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Quantum Genetic LMI-based H_∞ Control with Time Delay

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ABSTRACT

One of the main problems underlying most optimization theories is a local optimum. When time delays are presented, this issue becomes much more problematic. In such conditions, evolutionary optimization algorithms are proven to be helpful. In this paper, the quantum genetic algorithm (QGA) has been used to tackle the stated problem in the framework of delay-dependent linear matrix inequality (LMI) robust H_∞ control. QGA is employed to find suitable feedback gains and delay-dependent LMI solvers are concerned to resolve stability issues. In addition, to provide more balance between exploration and exploitation, to increase convergence rate as well as to prevent premature convergence, it is proposed that particle swarm optimization (PSO) is augmented with QGA. Simulation is dealt with LMI-based H_∞ control scheme of the QGA and QGA-PSO optimization space from the design point of one-degree freedom single link scara robot. The whole controller satisfies the desired properties for the uncertain-but-known constant bounded time delay. Furthermore, one of the drawbacks found in tests of most hybrid global-local strategies, i.e., premature convergence, has been canceled by the proposed scheme of QGA and PSO.

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I. INTRODUCTION

The idea of H_∞ optimization goes back to the early days of robust control theory as early as 1979 [1]. The principal objective of robust control is considering plant uncertainty explicitly to achieve the specified level of performance in which a trade-off is sought between robustness and optimality. Various works have been made in this area. Some consider a range of operating points with representing a variety of uncertainty structure such as μ synthesis and LMI-based H_∞ control. The μ synthesis is suitable when uncertainty is in diagonal block structure of variables, and the latter uses a map of the operating points into the uncertainty space in the controller to minimize an upper bound of the robust performance index. LMI-based methods offer a formation for a large variety of control problems; including H_∞ [2], the mixed H_2/H_∞ [3], the gain-scheduling [4] and so forth. Yet, the success of all these methods hinges on characterizing the global minimum of the index norm and the problem of stability. Many researchers have focused on this subject [5]. Considering the bilinear aspect of constraints in the resulting optimization problem leads to the idea of integrating evolutionary algorithms, where the well-known one is genetic algorithms [6]. In [7], genetic algorithm manages the PI controller subject to the H_∞ index in terms of LMI. In [8], the fitness function of genetic algorithm is computed by linear matrix inequality solver in the framework of H_2/H_∞ robust control. The method proposed in [9] retains GA to linearize a nonlinear channel and exploits LMI to resolve the estimated error.

Additionally, a method suggested in [10] that hosts output feedback using H_∞ control problem in continuous-time linear systems subject to time domain constraints. The combined GA and LMI in low order suboptimal H_∞ control by considering the matrix rank minimization problem is proposed in [11]. Other attempts can be found in [12]-[14], where a hybrid approach is used to solve non-convex optimization in the mixed H_2/H_∞ control domain. Some

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researchers tried to solve the time delay problem associated with the H_∞ control with hybrid approach [15]. In [16], mixed H_2/H_∞ control is proposed for Two-Time-Scale Markovian Switching Production-Inventory Systems.
However, time delay restricts the controller functioning, which may be led to the instability of the whole system. Consequently, it is imperative to analyze time-delay effects for control systems. Using Type-II fuzzy logic to deal with time delay is proposed in [17]. In [18], a robust genetic control design is proposed that applied for a class of uncertain neutral delay system. In [19], H_∞ control is used for an uncertain mechanical system with input time delay. The asymptotic stability analysis problem in [20] is considered where each subsystem of large-scale linear systems has time delay as well as the delay interconnections of subsystems is also considered. In [21], H_∞ performance for building structures is described by the delay-dependent feedback control, using a hybrid of GA and LMI.
GA, the simplest random-based approaches, is effective in a situation with little knowledge about the search space and is best for those problems with a clear way to evaluate fitness such as in our situation. In GA, we evolve solutions over the iterations/generations. However, there are shortages associated with GA such as slow convergence rate and premature convergence. Dealing with these deficiencies is an interesting field of research.
Some recent works have addressed approaches requiring genetic algorithm runs several times [6, 22], while more interesting techniques aim to find properly tuned parameters of GA during any attempt of search [22]-[24]. Another insight is using the heuristic approaches to improve the performance of genetic algorithm such as those are done in [25] where particle swarm optimization (PSO) is applied to enhance GA's individuals. Over the last decade, the possibility to emulate a quantum computing has led to a new class of GAs known as "Quantum Genetic Algorithms" (QGAs) [26]. In the comparison of QGA with GA, it has been shown that QGA is described by less population size, higher convergence rate, and strong global search capability [27]. From the recent successful application of QGA, one can mention task sequence planning [28] that the efficiency, stableness, and cost of a complex assembly system depends on it. An improved version of quantum genetic algorithm is suggested by considering frog-leaping algorithm, simulated annealing, and the quantum variation [29].

