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Refueling problem of alternative fuel vehicles under intuitionistic fuzzy refueling waiting times: a fuzzy approach

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Abstract

Using alternative fuel vehicles is one of the ways to reduce the consumption of fossil fuels which have many negative environmental effects. An alternative fuel vehicle needs special planning for its refueling operations because of some reasons, e.g. limited number of refueling stations, uncertain refueling queue times in the stations, variable alternative fuel prices among the stations, etc. In this paper, a new problem as refueling an alternative fuel vehicle on a given path is formulated to minimize the cost of refueling and waiting times in the stations for refueling operations, simultaneously. To be more close to real-world situations, the waiting times are considered as intuitionistic fuzzy numbers in order to reflect uncertainty as well as hesitation due to various uncontrollable factors. To cope with the uncertainty of the problem, an intuitionistic fuzzy chance constrained method based on credibility measure is proposed to convert the fuzzy formulation to a crisp model. In order to tackle the bi-objective crisp formulation, a new interactive fuzzy solution method is proposed. A computational study on a real case from Turkey shows that the performance of the presented method is either better or the same as the approaches of the literature.

Keywords: Refueling problem, Alternative fuel vehicle, Multi-objective programming, Intuitionistic fuzzy number, Credibility measure.

1 Introduction

The recent popularity of the use of fossil fuels in the transportation sector has caused significant concerns about the future of mankind and environment. The European Union collectively spends up to one billion Euros per day for importing oil to be used for more than 90% of its transportation activities [20]. Using alternative fuel vehicles (AFVs) that run on compressed natural gas (CNG), liquid petroleum gas (LPG or propane), hydrogen, ethanol or electricity is one of the ways to deal with the problems associated with oil importation and consumption [30]. The limited driving range is one of the most important shortcomings of AFVs [17, 10]. Limitation of fuel tank capacities (ranges) causes frequent refuelings during a trip. To extend the use of AFVs, we need to make customers eager to buy these types of vehicles. To achieve this goal, planners should focus on two issues to promote the use of AFVs.

1. The establishment of infrastructures for AFVs.
2. Planning for the users of AFVs based on the existing infrastructures.

Many studies have been done in context of the establishment of infrastructures for AFVs. Most studies addressed the problem of refueling stations by maximum covering or set covering approaches [25]. The objective of most of these problems is to maximize the total flow that passes through refueling stations. Hodgson [16] proposed the flow capturing location model (FCLM). Hodgson [16] was the first researcher who applied the concept of maximum covering problem for locating alternative fuel stations. The objective of the FCLM is to locate a predetermined number of refueling stations in order to cover the maximum flow. In the FCLM, it is assumed that if the flow passes through a refueling station it is "captured". The term "capture" is used by Hodgson to describe a form of covering the flow. The FCLM

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motivated Kuby and Lim [21] to develop it by considering the basic issue of *limited driving range*. They presented a maximum coverage model which is named the flow refueling location model (FRLM). The aim of the FRLM is to locate the specified number of stations to maximize the covered flow by incorporating the vehicle range limitation. Other researches about the maximum flow covering models and their solution approach have been done by Lim and Kuby [24] and Capar and Kuby [2]. In addition to the mentioned studies for locating refueling stations, considering the sparse distribution of stations is another important issue. In fact, the sparse distribution of refueling stations may cause some vehicles to deviate from their path to visit a station. To cope with this problem, Kim and Kuby [18] proposed a deviation flow refueling location model (DFRLM). The goal of the DFRLM is to locate stations to maximize the total covered flow on deviation paths (see also [23] and [19]). In addition to the maximum flow coverage models, there is another class of flow-based models based on the concept of set covering problems. Wang and Lin [36] introduced a flow-based set covering model considering the limitation of driving range using the data of distance matrix instead of the flow, which was easier to obtain. MirHassani and Ebrazi [26] reformulated the flow-based model proposed by Wang and Lin [36] as a flexible refueling station location problem.

Based on the literature, no study has been done on planning for the user of AFVs based on the existing infrastructures of AFVs. Time and cost of refueling are two important factors for users of vehicles. These factors have not been paid attention up to now. According to different fuel costs in various refueling stations, it is expected that refueling stations with cheaper fuel have more customers. This issue caused long queues at these stations. Therefore, having the minimum cost of refueling as well as minimum waiting time in queues are AFV's users' preferences. To meet these requirements of users, the problem of refueling is considered with two mentioned criteria in this research. Suppose a vehicle wants to have a trip on the path between a fixed origin and a specified destination without running out of fuel. Obviously, if the amount of fuel that the vehicle needs to travel the path, is greater than its fuel tank capacity, some refuelings are needed. Therefore, this paper contributes to the literature by introducing, formulating, and solving a new refueling problem considering above-mentioned issues. We propose a new bi-objective refueling problem in order to (1) choose in which stations, refuelings are done to have the least possible refueling costs, (2) have the least refueling waiting time in the stations. To be more close to real-world situations, the waiting times are considered as intuitionistic fuzzy numbers in order to reflect uncertainty as well as hesitation due to various uncontrollable factors. This property is an advantage of using intuitionistic fuzzy numbers instead of a regular fuzzy number (For more applications of intuitionistic fuzzy (IF) numbers the studies of [15, 22, 28, 31, 32, 34] can be referred). To cope with the uncertainty of the problem, an intuitionistic fuzzy chance constrained method based on credibility measure is proposed to convert the fuzzy formulation to a crisp model. In order to tackle the bi-objective crisp formulation, a new interactive fuzzy solution method is proposed. A computational study on a real case from Turkey shows that the performance of the presented method is either better or the same as the approaches of the literature such as Torabi and Hassini [35] (TH approach) and Demirli and Yimer [9] (DY approach). The obtained solution of the proposed problem can be given to an intelligent system installed on the vehicles to do intelligent planning for where to refuel the vehicle.

The rest of the paper is organized as follows: Section 2 describes the problem and proposes its mathematical programming model. In Section 3, a two-phase solution approach is proposed. Case study and computational results are stated in Section 4. Finally, Section 5 is devoted to the conclusion.

2 Problem statement and mathematical formulation

The intended problem of this paper is explained and formulated here. Let $\mathfrak{p}$ be the path from node O to D containing the set of nodes N (including O and D) and the set of arcs E. Nodes of the set N display the existing refueling stations. For any pair of nodes i and j on path $\mathfrak{p}$, $d(i,j)$ shows the length of the sub-path between them and $f(i,j)$ denotes the amount of fuel that is consumed by a vehicle while traveling the sub-path. Now, consider a vehicle with the range (fuel tank capacity) R tripping from the origin, O, to the destination, D, on path $\mathfrak{p}$. If the vehicle refuels at the origin, it can be assumed that the vehicle has a full tank at the beginning. If not, it is not reasonable that the vehicle could have a full or empty tank because, on the previous trip, the vehicle has certainly traveled over some distance from the last station to reach the origin; therefore it would not have a full tank. On the other hand, drivers usually consider a fair amount of fuel to be able to reach the first station on their next trip; thus an empty tank assumption in the origin does not appear to be reasonable[26, 21]. Thus, the vehicle may have missed any amount of fuel e.g. M at the origin where $M > 0$. Therefore, we assumed that the vehicle has a $(R - M)$-full tank of fuel at the origin. Regarding the limited fuel tank capacity of the vehicle, it may run out of fuel before reaching the destination. Therefore, it should refuel in some of the existing refueling stations that are located in nodes of path $\mathfrak{p}$. Fuel costs are different in various stations. Obviously, the cost of refueling is one of the major disturbances of drivers. Drivers want to have refuelings with minimum total cost. Therefore, it is expected that refueling stations with cheaper fuel have more customers.

