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A new method for solving fuzzy multi-objective linear programming problems

J. Y. Dong[1] and S. P. Wan[2]

[1] College of Statistics, Jiangxi University of Finance and Economics, Nanchang 330013, China
[2] College of Information Technology, Jiangxi University of Finance and Economics, Nanchang 330013, China

jiuyingdong@126.com, shupingwan@163.com

Abstract

The purpose of this paper is to develop a new two-stage method for fuzzy multi-objective linear program and apply to engineering project portfolio selection. In the fuzzy multi-objective linear program, all the objective coefficients, technological coefficients and resources are trapezoidal fuzzy numbers (TrFNs). An order relationship for TrFNs is introduced by using the interval expectation of TrFNs. In the first stage, the fuzzy multi-objective linear program with TrFNs is transformed into an interval multi-objective linear program according to the order relationship of TrFNs. Combining the ranking order relation between intervals with the satisfactory crisp equivalent forms of interval inequality relations, the interval multi-objective linear program is further transformed into a crisp multi-objective linear program. In the second stage, the positive and negative ideal solutions are calculated as well as the closeness degrees from the positive ideal solution to all objectives on the basis of the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution). Then, using the closeness degrees, we convert the crisp multi-objective linear program into mono-objective program to solve. The proposed method is not only mathematically rigorous, but also can adequately consider the acceptance degree of decision maker that the fuzzy constraints may be violated. The other possible cases of the fuzzy multi-objective linear program are also discussed. The proposed method is illustrated by means of a project portfolio selection problem.

Keywords: Fuzzy multi-objective linear program; Trapezoidal fuzzy number; Project portfolio selection; Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)

1 Introduction

In traditional mathematical program, the coefficients of the problems are always treated as deterministic and crisp values[9,35]. However uncertainty always exits in practical engineering problems. In order to deal with the uncertain optimization problems, fuzzy parameters are often introduced to capture the imprecise characteristics. As a result, the fuzzy mathematical program has attracted considerable attention[1, 36-38, 43].

Many decision problems in engineer management or finance as well as various other areas require the simultaneous consideration of several criteria and, thus, are often modeled and solved using methods from multi-objective program. Since the multiple objectives are usually conflicting and fraught with uncertainties, the fuzzy multi-objective linear programming problems appear. Heretofore, a great variety of methods have been proposed to solve the fuzzy multi-objective linear programming problems[2,4-6,8,12,15-17,19,20,23,26,31,32].

Hannan[17] quantified fuzzy or imprecise aspirations of the decision maker (DM) through the use of piecewise linear and continuous functions. He constructed the fuzzy goal programming models with preemptive priorities, with Archimedean weights, and with the maximization of the membership function corresponding to the minimum goal. Dhamar et al[8] used arithmetic addition to aggregate the fuzzy goals to construct the relevant decision function. Cardinal and ordinal weights for nonequivalent fuzzy goals were also incorporated in their method. Arenas et al[4] studied

Corresponding Author: S. P. Wan
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the possibilistic multiobjective linear programming problem and developed two different models from the initial solution and based on Goal Program: an Interval Goal Programming Problem if we define the relation as accurate as possible based on the expected intervals of fuzzy numbers, and an ordinary goal program based on the expected values of the fuzzy numbers that defined the goals. Gao et al[15] designed a λ-cut and goal-programming-based algorithm for fuzzy linear multiple objective bilevel optimization. Luhandjula[31] incorporated possibilistic data into a multiple objective linear programming framework. He specified the solution concept from the standpoints of feasibility and efficiency and established a necessary and sufficient condition for such a solution. Inuiguchi and Sakawa[23] extended the concept of efficient solutions to the conventional multiobjective linear programming problems to the fuzzy (possibilistic) coefficients case. They proposed two kinds of efficient solution sets, i.e., a set of possibly efficient solutions and a set of necessarily efficient solutions as fuzzy sets whose membership grades represent the possibility and necessity degrees to which the solution is efficient. Arenas et al[3] proposed a new Pareto optimal solution concept for fuzzy multiobjective programming problems based on the extension principle and the joint possibility distribution of the fuzzy parameters of the problem. The method relies on a-cuts of the fuzzy solution to generate its possibility distributions. Gupta and Mehlawat[16] developed a new possibilistic programming approach for solving fuzzy multiobjective assignment problem. Cadenas and Verdegay[5] proposed solution methodologies for multiobjective fuzzy mathematical programming problems using different ordering methods ranking fuzzy numbers. Arenas et al[2] developed fuzzy compromise programming approach to solving a multiobjective possibilistic problem. Chang et al[6] constructed a fuzzy multi-objective programming model for the evaluation of sustainable management strategies of optimal land development in the reservoir watershed. Wu[37] investigated the Karush.®Kuhn.®Tucker optimality conditions in multiobjective programming problems with interval-valued objective functions. Luhandjula and Rangoaga[32] presented a new approach for dealing with a multiobjective programming problem with fuzzy-valued objective functions based on the nearest interval approximation operator. Hassanzadeh et al[19] proposed a robust optimization method for interactive multiobjective program with imprecise information and applied to R&D (research and development) project portfolio selection. Li and Hu[26] studied an interactive satisfying method based on alternative tolerance for multiple objective optimization with fuzzy parameters. Dubey and Mehra[12] developed a bipolar approach to solving fuzzy multi-objective linear program. Rivaz and Yaghoobi[33] investigated the minimax regret solution to multiobjective linear programming problems with interval objective functions coefficients. Rivaz and Yaghoobi[34] put forward a weighted sum of maximum regrets in an interval multiobjective linear program.

According to the types of fuzzy parameters, all these fuzzy multi-objective linear programming models can be divided into four groups: the first is fuzzy multi-objective linear program with intervals[37,19,33,34], the second is fuzzy multi-objective linear program with triangular fuzzy numbers (TFNs)[2,4,5], the third is fuzzy multi-objective linear program with trapezoidal fuzzy numbers (TrFNs)[26], the fourth is fuzzy multi-objective linear program with fuzzy numbers[3,17,23,32]. The aforementioned methods seem to be effective to solve fuzzy multi-objective linear program. However, none of them considered the acceptance degrees of decision maker (DM) that the fuzzy constraints may be violated. In reality, some fuzzy constraints sometimes may not be satisfied while DM can accept them with some acceptance degree. Therefore, it is very necessary and reasonable to take the acceptance degrees of DM into account during the process of solving fuzzy mathematical program.

In this paper, combining the satisfactory crisp equivalent form of an interval inequality relation[27] with the order relation of TrFNs, we propose a new two-stage method for fuzzy multi-objective linear program with TrFNs. The order relationship for TrFNs is firstly defined through defining the interval expectation of TrFNs. Utilized the order relationship of TrFNs, the fuzzy multi-objective linear program with TrFNs is transformed into an interval multi-objective linear program. Combining the ranking order relation between intervals with interval objective program, the interval multi-objective linear program is further transformed into a crisp multi-objective linear program. Then, based on TOPSIS (Technique for Order Preference by Similarity to Ideal Solution[22]), the crisp multi-objective linear program is converted into a mono-objective program for resolution. The proposed method in this paper has the following outstanding features:

(1) The transforming process from the fuzzy multi-objective linear program with TrFNs to the crisp multi-objective linear program is mathematically rigorous and logical since it ingeniously combines the interval order relation[21] with interval objective program[24].

(2) To solve the interval multi-objective linear program, we sufficiently consider the acceptance degrees of DM that interval fuzzy constraints may be violated. The acceptance degree is capable of characterizing the risk attitude of DM and can make the decision result more consistent with real situations.

(3) Utilized the TOPSIS, the crisp multi-objective linear program is converted into a mono-objective program to solve. Especially, when the parameter q of norm function is equal to 1, the derived mono-objective program is a common linear programming model which can be readily solved by Simplex method.

(4) Choosing different acceptance degrees and the parameter of norm, DM can obtain different optimal solutions,

which greatly enhances the flexibility of decision making.

The rest of this paper is planned as follows. In Section 2, we briefly review the interval order relation and interval objective program and thereby define the order relation for TrFNs. In Section 3, a new two-stage method for fuzzy multi-objective linear program with TrFNs is developed. Section 4 is devoted to a R&D project portfolio selection example for the sake of illustration. The other potential cases on the fuzzy multi-objective linear program are further discussed in Section 5. Section 6 ends the paper with some concluding remarks.

