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A fuzzy reasoning method based on compensating operation and its application to fuzzy systems

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Abstract

In this paper, we present a new fuzzy reasoning method based on the compensating fuzzy reasoning (CFR). Its basic idea is to obtain a new fuzzy reasoning result by moving and deforming the consequent fuzzy set on the basis of the moving, deformation, and moving-deformation operations between the antecedent fuzzy set and observation information. Experimental results on real-world data sets show that proposed method significantly improve the accuracy and time performance of fuzzy neural network learning.

Keywords: Compensating Fuzzy Reasoning; Moving-Deformation Operation; Fuzzy System; Fuzzy Modus Ponens; Fuzzy Modus Tollens.

1 Introduction

Since Zadeh [25] has made the conception of fuzzy sets and membership function(1965), it has been widely used in various application fields, such as fuzzy control, fuzzy data mining, fuzzy expert system, and so on. The research of fuzzy reasoning aimed at more closely simulation of the ability of human thinking becomes a fundamental and essential part in the fuzzy theory. So far, a number of authors have proposed various principles and methods of the fuzzy reasoning based on different ideas. The most basic fuzzy reasoning models are fuzzy modus ponens (FMP) and fuzzy modus tollens (FMT).

FMP: Given the input "x *is* A' " and fuzzy rule "*if* x *is* A *then* y *is* B ", try to deduce a reasonable output "y *is* B' ".

FMT: Given the input "y *is* B' " and fuzzy rule "*if* x *is* A *then* y *is* B ", try to deduce a reasonable output "x *is* A' ".

Here A and A' are two fuzzy sets defined on an universe X, and B and B' are two fuzzy sets defined on another universe Y. The fuzzy reasoning method for solving FMP and FMT problems was first proposed by Zadeh [26], called the compositional rule of inference (CRI). In this method, a given fuzzy rule "*if* x *is* A *then* y *is* B " is expressed as a fuzzy relation through some fuzzy implication, and the reasoning result B' is calculated from the input A' by using the compositional operation of fuzzy relation. Although the CRI method has been successfully applied in many areas, it lacks clear logical basis and has some imperfections [21]. For example, the CRI method is not reductive [10]. In other words, if A' is equal to A , it does not always infer that B' is equal to B as we expect. To improve the CRI method, Wang [20] established the Triple Implication Principle(TIP) for fuzzy reasoning and proposed the full implication Triple I method based on R_0-implication operator. The TIP method can be considered as a complement of the CRI method, it also provides the criteria to choose the appropriate implication operator. Since the introduction of the TIP method, many studies have discussed the variants of Triple I method and its applications, including reverse Triple Implication method [27], Triple I based on first order logical system [13], Triple Implication method for interval-valued fuzzy reasoning [8], parametric Triple Implication method [9] and α-Triple Implication method [14], and so on.

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However, the TIP method and its variants cannot be applied to fuzzy control [3]. Mamdani [11] proposed a fuzzy reasoning method that uses the minimum operator and the sup-min composition instead of the implication operator in the classical boolean logic. Although the use of minimum operation is contrary to intuition, the Mamdani-type fuzzy reasoning has been very successfully applied to applications of fuzzy control. Takagi and Sugeno [16] proposed Takagi-Sugeno (T-S) fuzzy model, in which the antecedent consists with the fuzzy sets and the consequent is a linear functions of the fuzzy variables. This method has been widely used in fuzzy control and fuzzy identification so far. Some scholars investigated similarity-based fuzzy reasoning methods. Unlike the CRI method or Triple I method, it does not require the construction of a fuzzy relation between input and output fuzzy data, and it is conceptually clearer than CRI [19]. Turksen et al. [17, 18] proposed approximate analogical reasoning schema (AARS) based on similarity measure which exhibits the advantage of fuzzy sets theory and analogical reasoning. Chen [1, 2] presented two different methods for medical diagnosis problems based on the cosine angle between the two vectors. In the study of Yeung [23, 24], the similarity measure is based on the degree of subsethood between the input information and the antecedent. They also compared and analyzed six similarity-based fuzzy reasoning methods. Wang and Meng [19] defined the fuzzy similarity measure as a generalization of the similarity measure and proposed a novel fuzzy reasoning method, called fuzzy similarity reasoning. The most commonly used fuzzy reasoning methods in applications are still CRI method and TS fuzzy model, although they are very simple and have significant disadvantages [3]. However, these methods do not explicitly use the relation between the antecedent of rule and the observation (input) in the reasoning process. For example, in the FMP model, it is natural to believe $B^{'}$ should be close to B if $A^{'}$ is close to A. This means that the relation between A and $A^{'}$ should be also taken into account in the fuzzy reasoning process to get the reasoning result $B^{'}$. Although the similarity-based fuzzy reasoning methods deduce $B^{'}$ by modifying the consequent B with a modification function based on the similarity between A and $A^{'}$, the final reasoning results strongly depend on the similarity measure and the modification function. In 1992, Hans Hellendoorn proposed the generalized modus ponens (GMP) considered as a functional approach, in "Fuzzy Sets and Systems" 46(1): February (1992) 29-48. In [5], author mentioned that the GMP is a fuzzy reasoning rule, which is as follows. A lot of the criteria for this fuzzy reasoning rule were presented, there are 3 basic assumptions for dealing with the GMP, i.e., (1) the fuzzy rule is represented by a fuzzy relation, (2) Antecedent can be strengthened or weakened to obtain new conclusion $B^{'}$, and (3) the conclusion $B^{'}$ is obtained by the max-min compositional rule of inference. A number of theorems have shown that these 3 assumptions are not suitable with the criteria. Furthermore, construction of an implication rule to satisfy (2) and (3) is difficult. Therefore, (3) it must be modified into some functional relation.

In this paper, we intend to develop a new fuzzy reasoning method, called Compensating Fuzzy Reasoning (CFR).

This paper is organized as follows. In Section 2 we introduce the basic idea of compensating fuzzy reasoning method and formulate the FMP and FMT based on it. We also describe the logical properties of CFR method. And then we analyze about the mapping, linguistic modifier, and the role of moving, deformation, and moving-deformation operation. In Section 3 we compare proposed method with the previous methods. And Section 4 shows the experiment results of the proposed method through the fuzzy neural network on the precipitation data and security situation data, and conclusions are drawn in Section 5.

2 Principle of Compensating Fuzzy Reasoning

In this Section, we describe the reasoning model of FMP and FMT, which are the most basic form of fuzzy reasoning, from a new perspective, and consider the logical properties of them.

2.1 Basic Idea of New Compensating Fuzzy Reasoning

Given the fuzzy rules and the observation (input) data obtained from crisp input information, there are two kinds of relationships between the observation and the antecedent (consequent) of the fuzzy rule, i.e., moving and deformation. These relationships can be reflected in the antecedent (consequent) of the fuzzy rule to obtain a new consequent (antecedent) fuzzy set. We call this idea the Compensating Fuzzy Reasoning (CFR) or the Compensating rule of fuzzy reasoning. This consists of Moving method, Deformation method, and, Moving and Deformation method. Unlike the consequent fuzzy set obtained by using Zadeh's compositional rule of inference (CRI) is a non-regular convex set, our reasoning method derives a regular convex set by the compensating operation. In general, the simplest fuzzy rule is expressed in the form of formula (1) for FMP (resp. FMT). Given the observation fuzzy set $A^{'}$ (resp. $B^{'}$), the process to get the fuzzy reasoning result $B^{'}$ (resp. $A^{'}$) can be written as formula (2) for FMP (resp. FMT).

$$\textit{FMP-Rule: if } x \textit{ is } A \textit{ then } y \textit{ is } B \text{ and } \textit{FMT-Rule: if } y \textit{ is } \overline{B} \textit{ then } x \textit{ is } \overline{A} \tag{1}$$

$$\text{FMP-CFR:}(A \leftrightarrow A') \Rightarrow (B \leftrightarrow B') \text{ and } \text{FMT-CFR:}(B \leftrightarrow B') \Rightarrow (A \leftrightarrow A') \tag{2}$$

where symbol "$\leftrightarrow$ " indicates moving and deformation operation of the fuzzy set. Formula (2) implies that a new fuzzy reasoning result B' (resp. A') is obtained by the operations of moving and deforming the consequent fuzzy set B (resp. A) in response to the moving and deformation relationships between the antecedent fuzzy set A (resp. B) and the observation fuzzy set A' (resp. B') for FMP(resp. FMT). Note that A' (resp. B') is observation convex and normal fuzzy set for FMP (resp. FMT). This is obtained by moving and deformation based on input information x_0 (resp. y_0) for FMP(resp. FMT). The relationship between the antecedent fuzzy set and the observation includes the following three cases: the deformation without moving, the moving without deformation, and the combination of the moving and deformation. The type of operation is determined according to above relationship. Compositional rule of inference proposed by Zadeh has a great significance as a reasoning reflecting the human thinking. This type of reasoning is based on the principle that the fuzzy reasoning result is calculated by the compositional operation according to the fuzzy implication as shown in formula (3).