This issue caused long queues at these stations. As a result, waiting time in queues of fuel stations is another concern for drivers. On the other hand, the exact values of waiting time in queues cannot be determined. In fact, values of waiting time are vague and insufficient. Therefore, the values of waiting time can be more realistic to be considered as intuitionistic fuzzy numbers. In order to formulate the explained problem mathematically, a new intuitionistic fuzzy bi-objective programming model based on the network introduced by MirHassani and Ebrazi [26] for path $\mathfrak{p}$ is proposed in this research. The objectives of this model are to decide on where to refuel the vehicle on the path from O to D, in order to have the minimum total cost of refueling as well as minimum IF waiting time such that the vehicle does not run out of fuel during its trip.

In the following, valid combinations of refueling nodes on path $\mathfrak{p}$ (subsets of N that contain possible refueling stations) are found by a network denoted by $\mathcal{G} = \mathcal{G}(\mathcal{N}, \mathcal{E})$ [26]. Sets of $\mathcal{N}$ and $\mathcal{E}$ include nodes and arcs existing in the network $\mathcal{G}$. To construct the network $\mathcal{G}$, let $ord(i)$ be as an ordering index of node $i \in \mathcal{N}$ in the path sequence $\mathfrak{p}$ which keeps track of the order of the nodes on path $\mathfrak{p}$. In order to construct the network $\mathcal{G}$, the following steps are used.

Step 1. Two virtual nodes, s and t are added before the origin and after the destination on path $\mathfrak{p}$, respectively. Then node s is connected to the origin and the destination is connected to node t. So, $\mathcal{N}$ contains the nodes of path $\mathfrak{p}$ union nodes s and t. Moreover, arcs (s, O) and (D, t) are added to the empty set $\mathcal{E}$.

Step 2. Node s is connected to any other node say, $i \in \mathcal{N}$, if it is possible to begin at the origin node and arrive at node i with the existed amount of fuel in the tank or less. This means that,

$$\forall i \in \mathcal{N} \ni f(O, i) \leqslant R - M \quad \Rightarrow \quad (s, i) \in \mathcal{E}.$$

Step 3. Any node $i \in N$ is connected to any other node $j \in \mathcal{N}$ if the ordering index of i is less than the ordering index of j in the path sequence $\mathfrak{p}$, and the vehicle is able to start from node i with a full tank of fuel and arrive at node j. Hence,

$$\forall i \in N, j \in \mathcal{N} \ni (ord(i) < ord(j)) \ \& \ f(i, j) \leqslant R \quad \Rightarrow \quad (i, j) \in \mathcal{E}.$$

In $\mathcal{G}$, each arc corresponds to the consecutive valid refueling and each directed path from s to t shows a valid combination of refueling stations that can refuel path $\mathfrak{p}$ [26]. Notably, if in this process a node in the set N could not connect to the next node with larger ordering index in the set N , then path $\mathfrak{p}$ is considered invalid. Hence, this path could not be traveled due to the fuel capacity restriction.

Before introducing the new model, the definition of intuitionistic fuzzy number is expressed as follow.

Definition 2.1. *Let $a'_1 \leqslant a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4 \leqslant a'_4$. A trapezoidal intuitionistic fuzzy number (TIFN) in $\mathbb{R}$ is denoted as $\tilde{A}^I = (a_1, a_2, a_3, a_4)(a'_1, a_2, a_3, a'_4)$ with the following membership function $\mu_{\tilde{A}^I}(x)$, and non-membership function $\nu_{\tilde{A}^I}(x)$ (Fig. 2.1)* [6].

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & if \ \ a_1 \leqslant x \leqslant a_2 \\ 1 & if \ \ a_2 < x \leqslant a_3 \\ \frac{a_4-x}{a_4-a_3} & if \ \ a_3 < x \leqslant a_4 \\ 0 & if \ \ otherwise \end{cases} \tag{1}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1} & if \ \ a'_1 \leqslant x \leqslant a_2 \\ 0 & if \ \ a_2 < x \leqslant a_3 \\ \frac{x-a_3}{a'_4-a_3} & if \ \ a_3 < x \leqslant a'_4 \\ 1 & if \ \ otherwise \end{cases} \tag{2}$$

One of the main properties of the fuzzy numbers is that the non-membership degree of the elements is obtained by one minus the membership degree. But, in real-world situations as the information may have vagueness or insufficiency, the sum of membership and non-membership degrees can be a value less than one. In such cases, fuzzy numbers are not suitable as in these numbers the sum of membership and non-membership degrees are exactly equal to one. To overcome such difficulty, Atanassov [1] introduced the theory of intuitionistic fuzzy set (IFS) that is an extension of fuzzy theory and is highly useful for real-life problems to deal with vague information. The most important advantage of IFS comparing to fuzzy set is that it isolates the membership and non-membership degrees of a number of the set in a way that for an element the sum of these degrees is less than or equal to one. Therefore, this theory seems to be very applicable when considering the vagueness in the estimation of parameters by the decision maker. In the case of transportation problem some available information of waiting time in the queue of refueling may be vague

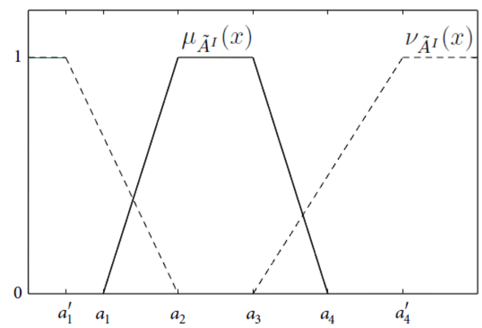

Figure 1: Membership and non-membership functions of a TIFN.

or insufficient in the estimation procedure. Therefore, estimating exact membership functions and also exact non-membership functions may not be possible, where, some hesitation still remains. For this reason, use of IFS may be more realistic than fuzzy set. For more applications of intuitionistic fuzzy numbers the studies of [15, 22, 28, 31, 32, 34] can be referred.

Now, by considering the constructed network $\mathcal{G}$, the intuitionistic fuzzy bi-objective programming model with the objectives of minimizing the total cost of refueling and minimizing the IF waiting time in queues of refueling, can be offered. The following parameters and decision variables are applied in the formulation of the proposed model.

Parameters

c_j: Fuel cost at the station located in node j;

$\tilde{t}_j^I$: Intuitionistic fuzzy waiting time in queue for refueling at station located in node$j$;

v: Average speed of the vehicle;

T: Total time for traveling the path.

Decision variables

y_j: A binary variable which takes value of 1 if refueling at station located in node $j \in N$ is done; otherwise it takes value of 0;

x_{ij}: A binary variable which takes value of 1 if arc $(i,j) \in \mathcal{E}$ is traveled by the vehicle; otherwise its value is 0.