2 Preliminaries

In this section, the interval order relation and interval objective program are recalled. Then, the order relation of TrFNs is defined.

2.1 Ranking order relation between intervals

An interval can be denoted by $\tilde{u} = [\underline{u}, \bar{u}]$, where $\underline{u}$ and $\bar{u}$ are the lower and upper bounds of $\tilde{u}$, respectively, satisfying $-\infty < \underline{u} \leqslant \bar{u} < +\infty$. The symbols $m(\tilde{u}) = (\underline{u} + \bar{u})/2$ and $r(\tilde{u}) = (\bar{u} - \underline{u})/2$ mean the middle point and radius of the interval $\tilde{u}$. Apparently, if $\underline{u} = \bar{u}$, then $r(\tilde{u}) = 0$, the interval $\tilde{u} = [\underline{u}, \bar{u}]$ degenerates into a real number.

Let $\tilde{u}_i = [\underline{u}_i, \bar{u}_i]$ $(i = 1, 2)$ be two intervals. The symbol $\tilde{u}_1 \preceq_I \tilde{u}_2$ represents that the interval $\tilde{u}_1$ is not greater than the interval $\tilde{u}_2$. According to the fuzzy set theory[40], the relation $\tilde{u}_1 \preceq_I \tilde{u}_2$ can be viewed as a fuzzy relation between $\tilde{u}_1$ and $\tilde{u}_2$. In this regard, a fuzzy partial order relation for intervals defined by Hu et al[21] is proven to be mathematical rigorous up to date.

Definition 2.1. [21] *The premise $\tilde{u}_1 \preceq_I \tilde{u}_2$ is viewed as a fuzzy set with membership degree as follows:*

$$\varphi(\tilde{u}_1 \preceq_I \tilde{u}_2) = \begin{cases} 1 & \text{if } \bar{u}_1 \leqslant \underline{u}_2 \\ 1^- & \text{if } \underline{u}_1 \leqslant \underline{u}_2 \leqslant \bar{u}_1 \leqslant \bar{u}_2 \text{ and } r(\tilde{u}_1) > 0 \\ \frac{\bar{u}_2 - \bar{u}_1}{2(r(\tilde{u}_2) - r(\tilde{u}_1))} & \text{if } \underline{u}_2 \leqslant \underline{u}_1 \leqslant \bar{u}_1 \leqslant \bar{u}_2 \text{ and } r(\tilde{u}_2) > r(\tilde{u}_1) \\ 0.5 & \text{if } r(\tilde{u}_1) = r(\tilde{u}_2) \text{ and } \underline{u}_1 = \underline{u}_2 \end{cases}$$

where 1^- represents a fuzzy number being less than one, which indicates that the interval $\tilde{u}_1$ is weakly not greater than the interval $\tilde{u}_2$.

Clearly, $0 \leqslant \varphi(\tilde{u}_1 \preceq_I \tilde{u}_2) \leqslant 1$. The membership degree $\varphi(\tilde{u}_1 \preceq_I \tilde{u}_2)$ expresses the acceptability degree of the fuzzy relation $\tilde{u}_1 \preceq_I \tilde{u}_2$. In particular, $\varphi(\tilde{u}_1 \preceq_I \tilde{u}_2) = 0$ shows that the premise $\tilde{u}_1 \preceq_I \tilde{u}_2$ is not accepted by the DM; $0 < \varphi(\tilde{u}_1 \preceq_I \tilde{u}_2) < 1$ implies that the DM accepts the premise $\tilde{u}_1 \preceq_I \tilde{u}_2$ with different satisfactory degrees within[0, 1]; $\varphi(\tilde{u}_1 \preceq_I \tilde{u}_2) = 1$ indicates that the DM is absolutely satisfied with the premise $\tilde{u}_1 \preceq_I \tilde{u}_2$. In other words, the DM deems that the premise $\tilde{u}_1 \preceq_I \tilde{u}_2$ holds definitely.

Analogously, the symbol $\tilde{u}_1 \succeq_I \tilde{u}_2$ denotes that the interval $\tilde{u}_1$ is not less than the interval $\tilde{u}_2$. It also can be viewed as a fuzzy relation.

Definition 2.2. [21] *The premise $\tilde{u}_1 \succeq_I \tilde{u}_2$ is regarded as a fuzzy set, whose membership degree is defined as $\varphi(\tilde{u}_1 \succeq_I \tilde{u}_2) = 1 - \varphi(\tilde{u}_1 \preceq_I \tilde{u}_2)$, i.e.,*

$$\varphi(\tilde{u}_1 \succeq_I \tilde{u}_2) = \begin{cases} 0 & \text{if } \bar{u}_1 \leqslant \underline{u}_2 \\ 0^- & \text{if } \underline{u}_1 \leqslant \underline{u}_2 \leqslant \bar{u}_1 \leqslant \bar{u}_2 \text{ and } r(\tilde{u}_1) > 0 \\ (\underline{u}_1 - \underline{u}_2)/[2(r(\tilde{u}_2) - r(\tilde{u}_1))] & \text{if } \underline{u}_2 \leqslant \underline{u}_1 \leqslant \bar{u}_1 \leqslant \bar{u}_2 \text{ and } r(\tilde{u}_2) > r(\tilde{u}_1) \\ 0.5 & \text{if } r(\tilde{u}_1) = r(\tilde{u}_2) \text{ and } \underline{u}_1 = \underline{u}_2 \end{cases}$$

where 0^- denotes a fuzzy number being greater than zero, which shows that the interval $\tilde{u}_1$ is weakly not less than the interval $\tilde{u}_2$.

In fact, the symbol $\preceq_I$ is an interval version of the order relation $\leqslant$ in the set of real numbers. It can be linguistically interpreted as essentially not greater than. The symbols $\succeq_I$ and $=_I$ are interpreted similarly. If $\tilde{u}_1 \succeq_I \tilde{u}_2$ and $\tilde{u}_1 \preceq_I \tilde{u}_2$, then $\tilde{u}_1 =_I \tilde{u}_2$. Thus, $\succeq_I$ and $\preceq_I$ establish fuzzy partial orders for intervals[21]. Definitions 1 and 2 may provide quantitative methods to determine the exact degree of membership for comparing two intervals. Li et al[27] introduced the satisfactory crisp equivalent forms of interval inequality relations according to the fuzzy ranking index φ.

Definition 2.3. [27] *A satisfactory crisp equivalent form of an interval inequality relation* $\tilde{u}_1 x \preceq_I \tilde{u}_2$ *is defined as* $\bar{u}_1 x \leqslant \bar{u}_2$ *and* $\varphi(\tilde{u}_1 x \succeq_I \tilde{u}_2) \leqslant \alpha$, *where* $\alpha \in [0,1]$ *denotes the acceptance degree of the interval inequality constraint which may be violated. Similarly, a satisfactory crisp equivalent form of an interval inequality relation* $\tilde{u}_1 x \succeq_I \tilde{u}_2$ *is defined as follows:* $\underline{u}_1 x \geqslant \underline{u}_2$ *and* $\varphi(\tilde{u}_1 x \preceq_I \tilde{u}_2) \leqslant \alpha$.

Remark 2.4. *For the interval inequality relation* $\tilde{u}_1 x \preceq_I \tilde{u}_2$, *the satisfactory crisp equivalent form is* $\bar{u}_1 x \leqslant \bar{u}_2$ *with considering the acceptance degree of the fuzzy relation* $\tilde{u}_1 x \preceq_I \tilde{u}_2$ *violated. The acceptance degree* α *=0 shows that the DM completely does not allow the relation* $\tilde{u}_1 x \preceq_I \tilde{u}_2$ *to be violated, i.e., the membership degree of* $\tilde{u}_1 x \succeq_I \tilde{u}_2$ *is equal to 0; The acceptance degree* α *=1 implies that the DM completely allows the relation* $\tilde{u}_1 x \preceq_I \tilde{u}_2$ *to be violated, i.e., the membership degree of* $\tilde{u}_1 x \succeq_I \tilde{u}_2$ *is not greater than 1; The acceptance degree* $\alpha \in (0,1)$ *indicates that the DM may allow the relation* $\tilde{u}_1 x \preceq_I \tilde{u}_2$ *to be violated with some degree between 0 and 1, i.e., the membership degree of* $\tilde{u}_1 x \succeq_I \tilde{u}_2$ *is not greater than* α. *The satisfactory crisp equivalent form of* $\tilde{u}_1 x \succeq_I \tilde{u}_2$ *can be similarly explained. The satisfactory crisp equivalent forms of interval inequality relations are particularly useful to transform the fuzzy constraints into crisp constraints. The selection of the value of* α *depends on the acceptance level of the DM about the fuzzy relation* $\tilde{u}_1 x \preceq_I \tilde{u}_2$ *being violated. Different DMs can select different values of* α, *which can accurately reflect the flexibility of decision making process.*

2.2 Interval objective program

The interval objective program initiated by Ishibuchi and Tanaka[24] includes the maximization problem with the interval objective function and the minimization problem with the interval objective function. We only list the maximization problem with the interval objective function. The minimization problem is referred to Ishibuchi and Tanaka[24].

