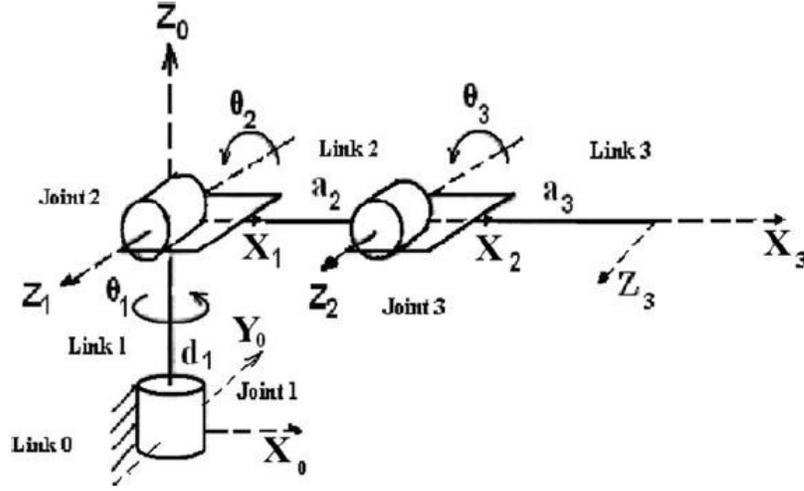
FIGURE 2. Symbolic Representation of Articulated Robot

$$I_i = \begin{bmatrix} I_{xx_i} & -I_{xy_i} & -I_{xz_i} \\ -I_{xy_i} & I_{yy_i} & -I_{yz_i} \\ -I_{xz_i} & -I_{yz_i} & I_{zz_i} \end{bmatrix} \tag{62}$$

The parameters of motors are given in Table 3. The desired position for every joint is given by

$$q_d = \frac{3\pi}{800}t^2 - \frac{\pi}{8000}t^3 \tag{63}$$

which is shown in Figure 3. All the controller have the same design parameters $K_p = 200, K_d = 5$. The gain γ for the compensator is set to $100, 10, 10$. Performance of control system is shown in Figure 4 while the joint tracking error is reduced well. The mean of integral of squared Error (MISE) was used as a performance index for evaluations and comparison. The MISE is defined as

$$\text{MISE} = \frac{1}{T} \int_0^T (e_1^2 + e_2^2 + e_3^2)dt \tag{64}$$

Where T is the time of operation, $e1, e2$ and $e3$ are the joint tracking errors for joint 1, 2 and 3, respectively. The MISE is about 1.057×10^{-5} which is ignorable. The control efforts are shown in Figure 5. The torque oscillates in the beginning due to fast response to high load and also due to compensating flexibility effect. The adapting parameters for compensator of second joint $\hat{y}$ are shown in Figure 6. It is noted that all nine adapting parameters reach to a constant value.

link	θ(rad)	$d(m)$	$a(m)$	α(rad)
1	θ_1	$d_1 = 280$	0	$\alpha_1 = \pi/2$
2	θ_2	0	$a_2 = 0.760$	0
3	θ_3	0	$a_3 = 0.930$	0

TABLE 2. The Denavit–Hartenberg Parameter

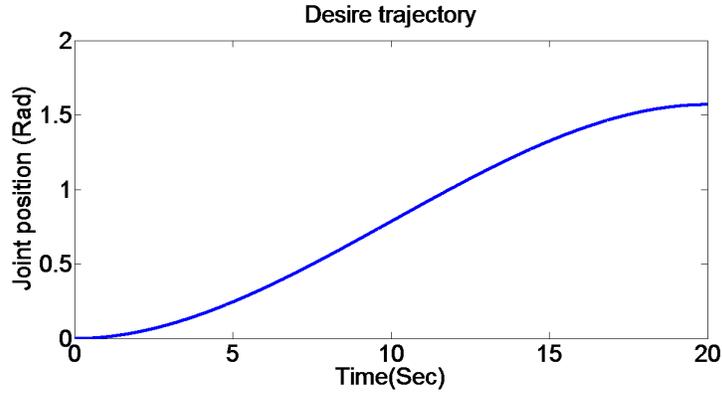|	x_i (m)	y_i (m)	z_i (m)	m_i (kg)	I_{xxi} (kgm^2)	I_{yyi} (kgm^2)	I_{zzi} (kgm^2)	I_{xyi} (kgm^2)	I_{xzi} (kgm^2)	I_{yzi} (kgm^2)
1	0	-0.22	0	19	0.34	0.36	0.31	0	0	0
2	-0.51	0	0	18.18	0.18	1.32	1.31	0	0	0
3	-0.67	0	0	10.99	0.07	0.92	0.93	0	0	0