$$\text{FMP-CRI: } A' \circ (A \to B) \Rightarrow B' \text{ and } \text{FMT-CRI: } (A \to B) \circ B' \Rightarrow A' \tag{3}$$

$$R = (A \to B) \tag{4}$$

As shown formula (3) and (4) the fuzzy implication $(A \to B)$ corresponds to a fuzzy relation R that may be defined in several ways. The fuzzy reasoning result depends on how the fuzzy relation is defined. When the center of the observation fuzzy set expressing input information is fixed, a well-defined fuzzy relation leads to a reasoning result that is consistent with human thinking. However, if the center of observation fuzzy set changes, it is difficult to obtain meaningful reasoning result. For example, let us consider the process of deriving the degree of ripeness from the color of the tomato. (See Figure 1)

Fuzzy Rule: If a tomato is A (red) then the tomato is B (ripe)
Observation: This tomato is A' and *Conclusion: This tomato is B'*

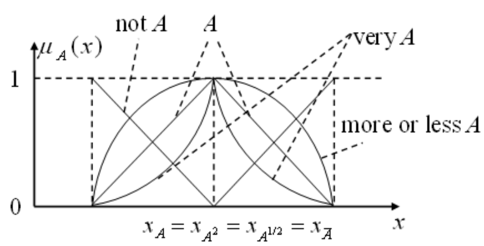


Figure 1: Observation of fuzzy sets with the same center

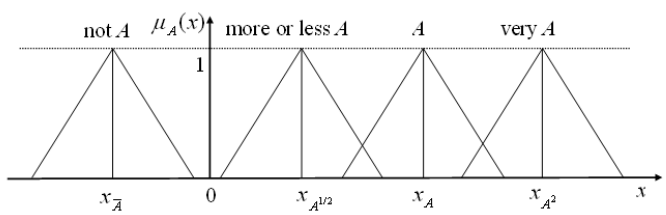


Figure 2: Observation of fuzzy sets with the different centers

It is desirable that when the observation A' takes "red A", "very red $A = A^2$ ", "more or less red $A = A^{1/2}$", and "not red $A = 1 - A$", the fuzzy reasoning result B' is obtained as "ripe B", "very ripe $B = B^2$", "more or less ripe $B = B^{1/2}$", and "not ripe $B = 1 - B$ ", respectively. For observation fuzzy sets whose center does not change, as shown in Figure 1, the fuzzy reasoning based on the compositional rule of inference gives us satisfactory reasoning results.

In the real world, it is more natural to express fuzzy sets from the viewpoint of the change of light wave length of the tomato color. That is, it is preferable that different observations are expressed by moving and deforming the centers of the fuzzy sets on the axis in the horizontal direction. (See Figure 2) Nonetheless, fuzzy reasoning based on

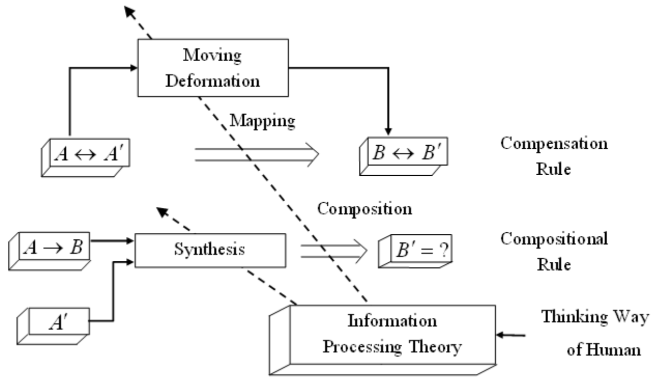


Figure 3: Zadeh's CRI and proposed CFR

the compositional rules does not yield meaningful results for observation fuzzy sets with different centers. Therefore, we propose a fuzzy reasoning method based on new perspective that reasoning result B' can be derived as a result of moving or deforming consequent fuzzy set B, because the observation A' is a moved or deformed version of the input fuzzy set A . The diagrammatic explanation of CRI and CFR is shown in Figure 3.

2.2 Fuzzy Modus Ponens and Fuzzy Modus Tollens based on CFR

2.2.1 FMP based on CFR Principle

The fuzzy reasoning scheme of the FMP of CFR based on the first formula of formula (1) and (2) is as follows.

Step 1: For the given observation fuzzy set A' obtained by crisp input information $x_0 \in X$, determine the moving and deformation operation F_X between the antecedent fuzzy set A and the observation A'.

$$\mu_{A'}(x) = F_X(\mu_A(x)) \tag{5}$$

where $A, A' \in F(X)$ are fuzzy set defined on the universe of discourse X and $x_0 \in X$ is antecedent variable, $F(X)$ is the set of all fuzzy sets on X , and F is a moving, deformation, and moving-deformation operation applied to the antecedent fuzzy set A to obtain the given Premise A' from the crisp input $x_0 \in X$.

Step 2: Calculate the fuzzy reasoning result B' by applying the moving-deformation operation F_Y to the consequent fuzzy set B .

$$\mu_B(y) = \begin{cases} F_Y(\mu_{B'}(y)), A \cap A' \neq \Phi \\ 0, A \cap A' = \Phi \end{cases} \tag{6}$$

where $B, B' \in F(Y)$ are fuzzy set defined on the universe of discourse Y, $y_0 \in Y$ is consequent variable, $F(Y)$ is the set of all fuzzy sets on Y, and F_Y is a moving, deformation, and moving-deformation operation applied to the consequent fuzzy set B to obtain the reasoning result B'. Note that both F_X and F_Y are the projections between the fuzzy sets. The moving-deformation operation F_Y may be defined in correspondence with the moving-deformation operation F_X in the antecedent part, which performs the compensating action in the consequent part. The reasoning results depend on how F_Y is defined. In practical applications, the moving-deformation operation F_Y can be defined in accordance with the actual situation, which makes the reasoning method valuable and flexible.

The FMP based on CFR has an important characteristic, which is as follows.

$$A' \cap A \neq \Phi \Rightarrow B' \neq \Phi, A' \cap A = \Phi \Rightarrow B' = \Phi \tag{7}$$

This means that if the observation does not match the antecedent of the fuzzy rule at all, it ignores the moving-deformation relationship in the antecedent and does not yield a valid reasoning result. The role of F_X and F_Y is considered in the subsection 2.5.

2.2.2 Fuzzy Modus Tollens based on CFR

The fuzzy reasoning scheme of FMT of CFR based on the second formula of formula (1) and (2) is represented as follows.

Step 1: For the fuzzy set B' obtained by the given crisp observation $y_0 \in Y$, determine the moving-deformation operation F_Y between the consequent fuzzy set B and the observation B' , where $B, B' \in F(Y)$.

$$\mu_{B'}(y) = F_Y(\mu_B(y)) \tag{8}$$

Step 2: Calculate the fuzzy reasoning result A' by applying the moving- deformation operation F_X to the antecedent fuzzy set A , where $A, A' \in F(X)$.

$$\mu_{A'}(x) = \begin{cases} F_X(\mu_A(y)), B \cap B' \neq \Phi \\ \quad 0, \qquad B \cap B' = \Phi \end{cases} \tag{9}$$

In formula (8) and formula (9), the meaning of each symbol is the same as in the FMP. The moving, deformation, and moving-deformation operation F_X performs the compensating action in the antecedent part. The role of moving-deformation operation F_X and F_Y is considered in the subsection 2.5. Similar to formula (7), the following fact holds also in the FMT.

$$B' \cap B \neq \Phi \Rightarrow A' \neq \Phi \quad or \quad B' \cap B = \Phi \Rightarrow A' = \Phi \tag{10}$$

Let us consider the reasoning process in detail about the FMT. Assume that the fuzzy sets $A, A' \in F(X)$ are normal and convex, and the following relation holds between them.

$$A' = \int_{A' \in F(X)} \mu_{(A')^\alpha}(x)/x = \int_{A \in F(X)} \mu_{A^\alpha}(x)/(x + \triangle x) \tag{11}$$