The proposed intuitionistic fuzzy bi-objective model is formulated as follows:

Model 1:

$$\min: \quad Z_1 = \sum_{\substack{(s,j)\in\mathcal{E}\\ j\in N}} c_j\,(M + f(s,j))\,x_{sj} + \sum_{\substack{(i,j)\in\mathcal{E}\\ i,j\in N}} c_j f(i,j)\; x_{ij}, \tag{3}$$

$$\min: \quad Z_2 = \sum_{j\in N} \tilde{t}_j^I\; y_j, \tag{4}$$

s.t.

$$\sum_{j\in N} \tilde{t}_j^I\; y_j + \sum_{(i,j)\in\mathcal{E}} \frac{d(i,j)}{v}\; x_{ij} \leqslant T, \tag{5}$$

$$\sum_{\{j|(i,j)\in\mathcal{E}\}} x_{ij} - \sum_{\{j|(j,i)\in\mathcal{E}\}} x_{ji} = \begin{cases} 1 & \text{if } i = s,\\ 0 & \text{if } i \neq s,t,\\ -1 & \text{if } i = t, \end{cases} \qquad \forall i \in \mathcal{N}, \tag{6}$$

$$\sum_{\{i|(i,j)\in\mathcal{E}\}} x_{ij} \leqslant y_j, \quad \forall j \in N, \tag{7}$$

$$x_{ij} \in \{0,1\}, \quad \forall (i,j) \in \mathcal{E}, \tag{8}$$

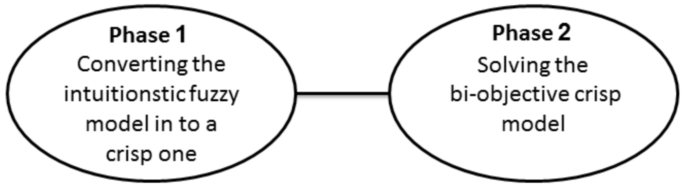

Figure 2: Schematic representation of the proposed solution approach

$$y_j \in \{0,1\}, \quad \forall j \in N. \tag{9}$$

The detailed description of the model is as follows: the first objective function minimizes the refueling costs. In fact, in calculating the refueling costs of the vehicle two cases should be considered. The first is that the vehicle starts its trip with a $(R-M)$-full tank of fuel from the virtual node s to arrive at the next node say j (which does not need successive node). In this sense, the refueling cost of the vehicle at node j is equal to $c_j\,(M+f(s,j))$. The second case is that the vehicle passes from other nodes, during the trip, on the path except s, say i. In this situation, since the vehicle has a full tank of fuel at node i then the refueling cost in the next node say j (which does not need successive node), is equal to $c_j\ f(i,j)$. Note that, s and t are virtual nodes and there is no station located on them. Moreover, $f(s,j)=f(O,j)$ (i.e. $f(s,O)=0), d(s,j)=d(O,j)$ and $f(j,D)=f(j,t), d(j,D)=d(j,t)$. In the second objective, the IF waiting time in queues for refueling during the trip is minimized. Constraint (5) states, the IF waiting time plus the time that the vehicle takes to travel the path by the average speed of v should be less than or equal to the total travel time T. Equations (6) are mass balance constraints, which demonstrate that the vehicle must start its trip from node s and should end at node t. Moreover, it shows that when the vehicle enters an intermediate node, it must also exit from that node. Constraint (7) ensures that when the vehicle enters a node, it should refuel. Constraints (8) and (9) enforce the binary integer restriction on the x_{ij} and y_j variables.

3 Solution approach

In order to solve Model 1, we are faced with challenging issues like the intuitionistic fuzziness and multi-objectivity of the model. To cope with such issues, a two-phase solution approach is proposed in this section. In the first phase, the intuitionistic fuzzy bi-objective Model 1 is converted to an equivalent crisp model. In the second phase, an efficient solution for the bi-objective crisp version is obtained (Fig. 2). These phases are explained in detail in the following sub-sections.

3.1 Phase 1: the equivalent crisp model

As mentioned in the literature, there are different methods to deal with a fuzzy event [5, 13, 14, 8, 33, 7, 11, 12]. Possibility theory is one of the most applicable methods for converting intuitionistic fuzzy events to their equivalent crisp forms. Possibility theory contains possibility (Pos), necessity (Nec) and credibility (Cr) measures. By considering, $\tilde{A}^I=(a_1,a_2,a_3,a_4)(a_1',a_2,a_3,a_4')$ and $\tilde{B}^I=(b_1,b_2,b_3,b_4)(b_1',b_2,b_3,b_4')$ with membership and non-membership functions of $\mu_{\tilde{A}^I}(x), \mu_{\tilde{B}^I}(x)$ and $\nu_{\tilde{A}^I}(x), \nu_{\tilde{B}^I}(x)$, the followings are introduced:

$$\mathrm{Pos}_\mu(\tilde{A}^I \leqslant \tilde{B}^I)=\sup\{\min(\mu_{\tilde{A}^I},\mu_{\tilde{B}^I}), x,y\in\mathbb{R}, x\leqslant y\},$$

$$\mathrm{Pos}_\nu(\tilde{A}^I \leqslant \tilde{B}^I)=\sup\{\min(\nu_{\tilde{A}^I},\nu_{\tilde{B}^I}), x,y\in\mathbb{R}, x\leqslant y\},$$

$$\mathrm{Nec}_\mu(\tilde{A}^I \leqslant \tilde{B}^I)=\inf\{\max(\mu_{\tilde{A}^I},\mu_{\tilde{B}^I}), x,y\in\mathbb{R}, x\leqslant y\},$$

$$\mathrm{Nes}_\nu(\tilde{A}^I \leqslant \tilde{B}^I)=\inf\{\max(\nu_{\tilde{A}^I},\nu_{\tilde{B}^I}), x,y\in\mathbb{R}, x\leqslant y\}.$$

Where, Pos_μ and Pos_ν define the possibility of membership and non-membership function, respectively, and Nes_μ and Nes_ν show the necessity of membership and non-membership function, respectively [6].

The intuitionistic fuzzy measures of $\tilde{A}^I$ for membership and non-membership function are as follows:

$$\mathrm{Me}_\mu\{\tilde{A}^I \leqslant \tilde{B}^I\}=\tau\mathrm{Pos}_\mu\{\tilde{A}^I \leqslant \tilde{B}^I\}+(1-\tau)\mathrm{Nes}_\mu\{\tilde{A}^I \leqslant \tilde{B}^I\},$$

$$\mathrm{Me}_\nu\{\tilde{A}^I \leqslant \tilde{B}^I\} = \tau \mathrm{Pos}_\nu\{\tilde{A}^I \leqslant \tilde{B}^I\} + (1-\tau)\mathrm{Nes}_\nu\{\tilde{A}^I \leqslant \tilde{B}^I\}.$$

Where $0 \leqslant \tau \leqslant 1$ is an optimistic-pessimistic parameter to determine the combined attitude of a decision maker. For $\tau = 0.5$, the decision maker takes compromise attitude and the intuitionistic fuzzy measures are equal to credibility measure, i.e. $\mathrm{Me}_\mu\{\tilde{A}^I \leqslant \tilde{B}^I\} = \mathrm{Cr}_\mu\{\tilde{A}^I \leqslant \tilde{B}^I\}$ and $\mathrm{Me}_\nu\{\tilde{A}^I \leqslant \tilde{B}^I\} = \mathrm{Cr}_\nu\{\tilde{A}^I \leqslant \tilde{B}^I\}$ [6]. In fact, the credibility measure can be expressed by the average of the possibility and necessity measures as, $\mathrm{Cr}_\mu(\tilde{A}^I \leqslant \tilde{B}^I) = \frac{1}{2}\Big(\mathrm{Pos}_\mu(\tilde{A}^I \leqslant \tilde{B}^I) + \mathrm{Nes}_\mu(\tilde{A}^I \leqslant \tilde{B}^I)\Big)$ and $\mathrm{Cr}_\nu(\tilde{A}^I \leqslant \tilde{B}^I) = \frac{1}{2}\Big(\mathrm{Pos}_\nu(\tilde{A}^I \leqslant \tilde{B}^I) + \mathrm{Nes}_\nu(\tilde{A}^I \leqslant \tilde{B}^I)\Big)$.