Definition 2.5. [24] *Let* $\tilde{u}_i = [\underline{u}_i, \bar{u}_i]$ $(i = 1, 2, \cdots, n)$ *be intervals and* $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T$ *be the n-dimensional decision variable vector. The maximization problem with the interval objective function is stated as*

$$\max \ \{ \ \tilde{z}(\mathbf{x}) = \sum_{i=1}^{n} \tilde{u}_i x_i \} \quad s.t. \quad \mathbf{x} = (x_1, x_2, \cdots, x_n)^T \in X$$

where X *means a set of constraints in which the variable* x *should fulfill depending on the needs of real-life problem.*

Theorem 2.6. *The solution set of above problem can be obtained as the Pareto optimal solutions (i.e., noninferior or nondominated solutions) of the following multi-objective problem:*

$$\begin{aligned} &\max\{\underline{z}(\mathbf{x}) = \sum_{i=1}^{n} \underline{u}_i x_i\} \\ &\max\{s(\mathbf{x}) = \tfrac{1}{2} \sum_{i=1}^{n} (\underline{u}_i + \bar{u}_i) x_i\} \\ &s.t. \quad \mathbf{x} \in X \end{aligned}$$

2.3 Order relation for TrFNs

As a special fuzzy subset on the real number set, a TrFN $\tilde{p} = (p_1, p_2, p_3, p_4)$ has the following membership function:

$$\mu_{\tilde{p}}(x) = \begin{cases} (x - p_1)/(p_2 - p_1), & \text{if } p_1 \leqslant x < p_2 \\ 1, \text{ if } p_2 \leqslant x \leqslant p_3 \\ (p_4 - x)/(p_4 - p_3), & \text{if } p_3 < x \leqslant p_4 \\ 0, \text{ if } x < p_3 \text{ or } x > p_4 \end{cases}$$

where the closed interval $[p_2, p_3]$ represents the mode, p_1 and p_4 are the lower and upper limits of $\tilde{p}$, respectively[13]. The interval expectation $\mathrm{E}(\tilde{p})$ of a TrFN $\tilde{p} = (p_1, p_2, p_3, p_4)$ is defined as follows[14]:

$$\mathrm{E}(\tilde{p}) = \left[\tfrac{1}{2}(p_1 + p_2), \tfrac{1}{2}(p_3 + p_4)\right]. \tag{1}$$

Li and Wan[28] introduced an order relation for TrFNs below.

Definition 2.7. [28] *The order relation between two TrFNs* $\tilde{p}$ *and* $\tilde{t}$ *is defined as follows:*

1) $\tilde{p} \geqslant \tilde{t}$ *iff* $E(\tilde{p}) \succeq_I E(\tilde{t})$;
2) $\tilde{p} \leqslant \tilde{t}$ *iff* $E(\tilde{p}) \preceq_I E(\tilde{t})$;
3) $\tilde{p} = \tilde{t}$ *iff* $E(\tilde{p}) =_I E(\tilde{t})$.

3 A new method to solve fuzzy multi-objective linear program

In this section, we describe the fuzzy multi-objective linear program and develop a new two-stage method to solve it.

3.1 Fuzzy multi-objective linear program

A fuzzy multi-objective linear program can be stated as follows:

$$\max \ \{\ \tilde{z}_i(\mathbf{x}) = \tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x},\ i = 1, 2, \ldots, k\}$$
$$s.t.\quad \begin{cases} \tilde{\mathbf{A}}\mathbf{x} \leqslant \tilde{\mathbf{b}} \\ \mathbf{x} \geqslant \mathbf{0} \end{cases} \tag{2}$$

where $\tilde{\mathbf{A}} = (\tilde{a}_{ij})_{m\times n}$ is the technological coefficient matrix, $\tilde{\mathbf{b}} = (\tilde{b}_1, \tilde{b}_2, \cdots, \tilde{b}_m)^{\mathrm{T}}$ is the resource vector, and $\tilde{\mathbf{c}}_i = (\tilde{c}_{i1}, \tilde{c}_{i2}, \cdots, \tilde{c}_{in})^{\mathrm{T}}$ is the objective coefficient vector, $\mathbf{x} = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ is the decision variable vector, $\tilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$, $\tilde{c}_{ij} = (c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4})$ and $\tilde{b}_i = (b_{i1}, b_{i2}, b_{i3}, b_{i4})(i = 1, 2, \cdots, m;\ j = 1, 2, \cdots, n)$ are all known TrFNs, $x_j \geqslant 0(j = 1, 2, \cdots, n)$ are unknown, which need to be solved.

Remark 3.1. *In Equation (2), the objective functions, the technological coefficients, and resources are TrFNs. Thus, Equation (2) is called a fuzzy multi-objective linear program with TrFNs in this paper.*

There are several concepts of solutions for multi-objective linear programming model. The most used is Pareto optimal solution. For the fuzzy multi-objective linear program, we introduce the notion of Pareto optimal solution.

Definition 3.2. *Let Θ_1 be the set of feasible solutions of Equation (2). $\mathbf{x}^* \in \Theta_1$ is said to be a Pareto optimal solution of Equation (2) if there is no $\mathbf{x} \in \Theta_1$ such that $\tilde{z}_i(\mathbf{x}) \geqslant \tilde{z}_i(\mathbf{x}^*)\ \forall i$ with at least one $i \in \{1, 2, \ldots, k\}$ such that $\tilde{z}_i(\mathbf{x}) > \tilde{z}_i(\mathbf{x}^*)$.*

3.2 First stage: using interval ranking order relation and interval objective program

Corresponding to Equation (2), the interval multi-objective programming model is stated as follows:

$$\max\ Z_i(\mathbf{x}) = (\mathrm{E}(\tilde{\mathbf{c}}_i))^{\mathrm{T}}\mathbf{x}\ (i = 1,2,\cdots,k)$$
$$s.t.\quad \begin{cases} \mathrm{E}(\tilde{\mathbf{A}})\mathbf{x} \preceq_I \mathrm{E}(\tilde{\mathbf{b}}) \\ \mathbf{x} \geqslant \mathbf{0}, \end{cases} \tag{3}$$

where $\mathrm{E}(\tilde{\mathbf{c}}_i) = (\mathrm{E}(\tilde{c}_{i1}), \mathrm{E}(\tilde{c}_{i2}), \cdots, \mathrm{E}(\tilde{c}_{in}))^{\mathrm{T}}$, $\mathrm{E}(\tilde{\mathbf{b}}) = (\mathrm{E}(\tilde{b}_1), \mathrm{E}(\tilde{b}_2), \cdots, \mathrm{E}(\tilde{b}_m))^{\mathrm{T}}$ and $\mathrm{E}(\tilde{\mathbf{A}}) = (\mathrm{E}(\tilde{a}_{ij}))_{m\times n}$ are interval expectation vectors and matrix of corresponding TrFNs.