We call the quantities $\triangle x$ and α as the moving amount and the deformation index of the antecedent, respectively. Assume that $B, B' \in F(Y)$ are also normal and convex. Then, the fuzzy set B' of fuzzy reasoning result is gained by a moving-deformation operation on the consequent fuzzy set B , for FMP, as shown in formula (12).

$$B' = \int_{B' \in F(Y)} \mu_{(B')^\beta}(y)/y = \int_{B \in F(Y)} \mu_{B^\beta}(y)/(y + \triangle y) \tag{12}$$

where $\triangle y$ and β are the moving amount and the deformation index of the consequent, which are determined from $\triangle x$ and α , respectively. In the subsection 2.5, deformation index α and β for the FMP and FMT is considered. The moving amount of the consequent $\triangle y$ is calculated from $\triangle x$ as follows.

$$\triangle y = \begin{cases} 0, \triangle x = 0 \\ f(\triangle x), \triangle x \neq 0 \end{cases} \tag{13}$$

Where $f : X \to Y$ is the mapping defined according to the relationship between the universes of discourses X and Y . In general, mapping f is set so that moving amount $\triangle x$ and $\triangle y$ have a directly (or inverse) proportional or nonlinear relationship. In the subsection 2.5.1, we analyze for the mapping f for FMP and FMT.

2.3 Logical Properties of New CFR

In this subsection we consider logical property of qualitative criteria in the fuzzy reasoning method presented above. If any reasoning method is useful, it should satisfy the qualitative criteria that are consistent with human thinking. Mizumoto and Zimmermann [12] compared the fuzzy reasoning methods using qualitative criteria as shown in Table 1 for FMP and Table 2 for FMT. Our proposed method based on CFR satisfies the qualitative criteria of the fuzzy reasoning.

Theorem 2.1. *The FMP based on CFR satisfies the qualitative criteria of the fuzzy reasoning given in Table 1.*

Proof. Using formula (11) and (12), we can calculate the fuzzy reasoning result B' for a given observation fuzzy set A'. In the case of $A' = A$, we know that α=1 and $\triangle x$=0 since $A' = \int_{A\in F(X)} \mu_A(x)/x$. So it is obvious that β=1 and $\triangle y$=0, therefore the fuzzy reasoning result is $B' = \int_{B'\in F(Y)} \mu_{B^\beta}(y)/y = \int_{B\in F(Y)} \mu_{B^\beta}(y)/(y + \triangle y) = \int_{B\in F(Y)} \mu_B(y)/y = B$. Thus criterion 1 in Table 1 is satisfied. In Table 1, criterion 2 requires that either "*y is very B* " or "*y is B*" should be obtained as consequent from observation "*x is very A* ". In our proof, "*y is very B* " is obtained, so though *y is B* , criterion 2 is satisfied. In case of observation "*x is more or less A* ", if either " *y is more or less B*" or "*y is B*" is obtained, then criterion 3 is satisfied. In our method, when $\alpha = 1/2$, $\triangle x = 0$ then $\beta = 1/2, \triangle y = 0$. Thus the fuzzy reasoning conclusion B' is obtained, $B' = \int_{B'\in F(Y)} \mu_{B^\beta}(y)/(y + \triangle y) = \int_{B\in F(Y)} \mu_{B^{1/2}}(y)/y = \sqrt{B} = \textit{more or less } B$

From this, though "*y is B*", criterion 3 is satisfied. Lastly, in case of observation "*x is not A*", moving amount $\triangle x$=0, and deformation index α =not, so $A' = \int \mu_{A^\alpha}(x)/x = \int \mu_{A^{not}}(x)/x = \int \mu_{\bar{A}}(x)/x$. And the deformation index $\beta = \alpha = \textit{not}$, moving amount $\triangle y = \triangle x$=0, therefore, the fuzzy reasoning result is obtained as follows.

$$B' = \int_{B'\in F(Y)} \mu_{B^\beta}(y)/(y + \triangle y) = \int_{B\in F(Y)} \mu_{\bar{B}}(y)/(y + \triangle y) = \int_{B\in F(Y)} \mu_{\bar{B}}(y)/(y) \neq B$$

In case that A' *is not A* , if either "*y is not B* " or "*y is unknown*" is obtained, qualitative criteria must be satisfied. According to our method, "*y is not B* " is obtained, so criterion 4 is satisfied. Consequently, Theorem 2.1 satisfies qualitative criteria for FMP. Thus this Theorem 2.1 is true. □

Criteria	Observation A'	Conclusion B'
criterion 1	*x is A*	*y is B*
criterion 2	*x is very A*	*y is very B or y is B*
criterion 3	*x is more or less A*	*y is more or less B* or *y is B*
criterion 4	*x is not A*	*y is not B* or *y is unknown*

Table 1: Qualitative criteria for FMP

Criteria	Observation B'	Conclusion A'
criterion 1	*y is not B*	*x is not A*
criterion 2	*y is not very B*	*x is not A or x is not very A*
criterion 3	*y is not more or less B*	*x is not A* or *x is not more or less A*
criterion 4	*y is B*	*x is A* or *x is unknown*

Table 2: Qualitative criteria for FMT

Theorem 2.2. *The FMT based on CFR satisfies the qualitative criteria of the fuzzy reasoning given in Table 2.2.*

As known in the seconds of the formula (1) and (2), FMT is opposite to FMP. Thus we can easily get the proof of Theorem 2.2 similar to Theorem 2.1. Its proof is omitted here.

Now let us consider the logic validness of the CFR method based on our Theorems. First, the CFR method is based on the deductive reasoning like in the FMP and the FMT based on CRI. (See Figure 4) In the CRI method, when a fuzzy system consisting of fuzzy rules is given and an input is entered into the system, the new knowledge (reasoning result) is derived by using the form of the logic positive and negative thought. Here, the fuzzy reasoning result is determined by how to express the relationship between the input and output of the fuzzy system, and the way of expressing this relationship distinguishes each reasoning method. For the FMP based on the CFR, from the viewpoint that the input fuzzy set is the result of the moving and the deformation of the antecedent fuzzy set, the fuzzy reasoning result is also referred to as the moving and the deformation of the consequent fuzzy set. That is, the relationship between the input and the antecedent fuzzy set is deduced to the relationship between the reasoning result and the consequent fuzzy set. Second, the CFR method is based on the analogy reasoning. In the CFR method, the moving amount of the consequent has a directly (or inverse) relationship with that of the antecedent. Moreover, the fuzzy reasoning result obtained by the deformation of the consequent has a similarity with the input fuzzy set. This means that moving-deformation operation in the consequent of the fuzzy rule belongs to the form of the analogy reasoning. Therefore, it is possible to say that the CFR is a reasoning method embodying the human thinking way that combines the deductive reasoning and the analogy reasoning.

Let us consider a simple and typical example of fuzzy reasoning method based on CFR .

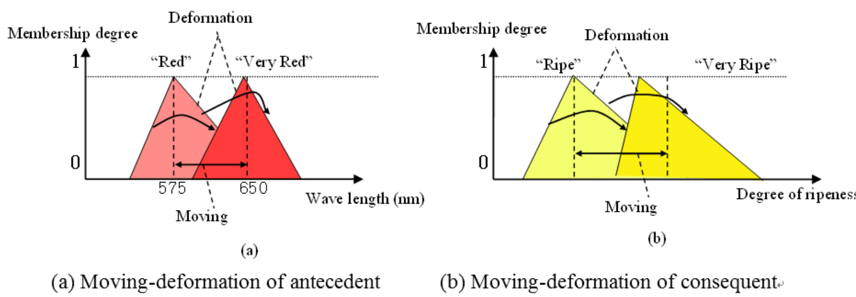


Figure 4: Example of fuzzy reasoning method based on CFR principle

Fuzzy Rule: If a tomato is red then the tomato is ripe
Observation: This tomato is very red
Conclusion: This tomato is very ripe

In Figure 4, the conclusion "tomato is very ripe" was obtained by reflecting of the moving-deformation relationship between the antecedent fuzzy set "red" of the fuzzy rule "If a tomato is red then the tomato is ripe" and the observation fuzzy set "very red" in the consequent part. Hence, there is the similarity between the fuzzy set "very red" in the antecedent and the fuzzy set "very ripe" in the consequent.