To cope with the fuzziness of the proposed problem, a credibility-based chance operator for the intuitionistic fuzzy programming model is used in this paper. To convert a fuzzy chance operator to its equivalent crisp form the method of credibility-based chance constraint programming is used. In the case of intuitionistic chance operator, the decision maker should satisfy two chance constraints with (at least/at most) confidence levels. The chance constraints and their equivalent crisp forms are shown by relations (10) and (11), where α and β are the confidence levels [6].

$$\mathrm{Cr}_\mu(\tilde{A}^I \leqslant \tilde{B}^I) \geqslant \alpha \iff \frac{b_4 - a_1}{2(b_4 - b_3 + a_2 - a_1)} \geqslant \alpha,\ \frac{a_4 - 2a_3 + 2b_1 - b_2}{2(a_4 - a_3 - b_2 + b_1)} \geqslant \alpha, \tag{10}$$

$$\mathrm{Cr}_\nu(\tilde{A}^I \leqslant \tilde{B}^I) \leqslant \beta \iff \frac{2a_2 - 2b'_3 + b'_4 - a'_1}{2(a_2 - a'_1 + b'_4 - b_3)} \leqslant \beta,\ \frac{a'_4 - b_2}{2(a'_4 - a_3 - b_2 + b'_1)} \leqslant \beta. \tag{11}$$

Obviously, in a case that the second side of inequality is not fuzzy ($B \in \mathbb{R}$) the above relations would be as follows,

$$\mathrm{Cr}_\mu(\tilde{A}^I \leqslant B) \geqslant \alpha \iff \frac{B - a_1}{2(a_2 - a_1)} \geqslant \alpha,\ \frac{a_4 - 2a_3 + B}{2(a_4 - a_3)} \geqslant \alpha, \tag{12}$$

$$\mathrm{Cr}_\nu(\tilde{A}^I \leqslant B) \leqslant \beta \iff \frac{2a_2 - B - a'_1}{2(a_2 - a'_1)} \leqslant \beta,\ \frac{a'_4 - B}{2(a'_4 - a_3)} \leqslant \beta. \tag{13}$$

Considering the above-mentioned explanations, a three-step method of Chakraborty et al. [6] for converting an intuitionistic fuzzy single objective problem into a crisp problem is extended for converting the intuitionistic fuzzy bi-objective Model 1 into its equivalent crisp one. The three steps are given below.

Step 1. Apply chance operator credibility for the intuitionistic fuzzy bi-objective Model 1. Therefore, the credibility-based chance constrained programming version of Model 1 is Model 2.

Model 2:

$$\min: \quad Z_1 = \sum_{\substack{(s,j)\in\mathcal{E}\\ j\in N}} c_j\,(M + f(s,j))\,x_{sj} + \sum_{\substack{(i,j)\in\mathcal{E}\\ i,j\in N}} c_j f(i,j)\ x_{ij}, \tag{14}$$

$$\min: \quad Z_2 = g_1 + g_2, \tag{15}$$

s.t.

$$Cr_\mu\Big\{\sum_{j\in N} \tilde{t}^I_j\ y_j \leqslant g_1\Big\} \geqslant \alpha, \tag{16}$$

$$Cr_\nu\Big\{\sum_{j\in N} \tilde{t}^I_j\ y_j \leqslant g_2\Big\} \leqslant \beta \tag{17}$$

,

$$Cr_\mu\Big\{\sum_{j\in N} \tilde{t}^I_j\ y_j + \sum_{(i,j)\in\mathcal{E}} \frac{d(i,j)}{v}\ x_{ij} \leqslant T\Big\} \geqslant \lambda, \tag{18}$$

$$Cr_\nu\Big\{\sum_{j\in N} \tilde{t}^I_j\ y_j + \sum_{(i,j)\in\mathcal{E}} \frac{d(i,j)}{v}\ x_{ij} \leqslant T\Big\} \leqslant \phi, \tag{19}$$

$$\sum_{\{j|(i,j)\in\mathcal{E}\}} x_{ij} - \sum_{\{j|(j,i)\in\mathcal{E}\}} x_{ji} = \begin{cases} 1 & \text{if } i=s, \\ 0 & \text{if } i\neq s,t, \\ -1 & \text{if } i=t, \end{cases} \quad \forall i\in\mathcal{N}, \tag{20}$$

$$\sum_{\{i|(i,j)\in\mathcal{E}\}} x_{ij} \leqslant y_j, \quad \forall j\in N, \tag{21}$$

$$x_{ij}\in\{0,1\}, \qquad \forall (i,j)\in\mathcal{E}, \tag{22}$$

$$y_j\in\{0,1\}, \qquad \forall j\in N, \tag{23}$$

$$g_1, g_2 \geq 0, \tag{24}$$

$$0\leqslant \alpha+\beta\leqslant 1,\ 0\leqslant \lambda+\phi\leqslant 1. \tag{25}$$

In Model 2, α,β,λ, and ϕ are confidence levels that are determined by the decision maker.
Step 2. Let $\tilde{t}_j^I=(t_{j_{(1)}},t_{j_{(2)}},t_{j_{(3)}},t_{j_{(4)}})(t'_{j_{(1)}},t_{j_{(2)}},t_{j_{(3)}},t'_{j_{(4)}})$. Using (12) and (13), relations (18) and (19) are converted into their equivalent crisp versions. Also the relations (16) and (17) are transformed into their crisp forms, then using these obtained crisp forms and with the help of some computational work the inequality of $g_1+g_2\geqslant W$ is obtained as follow. Using (12) and (13) the inequality $Cr_\mu\Big\{\sum_{j\in N}\tilde{t}_j^I y_j\leqslant g_1\Big\}\geqslant\alpha$ is converted to (26) and (27). Moreover, $Cr_\nu\Big\{\sum_{j\in N}\tilde{t}_j^I y_j\leqslant g_2\Big\}\leqslant\beta$ is converted to (28) and (29).