Definition 3.3. *Let Θ_2 be the set of feasible solutions to Equation (3). $\mathbf{x}^* \in \Theta_2$ is said to be a Pareto optimal solution of Equation (3) if there is no $\mathbf{x} \in \Theta_2$ such that $Z_i(\mathbf{x}) \succeq_I Z_i(\mathbf{x}^*)\forall i$ with at least one $i \in \{1, 2, \ldots, k\}$ such that $Z_i(\mathbf{x}) \succ_I Z_i(\mathbf{x}^*)$, where symbol $\succ_I$ means greater than.*

Theorem 3.4. *Equation (2) is equivalent to the above interval multi-objective programming model (i.e., Equation (3)) in the sense of Definition 2.7.*

Proof. If $\mathbf{x} \in \Theta_2$, then $\sum\limits_{j=1}^{n} \mathrm{E}(\tilde{a}_{ij})x_j \preceq_I \mathrm{E}(\tilde{b}_i)$, i.e., $\mathrm{E}(\sum\limits_{j=1}^{n} \tilde{a}_{ij}x_j) \preceq_I \mathrm{E}(\tilde{b}_i)$. It is derived from Definition 2.7 that $\sum\limits_{j=1}^{n} \tilde{a}_{ij}x_j \leqslant \tilde{b}_i(i = 1, 2, \cdots, m)$, i.e., $\tilde{\mathbf{A}}\mathbf{x} \leqslant \tilde{\mathbf{b}}$. Thus, $\mathbf{x} \in \Theta_1$. That is, $\Theta_2 \subseteq \Theta_1$.

In a similar way, we can prove $\Theta_2 \supseteq \Theta_1$, i.e., if $\mathbf{x} \in \Theta_1$, then $\mathbf{x} \in \Theta_2$. Thus, we have $\Theta_1 = \Theta_2$.

Suppose that $\mathbf{x}^*$ is a Pareto optimal solution of Equation (3). Then, there is no $\mathbf{x} \in \Theta_2$ such that $Z_i(\mathbf{x}) \succeq_I Z_i(\mathbf{x}^*)\forall i$ with at least one $i \in \{1, 2, \ldots, k\}$ such that $Z_i(\mathbf{x}) \succ_I Z_i(\mathbf{x}^*)$. Since $Z_i(\mathbf{x}^*) = \sum\limits_{j=1}^{n} \mathrm{E}(\tilde{c}_{ij})x_j^*$ and $Z_i(\mathbf{x}) = \sum\limits_{j=1}^{n} \mathrm{E}(\tilde{c}_{ij})x_j$, $Z_i(\mathbf{x}) \succeq_I Z_i(\mathbf{x}^*)$ is equivalent to $\mathrm{E}(\sum\limits_{j=1}^{n} \tilde{c}_j x_j) \succeq_I \mathrm{E}(\sum\limits_{j=1}^{n} \tilde{c}_j x_j^*)$, which is equivalent to $\sum\limits_{j=1}^{n} \tilde{c}_{ij}x_j \geqslant \sum\limits_{j=1}^{n} \tilde{c}_{ij}x_j^*$ by Definition 2.7, i.e., $\tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x} \geqslant \tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x}^*$, $\tilde{z}_i(\mathbf{x}) \geqslant \tilde{z}_i(\mathbf{x}^*)$. Similarly, $Z_i(\mathbf{x}) \succ_I Z_i(\mathbf{x}^*)$ is equivalent to $\tilde{z}_i(\mathbf{x}) = \tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x} > \tilde{z}_i(\mathbf{x}^*) = \tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x}^*$. Due to $\Theta_1 = \Theta_2$, there is no $\mathbf{x} \in \Theta_1$ such that $\tilde{z}_i(\mathbf{x}) \geqslant \tilde{z}_i(\mathbf{x}^*)\forall i$ with at least one $i \in \{1, 2, \ldots, k\}$ such that $\tilde{z}_i(\mathbf{x}) > \tilde{z}_i(\mathbf{x}^*)$. Hence, $\mathbf{x}^*$ is a Pareto optimal solution of Equation (2).

In a similar way, we can prove that $\mathbf{x}^*$ is a Pareto optimal solution of Equation (3) if it is a Pareto optimal solution of Equation (2). Hence, the proof of Theorem 3.4 is completed. □

The objective functions and constraints of Equation (3) include intervals. In what follows, Definition 2.5 is used to propose a solving strategy for maximizing Z_i, Definition 2.3 is employed to deal with the system of interval inequalities, i.e., $\mathrm{E}(\tilde{\mathbf{A}})\mathbf{x} \preceq_I \mathrm{E}(\tilde{\mathbf{b}})$.

Theorem 3.5. *Equation (3) is equivalent to the crisp multi-objective linear program as follows:*

$$\max\ Z_{i1}(\mathbf{x}) = \tfrac{1}{2}\sum_{j=1}^{n}(c_{ij1}+c_{ij2})x_j\ (i=1,2,\cdots,k)$$

$$\max\ Z_{i2}(\mathbf{x}) = \tfrac{1}{4}\sum_{j=1}^{n}(c_{ij1}+c_{ij2}+c_{ij3}+c_{ij4})x_j\ (i=1,2,\cdots,k)$$

$$s.t.\ \begin{cases}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant (b_{i3}+b_{i4})\ (i=1,2,\cdots,m)\\ (1-\alpha)\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j+\alpha\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant\\ (1-\alpha)(b_{i1}+b_{i2})+\alpha(b_{i3}+b_{i4})\ (i=1,2,\cdots,m)\\ \mathbf{x}\geqslant\mathbf{0}\end{cases} \tag{4}$$

Proof. By Equation (1), it is easy to calculate the interval expectation as follows:

$$(\mathrm{E}(\tilde{\mathbf{c}}_i))^{\mathrm{T}}\mathbf{x} = \sum_{j=1}^{n}\mathrm{E}(\tilde{c}_{ij})x_j = [\tfrac{1}{2}\sum_{j=1}^{n}(c_{ij1}+c_{ij2})x_j, \tfrac{1}{2}\sum_{j=1}^{n}(c_{ij3}+c_{ij4})x_j],$$

$$\sum_{j=1}^{n}\mathrm{E}(\tilde{a}_{ij})x_j = [\tfrac{1}{2}\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j, \tfrac{1}{2}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j]$$

and $\mathrm{E}(\tilde{b}_i) = [\frac{1}{2}(b_{i1}+b_{i2}), \frac{1}{2}(b_{i3}+b_{i4})]$. Then, by using Definition 2.5, the interval objective function of Equation (3),

$$\max\ \{\ Z_i(\mathbf{x}) = [\tfrac{1}{2}\sum_{j=1}^{n}(c_{ij1}+c_{ij2})x_j, \tfrac{1}{2}\sum_{j=1}^{n}(c_{ij3}+c_{ij4})x_j]\},$$

is equivalent to $\max\ \{\ Z_{i1}(\mathbf{x}) = \frac{1}{2}\sum_{j=1}^{n}(c_{ij1}+c_{ij2})x_j\}$ and $\max\ \{\ Z_{i2}(\mathbf{x}) = \frac{1}{4}\sum_{j=1}^{n}(c_{j1}+c_{j2}+c_{j3}+c_{j4})x_j\}$, i.e., the two objective functions of Equation (4).

The first constraint of Equation (3), $\mathrm{E}(\tilde{\mathbf{A}})\mathbf{x} \preceq_I \mathrm{E}(\tilde{\mathbf{b}})$, is equivalent to

$$[\tfrac{1}{2}\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j, \tfrac{1}{2}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j] \preceq_I [\tfrac{1}{2}(b_{i1}+b_{i2}), \tfrac{1}{2}(b_{i3}+b_{i4})]$$

$(i=1,2,\cdots,\mathrm{m})$, which are equivalent to $\frac{1}{2}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant \frac{1}{2}(b_{i3}+b_{i4})$ and $\varphi(\mathrm{E}(\sum_{j=1}^{n}\tilde{a}_{ij}x_j) \succeq_I \mathrm{E}(\tilde{b}_i)) \leqslant \alpha (i=1,2,\cdots,\mathrm{m})$ according to Definition 2.3. The former is $\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant (b_{i3}+b_{i4})$.

By Definition 2.2, $\varphi(\mathrm{E}(\sum_{j=1}^{n}\tilde{a}_{ij}x_j) \succeq_I \mathrm{E}(\tilde{b}_i)) \leqslant \alpha$ is transformed into the following:

$$\frac{\frac{1}{2}\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j - \frac{1}{2}(b_{i1}+b_{i2})}{\frac{1}{2}(b_{i3}+b_{i4}) - \frac{1}{2}(b_{i1}+b_{i2}) - [\frac{1}{2}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j - \frac{1}{2}\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j]} \leqslant \alpha \tag{5}$$

After rearranging, Equation (5) is rewritten as follows: $(1-\alpha)\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j + \alpha\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant (1-\alpha)(b_{i1}+b_{i2})+\alpha(b_{i3}+b_{i4})$, which is just the second constraint of Equation (4). Therefore, Theorem 3 is proven. □

Theorems 3.4 and 3.5 signify that the Pareto optimal solutions of fuzzy multi-objective linear program Equation (2) can be obtained by the Pareto optimal solutions of crisp multi-objective linear program Equation (4).