2.4 Computational Example for Different Input Information

In this subsection, we analyze the fuzzy reasoning results obtained by the FMP based on the Mamdani method and proposed CFR. In both methods the input information is placed on the left and right hand in the center of the antecedent fuzzy set. (See Figure 5)

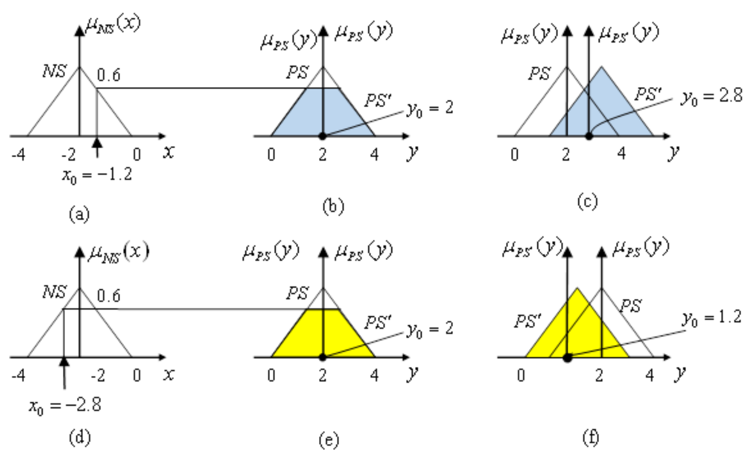


Figure 5: Fuzzy reasoning for different input information

Assume that the following fuzzy control rule is given: "If a temperature deviation x of the reaction tank is negative small (NS), then turn the motor valve y slightly clockwise (PS)". Let us consider the reasoning result of Mamdani-type fuzzy reasoning method. As shown in Figure 5(a), if the input information x_0 (temperature deviation) is -1.2 , then $\mu_{NS}(x_0) = \mu_{NS}(-1.2) = 0.6$. Therefore, the reasoning result PS' is the shaded area of Figure 5(b) and the defuzzified crisp value of the output y_0 (i.e., opening degree of motor valve) is 2. For the crisp input information $x_0 = -2.8$, since $\mu_{NS}(x_0) = \mu_{NS}(-2.8) = 0.6$, the fuzzy reasoning result PS' is the shaded area of Figure 5(e) and its defuzzified crisp value y_0 is also 2. In this case, the Mamdani-type fuzzy reasoning method gives the same reasoning result for different input. However, the CFR method differs from it. When the crisp input information x_0 is -1.2 , then , the CFR method gives the crisp output reasoning result $y_0 = y_{PS} + \triangle y = y_{PS} + \triangle x = 2 + 0.8 = 2.8$, and if crisp input information $x_0 = -2.8$, then $y_0 = y_{PS} + \triangle y = 2 - \triangle x = 2 - 0.8 = 1.2$. (See Figure 5(c) and Figure 5(f)). That is, different inputs yield different results. In the Mamdani-type fuzzy reasoning method, the same reasoning results are always obtained for arbitrary inputs belonging to the interval [-4, 0]. Using the CFR method, in contrast, leveled by -2, the bigger the input information, the bigger the defuzzified crisp value of PS' , and the smaller the input information, the smaller it becomes. In general, it is true that the several rules work to get the reasoning result for the given input information of any time in fuzzy control or fuzzy expert system. Since the reasoning result is obtained from several rules, the logical contradiction that different inputs have the same result for one rule was not questioned. However, it is more reasonable that different results are obtained for different inputs. Our CFR method is an improved general fuzzy reasoning method by which the different reasoning results are obtained from different input information.

2.5 Analysis of New Fuzzy Reasoning Method

In this section we analyze mapping f, deformation index α and β , and the role of moving, deformation, and moving-deformation operation presented in subsection 2.2 of this paper.

2.5.1 Analysis of Moving Amount $\triangle x$ and $\triangle y$ by Mapping f

Let us analyze the mapping $f : X \to Y$ in formula (13). In the case $\triangle x \neq 0$ in formula (13), $\triangle y = f(\triangle x)$ may be illustratively defined as follows.

$$\triangle y = f(\triangle x) = k \cdot \triangle x \tag{14}$$

where $\triangle y$ and $\triangle x$ are the same as mentioned above, and k is the proportional coefficient in the closed interval $[0, \ell]$, and ℓ is the maximum of $\triangle x$. For example, in case $k = 0.3, 1, 1.2$, $\triangle y = 0.3 \cdot \triangle x$, $\triangle y = 1 \cdot \triangle x$, $\triangle y = 1.5 \cdot \triangle x$. In the practical application, k is determined by designer or engineer according to the characteristics of the object and spot experience. Generally if $k > 1$, moving amount $\triangle y$ is increased, $k = 1$, $\triangle y = \triangle x$, and if $0 < k < 1$, it is decremented. Other definition of $\triangle y$ can be illustratively described as follows.

$$\triangle y = f(\triangle x) = (\triangle x)^k, \tag{15}$$

where k means increase or decrease of $\triangle x$. For example, in case $k = 0, 1, 2, 3$, $\triangle y = 1, \triangle y = \triangle x, \triangle y = (\triangle x)^2$, and $\triangle y = (\triangle x)^3$. In case $k = 0, \triangle y = 1$ means the moving amount of consequent is 1 though the antecedent fuzzy set has moving amount $\triangle x$. From this definition, it can be shown that in case $0 < \triangle x < 1$, $\triangle y$ decreases according to the increase of k, and in case $\triangle x > 1, \triangle y$ increases according to the increase of k. From the above two definitions, k can be determined with the experimental method. The definition of mapping f can be made variously according to the characteristics of system besides the above two methods. The above definition of moving amount may be applied to several branches such as water level control of water tank, stabilization control of inverted equilibrium, image processing, expert system, temperature prediction of furnace, medical diagnosis, and pattern recognition, and so on.

2.5.2 Analysis of the deformation index α and β for FMP and FMT

Let us analyze the deformation index α and β in the FMP and FMT. In fact, α of formula (11) is the deformation index or hedge for the observation fuzzy set A' , while β of formula (12) is that for the reasoning result set B' . Generally, it is taken as $\beta \neq \alpha$. If the antecedent fuzzy set A and consequent fuzzy set B is scaled in the closed interval, then we can consider as $\beta = \alpha$ for FMP. The fuzzy reasoning result B' is determined only by the moving operation when $\alpha \neq 1$ and $\triangle x \neq 0$, and is determined only by the deforming operation when $\alpha \neq 1$ and $\triangle x = 0$. On the other hand, if $\alpha \neq 1$ and $\triangle x \neq 0$, the moving and deforming operations can be applied together to obtain the fuzzy reasoning result. Consider the relation between α and β in FMP. In formula (11), α is the deformation index of the antecedent fuzzy set A . In other words, observation fuzzy set A' is obtained by the deformation of antecedent fuzzy set A by using α . Let $\beta = m \cdot \alpha$ for FMP, where m is called proportional coefficient. For example. If $m = 0, 1, 2$ then the deformation index $\beta = 0, \beta = \alpha$, and $\beta = 2\alpha$, therefore the new consequent fuzzy set B' in FMP, that is, fuzzy reasoning result is obtained variously as follows, respectively.

$$B' = \int_{B' \in F(Y)} \mu^0_{B'}(y)/y = \int_{B \in F(Y)} 1/(y + \triangle y)$$

$$B' = \int_{B' \in F(Y)} \mu^\beta_B(y)/y = \int_{B \in F(Y)} \mu_{B^\alpha}(y)/(y + \triangle y)$$

$$B' = \int_{B' \in F(Y)} \mu_B^{\beta}(y)/y = \int_{B \in F(Y)} \mu_{B^{2\alpha}}(y)/(y + \triangle y)$$

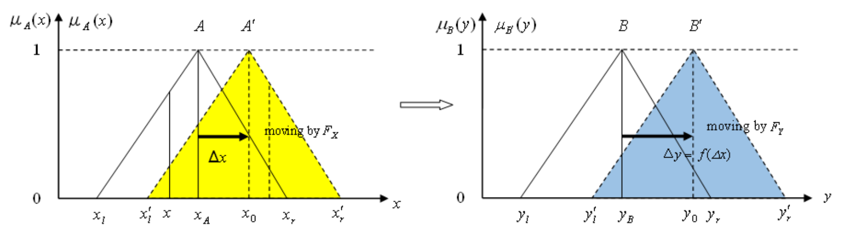
Then, the relation between the deformation index β and α in FMT is similar to that in FMP, so it is omitted here. In general, the deformation index $\alpha \neq \beta$, it can be illustratively defined as $\alpha = m \cdot \beta$ for FMT, where m is also called proportional coefficient. Therefore the designer should determine the reasonable α according to the characteristics of the object. Consequently in FMP and FMT, in case that fuzzy set A, B and A' (resp. B') of the fuzzy rule and antecedent(the given premise) are scaled in closed unit interval, it can be simply treated as $\beta = \alpha$ (resp. $\alpha = \beta$) for FMP(resp. FMT).