TABLE 3. The Dynamic Parameters

FIGURE 3. The Desire Trajectory

Simulation 2: the control laws (46) and (18) are simulated and compared with feedback linearization. The gain α and γ for adapting the controllers 1, 2, 3 are set to 5000. The control performance in Figure 7 shows that tracking error is very small and the performance of tracking is very well. The MISE of the adaptive fuzzy controller is about 2.22×10^{-5} which is small and close to the MISE of feedback linearization. Figure 8 shows the control signal torque. The effects of the flexibility in joints appeared in the first part of responses are reduced well. The parameters $\hat{y}$ and $\hat{f}$ are adapted well as shown in Figure 9.
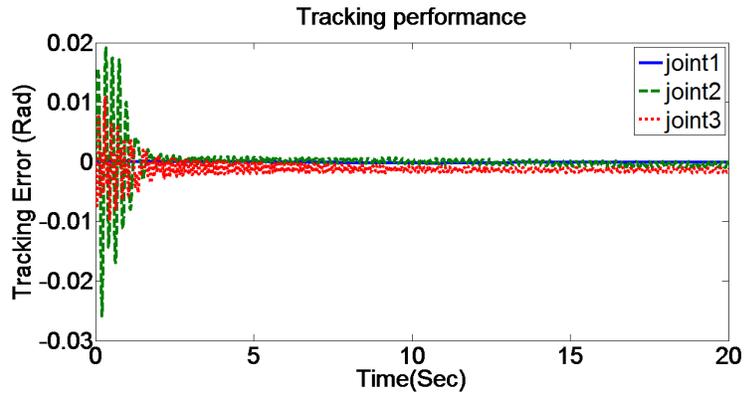
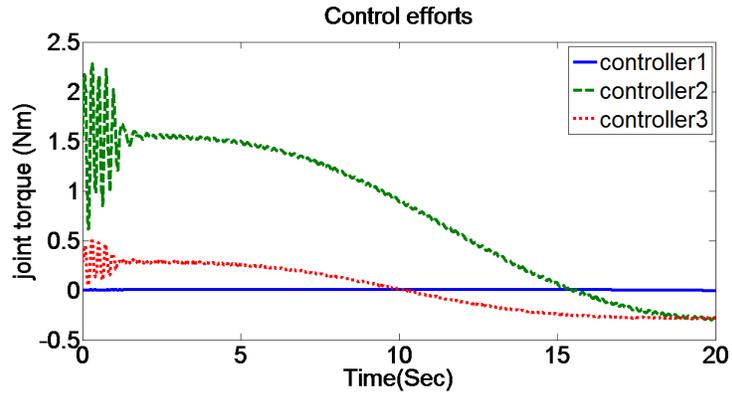FIGURE 4. Performance of Tracking Error
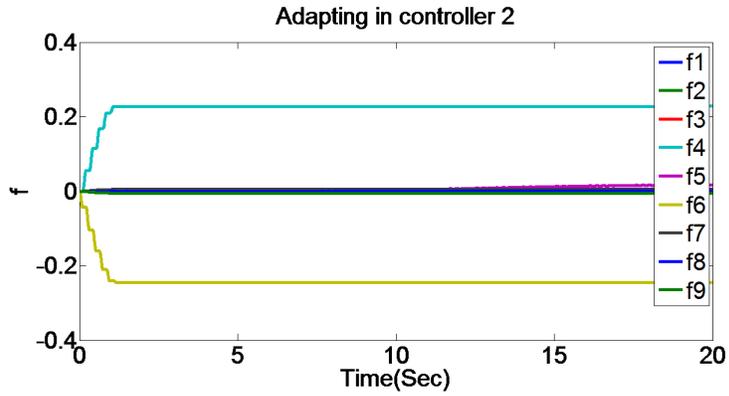