$$\frac{g_1-\sum_{j\in N}t_{j_{(1)}}y_j}{2(\sum_{j\in N}t_{j_{(2)}}y_j-\sum_{j\in N}t_{j_{(1)}}y_j)}\geqslant\alpha \tag{26}$$

$$\frac{\sum_{j\in N}t_{j_{(4)}}y_j-2\sum_{j\in N}t_{j_{(3)}}y_j+g_1}{2(\sum_{j\in N}t_{j_{(4)}}y_j-\sum_{j\in N}t_{j_{(3)}}y_j)}\geqslant\alpha \tag{27}$$

$$\frac{2\sum_{j\in N}t_{j_{(2)}}y_j-\sum_{j\in N}t'_{j_{(1)}}y_j-g_2}{2(\sum_{j\in N}t_{j_{(2)}}y_j-\sum_{j\in N}t'_{j_{(1)}}y_j)}\leqslant\beta \tag{28}$$

$$\frac{\sum_{j\in N}t'_{j_{(4)}}y_j-g_2}{2(\sum_{j\in N}t'_{j_{(4)}}y_j-\sum_{j\in N}t_{j_{(3)}}y_j)}\leqslant\beta \tag{29}$$

Obviously, relations (26)-(29) can be written as (30)-(33), respectively.

$$g_1\geqslant 2\alpha(\sum_{j\in N}t_{j_{(2)}}y_j-\sum_{j\in N}t_{j_{(1)}}y_j)+\sum_{j\in N}t_{j_{(1)}}y_j \tag{30}$$

$$g_1\geqslant 2\alpha(\sum_{j\in N}t_{j_{(4)}}y_j-\sum_{j\in N}t_{j_{(3)}}y_j)-\sum_{j\in N}t_{j_{(4)}}y_j+2\sum_{j\in N}t_{j_{(3)}}y_j \tag{31}$$

$$g_2\geqslant -2\beta(\sum_{j\in N}t_{j_{(2)}}y_j-\sum_{j\in N}t'_{j_{(1)}}y_j)-\sum_{j\in N}t'_{j_{(1)}}y_j+2\sum_{j\in N}t_{j_{(2)}}y_j \tag{32}$$

$$g_2 \geqslant -2\beta(\sum_{j\in N} t'_{j_{(4)}} y_j - \sum_{j\in N} t_{j_{(3)}} y_j) + \sum_{j\in N} t'_{j_{(4)}} y_j. \tag{33}$$

Hence, by adding the inequalities (30)-(33) the relation W is obtained as follows.

$$\begin{aligned} W = \frac{1}{2}\Big(& 2\alpha(\sum_{j\in N} t_{j_{(2)}} y_j - \sum_{j\in N} t_{j_{(1)}} y_j + \sum_{j\in N} t_{j_{(4)}} y_j - \sum_{j\in N} t_{j_{(3)}} y_j) + \sum_{j\in N} t_{j_{(1)}} y_j - \sum_{j\in N} t_{j_{(4)}} y_j \\ & - 2\beta(\sum_{j\in N} t_{j_{(2)}} y_j - \sum_{j\in N} t'_{j_{(1)}} y_j + \sum_{j\in N} t'_{j_{(4)}} y_j - \sum_{j\in N} t_{j_{(3)}} y_j) - \sum_{j\in N} t'_{j_{(1)}} y_j + \sum_{j\in N} t'_{j_{(4)}} y_j \\ & + 2(\sum_{j\in N} t_{j_{(3)}} y_j + \sum_{j\in N} t_{j_{(2)}} y_j)\Big) \end{aligned}$$

As a result Model 2 is written as Model 3.

Model 3:

$$\begin{aligned} \min: \quad & Z_1 = \sum_{\substack{(s,j)\in\mathcal{E} \\ j\in N}} c_j\,(M + f(s,j))\,x_{sj} + \sum_{\substack{(i,j)\in\mathcal{E} \\ i,j\in N}} c_j f(i,j)\; x_{ij}, \\ \min: \quad & Z_2 = g_1 + g_2, \end{aligned}$$

s.t.

$$g_1 + g_2 \geqslant W, \tag{34}$$

$$\frac{T - \sum_{(i,j)\in\mathcal{E}} \frac{d(i,j)}{v}\; x_{ij} - \sum_{j\in N}\; t_{j_{(1)}} y_j}{2(\sum_{j\in N}\; t_{j_{(2)}} y_j - \sum_{j\in N}\; t_{j_{(1)}} y_j)} \geqslant \lambda, \tag{35}$$

$$\frac{\sum_{j\in N}\; t_{j_{(4)}} y_j - 2\sum_{j\in N}\; t_{j_{(3)}} y_j + T - \sum_{(i,j)\in\mathcal{E}} \frac{d(i,j)}{v}\; x_{ij}}{2(\sum_{j\in N}\; t_{j_{(4)}} y_j - \sum_{j\in N}\; t_{j_{(3)}} y_j)} \geqslant \lambda, \tag{36}$$

$$\frac{2\sum_{j\in N}\; t_{j_{(2)}} y_j - T + \sum_{(i,j)\in\mathcal{E}} \frac{d(i,j)}{v}\; x_{ij} - \sum_{j\in N}\; t'_{j_{(1)}} y_j}{2(\sum_{j\in N}\; t_{j_{(2)}} y_j - \sum_{j\in N}\; t'_{j_{(1)}} y_j)} \leqslant \phi, \tag{37}$$

$$\frac{\sum_{j\in N}\; t'_{j_{(4)}} y_j - T + \sum_{(i,j)\in\mathcal{E}} \frac{d(i,j)}{v}\; x_{ij}}{2(\sum_{j\in N}\; t'_{j_{(4)}} y_j - \sum_{j\in N}\; t_{j_{(3)}} y_j)} \leqslant \phi, \tag{38}$$

$$(20) - (25). \tag{39}$$

Step 3. According to Chakraborty et al. [6], write the equivalent version of Model 3 as below.

Model 4:

$$\begin{aligned} \min: \quad & Z_1 = \sum_{\substack{(s,j)\in\mathcal{E} \\ j\in N}} c_j\,(M + f(s,j))\,x_{sj} + \sum_{\substack{(i,j)\in\mathcal{E} \\ i,j\in N}} c_j f(i,j)\; x_{ij}, \\ \min: \quad & Z_2 = W, \end{aligned}$$

s.t.

$$(35) - (39).$$

In the sequel, for solving the bi-objective crisp Model 4, the proposed method in the next subsection is applied.

3.2 Phase 2: multi-objective solution approach

The obtained Model 4 is a crisp bi-objective problem. Multi-objective problems have gained many researchers' attention [3, 4, 27]. In any multi-objective optimization problem, it is rarely possible to find a single solution which optimizes each objective function simultaneously. In this case, the objective functions are said to be conflicting and the concept of the efficient solution is used. Solving multi-objective problems is not as straightforward as single objective optimization problems. There are different methods for solving multi-objective models. Max-min is a well-known operator in fuzzy programming approach [37]. Nevertheless, sometimes this approach could not find efficient solutions for a multi-objective problem [29]. In order to improve the max-min operator in fuzzy programming approach, Torabi and Hassini [35], Demirli and Yimer [9], Ferdowsi et al. [11], etc. presented some modified solution methods. In this paper, a new approach is also developed to solve the bi-objective crisp formulation (Model 4). The steps of this new fuzzy interactive method are as follows.

Step 1. Ask decision maker to select values of the acceptable confidence levels α, β, λ and ϕ.