Remark 3.6. *In Equation (4), we introduce the parameter α to characterize the acceptance degree of the interval constraint $E(\tilde{\mathbf{A}})\mathbf{x} \preceq_I E(\tilde{\mathbf{b}})$ which may be violated. Although this paper utilizes the interval expectation to transform trapezoidal fuzzy constraint $\tilde{\mathbf{A}}\mathbf{x} \leqslant \tilde{\mathbf{b}}$ into the interval constraint $E(\tilde{\mathbf{A}})\mathbf{x} \preceq_I E(\tilde{\mathbf{b}})$, there exists other methods to transform fuzzy constraint into the interval constraint, such as the cut sets and interval approximation. Therefore, any fuzzy constraints always can be transformed into the interval constraints. From this point of view, the parameter may be regarded as the acceptance degree of the fuzzy constraint $\tilde{\mathbf{A}}\mathbf{x} \leqslant \tilde{\mathbf{b}}$ which may be violated.*

Remark 3.7. *Due to the uncertainty inherent in the fuzzy constraint, the fuzzy constraint cannot always hold. Even though the fuzzy constraint is violated, the DM may accept it within some scope of acceptance. The acceptance degree parameter α is just the suitable tool to describe such scope. Consequently, it is reasonable and necessary to consider the acceptance degree of fuzzy constraint violated when solving fuzzy mathematical program.*

3.3 Second stage: using TOPSIS

There are several efficient methods to solve crisp multi-objective linear programs, such as the weighted sum approach, constraint method, TOPSIS method, and so on[7]. However, choosing one of them as the best is not easily possible[7]. In this paper, we suggest using of TOPSIS method to solve the resulted crisp multi-objective program because of its simple computation and understandability. To preserve the completeness of the proposed two-stage method, the main counterparts of TOPSIS are listed below.

Let Z_{it}^{+} be the optimal value of the single objective program $\max\{Z_{it}(x)\}$ ignoring other objectives in Equation (4), Z_{it}^{-} be the optimal value of the single objective program $\min\{Z_{it}(x)\}$ ignoring other objectives in Equation (4). Denote the positive ideal solution (PIS) by $Z^{+} = (Z_{11}^{+}, Z_{21}^{+}, \cdots, Z_{k1}^{+}, Z_{12}^{+}, Z_{22}^{+}, \cdots, Z_{k2}^{+})$ which consists of individual best feasible solutions for all objectives, the negative ideal solution (NIS) by $Z^{-} = (Z_{11}^{-}, Z_{21}^{-}, \cdots, Z_{k1}^{-}, Z_{12}^{-}, Z_{22}^{-}, \cdots, Z_{k2}^{-})$ which consists of individual worst feasible solutions for all objectives. By using Minkowskis -metric, the distances from the PIS and NIS to all objectives are defined:

$$d_q^{+}(\mathrm{x}) = \{\sum_{i=1}^{k}[w_{i1}\frac{Z_{i1}^{+}-Z_{i1}(x)}{Z_{i1}^{+}-Z_{i1}^{-}}]^q + \sum_{i=1}^{k}[w_{i2}\frac{Z_{i2}^{+}-Z_{i2}(x)}{Z_{i2}^{+}-Z_{i2}^{-}}]^q\}^{1/q} \tag{6}$$

$$d_q^{-}(\mathrm{x}) = \{\sum_{i=1}^{k}[w_{i1}\frac{Z_{i1}(x)-Z_{i1}^{-}}{Z_{i1}^{+}-Z_{i1}^{-}}]^q + \sum_{i=1}^{k}[w_{i2}\frac{Z_{i2}(x)-Z_{i2}^{-}}{Z_{i2}^{+}-Z_{i2}^{-}}]^q\}^{1/q} \tag{7}$$

where w_{it} is the relative importance (weight) of objective function $Z_{it}(x)$ satisfying $w_{it} \in [0,1] (t = 1,2;\ i = 1,2,\cdots,k)$ and $\sum_{i=1}^{k}(w_{i1}+w_{i2}) = 1$, and $q = 1, 2, \cdots, +\infty$ is the parameter of norm functions. There are several methods to determine the values of weights $w_{it} \in [0,1] (t = 1,2;\ i = 1,2,\cdots,k)$ for different objective functions in crisp multi-objective decision making theory (please refer to Chankong and Haimes[7] for more details).

The closeness degree from the PIS to all objectives is defined as follows:

$$\eta_q(\mathrm{x}) = \frac{(d_q^{-}(\mathrm{x}))^q}{(d_q^{+}(\mathrm{x}))^q + (d_q^{-}(\mathrm{x}))^q} \tag{8}$$

Obviously, $0 \leqslant \eta_q(x) \leqslant 1$. The bigger $\eta_q(x)$, the better the objectives.

Instead of the original $2k$ objectives in Equation (4), Equation (4) can be transformed into the following mono-objective programming model:

$$\max \eta_q(x) = \frac{(d_q^{-}(\mathrm{x}))^q}{(d_q^{+}(\mathrm{x}))^q + (d_q^{-}(\mathrm{x}))^q}$$
$$s.t. \begin{cases} \sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant (b_{i3}+b_{i4})\ (i = 1,2,\cdots,m) \\ (1-\alpha)\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j + \alpha\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant \\ (1-\alpha)(b_{i1}+b_{i2}) + \alpha(b_{i3}+b_{i4})\ (i = 1,2,\cdots,m) \\ \mathrm{x} \geqslant \mathbf{0} \end{cases}$$

(9)

Remark 3.8. *Using existing nonlinear programming methods[29,30,41] and some heuristic and meta-heuristic approaches such as Tabu Search, Simulated Annealing, Genetic Algorithms, Evolution Program (EP), Evolution Strategies (ES) and Particle Swarm Optimization (PSO) as well as Ant Colony Optimization (ACO)[10,11,25], Equation (9) can be solved.*

Especially, for $q=1$, Equation (9) with $q=1$ can be rewritten as follows:

$$\max \eta_1(\mathrm{x}) = \sum_{i=1}^{k}\sum_{j=1}^{n}\left[\frac{w_{i1}}{2(Z_{i1}^{+}-Z_{i1}^{-})}(c_{ij1}+c_{ij2})+\frac{w_{i2}}{4(Z_{i2}^{+}-Z_{i2}^{-})}(c_{ij1}+c_{ij2}+c_{ij3}+c_{ij4})\right]x_j-\sum_{i=1}^{k}\left(\frac{w_{i1}Z_{i1}^{-}}{Z_{i1}^{+}-Z_{i1}^{-}}+\frac{w_{i2}Z_{i2}^{-}}{Z_{i2}^{+}-Z_{i2}^{-}}\right)$$

$$s.t.\begin{cases}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant (b_{i3}+b_{i4}) \ (i=1,2,\cdots,m);\\ (1-\alpha)\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j+\alpha\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j \leqslant \\ (1-\alpha)(b_{i1}+b_{i2})+\alpha(b_{i3}+b_{i4}) \ (i=1,2,\cdots,m);\\ \mathrm{x}\geqslant \mathbf{0}.\end{cases} \tag{10}$$

It is easily seen that Equation (10) is a common linear program which can be easily solved by the Simplex method.

Remark 3.9. *Among all q values, the case of $q=1$ is operationally and practically important, which provides better credibility than others in the measuring concept and emphasizes the sum of individual distances (regrets for $d_q^{+}(\mathrm{x})$ and rewards for $d_q^{-}(\mathrm{x})$ in the utility concept. For the case of $q=1$, we derive the above linear programming model (i.e., Equation (10)).*

3.4 A new two-stage method for fuzzy multi-objective linear program with TrFNs

On the basis of the aforementioned analysis, a new two-stage method for fuzzy multi-objective linear program with TrFNs is summarized as follows:

First stage:

Step 1: Transform the fuzzy multi-objective linear program Equation (2) into interval multi-objective program Equation (3).

Step 2: Convert interval multi-objective program Equation (3) into the crisp multi-objective linear program Equation (4).

Second stage:

Step 3: Determine the PIS Z^{+} and NIS Z^{-} for Equation (4).