2.5.3 Analysis of the Role of Moving-Deformation Operation

Let us analyze the role of moving-deformation operation F_X and F_Y . Now, consider the formula (5), (6), (8) and (9). Formula (5) and (6) is applied in FMP, and formula (8) and (9) in FMT. First, consider the role of F_X for FMP in formula (5). In formula (5), observation fuzzy set A' is obtained by deformation, moving, or moving-deformation of antecedent fuzzy set A . That is, by the fuzzy set theory in paper [25], observation fuzzy set A' corresponds to the fuzzification of crisp input information. In formula (2), $A \leftrightarrow A'$ can be written as $A(x) \leftrightarrow A'(x)$ in detail. Consider the role of F_X in case that A moves to A' .

Proposition 2.3. *Suppose that input information $x_0 \in X$ is away from the center x_A of triangular-shaped fuzzy set A as much as $\triangle x$. When x_0 is crisp input information, A' can be obtained by moving A as much as $\triangle x = |x_0 - x_A|$.*

Proof. For this proof, role of F_X and F_Y in case of moving for FMP is illustratively shown in Figure 6.



(a) Role of F_X in case that A moves to A' (b) Role of F_Y in case that B moves to B'

Figure 6: Role of F_X and F_Y in case of moving for FMP

In Figure 6(a), x_A is the coordinate value of the center of triangular-shaped fuzzy set A, and x_0 is the coordinate value of new input information. On the basis of the idea in subsection 2.1, it can be shown that A moves to A' linearly. Using the distance between x_0 and x_A , and left width $w_{x\ell}$, right width w_{xr} of A , from formula (5) A' can be written as follows.

$$\mu_{A'}(x)=F_X(\mu_A(x))=\int_{A\in F(X)} \mu_A(x)/(x+\triangle x)=\int_{A\in F(X)} \mu_{A'}(x)/x_0$$
$$=\int_{x\in[x'_\ell,x_0]}(\tfrac{x-x'_\ell}{x_0-x'_\ell})/x \bigcup \int_{x\in[x_0,x'_r]}(\tfrac{x'_r-x}{x'_r-x_0})/x$$
$$=\int_{x\in[x_\ell,x_0]}(1+\tfrac{x-x_0}{w_{x\ell}})/(x+\triangle x) \bigcup \int_{x\in[x_0,x_r]}(1+\tfrac{x_0-x}{w_{xr}})/(x+\triangle x) \quad (16)$$

Therefore we can know that this Proposition 2.3 is right. □

Proposition 2.4. *For triangular-shaped fuzzy set A' obtained by the formula (16), fuzzy reasoning result B' is concluded by formula (17).*

$$\mu_{B'}(y)=\int_{y\in[y_\ell,y_B]}(1+\tfrac{y-y_B-\triangle x}{w_{y\ell}})/(y+\triangle x) \bigcup \int_{y\in[y_\ell,y_B]}(1+\tfrac{y_B-y+\triangle x}{w_{yr}})/(y+\triangle x) \quad (17)$$

Proof is abbreviated.(See Figure 6(b))

Next, let us consider the role of F_X in formula (5) in the case that observation A is deformed to fuzzy set A' obtained by crisp input x_0 as shown in Figure 7 (a).

Proposition 2.5. *For the given premise $A' = very\ A$, deformed fuzzy set A' is described by formula (18).*

$$\mu_{A'}(x) = \int_{x\in[x_\ell,x_0]}(1+\tfrac{x-x_0}{w_{x\ell}})^2/(x+\triangle x) \bigcup \int_{x\in[x_0,x_r]}(1+\tfrac{x_0-x}{w_{xr}})^2/(x+\triangle x) \quad (18)$$

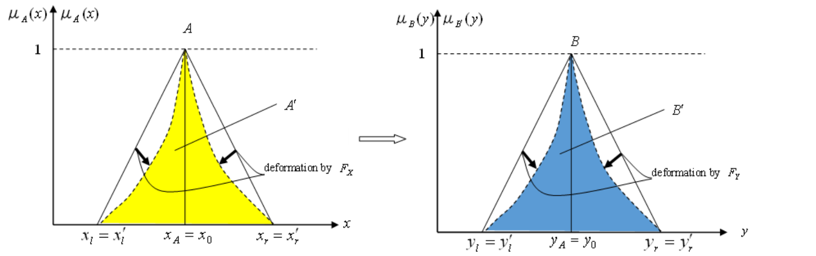


Figure 7: Role of F_X and F_Y in case of moving for FMP

Proof is abbreviated.(See Figure 7 (a))

Proposition 2.6. *For the given premise $A' = veryA$, fuzzy reasoning result, i.e., deformed fuzzy set B' is concluded by formula (19).*

$$\mu_{B'}(y)=\int_{y\in[y_\ell,y_B]}(1+\frac{y-y_B-\triangle x}{w_{y\ell}})^2/(y+\triangle x)\bigcup\int_{y\in[y_B,y_r]}(1+\frac{y_B-y+\triangle x}{w_{yr}})^2/(y+\triangle x) \tag{19}$$

Proof is abbreviated.(See Figure 7 (b)).

Then, consider the role of F_X in case that deformation and moving exist at the same time in formula (5).

Proposition 2.7. *For the given premise A'=more or less A , deformed fuzzy set A' is described as formula (20).*

$$\mu_{A'}(x)=\int_{x\in[x_\ell,x_0]}(1+\frac{x-x_0}{w_{x\ell}})^{\frac{1}{2}}/(x+\triangle x)\bigcup\int_{x\in[x_0,x_r]}(1+\frac{x_0-x}{w_{xr}})^{\frac{1}{2}}/(x+\triangle x) \tag{20}$$

Proof is abbreviated.(See Figure 8 (a))

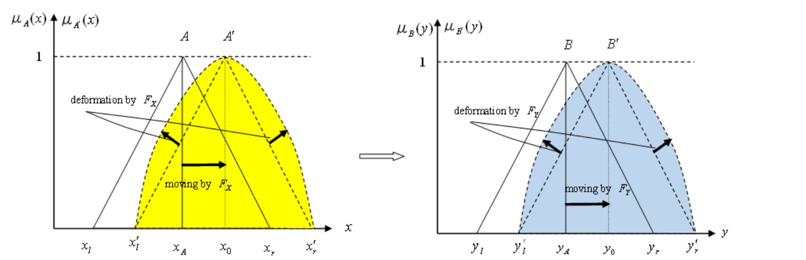


Figure 8: Role of F_X and F_Y in case of moving for FMP

Proposition 2.8. *For the given premise $A' = veryA$, the fuzzy reasoning result B' is concluded by formula (21).*

$$\mu_{B'}(y)=\int_{y\in[y_\ell,y_B]}(1+\frac{y-y_B-\triangle x}{w_{y\ell}})^{\frac{1}{2}}/(y+\triangle x)\bigcup\int_{y\in[y_B,y_r]}(1+\frac{y_B-y+\triangle x}{w_{yr}})^{\frac{1}{2}}/(y+\triangle x) \tag{21}$$

Proof is abbreviated.(See Figure 8 (b))

Next, let us analyze the role of F_Y and F_X for FMT presented by formula (8) and (9). This can be analyzed similar to the formula (5) and (6) for FMP. That is, F_Y plays a role that obtains the observation B' by moving and deformation, or moving-deformation of consequent fuzzy set B . This corresponds to the fuzzification of crisp input information in the general FMT. And the role of F_X in formula (9) can be analyzed similar to in formula (6). That is, in formula (9), The moving, deformation, and moving-deformation operation F_X plays a role that infer a new conclusion fuzzy set A' from A by moving amount $\triangle y$ and deformation of B' from B , where B' is obtained on the basis of F_Y in formula (8). In this paper, since the role of F_Y and F_X for FMT is similar for FMP, concrete expression is omitted. So far, we analyzed the role of F_X (resp. F_Y) and F_Y (resp. F_X) in FMP(resp. FMT) according to the of fuzzy reasoning based on compensating principle in case that input information fuzzy set A' (resp. B') is obtained by the moving, deformation, and moving-deformation operations of A (resp. B).