Step 2. Calculate the positive and negative ideal solutions for each objective function (Z_k^{PIS} and Z_k^{NIS} for $k = 1, 2$) by solving models (40) and (41), respectively.

$$\begin{aligned} &\min_{k=1,2} \; Z_k \\ &s.t. \\ &(35) - (39). \end{aligned} \tag{40}$$

$$\begin{aligned} &\max_{k=1,2} \; Z_k \\ &s.t. \\ &(35) - (39). \end{aligned} \tag{41}$$

Step 3. Specify a linear membership function for each objective function ($k = 1, 2$) as below.

$$\mu_k(x) = \begin{cases} 1 & \text{if } \; Z_k < Z_k^{\text{PIS}}, \\ \dfrac{Z_k^{\text{NIS}} - Z_k}{Z_k^{\text{NIS}} - Z_k^{\text{PIS}}} & \text{if } \; Z_k^{\text{PIS}} \leqslant Z_k \leqslant Z_k^{\text{NIS}}, \\ 0 & \text{if } \; Z_k > Z_k^{\text{NIS}}, \end{cases}$$

Step 4. Transform the bi-objective crisp model to the following proposed model by using the membership functions obtained from Step 3. Then solve it to find an efficient solution for the bi-objective crisp formulation.

$$\begin{aligned} &\max : \lambda(x) = \lambda_0 + \sum_{k=1}^{2} \theta_k \mu_k(x) \\ &s.t. \\ &\lambda_0 + \theta_k \leqslant \sum_{k=1}^{2} \theta_k \mu_k(x) \quad k = 1, 2, \\ &\lambda_0 \in [0, 1] \\ &(35) - (39) \end{aligned} \tag{42}$$

In the model (42), θ_k is the weight that represents the importance of kth objective function. The weight values are determined by the decision maker so that $\sum_{k=1}^{2} \theta_k = 1$, $\theta_k > 0$.

Some advantages of the single objective model of the proposed approach (42) compared to the mentioned approaches in the literature are as follows:

- Membership function values are not only applied in the objective function.
- Being independent of the parameters that used in other solution approaches, like γ in the TH approach.

Step 5. If the obtained efficient solution satisfies decision maker, stop. Otherwise, return to Step 1 and change the required parameters such as $\alpha, \beta, \lambda, \phi$ and θ_k to reach a satisfactory solution for the decision maker.

The explained solution procedure is summarized in Fig. 3.

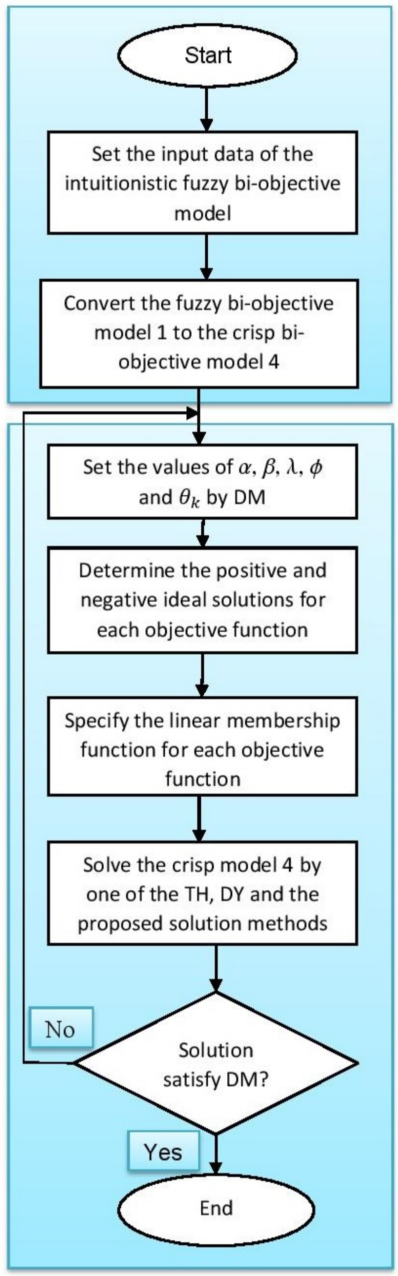

Figure 3: A flowchart covering the proposed solution approach and previous approaches of the literature.

4 Computational experiments

To study the performance of the proposed solution approach, a case study is considered in this section. The required models of the proposed solution approach are solved using CPLEX solver of GAMS. Experiments are done on a laptop equipped with Core i5 Processor (2.40 GHz) and 4.00 GB RAM. The case is taken from Turkey. Petrol Ofisi (PO), OPET, Shell and Aytemiz are four different fuel distributors in Turkey are presenting different fuels, including liquefied petroleum gas (LPG) in different costs for consumers of alternative fuel based vehicles. Fuel stations between two cities, Istanbul and Van are considered in the case study. The LPG has different costs in these stations. Table 1 shows the name of fuel stations, their corresponding node number on the network connecting Istanbul to Van (Fig. 4) and the offered costs of LPG. Figure 4 demonstrates a network topology of the fuel stations of Table 1.

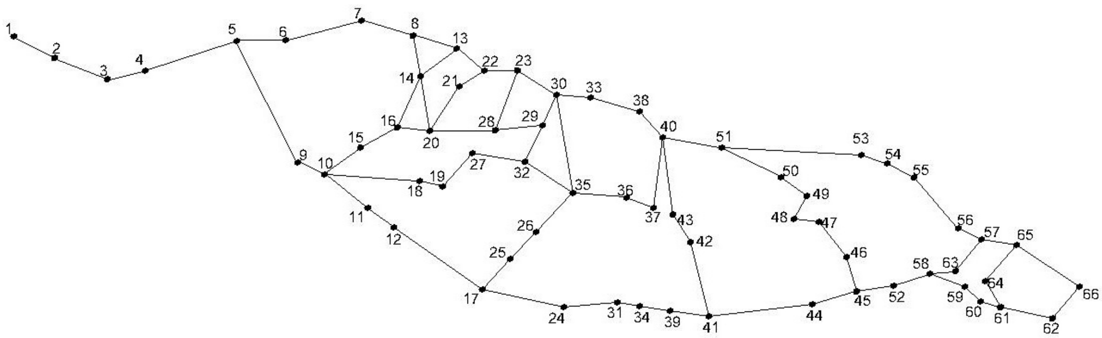

Figure 4: Network of fuel stations of case study.

Network node number	Fuel station name	Cost of LPG	Network node number	Fuel station name	Cost of LPG
1	Istanbul	2.87	34	Darende	2.79
2	Gebze	2.86	35	Sivas	2.97
3	Sapanca	2.79	36	Zara	3.01
4	Hendek	2.79	37	Imranli	3.01
5	Gerede	2.89	38	Susehri	2.97
6	Cerkes	2.89	39	Kurecik	2.94
7	Tosya	2.83	40	Refahiye	2.95
8	Osmancik	2.79	41	Malatya	2.79
9	Elmadag	2.85	42	Arapkir	2.99
10	Kirikkale	2.84	43	Kemaliye	3.04
11	Kaman	2.93	44	Palu	3.01
12	Kirsehir	2.69	45	Bingol	2.69
13	Merzifon	2.95	46	Sarigumus	2.69
14	Corum	2.79	47	Kigi	3.01
15	Delice	2.91	48	Yayladere	2.93
16	Sungurlu	2.79	49	Guzelpinar	2.93
17	Kayseri	2.79	50	Pulumur	3.01
18	Yozgat	2.79	51	Erzincan	2.95
19	Sorgun	2.79	52	Solhan	2.69
20	Alaca	2.99	53	Erzurum	2.89
21	Mecitozu	2.94	54	Pasinler	3.03
22	Amasya	2.95	55	Horasan	3.07
23	Tasova	2.99	56	Tutak	3.02
24	Pinarbasi	3.00	57	Patnos	3.02
25	Sarioglan	3.00	58	Mus	2.74
26	Sarkisla	2.97	59	Haskoy	3.02
27	Akdagmadeni	3.00	60	Guroymak	3.02
28	Zile	2.89	61	Tatvan	3.02
29	Tokat	2.89	62	Gevas	3.02
30	Niksar	2.89	63	Bulanik	2.97
31	Gurun	2.99	64	Ahlat	3.02
32	Yildizeli	3.01	65	Ercis	2.98
33	Resadiye	3.01	66	Van	2.77

Table 1: Fuel stations and cost of LPG.