Here, we investigate a method consist of quantum genetic algorithm (QGA) and linear matrix inequality (LMI) in H_∞ control framework, to explore the bilinear relation between the controller gain and lyapunove matrix. In our method, the stability conditions in the presence of input time delay are formulated by linear matrix inequalities; performance index is regarded as optimization objective and searching for output feedback gain, the optimal result, is done by QGA. Specifically, we propose a framework of quantum genetic LMI-based H_∞ control. We also suggest a new combination QGA and PSO applied it to LMI-based H_∞ control. According to the obtained results, it has been provided outstanding performance. We provide two flowcharts for the proposed structure as well as its combination with PSO. This framework uses different update rules, advanced techniques like combination with local search, and a new generational scheme. Indeed, the proposed approach here is rather a practical one to overcome the deficiencies of GA. Specifically, one of the problems associated with many hybrid global-local strategies including GA is premature convergence. Simulation results show that this issue has been extensively canceled by the proposed scheme of QGA and PSO.
The arrangement of this paper after this introduction is as follows, and the second section is dedicated to preliminary assumptions about the system and the formulation of the H_∞ control problem in linear matrix inequality framework. A brief explanation about QGA is drawn in Section III. The proposed method allows computing a control law by output feedback gain which genetic algorithm globally searches it and implicitly involving stability conditions in the presence of time delay with taking advantage of linear matrix inequalities is described in the fourth section. The hybrid algorithm of QGA with particle swarm optimization for the proposed LMI-based H_∞ control scheme is also explained in Section IV. Section V illustrates simulation results of the proposed frameworks that are done by Matlab. Finally, the conclusion appears in Section VI.

II. PROBLEM STATEMENT

Consider a linear time-invariant plant $P(s)$ which is termed by:

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = P(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \tag{1}$$

where w is exogenous input u is control input, z is controlled output, and y is measured output subject to physical constraints. H_∞ control problem finds the output feedback control law, $u=ky$ such that for a given scalar $\gamma>0$ the closed-loop transfer function from w to z follows as:

$$\|T_{zw}\|_\infty < \gamma \tag{2}$$

where $\| \|_\infty$ denotes the H_∞ norm. The framework of the plant and controller are shown in Fig. 1-A and the block diagram of the augmented system reflecting the uncertainty of the model is shown in Fig. 1-B.

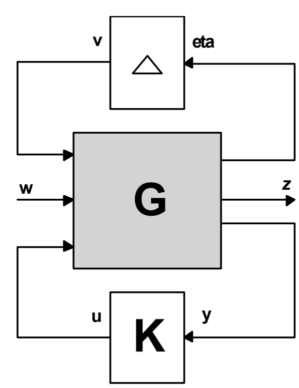

Fig. 1(a). Uncertainty structure

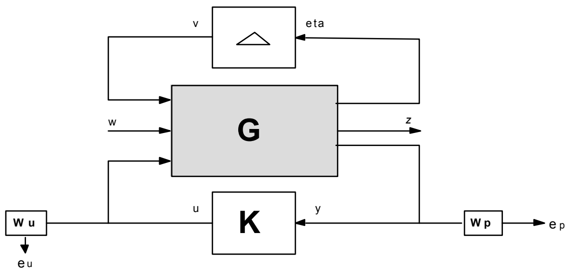

Fig. 1(b). Augmented system

Consider the minimal realization of plant *P(s)* as follows [30]:

$$P(s) = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} (SI - A)^{-1}(B_1, B_2) \qquad (3)$$
$$A \in R^{n\times n} \quad D_{11} \in R^{p_1\times m_1}, D_{22} \in R^{p_2\times m_2}$$

We assume: *i) (A,B₂) is stablizable and (A,C₂) is detectable ii)D₂₂=0*

By defining the following controller *k(s)*:

$$K(s) = D_K + C_K(SI - A_K)^{-1}B_k \qquad (4)$$

The closed transfer function from *w* to *z* is:

$$T_{zw} = D_{cl} + C_{cl}(SI - A_{cl})^{-1}B_{cl},$$
$$A_{cl} = \begin{bmatrix} A + B_2D_kC_2 & B_2C_k \\ B_2C_k & A_k \end{bmatrix}$$
$$B_{cl} = \begin{bmatrix} B_1 + B_2D_kD_{21} \\ B_kD_{21} \end{bmatrix}, \qquad (5)$$
$$C_{cl} = [C_1 + D_{12}D_kC_2 \quad D_{12}C_k],$$
$$D_{cl} = D_{11} + D_{12}D_kD_{21}$$