3 Adaption and Comparison to Several Fuzzy Systems

3.1 Mamdani Fuzzy System Based on CFR

In this subsection, we apply our CFR for the Mamdani fuzzy system [13], in which the consequents consist of fuzzy sets. Assume that the fuzzy system is given as formula (22).

R_j: if x_1 is P_{j1} ... , x_i is P_{ji}, ... , x_s is P_{js} then z_j=Q_j
Input: x_1 is P_1, ... , x_i is P_i, ..., x_s is P_s
Conclusion z is z^0=? (22)

where $x_i \in X_i$, $i = 1, 2, ..., s$ is the input variable, z, $z_0 \in Z$ is the output variable, $p_{ji} \in F(X_i)$, $i = 1, 2, ..., s$, $j = 1, 2, ..., n$ is the antecedent fuzzy set of j^{th} rule,$Q_j \in F(Z), j = 1, 2, ..., n$ is the consequent fuzzy set of j^{th} rule, $P_i \in F(X_i), i = 1, 2, ..., s$ is the i^{th} input fuzzy set, $Q' \in F(Z)$ is the conclusion fuzzy set, s is the number of inputs, and n is the number of rules. Without loss of generality, we assume that all of the fuzzy sets are triangular-shaped function. (See Figure 9) Our method consists of the following three steps:

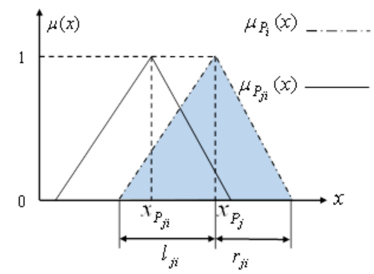


Figure 9: Moving operation of triangluar-shaped fuzzy sets

Step 1: Determine the moving amounts of antecedent for all fuzzy rules. First, calculate the moving amount between the i^{th} input triangular-shaped fuzzy set P_i and antecedent triangular-shaped fuzzy set p_{ji} of rule R_j as follows.

$$\triangle x_{ji} = \begin{cases} (x_{P_{ji}} - x_{P_i})/\ell_{ji}, x_{P_i} \leq x_{P_{ji}} \\ (x_{P_{ji}} - x_{P_i})/r_{ji}, x_{P_{ji}} \leq x_{P_i} \end{cases} \quad i = 1, 2, ..., s, j = 1, 2, ..., n \tag{23}$$

where ℓ_{ji} and r_{ji} are the left width and right width of antecedent fuzzy set P_{ji} , x_{p_i} and $x_{p_{ji}}$ are the centers of P_i and P_{ji} , respectively, as shown in Figure 9, and $\triangle x_{ji}$ denotes the moving amount between P_i and P_{ji} .

Next, determine the moving amount of antecedent as follows.

$$\triangle x_j = \frac{1}{S} \sum_{i=1}^{s} \varphi_i \triangle x_{ji}, j = 1, 2, ..., n \tag{24}$$

where φ_i is sign that reflects the proportional relationship between the i^{th} input variable x_i and the output variable z. If z is direct proportion to x_i , then φ_i=1 , else φ_i=-1 . For some j, if $|\triangle x_j| > 1$, then we assume that the input information does not match the j^{th} rule and exclude it from the calculation of the final reasoning result.

Step 2: Calculate the reasoning results for all fuzzy rules. Determine the moving amounts of consequent from moving amount $\triangle x_j, j = 1, 2, ..., n$ as follows.

$$\triangle z_j = F(\triangle x_j), j = 1, 2, ..., n \tag{25}$$

where F is a pre-defined function that reflects the moving action on the consequent and $\triangle z_j$ is the moving amount the consequent in j^{th} rule. Move the consequent fuzzy set Q_j to get the fuzzy reasoning result Q'_j for j^{th} rule.

$$Q'_j = \int_{Q'_j \in F(Z)} \mu_{Q'_j}(z)/z = \int_{Q_j \in F(Z)} \mu_{Q_j}(z)/(z + \triangle z_j), j = 1, 2, ..., n \tag{26}$$

Step 3: Calculate the final fuzzy reasoning result Q' . The final fuzzy reasoning result Q' is determined by the union of $Q'_j, j = 1, 2, ..., n$ shown as formula (27). The defuzzified crisp value of the fuzzy reasoning result can be calculated as formula (28).

$$Q' = \bigcup_{j=1|\triangle x_j|\le 1}^{n} Q'_j \tag{27}$$

$$z^0 = \int_{Q'\in F(Z)} z \cdot \mu_{Q'}(z)dz \Big/ \int_{Q'\in F(Z)} \mu_{Q'}(z)dz \tag{28}$$

For simplicity, the defuzzified value z^0 can be simply calculated as the arithmetic average of the centers of Q'_j.

$$z^0 = \frac{1}{n'} \sum_{i=1|\triangle x_j|\le 1}^{n'} (Z_{Q_j} + \triangle z_j) \tag{29}$$

where, n' denotes the number of rules participating in the computation of the reasoning result and z_{Q_j} denotes the center of Q_j . For example, let us consider two fuzzy rules with three inputs and one output.

R_1 : *if* x_1 *is* P_{11}, x_2 *is* P_{12}, x_3 *is* P_{13} *then* z *is* Q_1
R_2 : *if* x_1 *is* P_{21}, x_2 *is* P_{22}, x_3 *is* P_{23} *then* z *is* Q_2
Input : x_1 *is* P_1, x_2 *is* P_2, x_3 *is* P_3
Conclusion : z *is* z^0 =?

The fuzzy reasoning process is as follows. Using formula (23), we compute the moving amounts $\triangle x_{11}$, $\triangle x_{12}$ and $\triangle x_{13}$ between the antecedent fuzzy sets and the inputs for the rule R_1, where x_{p_1}, x_{p_2} and x_{p_3} are crisp input information, $x_{p_{11}}$, $x_{p_{12}}$ and $x_{p_{13}}$ are center of fuzzy sets P_{11}, P_{12} and P_{13} , respectively.

$$\triangle x_{11} = \begin{cases} (x_{P_{11}} - x_{P_1})/r_{11}, x_{P_1} \ge x_{P_{11}} \\ (x_{P_{11}} - x_{P_1})/\ell_{11}, x_{P_1} < x_{P_{11}} \end{cases} \tag{30}$$

$$\triangle x_{12} = \begin{cases} (x_{P_{12}} - x_{P_2})/r_{12}, x_{P_2} \ge x_{P_{12}} \\ (x_{P_{12}} - x_{P_2})/\ell_{12}, x_{P_2} < x_{P_{12}} \end{cases} \tag{31}$$

$$\triangle x_{13} = \begin{cases} (x_{P_{13}} - x_{P_3})/r_{13}, x_{P_3} \ge x_{P_{13}} \\ (x_{P_{13}} - x_{P_3})/\ell_{13}, x_{P_3} < x_{P_{13}} \end{cases} \tag{32}$$

Likewise, compute the moving amounts $\triangle x_{21}$, $\triangle x_{22}$, and $\triangle x_{23}$ for rule R_2 . Next, from formula (15), compute the moving amounts of antecedent $\triangle z_1$ and $\triangle z_2$.

$$\triangle z_1 = \frac{1}{3}(\pm\triangle x_{11} \pm \triangle x_{12} \pm \triangle x_{13}), \triangle z_2 = \frac{1}{3}(\pm\triangle x_{21} \pm \triangle x_{22} \pm \triangle x_{23}) \tag{33}$$

Using formula (26) and formula (29), compute final defuzzified reasoning result z^0, where Z_{Q_1} and Z_{Q_2} are center of fuzzy sets Q_1 and Q_2 , respectively.

$$z^0_1 = z_{Q_1} + \triangle z_1, z^0_2 = z_{Q_2} + \triangle z_2 \tag{34}$$

$$z^0 = \frac{1}{2}(z^0_1 + z^0_2) \tag{35}$$

Our method does not require complicated calculations and the computational complexity is $O(n)$. It does not include logical operations such as max or min, so mathematical analysis is convenient and easy to combine with other methods.

3.2 T-S Fuzzy System based on CFR

3.2.1 T-S Fuzzy Reasoning Process based on CFR

T-S fuzzy system model with s inputs are given as formula (36).

$$R_j: \text{ if } x_1 \text{ is } A_{j1}\ x_2 \text{ is } A_{j2}\ \ldots\ x_s \text{ is } A_{js} \text{ then } y_j = c_{j0} + c_{j1}\cdot x_1 + \ldots + c_{ji}\cdot x_j + \ldots + c_{js}\cdot x_s \quad (36)$$

where A_{ji} is the i^{th} antecedent triangular-shaped fuzzy set of the j^{th} rule, c_{i0} and c_{ij} are the coefficients of the consequent linear function $(i = 1, 2, ..., s, j = 1, 2, ..., n)$. Without loss of generality, we suppose that inputs x_i^0 of fuzzy system are crisp values.