To travel from Istanbul to Van the path, $1-2-3-4-5-6-7-8-13-22-23-30-33-38-40-51-53-54-55-56-57-65-66$ is considered. Length of arcs on the path and values of fuel consumption for traveling the arcs between nodes which are located on this path are provided in Table 2. The fuel consumption between the nodes is computed by an assumption that, in average 10 liters of gas is needed for traversing 120 *km*. Intuitionistic fuzzy values of waiting time for refueling stations on the considered path are also presented in Table 3. Suppose that a vehicle with the fuel tank capacity of $R = 25$ *liters* is going to trip on the path from Istanbul to Van. Moreover, it is assumed that the vehicle has a half-full tank of fuel at the origin, Istanbul. Due to the limited tank capacity, it needs some refuelings for reaching the destinations. To find appropriate stations according to mentioned criteria in previous sections, Model 4 is solved by the new proposed solution method and the results are compared to TH and DY solution methods of the literature [9, 35]. To this aim, some parameters like $\gamma, \theta_1, \theta_2, \phi$, α, β, λ, v and T should be tuned by the decision maker to attain a satisfactory solution. The results in Tables 4-7 are obtained by considering $\gamma = 0.4$, $\theta_2 = 0.4$, $v = 78$ and $T = 1100$. In Table 4, λ and ϕ are fixed to values of 0.1 and 0.8 and in Table 5 are fixed to 0.8 and 0.1 and they are examined over different values of α and β ($\alpha, \beta \in \{0.2, 0.4, 0.6, 0.8\}$). Furthermore, in Table 6, α and β are fixed to values of 0.1 and 0.8 and in Table 7 are fixed to 0.8 and 0.1 and they are examined over different values of λ and ϕ ($\lambda, \phi \in \{0.2, 0.4, 0.6, 0.8\}$). In fact, if in any case the solution doesn't satisfy the decision maker, the changeable parameters such as $\alpha, \beta, \lambda, \phi$ and θ_k could vary to obtain a satisfactory solution. In these tables, the values of the objective functions and their corresponding satisfaction degrees are obtained by TH, DY and the proposed solution methods. In order to compare these fuzzy approaches, a well-known distance measure i.e., $D(x) = \sum_k \theta_k(1 - \mu_k(x))$ is defined for determining the degree of closeness of each solution to the corresponding ideal solution [35].

Arc	Distance	Fuel consumption	Arc	Distance	Fuel consumption
(1,2)	63	5.25	(30,33)	53	4.42
(2,3)	80	6.66	(33,38)	78	6.5
(3,4)	53	4.42	(38,40)	72	6
(4,5)	141	11.75	(40,51)	73	6.08
(5,6)	68	5.66	(51,53)	210	17.5
(6,7)	113	9.42	(53,54)	40	3.33
(7,8)	75	6.25	(54,55)	46	3.83
(8,13)	63	5.25	(55,56)	104	8.67
(13,22)	46	3.83	(56,57)	40	3.33
(22,23)	50	4.17	(57,65)	50	4.17
(23,30)	71	5.92	(65,66)	156	13

Table 2: Values of fuel consumption and length of arcs on the path.

Station	IF waiting time	Station	IF waiting time
1	(5,6,7,8)(3,6,7,9)	33	(1,3,4,6)(0.5,3,4,7)
2	(5,6,7,8.5)(4,6,7,10)	38	(1,3,4,6)(0.5,3,4,7)
3	(6,8,10,12)(5,8,10,13)	40	(3,4,5,8)(2,4,5,10)
4	(6,8,10,12)(5,8,10,13)	51	(3,4,5,8)(2,4,5,10)
5	(3,5,6,7)(2,5,6,9)	53	(3,5,6,7)(2,5,6,9)
6	(3,5,6,7)(2,5,6,9)	54	(1,2,3,4)(0.5,2,3,5)
7	(6,8,9,11)(4,8,9,14)	55	(0,0.5,1,2)(0,0.5,1,4)
8	(6,8,10,12)(5,8,10,13)	56	(1.5,2,3,4.5)(1,2,3,6)
13	(3,4,5,8)(2,4,5,10)	57	(1.5,2,3,4.5)(1,2,3,6)
22	(3,4,5,8)(2,4,5,10)	65	(3,4,5,7)(1,4,5,9)
23	(1.5,3,4,7)(0.5,3,4,8)	66	(7,9,11,13)(5,9,11,14)
30	(3,5,6,7)(2,5,6,9)		

Table 3: Intuitionistic fuzzy values of waiting time in queue for refueling stations located on the path.

The results in Table 4 show that for different values of α and β the performance of TH and the proposed solution methods is better than the DY method in terms of distance measure. In fact, based on the definition of D, the approach with the lower value of distance measure has better performance. For $\alpha = 0.2, \beta = 0.8$, TH and the proposed methods have the same results and in comparison to DY method, they have fewer values of objective functions. In other cases for $\alpha = 0.4, \beta = 0.6$; $\alpha = 0.6, \beta = 0.4$ and $\alpha = 0.8, \beta = 0.2$ the proposed method has better performance than TH and DY approaches.

The results in Table 5 are obtained in case $\lambda = 0.8$ and $\phi = 0.1$ for different values of α and β. In this table, DY method has more distance measure than TH and the proposed methods. Therefore, the performance of TH and the proposed solution methods is better than DY method in all cases. For $\alpha = 0.2$, $\beta = 0.8$ and $\alpha = 0.4$, $\beta = 0.6$ the obtained values of the first and the second objective functions by TH and the proposed approaches are the same. While in other cases, the proposed method has better performance than TH approach. In fact, the proposed method can obtain solutions which are closer to ideal solutions.

The results in Table 6 are concluded from the fixed values of $\alpha = 0.1$ and $\beta = 0.8$ over different values of λ and ϕ. In all cases, the performance of TH and the proposed solution methods is the same. Moreover, their obtained solutions are better than DY approach in these cases. As can be seen in Table 7, the results of TH and the proposed methods are

α	β	TH			DY			The proposed		
		$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D
0.2	0.8	0.932,0.943	400.004,49.9	0.064	0.898,0.752	402.402,76.25	0.160	0.932,0.943	400.004,49.9	0.064
0.4	0.6	0.932,0.947	400.004,63.3	0.062	0.898,0.748	402.402,96.75	0.162	0.994,0.868	395.614,76.6	0.056
0.6	0.4	0.932,0.949	400.004,76.7	0.061	0.898,0.744	402.402,117.25	0.164	0.994,0.885	395.614,89.4	0.050
0.8	0.2	0.967,0.923	397.523,96.45	0.051	0.898,0.742	402.402,137.75	0.164	0.994,0.898	395.614,102.2	0.044

Table 4: The results of the different solution methods when $\lambda = 0.1$, $\phi = 0.8$, $\gamma = 0.4$, $T = 1100$, $v = 78$ and $\theta_2 = 0.4$.