With taking advantage of the bounded real lemma (BRL) [31], the H∞ norm constraint $\|T_{zw}\|_\infty < \gamma$, can be convinced by the feasibility of the following matrix inequality:

$$\begin{bmatrix} A_{cl}^TP + PA_{cl} & PB_{cl} & C_{cl}^T \\ * & -\gamma I & D_{cl}^T \\ * & * & -\gamma I \end{bmatrix} < 0 \qquad (6)$$

The notation * indicates terms that can be induced by symmetry.

Matrix inequality (6) is LMI if the controller parameters A_{cl}, B_{cl}, C_{cl}, D_{cl} are known. Now, let us consider the state space description of time-invariant system with an input time delay as:

$$y = C_yx + D_{yw}w + D_{yu}u(t-\tau)$$
$$\dot{x} = Ax + Ew + Bu(t-\tau)\; z = C_zx + D_{zu}u(t-\tau) \qquad (7)$$

where x is the state.

Consider the control signal as follows:

$$u = Fy = FCx \qquad (8)$$

The following theorem affords sufficient conditions in the presence of delay for feedback LMI-based H∞ control.

Theorem 1 [21]."Given a scalar τ_1 the closed-loop system is asymptotically stable with H∞ performance index, *γ>0*, for any constant time-delay satisfying $0<\tau<\tau_1$, if there exist matrices *F,P>0,Q>0 , Z>0, X* and *Y* satisfying the matrix inequalities Equ. (9).

This theorem states that the stability within maximum practicable delay bound is guaranteed and designing a delay-dependent feedback controller based on LMI is promising.

To prove this theorem, we choose the following Lyapunov function [21] $V(q(t)) = q^TPq + \int_{-\tau}^{0}\int_{t+\beta}^{t}\dot{q}^TZ\dot{q}\,dtd\beta + \int_{t-\tau}^{0}\int_{t+\beta}^{t} q^TQq\,dtd\beta$, where $P = P^T > 0, Z = Z^T > 0, Q = Q^T > 0$ are need to be chosen first. By taking the derivative of this Lyapunov function along the state trajectory and using Leibniz-Newton formula, we found that to guarantee stability and convergence, Equ. (9) must be satisfied. Therefore, for any gain we need to examine Equ. (9). Note that LMI Equ. (9) is said to be convex if the output feedback gain *F* is given.

III. Quantum Genetic Algorithm

Genetic algorithms (GAs) are popular heuristic optimization methods inspired by Darwinian natural selection that are based on mutation, crossover, selection, etc. The possibility to emulate a quantum computing has led to a new class of GAs known as "Quantum Genetic Algorithms" (QGAs) [26]. Quantum Genetic Algorithm (QGA) uses quantum bit coding and quantum gate as operators to complete the evolution process.

Based on quantum mechanics, the evolution of an isolated quantum system is engaged by the Schrödinger equation: $i\hbar\frac{\partial}{\partial t}|\psi(t)> = H(t)|\psi(t)>$, where i presents the imaginary number $\sqrt{-1}$ and $\hbar$ a Planck constant. The smallest unit of information in quantum computing is addressed in a two-state is called a qubit that may be in the "1" state, in the "0" state, or in any superposition of the two as follows:

$$\begin{bmatrix} A_{cl}^TP + PA_{cl} + \tau_1X + Y + Y^T + Q & PBKC - Y & PE & \tau_1A^TZ & C_1^T \\ * & -Q & 0 & \tau_1(KC)^TB_1^TZ & (KC)^TD_{12}^T \\ * & * & -\gamma^2I & \tau_1B_1^TZ & 0 \\ * & * & * & -\tau_1Z & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \qquad (9)$$
$$P > 0,$$
$$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0$$