The fuzzy reasoning process based on CFR for the T-S fuzzy system is presented as follows.

Step 1: Calculate the moving amounts $d_{ji}, i = 1, 2, ..., s, j = 1, 2, ..., n$ between the center of the antecedent fuzzy set A_{ji} and the input information x_i^0 by using formula (37).(See Figure 10)

$$d_{ji} = \begin{cases} (x_i^0 - x_{ji}^c)/(x_{ji}^r - x_{ji}^c), & x_{ji}^r > x_i^0 \geq x_{ji}^c \\ (x_{ji}^c - x_i^0)/(x_{ji}^c - x_{ji}^\ell), & x_{ji}^\ell < x_i^0 < x_{ji}^c \\ 1, & x_i^0 \leq x_{ji}^\ell \quad or \quad x_{ji}^r \leq x_i^0 \end{cases} \quad (37)$$

where x_{ji}^c , x_{ji}^r and x_{ji}^ℓ denote the center, right endpoint, and left endpoint of the triangular-shaped fuzzy set A_{ji},$(i = 1, 2, ..., s, j = 1, 2, ..., n)$ respectively, and x_i^0 is the crisp input value corresponding to i^{th} input variable as shown in Figure 10.

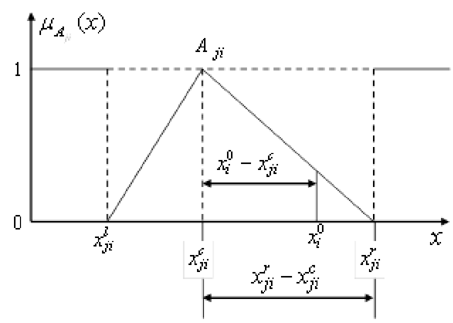


Figure 10: Calculation of the moving amounts with crisp input value

Step 2: Calculate the moving degree d_j in the j^{th} fuzzy rule for $j = 1, 2, ..., n$.

$$d_j = 1 - [d_{j1} \wedge d_{j2} \wedge \ldots \wedge d_{js}] \quad (38)$$

where, symbol $\wedge$ denotes the minimum or product operator.

Step 3: Construct a fuzzy rule subset I_{act} to participate in the calculation of the fuzzy reasoning result.

$$I_{act} = j|d_j \geq \varepsilon, 1 \leq j \leq n \quad (39)$$

where $\varepsilon \in [0, 1]$ is predefined threshold that allows rules that do not match the input to be excluded from the calculation of the fuzzy reasoning result.

Step 4: Calculate the final defuzzified crisp reasoning result y^0.

$$y^0 = (\sum_{j\in I_{act}} d_j \cdot y_j)/\sum_{j\in I_{act}} d_j = (\sum_{j\in I_{act}} d_j(c_{j0} + c_{j1}\cdot x_1 + \ldots + c_{ji}\cdot x_j + \ldots + c_{js}\cdot x_s)/\sum_{j\in I_{act}} d_j \quad (40)$$

Our method is very similar to Sugeno's method [15] and Wang's method [22], and Hellendoorn's method [5], but our method has essentially distinct differences. It is shown in the next subsection 3.2.2, 3.2.3, and subsection 3.3, respectively.

3.2.2 Comparison of Ours with Sugeno's Method

Sugeno's method is a matching degree based fuzzy reasoning method applied to the T-S fuzzy model, in which the final reasoning result is calculated by weighted average based on the matching degree between the antecedent membership function and the input values. Figure 11 shows the relationship between Sugeno's method and proposed CFR method when fuzzy system has two inputs. From the formula (37), (38), and (41), it is clear that the larger matching degree of input information to fuzzy set, the smaller difference between input information and central point of fuzzy set, so the smaller moving degree. However, our method is essentially different from the previous method because it examines the degree of matching between rules and input from the perspective of the moving.

$$w_{ji} = \begin{cases} (x_{ji}^r - x_i^0)/(x_{ji}^r - x_{ji}^c), & x_{ji}^c \le x_i^0 < x_{ji}^r \\ (x_i^0 - x_{ji}^i)/(x_{ji}^c - x_{ji}^\ell), & x_{ji}^\ell < x_i^0 < x_{ji}^c \\ 0, & x_i^0 \le x_{ji}^\ell \quad or \quad x_{ji}^r \le x_i^0 \end{cases} \tag{41}$$

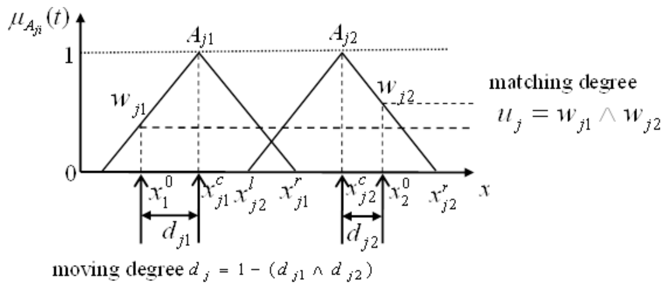


Figure 11: Relationship between CFR method and Sugeno's method

In the case of Sugeno's method, even if one input is not matched with the antecedent, the corresponding rule should be not participate in reasoning. Unlike, the proposed method allows such rules to participate in reasoning by appropriately setting the threshold value ε. By using the appropriate threshold, we can control the number of rules that participate in reasoning. In particular, since the proposed method does not require the calculation of the membership function for the input, the computation time required for the reasoning is much smaller than the previous methods.

3.2.3 Comparison of Our Method with Distance-Type Fuzzy Reasoning

Wang's method [22] is one kind of the distance-type fuzzy reasoning that derives the conclusion based on the distance between the consequent and input fuzzy set. The distance d_j between the consequent and input fuzzy sets for the rule R_j , represented by formula (42), is calculated as follows. In formula (42) $A_{io}, i = 1, 2, ..., n$ is the input fuzzy set corresponding to i^{th} input variable. The crisp reasoning result y^0 is obtained by the following formula (43).

$$d_j = \sum_{i=1}^{n} d(A_{ji}, A_{i0}) \tag{42}$$

$$y^0 = \sum_{j=1}^{m} y_j \cdot \prod_{j \neq k} d_k / \sum_{j=1}^{m} \prod_{j \neq k} d_k \tag{43}$$

From formula (42) and formula (43), it can be known that Wang's method is much more complicated than our method in terms of computational complexity. Moreover, in the Wang's method, unnecessary distance calculation is performed even if all input fuzzy sets do not match with the corresponding antecedent (i.e., the corresponding rule does not affect the calculation of reasoning result). This way of the reasoning is incompatible with human thinking. If some information is obtained, the person does not use all his knowledge to obtain the result but uses only some appropriate knowledge. For CFR method, the moving degree of the fuzzy rule is 0 and it is excluded from the calculation of the reasoning result in above case. Therefore, our CFR method is simpler than the distance-type method.

3.3 Checking of [5]'s and Our Proposed Method

In this paper, our proposed method is very similar to [5], which is different from [5] in several aspects. In this section, we check about [5]'s method and ours.
The sameness of [5]'s and ours is as follows.
1) Powered hedges in [5] are the same as $\mu_{A^\alpha}(x)$ in formula (11) in this paper, where α is deformation index, $\alpha > 0$. When $\alpha > 1$ then $\mu_{A^\alpha}(x) = \mu_A^m(x)$, and when $0 < \alpha < 1$ then $\mu_{A^\alpha}(x) = \mu_A^n(x)$.
2) As shown in formula (11), (12), and (13), the moving amount $\triangle x$ and $\triangle y$ in our method are equivalent to [5]'s shifted hedges A^+ and A^-, respectively.
3) For solving the fuzzy reasoning conclusion B' , information $[a, b]$, $[c, d]$, and $[e, f]$ are all used in [5] and our method.
The differences of [5]'s and ours are as follows.
1) [5] deals with a reasoning method of the case that input information is given as interval fuzzy set, our method deals with a reasoning method based on the observed crisp input. That is, in our method, new conclusion B' is obtained by crisp input x_0 , for example, as shown in formula (16) and (17), moving amount $\triangle x$ is obtained from center x_A of fuzzy set A and input information x_0 , reasoning result B' is obtained by moving amount $\triangle y$ based on $\triangle x$. In [5] new conclusion B' is obtained based on $(x - p)$ and $(x - q)$.
2) [5] deals with the case that two endpoints g, h and center m_B of new fuzzy reasoning result B' are obtained by using of the measure of overlap. Our method deals with the case that the new reasoning result B' is obtained by the moving amount $\triangle x$ and $\triangle y$, as shown in formula (13).
3) Our method deals with the fuzzification as shown in formula (16), (18) and (20), and subsection 2.4, and defuzzification as shown in formula (5), (8), (28), (29), (35) and (40). And [5] deals with the measure of overlap for 2 fuzzy sets and its center.