FIGURE 5. Joint Torque of Articulated Robot

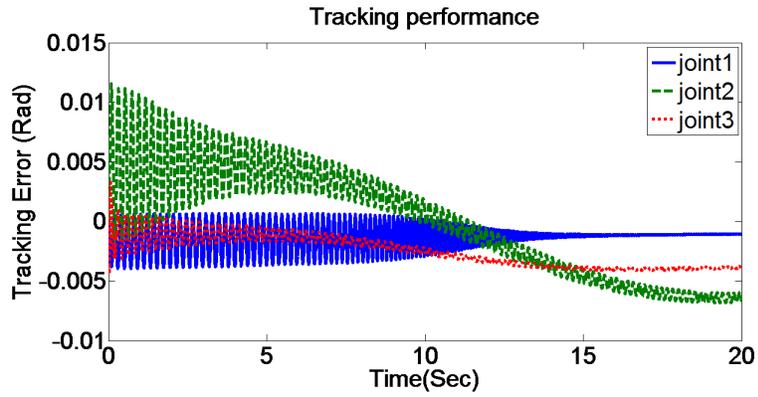FIGURE 6. Adaption Parameters of Joint 2
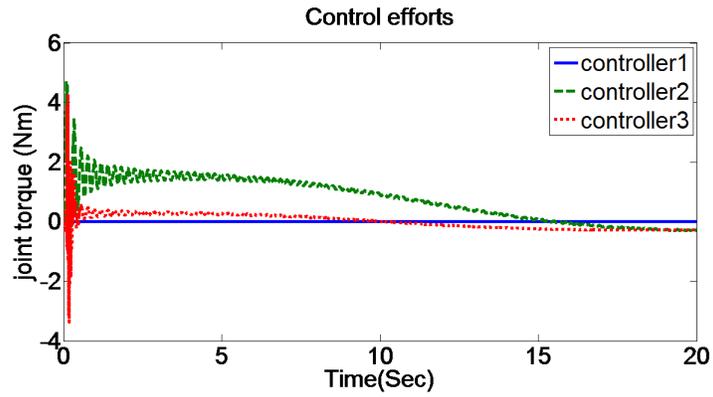
FIGURE 7. Performance of Tracking Error

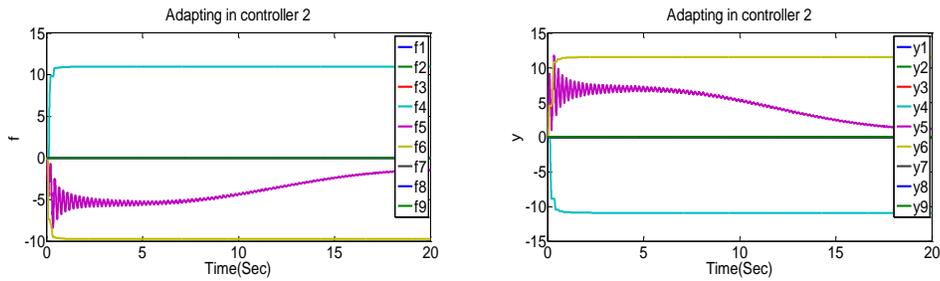FIGURE 8. Joint torque of Articulated Robot

FIGURE 9. Adaptation Parameters in Compensator (18) and
Controller (46) for Joint 2

7. **Conclusion**

A novel fuzzy controller has been designed for flexible-joint robots based on rigid model of robot manipulators. In order to compensate flexibility in joints, adaptive fuzzy controllers were designed for each joint. The proposed controllers are in the class of decentralized controllers. Feedback linearization method is applied for flexible-joint robot manipulators. Simulation results show the effectiveness of this method. Then, to overcome complexity of system, adaptive fuzzy controller has been designed for tracking a desire trajectory. Stability analysis has verified the proposed methods and simulation results show their effectiveness.

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