Step 4: Calculate the closeness degrees from the PIS to all objectives by using Equations (6)-(8).

Step 5: Transform Equation (4) into the mono-objective program Equation (9) to solve.

For different parameters q of norm functions, DM can obtain different optimal solutions, which can greatly enhance the flexibility of solving process. In particular, for the case of $q=1$, Equation (9) is reduced to a common linear program Equation (10).

Remark 3.10. *In terms of Definitions 2.1-3.3, we develop the above two-stage method to solve fuzzy multi-objective linear program with TrFNs. This method is logical and rigorous since the interval order relation, interval objective program and TrFN order relation are mathematical rigor.*

Remark 3.11. *The parameter α can be provided by the DM according to his/her risk preference. We can also perform post optimality analysis to examine the effect of small perturbation in the value of this parameter on the optimal objective value and optimal solution. Since Equation (10) is a linear programming model, performing a post optimality analysis is a straightforward task.*

4 Application to R&D project portfolio selection

In this section, a project portfolio selection problem is analyzed and the comparative analysis with other methods is conducted.

4.1 A project portfolio selection problem

To illustrate the potential application of the proposed method in this paper, let us consider a project portfolio selection problem in this section. Over the past two decades, expenditure on research and development (R&D) has increased dramatically. In many industries, R&D is investment intensive and the required resources can be scarce in narrow fields, while markets are changing faster and faster. As a result, the available information based on which R&D decisions are made is highly uncertain and often very inaccurate[18].

An IT company faces the selection of a portfolio from a total of three projects where data on costs, benefits, market shares, and other related information for these projects are estimated. Existing cost interdependencies and synergistic benefits among projects are also identified. Due to the use of decimal truncation or rough estimation of the parameters by DM, all relevant data are imprecise. Since TrFN permits two parameters to represent the most possible values, while TFN uses the single parameter to represent the most possible value, TrFN has stronger ability to capture the fuzziness and uncertainty than TFN. Therefore, the evaluation of these relevant data provided by DM may be represented as TrFNs. Tables 1 and 2 present the original problem data. There are two resource constraints for hardware costs and software costs of the projects that must be satisfied. The problem has two objectives: maximization of total benefits and maximization of total market shares. The total hardware and software budgets are (16,18,20,22) and (26,27,30,38), respectively. The problem which the IT company considers is how to allocate the numbers (units) to each projects. Suppose that the numbers allocated to the project i is x_i (i=1, 2, 3). Then, the corresponding continuous project

	Project 1	Project 2	Project 3
Market share (%/unit)	(2,3,4,5)	(1,4,6,7)	(3,4,5,6)
Benefit (Thousand $/unit)	(7,8,12,13)	(13,14,16,17)	(10,11,12,13)

Table 1: Estimates for market shares and benefits

	Project 1	Project 2	Project 3	Budget
Hardware cost (Thousand $/unit)	(2,3,8,9)	(6,7,8,9)	(2,3,5,8)	(16,18,20,22)
Software cost (Thousand $/unit)	(7,9,12,14)	(11,12,13,15)	(14,18,19,23)	(26,27,30,38)

Table 2: Estimates for hardware and software costs and resource constraints

portfolio selection problem is formulated as follows:

$$
\begin{aligned}
&\max \tilde{z}_1(x) = (2,3,4,5)x_1 + (1,4,6,7)x_2 + (3,4,5,6)x_3\\
&\max \tilde{z}_2(x) = (7,8,12,13)x_1 + (13,14,16,17)x_2 + (10,11,12,13)x_3\\
&s.t. \begin{cases} (2,3,8,9)x_1 + (6,7,8,9)x_2 + (2,3,5,8)x_3 \leqslant (16,18,20,22)\\ (7,9,12,14)x_1 + (11,12,13,15)x_2 + (14,18,19,23)x_3 \leqslant (26,27,30,38)\\ x_i \geqslant 0 \ (i = 1,2,3) \end{cases}
\end{aligned} \tag{11}
$$

According to Equation (3), Equation (11) is equivalent to the following interval multi-objective program:

$$
\begin{aligned}
&\max Z_1(x) = [2.5,4.5]x_1 + [2.5,6.5]x_2 + [3.5,5.5]x_3\\
&\max Z_2(x) = [7.5,12.5]x_1 + [13.5,16.5]x_2 + [10.5,12.5]x_3
\end{aligned}
$$

s.t.

$$
\begin{aligned}
&[2.5,8.5]x_1 + [6.5,8.5]x_2 + [2.5,6.5]x_3 \preceq_I [17,21]\\
&[8,13]x_1 + [11.5,14]x_2 + [16,21]x_3 \preceq_I [26.5,34] \text{ and } x_i \geqslant 0 \ (i = 1,2,3)
\end{aligned} \tag{12}
$$

By Equation (4), Equation (12) is converted to the crisp multi-objective linear program as follows:

$$
\begin{aligned}
&\max Z_{11}(x) = 2.5x_1 + 2.5x_2 + 3.5x_3\\
&\max Z_{12}(x) = 3.5x_1 + 4.5x_2 + 4.5x_3\\
&\max Z_{21}(x) = 7.5x_1 + 13.5x_2 + 10.5x_3\\
&\max Z_{22}(x) = 10x_1 + 15x_2 + 11.5x_3\\
&s.t. \begin{cases} 17x_1 + 17x_2 + 13x_3 \leqslant 42\\ 26x_1 + 28x_2 + 42x_3 \leqslant 68\\ (1-\alpha)(5x_1 + 13x_2 + 5x_3) + \alpha(17x_1 + 17x_2 + 13x_3) \leqslant 34(1-\alpha) + 42\alpha\\ (1-\alpha)(16x_1 + 23x_2 + 32x_3) + \alpha(28x_1 + 28x_2 + 42x_3) \leqslant 53(1-\alpha) + 68\alpha\\ x_i \geqslant 0 \ (i = 1,2,3) \end{cases}
\end{aligned}
$$

(13)

Set $\alpha = 0.5$. Solving the single objective program $\max\{Z_{it}(\mathrm{x})\}$ ignoring other objectives in Equation (13), we obtain the optimal values as follows:

$$Z_{11}^{+} = 6.4468,\ Z_{12}^{+} = 10.6765,\ Z_{21}^{+} = 32.0294,\ Z_{22}^{+} = 35.5882$$

Similarly, solving the single objective program $\min\{Z_{it}(\mathrm{x})\}$ ignoring other objectives in Equation (13), we obtain the optimal values as follows: $Z_{11}^{-} = Z_{12}^{-} = Z_{21}^{-} = Z_{22}^{-} = 0$ Thus, the PIS Z^{+} and NIS Z^{-} are respectively $Z^{+} = (6.4468, 10.6765, 32.0294, 35.5882)$ and $Z^{-} = (0, 0, 0, 0)$.

In terms of Equations (6) and (7), the distances from the PIS and NIS to all objectives are respectively computed as follows:
$d_q^{+}(\mathrm{x}) = \{[\frac{w_{11}}{6.4468}(6.4468 - Z_{11}(x))]^{\mathrm{q}} + [\frac{w_{11}}{10.6765}(10.6765 - Z_{12}(\mathrm{x}))]^{\mathrm{q}} + [\frac{w_{21}}{32.0294}(32.0294 - Z_{21}(\mathrm{x}))]^{\mathrm{q}} + [\frac{w_{22}}{35.5882}(35.5882 - Z_{22}(\mathrm{x}))]^{\mathrm{q}}\}^{1/q}$
$d_q^{-}(\mathrm{x}) = \{[\frac{w_{11}}{6.4468}Z_{11}(\mathrm{x})]^{\mathrm{q}} + [\frac{w_{11}}{10.6765}Z_{12}(\mathrm{x})]^{\mathrm{q}} + [\frac{w_{21}}{32.0294}Z_{21}(\mathrm{x})]^{\mathrm{q}} + [\frac{w_{22}}{35.5882}Z_{22}(\mathrm{x})]^{\mathrm{q}}\}^{1/q}$
By Equation (9), Equation (13) can be transformed into the following non-linear programming model:

$$\max c_q(\mathrm{x}) = \frac{(d_q^{-}(\mathrm{x}))^q}{(d_q^{+}(\mathrm{x}))^q + (d_q^{-}(\mathrm{x}))^q}$$
$$s.t. \begin{cases} 17x_1 + 17x_2 + 13x_3 \leqslant 42; \\ 26x_1 + 28x_2 + 42x_3 \leqslant 68; \\ \ (1-0.5)(5x_1 + 13x_2 + 5x_3) + 0.5(17x_1 + 17x_2 + 13x_3) \leqslant \\ 34(1-0.5) + 42 \times 0.5; \\ (1-0.5)(16x_1 + 23x_2 + 32x_3) + 0.5(28x_1 + 28x_2 + 42x_3) \leqslant \\ 53(1-0.5) + 68 \times 0.5; \\ x_i \geqslant 0\ (i = 1,2,3); \end{cases} \quad (14)$$