4 The Improved Learning Algorithm of Fuzzy Neural Network based on CFR

In order to verify the effectiveness of the proposed fuzzy reasoning method based on CFR , we carried out the learning experiments of the fuzzy neural network using CFR method and compared it with the Sugeno's method.

4.1 Configuration of *pi-sigma* Fuzzy Neural Network

The *pi-sigma* neural network is the neural network that has addition neuron and production neuron [10]. The output of *pi-sigma* neural network is as follows.

$$y^0 = \prod_{j=1}^{k} y_j = \prod_{j=1}^{k} \sum_{i=1}^{2} c_{ji} \cdot x_i \tag{44}$$

where c_{ji} is the weight in neural network. As can be known from the structure of neural network, network output corresponds to the fuzzy reasoning output for the T-S system with two inputs, expressed as formula (44). Therefore, for T-S system, fuzzy reasoning method can be fully realized by using above neural structure. This neural network is called *pi-sigma* fuzzy neural network. In fuzzy neural networks, there are not only addition and multiplication operation but also fuzzy operation (e.g., maximum and minimum operation). For convenience of neural network learning, all membership functions of every antecedent fuzzy sets are taken as Gaussian functions (45). The estimation function is defined as formula (46). *pi-sigma* neural network consists of two input neurons and k hidden layers, where S denotes addition neuron and P denotes production neuron.

$$\mu_{A_{ji}}(x_i) = exp\lfloor -(x_i - a_{ji})^2 / b_{ji} \rfloor \tag{45}$$

$$E = (y^d - y^0)^2/2 \tag{46}$$

In formula (45), a_{ji} and b_{ji} denote the center and width of fuzzy set A_{ji} , respectively, and in formula (46), y^0 is output of *pi-sigma* fuzzy neural network, and y^d is the goal output of neural network. By the learning of fuzzy neural network, the centers of antecedent membership function and the coefficients of consequence linear function are decided, where the evaluation function E has the minimum value by using gradient method as formula (47), (48), and (49).

$$c_{ji}(t+1) = c_{ji}(t) - \eta \frac{\partial E}{\partial c_{ji}}|_{c_{ji}=c_{ji}(t)} \tag{47}$$

$$a_{ji}(t+1) = a_{ji}(t) - \eta \frac{\partial E}{\partial a_{ji}}|_{a_{ji}=a_{ji}(t)} \tag{48}$$

$$b_{ji}(t+1) = b_{ji}(t) - \eta \frac{\partial E}{\partial b_{ji}}|_{b_{ji}=b_{ji}(t)} \tag{49}$$

4.2 Learning Experiment of Fuzzy Neural Network for Case Data Sets

The precipitation data set used in this experiment is obtained from the precipitation data during 1952 to 1977 provided at China Tianjin city Weather Service.

The experiment were carried out on precipitation data and computer network security situation data, under the computational environment of R2012a MATLAB platform on a Window 8.1 (Intel(R) Core(TM) i7 CPU, 2.78 GHz processing, and 16 GB RAM). The 36 fuzzy rules have been made out for the experiment by setting 6 fuzzy sets PL, PM, PS, NS, NM, and NL to every input variable $x_i (i = 1, 2)$.

R_1: *if* x_1 *is NL and* x_2 *is NL then* $y_1 = c_{10} + c_{11}x_1 + c_{12}x_2$
R_2: *if* x_1 *is NL and* x_2 *is NM then* $y_2 = c_{20} + c_{21}x_1 + c_{22}x_2$
... ...
R_{36}: *if* x_1 *is PL and* x_2 *is PL then* $y_{36} = c_{360} + c_{361}x_1 + c_{362}x_2$

Table 3 shows the change of error according to learning iterations. Prediction of network security situation evaluates security situation by learning data in the past and predicts the situation in the future. This method does prediction and studying by using fuzzy neural networks [7]. The output of *pi-sigma* neural network in experiment is calculated as formula (44). We experiment learning performance of neural network based on security situation data using our method. And Table 4 shows the experiment results for iteration of learning. In Table 3 and 4, LI, LT (s), and LE (%) mean Learning Iteration, Learning Time, and Learning Error, respectively. For a simple comparison experiment, fuzzy level PL, PM and PS are chosen and 3 input variable are used. The experiment result shows that the learning process of precipitation data was about 5.2 times faster and the learning process of security situation date was about 1.37 times faster compared to Sugeno's method. Computational time difference of the two methods will get bigger as fuzzy variables and rules are added. Learning accuracy of precipitation prediction has improved by 7.35% but prediction accuracy of security situation has decreased by 0.937%. The accuracy of learning will improve as the number of fuzzy partitions increases. The reason why the learning time is reduced for the proposed method is that it uses only the distance obtained from the membership function and the input information, and eliminates the unnecessary calculations in reasoning process by using threshold. In addition, Sugeno's method needs to calculate the membership functions with the form of exponential function while reasoning process. Unlike, the CFR method does not require such a time-consuming calculation. It can be easily known that the proposed method has higher efficiency compared to the previous method.

5 Conclusions

In this paper we proposed a new fuzzy reasoning, called Compensating Fuzzy Reasoning (CFR) method, by deducing the moving, deformation, and moving-deformation relationship between the antecedent and input to the relationship between the consequent and conclusion, for FMP and FMT with single input and single output. The method works by first evaluating the moving-deformation relationship between the antecedent of fuzzy rule and input (the given observations), and then converting the consequent of fuzzy rule to the conclusion using the moving and deforming operations. A new model for the FMP and FMT representing the if-then fuzzy rule has proposed based on the compensating operation of the moving-deformation relations. The CFR method is consistent with human thinking and satisfies logical reductive. Our method was illustratively compared with Zadeh's, Sugeno's, Wang's, and Hellendoorn's method, respectively, which has some sameness, differences and independent property. The proposed method has applied to two fuzzy systems, i.e., Mamdani's one with s input 1 output, and T-S fuzzy neural network's one with s input linear function output, and then has compared with their previous methods. And the proposed method

LI	[16]'s LT (s)	[16]'s LE (%)	Our LT (s)	Our LE (%)
100	0.7	94.710	0.6	68.114
500	4.9	77.685	1.4	11.870
1 000	9.8	67.205	2.1	10.803
1 500	14.7	59.911	3.5	9.370
2 000	19.6	53.567	4.2	9.253
4 000	38.5	39.594	8.4	9.531
8 000	70.7	25.427	16.8	8.582
10 000	97.3	23.184	20.3	8.516
15 000	147.0	20.478	31.5	8.832
18 000	175.7	19.163	37.1	8.078
20 000	192.5	18.340	41.3	8.640
25 000	237.3	16.899	51.8	7.887
30 000	284.2	15.799	61.6	8.318
32 670	516.6	15.260	67.2	7.911

Table 3: Experiment results on precipitation data by two fuzzy neural network learning

LI	[16]'s LT (s)	[16]'s LE (%)	Our LT (s)	Our LE (%)
100	2.8	25.558	2.10	37.169
500	11.9	12.001	9.10	18.366
1 000	23.8	10.231	18.2	9.148
1 500	34.3	8.919	26.6	6.695
2 000	44.8	7.854	35.7	5.772
4 000	90.3	5.375	69.3	2.968
8 000	179.9	3.757	135.8	4.299
10 000	225.4	3.523	168.7	4.090
15 000	338.1	3.072	251.3	3.565
18 000	448.0	2.850	307.3	2.876
20 000	450.8	2.727	335.3	2.914
25 000	569.1	2.476	417.2	3.065
30 000	674.1	2.279	492.8	2.642
32 670	751.1	2.191	533.4	2.383

Table 4: Experiment results on security situation by two fuzzy neural network learning

is computationally simple and does not involve strict logical operations, so it is easy to handle mathematically. Experimental evaluations of the proposed method through the fuzzy neural network indicate that it's learning accuracy and time performance are clearly improved and compared with previous Sugeno's method.

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