α	β	TH			DY			The proposed		
		$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D
0.2	0.8	0.932,0.928	400.004,49.9	0.069	0.595,0.711	423.754,73.35	0.359	0.932,0.928	400.004,49.9	0.069
0.4	0.6	0.932,0.928	400.004,63.3	0.070	0.898,0.659	402.402,96.75	0.198	0.932,0.928	400.004,63.3	0.070
0.6	0.4	0.932,0.929	400.004,76.7	0.069	0.898,0.641	402.402,117.25	0.205	0.994,0.839	395.614,89.4	0.068
0.8	0.2	0.912,0.957	401.395,85.75	0.070	0.898,0.627	402.402,137.75	0.210	0.994,0.853	395.614,102.2	0.063

Table 5: The results of the different solution methods when $\lambda = 0.8$, $\phi = 0.1$, $\gamma = 0.4$, $T = 1100$, $v = 78$ and $\theta_2 = 0.4$.

also better than DY approach. In this table, the proposed approach has better performance than TH and DY methods for different values of λ and ϕ.

λ	ϕ	TH			DY			The proposed		
		$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D
0.2	0.8	0.932,0.944	400.004,47.3	0.063	0.898,0.753	402.402,72.65	0.16	0.932,0.944	400.004,47.3	0.063
0.4	0.6	0.932,0.944	400.004,47.3	0.063	0.898,0.753	402.402,72.65	0.16	0.932,0.944	400.004,47.3	0.063
0.6	0.4	0.932,0.942	400.004,47.3	0.064	0.898,0.747	402.402,72.65	0.162	0.932,0.942	400.004,47.3	0.064
0.8	0.2	0.932,0.933	400.004,47.3	0.068	0.898,0.706	402.402,72.65	0.179	0.932,0.933	400.004,47.3	0.068

Table 6: The results of the different solution methods when $\alpha = 0.1$, $\beta = 0.8$, $\gamma = 0.4$, $T = 1100$, $v = 78$ and $\theta_2 = 0.4$.

In order to study the effect of different values of θ_k, the study case is solved by TH, DY and the proposed solution approaches when $\alpha = 0.1, \beta = 0.8, \lambda = 0.4, \phi = 0.6$, $\gamma = 0.4$, $v = 78$ and $T = 1100$. The results are presented in Table 8. As seen, the results of TH and DY approaches, except in one case, are not sensitive to the changes made in the value of θ_k, which is not desirable for the decision maker. In fact, just for $\theta_2 = 0.9$ TH and DY methods give different values for Z_1 and Z_2 compared to what obtained by other weight values. In contrast, the proposed approach is sensitive to the different values of the weights. Moreover as can be seen in this table, the proposed method has better performance than DY method in terms of distance measure for different values of weights. Furthermore, the proposed method has the same performance with that of TH approach for $\theta_2 = 0.4$ and $\theta_2 = .05$ and for other values of the weights it has better performance.

By the above analysis, we may conclude that the proposed solution method in all cases, has better performance than DY approach. Also, it obtains similar solutions to TH approach in some cases, and in other cases, it has better performance than that of TH approach. Moreover, the proposed approach is more sensitive to θ_k changes. In addition to the above descriptions, the proposed method is not dependent on the parameter like γ. Therefore, despite TH approach, it doesn't need to search for finding the best value of γ.

λ	ϕ	TH			DY			The proposed		
		$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D
0.2	0.8	0.967,0.926	397.523,100.45	0.049	0.898,0.741	402.402,144.4	0.165	0.994,0.901	395.614,106.3	0.043
0.4	0.6	0.967,0.926	397.523,100.45	0.049	0.898,0.741	402.402,144.4	0.165	0.994,0.901	395.614,106.3	0.043
0.6	0.4	0.967,0.921	397.523,100.45	0.051	0.898,0.723	402.402,144.4	0.172	0.994,0.894	395.614,106.3	0.046
0.8	0.2	0.912,0.963	401.395,89.5	0.062	0.898,0.659	402.402,144.4	0.198	0.994,0.870	395.614,106.3	0.056

Table 7: The results of the different solution methods when $\alpha = 0.8$, $\beta = 0.1$, $\gamma = 0.4$, $T = 1100$, $v = 78$ and $\theta_2 = 0.4$.

θ_2 ($\theta_1 = 1 - \theta_2$)	TH			DY			The proposed		
	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D	$\mu(Z_1),\mu(Z_2)$	Z_1,Z_2	D
0.1	0.932,0.944	400.004,47.3	0.067	0.898,0.753	402.402,72.65	0.116	0.994,0.838	395.614,61.35	0.022
0.2	0.932,0.944	400.004,47.3	0.066	0.898,0.753	402.402,72.65	0.131	0.994,0.838	395.614,61.35	0.037
0.3	0.932,0.944	400.004,47.3	0.064	0.898,0.753	402.402,72.65	0.145	0.994,0.838	395.614,61.35	0.053
0.4	0.932,0.944	400.004,47.3	0.063	0.898,0.753	402.402,72.65	0.16	0.932,0.944	400.004,47.3	0.063
0.5	0.932,0.944	400.004,47.3	0.062	0.898,0.753	402.402,72.65	0.174	0.932,0.944	400.004,47.3	0.062
0.6	0.932,0.944	400.004,47.3	0.061	0.898,0.753	402.402,72.65	0.189	0.867,1	404.581,39.8	0.053
0.7	0.932,0.944	400.004,47.3	0.060	0.898,0.753	402.402,72.65	0.203	0.867,1	404.581,39.8	0.040
0.8	0.932,0.944	400.004,47.3	0.058	0.898,0.753	402.402,72.65	0.218	0.867,1	404.581,39.8	0.027
0.9	0.867,1	404.581,39.8	0.013	0.926,0.863	400.369,58.05	0.131	0.867,1	404.581,39.8	0.013

Table 8: The results of the different solution methods with different weights when $\alpha = 0.1$, $\beta = 0.8, \lambda = 0.4, \phi = 0.6$, $\gamma = 0.4$, $v = 78$ and $T = 1100$.

5 Conclusions

Due to the limited fuel tank capacity of alternative fuel vehicles, vehicles may need frequent refuelings during their trip. This paper introduced a new bi-objective intuitionistic fuzzy programming model to formulate the refueling alternative fuel vehicles problem. The model has two objective functions. Their aims are minimizing the total cost of refueling as well as minimizing the intuitionistic fuzzy refueling waiting time. To solve the proposed model, a new two-phase solution approach was suggested. In the first phase, the intuitionistic fuzzy model is transformed to a crisp model using the intuitionistic fuzzy chance constrained method based on the concept of credibility measure. In the second phase, the bi-objective crisp model is solved by a new interactive fuzzy solution method. In order to investigate the validity of the proposed solution approach, a real case from Turkey is solved. The obtained results show that the proposed method performs either better or the same as the solution methods of the literature.

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