Taking $w_{11} = w_{12} = w_{21} = w_{22} = 0.25$ and $q = 1$, we can deduce the following linear program:

$$\max c_1(\mathrm{x}) = 0.3077x_1 + 0.4131x_2 + 0.4038x_3$$
$$s.t. \begin{cases} 17x_1 + 17x_2 + 13x_3 \leqslant 42; \\ 26x_1 + 28x_2 + 42x_3 \leqslant 68; \\ \ (1-0.5)(5x_1 + 13x_2 + 5x_3) + 0.5(17x_1 + 17x_2 + 13x_3) \leqslant \\ 34(1-0.5) + 42 \times 0.5; \\ (1-0.5)(16x_1 + 23x_2 + 32x_3) + 0.5(28x_1 + 28x_2 + 42x_3) \leqslant \\ 53(1-0.5) + 68 \times 0.5; \\ x_i \geqslant 0\ (i = 1,2,3). \end{cases} \quad (15)$$

By using the Simplex method to solve Equation (15), the optimal solution is obtained as follows:

$$x_1 = 0,\ x_2 = 2.3725,\ x_3 = 0.$$

Similar to the case of $q = 1$, we can obtain the corresponding optimal solutions for different values of parameter q of norm functions, which are listed in Table 3.

Norm parameter	Optimal solution
q=1	(0, 2.3725, 0)
q=2	(0.14291, 2.2493, 0)
q=3	(0.2746, 2.1356, 0)
q=5	(0.3594, 2.0625, 0)
q=7	(0.2460, 2.1603, 0)
q=9	(0, 2.3725, 0)
q=13	(0.10368, 2.1941, 0.0313)
q=17	(0.2286, 1.7393, 0.2239)
q=20	(1.0664, 1.3686, 0.0464)

Table 3: Optimal solutions for different values of parameter q of norm functions

In addition, for the case of $q = 1$, we can obtain the corresponding optimal solutions for different values of acceptance degree α, which are listed in Table 4.

Acceptance degree	Optimal solution
α=0.0	(0.53333, 1.9333, 0)
α=0.1	(0.53333, 1.9333, 0)
α=0.2	(0, 2.3333, 0)
α=0.3	(0, 2.3469, 0)
α=0.4	(0, 2.36,0)
α=0.5	(0, 2.3725,0)
α=0.6	(0, 2.3846,0)
α=0.7	(0, 2.3962,0)
α=0.8	(0, 2.4074,0)
α=0.9	(0, 2.4182,0)
α=1.0	(0, 2.4286,0)

Table 4: Optimal solutions for different values of acceptance degree α

4.2 Comparison analysis with other methods

To illustrate the superiorities of the proposed method, we utilize the example in Arenas et al.[4] to make comparison with Arenas et al.'s method[4]. The numerical example is as follows:

$$\begin{aligned} &\max\ (40{,}50{,}80)x_1 + 100x_2 + 17.5x_3 \\ &\max\ 92x_1 + (70{,}75{,}90)x_2 + 50x_3 \\ &\max\ (10{,}20{,}70)x_1 + 100x_2 + 75x_3 \\ &\text{s.t.} \begin{cases} (6{,}12{,}14)x_1 + 17x_2 \leqslant 1400 \\ 3x_1 + 9x_2 + (3{,}8{,}10)x_3 \leqslant 1000 \\ 10x_1 + (7{,}13{,}15)x_2 + 15x_3 \leqslant 1750 \\ (4{,}6{,}8)x_1 + 16x_3 \leqslant 1325 \\ (7{,}12{,}19)x_2 + 7x_3 \leqslant 900 \\ 9.5x_1 + (3.5{,}9.5{,}11.5)x_2 + 4x_3 \leqslant 1075 \end{cases} \end{aligned}$$

Due to the fact that real number, interval, and triangular fuzzy number are the special cases of TrFN, the above example can be rewritten as a fuzzy multi-objective linear program with TrFNs. Therefore, by using the proposed method of this paper, when α=0.5 and the weights $w_{it} = \frac{1}{6}(t = 1, 2;\ i = 1,2,3)$, the optimal solutions for different parameter values of q of norm functions are listed in Table 5. Table 6 presents the optimal solutions for different values of parameter β obtained by using Arenas et al.'s method[4].

q	x_1	x_2	x_3	q	x_1	x_2	x_3
1	52.67456	36.70795	47.28954	6	1.234568	1.234568	1.234568
2	52.67456	36.70795	47.28954	7	1.234568	1.234568	1.234568
3	52.67456	36.70795	47.28954	8	1.234568	1.234568	1.234568
4	53.70712	35.66522	47.57438	9	1.234568	1.234568	1.234568
5	1.234568	1.234568	1.234568	10	1.234568	1.234568	1.234568

Table 5: Optimal solutions for different values of parameter q of norm functions

β	x_1	x_2	x_3
0	57	29	50
0.5	56	46	44
1	55	46	45

Table 6: Optimal solutions for different values of parameter β by Arenas et al.'s method

Compared Table 5 with Table 6, it is easily seen that the optimal solutions obtained by the proposed method of this paper are remarkably different with those obtained by Arenas et al.'s method[4]. Table 5 shows that the optimal solutions of x_1, x_2 and x_3 tend to the same value 1.234568 with the increase of q. In addition, when the acceptance degree α and the weights w_{it} $(t = 1, 2;\ i = 1,2,3)$ take different values, the optimal solutions obtained by the proposed method of this paper may be different. These observations not only verify the effectiveness of the proposed method but also reflect the flexibility of the proposed method.

The differences and superiorities of the proposed method in this paper over other methods are summarized as follows:

(1) Wu[39] proposed the KarushCKuhnCTucker optimality conditions in multi-objective programming problems. They only suit the multi-objective programming problems with interval-valued objective functions and cannot handle the multi-objective programming problems with TrFNs. On the contrary, if the fuzzy parameters are reduced to intervals, the proposed method in this paper still can solve the multi-objective programming problems with interval-valued objective functions.

(2) Arenas et al. [4] studied the possibilistic multiobjective linear program with TFNs. The method[4] is also unable to treat the multi-objective programming problems with TrFNs. Since TFN is a special TrFN, the proposed method in this paper is still valid for the possibilistic multiobjective linear program with TFNs.

(3) For the multi-objective programming problems with any types of fuzzy numbers (e.g.,[3,14,23,32]), we can compute the interval expectations of any fuzzy numbers through using cut sets[13] or nearest interval approximation[32]. Therefore, the proposed method of this paper can be used to solve the multi-objective programming problems with any types of fuzzy numbers although it aims at the multi-objective programming problems with TrFNs. Compared with nearest interval approximation[32], using interval expectation of TrFNs is simpler in computation and more easy to use for DM in real-life application.

(4) The methods[3,23] need to know the possibility distribution of involved fuzzy quantities in priori, which is not easy in real-life decision problems. The goal programming methods[8,17] need to give the DMs aspiration levels. However, quantification of imprecise DMs aspiration levels may be a huge task. The Kuhn and Tucker approach[39] requires differentiability and convexity of involved functions. In the fuzzy ranking method[6], there are several rankings, leaving the DM with a problem choice. The interactive method[26] leaves DM with great amount of computation.

(5) Since the acceptance degree reflects the subjective preference of DM on the fuzzy constraints, it is more reasonable and natural to incorporate the acceptance degree into the process of solving fuzzy multi-objective linear program. The proposed method in this paper is more useful and practical than the existing methods for real-life fuzzy decision problems.

5 Further discussions

In this section, we further discuss the other potential cases of the fuzzy multi-objective linear programming problems. The first case is the fuzzy multi-objective linear program with TrFNs as follows:

$$\begin{aligned} &\max\ \{\ \tilde{z}_i(\mathbf{x}) = \tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x},\ i=1,2,\ldots,k\} \\ &s.t.\quad \begin{cases} \tilde{\mathbf{A}}\mathbf{x} \geqslant \tilde{\mathbf{b}} \\ \mathbf{x} \geqslant \mathbf{0} \end{cases} \end{aligned} \tag{16}$$

Theorem 5.1. *Equation (16) is equivalent to a multi-objective linear programming model as follows:*

$$\begin{aligned} &\max\ \{\ Z_{i1}(\mathbf{x}) = \tfrac{1}{2}\sum_{j=1}^{n}(c_{ij1}+c_{ij2})x_j,\ i=1,2,\ldots,k\} \\ &\max\ \{\ Z_{i2}(\mathbf{x}) = \tfrac{1}{4}\sum_{j=1}^{n}(c_{ij1}+c_{ij2}+c_{ij3}+c_{ij4})x_j,\ i=1,2,\ldots,k\} \\ &s.t.\ \begin{cases} \sum\limits_{j=1}^{n}(a_{ij1}+a_{ij2})x_j \geqslant (b_{i1}+b_{i2})\ (i=1,2,\cdots,m); \\ (1-\alpha)\sum\limits_{j=1}^{n}(a_{ij3}+a_{ij4})x_j + \alpha\sum\limits_{j=1}^{n}(a_{ij1}+a_{ij2})x_j \geqslant \\ (1-\alpha)(b_{i3}+b_{i4}) + \alpha(b_{i1}+b_{i2})\ (i=1,2,\cdots,m); \\ \mathbf{x} \geqslant \mathbf{0}. \end{cases} \end{aligned} \tag{17}$$

Proof. The two objective functions of Equation (17) directly come from Theorem 3.5. The first constraint of Equation (16) is equivalent to $\mathrm{E}(\tilde{\mathbf{A}})\mathbf{x} \succeq_I \mathrm{E}(\tilde{\mathbf{b}})$, i.e.,

$$[\tfrac{1}{2}\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j, \tfrac{1}{2}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j] \succeq_I [\tfrac{1}{2}(b_{i1}+b_{i2}), \tfrac{1}{2}(b_{i3}+b_{i4})](i=1,2,\cdots,m),$$

which are equivalent to $\frac{1}{2}\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j \geqslant \frac{1}{2}(b_{i1}+b_{i2})$ and $\varphi(\mathrm{E}(\sum_{j=1}^{n}\tilde{a}_{ij}x_j) \preceq_I \mathrm{E}(\tilde{b}_i)) \leqslant \alpha (i=1,2,\cdots,m)$ according to Definition 2.3. The former is $\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j \geqslant b_{i1}+b_{i2}$. By Definition 2.1, $\varphi(\mathrm{E}(\sum_{j=1}^{n}\tilde{a}_{ij}x_j) \preceq_I \mathrm{E}(\tilde{b}_i)) \leqslant \alpha$ is transformed into the following:

$$\frac{\frac{1}{2}(b_{i3}+b_{i4}) - \frac{1}{2}\sum\limits_{j=1}^{n}(a_{ij3}+a_{ij4})x_j}{\frac{1}{2}(b_{i3}+b_{i4}) - \frac{1}{2}(b_{i1}+b_{i2}) - [\frac{1}{2}\sum\limits_{j=1}^{n}(a_{ij3}+a_{ij4})x_j - \frac{1}{2}\sum\limits_{j=1}^{n}(a_{ij1}+a_{ij2})x_j]} \leqslant \alpha \tag{18}$$

After rearranging, Equation (18) is rewritten as follows: $(1-\alpha)\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j+\alpha\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j \geqslant (1-\alpha)(b_{i3}+b_{i4})+\alpha(b_{i1}+b_{i2})$, which is just the second constraint of Equation (17). Therefore, Theorem 5.1 is proven.

□

The second case is the fuzzy multi-objective linear program with TrFNs as follows:

$$\min\ \{\ \tilde{z}_i(\mathbf{x})=\tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x},\ i=1,2,\ldots,k\}$$
$$s.t.\quad \begin{cases}\tilde{\mathbf{A}}\mathbf{x}\leqslant\tilde{\mathbf{b}}\\ \mathbf{x}\geqslant\mathbf{0}\end{cases} \tag{19}$$

Theorem 5.2. *Equation (19) is equivalent to a multi-objective linear programming model as follows:*

$$\min\ \{\ Z_{i1}(\mathbf{x})=\tfrac{1}{2}\sum_{j=1}^{n}(c_{ij3}+c_{ij4})x_j,\ i=1,2,\ldots,k\}$$
$$\min\ \{\ Z_{i2}(\mathbf{x})=\tfrac{1}{4}\sum_{j=1}^{n}(c_{ij1}+c_{ij2}+c_{ij3}+c_{ij4})x_j,\ i=1,2,\ldots,k\}$$
$$s.t.\ \begin{cases}\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j\leqslant(b_{i3}+b_{i4})\ \ (i=1,2,\cdots,m);\\ (1-\alpha)\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j+\alpha\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j\leqslant\\ (1-\alpha)(b_{i1}+b_{i2})+\alpha(b_{i3}+b_{i4})\ \ (i=1,2,\cdots,m);\\ \mathbf{x}\geqslant\mathbf{0}.\end{cases} \tag{20}$$

The third case is the fuzzy multi-objective linear program with TrFNs as follows:

$$\min\ \{\ \tilde{z}_i(\mathbf{x})=\tilde{\mathbf{c}}_i^{\mathrm{T}}\mathbf{x},\ i=1,2,\ldots,k\}$$
$$s.t.\quad \begin{cases}\tilde{\mathbf{A}}\mathbf{x}\geqslant\tilde{\mathbf{b}}\\ \mathbf{x}\geqslant\mathbf{0}\end{cases} \tag{21}$$

Theorem 5.3. *Equation (21) is equivalent to a multi-objective linear programming model as follows:*

$$\min\ \{\ Z_{i1}(\mathbf{x})=\tfrac{1}{2}\sum_{j=1}^{n}(c_{ij3}+c_{ij4})x_j,\ i=1,2,\ldots,k\}$$
$$\min\ \{\ Z_{i2}(\mathbf{x})=\tfrac{1}{4}\sum_{j=1}^{n}(c_{ij1}+c_{ij2}+c_{ij3}+c_{ij4})x_j,\ i=1,2,\ldots,k\}$$
$$s.t.\ \begin{cases}\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j\geqslant(b_{i1}+b_{i2})\ \ (i=1,2,\cdots,m);\\ (1-\alpha)\sum_{j=1}^{n}(a_{ij3}+a_{ij4})x_j+\alpha\sum_{j=1}^{n}(a_{ij1}+a_{ij2})x_j\geqslant\\ (1-\alpha)(b_{i3}+b_{i4})+\alpha(b_{i1}+b_{i2})\ \ (i=1,2,\cdots,m);\\ \mathbf{x}\geqslant\mathbf{0}.\end{cases} \tag{22}$$

Equations (17), (20), and (22) can be analogously solved by the TOPSIS method presented in Section 3.

6 Conclusions

This paper developed a new two-stage method for the fuzzy multi-objective linear program with TrFNs. The order relationship for TrFNs is firstly given by using the interval expectation of TrFNs. In the first stage, the fuzzy multi-objective linear program with TrFNs is transformed into the interval multi-objective linear program according to the order relationship of TrFNs. Combining the interval order relation with interval objective program, the interval multi-objective linear program is further transformed into the crisp multi-objective linear program. In the second stage, the crisp multi-objective linear program is converted into mono-objective program on the basis of TOPSIS. The notable characteristic of the proposed method in this paper is to sufficiently consider the acceptance degree of DM that the fuzzy constraints may be violated. The acceptance degree can accurately capture the risk attitude of DM and make the decision result more reasonable and consistent with the real-life situations.

A project portfolio selection problem is analyzed to illustrate the actual application of the proposed method. The comparison analysis with the other methods clearly verifies the effectiveness and flexibility of the proposed method. It is not only suitable for the project portfolio selection, but also can be used to the fuzzy knapsack problem, cargo loading, cutting stock, economic plan, transportation problem, capital budgeting, and engineering management, etc. For future research, we will extend the proposed method to multi-objective linear program under intuitionistic fuzzy environment, which is a brand new and an interesting issue.

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