$$|\Psi> = \alpha|0> + \beta|1> \tag{10}$$

where α and β are complex numbers that specify the probability amplitudes of the corresponding states. $|\alpha|^2$ and $|\beta|^2$ gives the probability that the qubit is in the "0" state and in the "1" state, respectively. Normalization of the state to unity guarantees $|\alpha|^2 + |\beta|^2 = 1$. Based on Dirac notation we have $|\Psi> = \binom{\alpha}{\beta}$, and therefore the qubit states would be $|0> = \binom{1}{0}$, $|1> = \binom{0}{1}$. Quantum gates (Q-gates) manage qubits based on unitary transformations. Two useful Q-gates are the Hadamard (*H* gate) and rotation gate.
Quantum evolutionary algorithm is a probabilistic algorithm that maintains a population of Q-bit individuals $Q(t) = \{q_1^v, q_2^v, \dots, q_n^v\}$, at generation v, where n is the size of population, and q_j^v is a Q-bit individual defined as

$$q_j^v = \begin{bmatrix} \alpha_{j1}^v & \dots & \alpha_{jm}^v \\ \beta_{j1}^v & \dots & \beta_{jm}^v \end{bmatrix} \quad j = (1,2,\dots,n) \tag{11}$$

where m is the number of Q-bits. To update a Q-bit individual $q\ (\alpha_i, \beta_i)$ of ith Q-bit, a rotation gate $U(\Delta\theta_i)$ is used as follows:

$$\begin{bmatrix} \alpha_i' \\ \beta_i' \end{bmatrix} = \begin{bmatrix} \cos(\Delta\theta_i) & -\sin(\Delta\theta_i) \\ sin(\Delta\theta_i) & cos(\Delta\theta_i) \end{bmatrix} \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \tag{12}$$

The plot of the rotation gate and Hadamard gate are depicted in Fig.2.

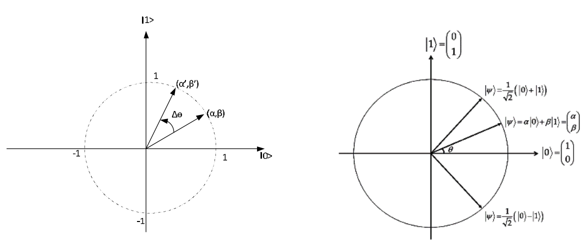

Fig.2. Polar plot of the rotation gate for Q-bit individuals [26] (left) and Hadamard gate (right)[32].

Q-bit representation has the advantage that it is able to represent a linear superposition of states. If there is, for instance, a three-Q-bit system with three pairs of amplitudes such as

$$q_j^v = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & \frac{\sqrt{2}}{2} \end{bmatrix} \tag{13}$$

then the states of the system can be represented as

$$\frac{1}{4}|000> + \frac{\sqrt{3}}{4}|001> - \frac{1}{4}|010> - \frac{\sqrt{3}}{4}|011> + \frac{1}{4}|100> + \frac{\sqrt{3}}{4}|101> - \frac{1}{4}|110> - \frac{\sqrt{3}}{4}|111>. \tag{14}$$

It should be noted that the probabilities of $|000>$،$|001>$،$|010>$،$|011>$،$|100>$،$|101>$،$|110>$،$|111>$ are 1/16, 3/16, 1/16, 3/16, 1/16, 3/16, 1/16, and 3/16, respectively. As a result, the three-Q-bit system contains information of eight states. Evolutionary computing with Q-bit representation has a higher population diversity in comparison. Only one Q-bit individual is enough to represent eight states; however, in binary representation, we need eight strings, (000), (001), (010), (011), (100), (101), (110), and (111) to do it.

A chromosome is simply a string of m qubits that forms a quantum register. A typical quantum chromosome is shown in Fig. 3.

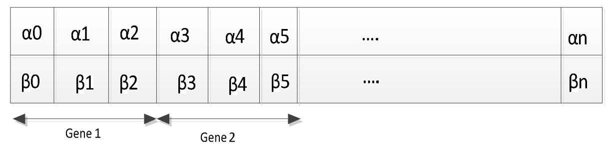

Fig.3. A typical quantum chromosome.

IV. Quantum Genetic Algorithm

A. Convex formulation

When feedback gain *F* is unknown, the convex algorithms could not be applied. QGA cannot give a solution for *F* and *P* at the same time of Equ. (9). But if *F* has been given, Equ. (9) would be convex and LMI solver can find *P* from Equ. (9). The proposed approach takes advantages of QGA to search *F* and then use LMI solver to solve *P* to preserve stability in the presence of time delay.

B. Proposed QGA-LMI

The QGA –LMI approach in the present steps is considered.

Step 1: Encoding: Each feedback gain vector *F* that QGA searches it, is considered as an individual. The range of gain is limited.

Step 2: Initialization: set $v = 0$ *and* initialize $Q(v)$ to $1/\sqrt{2}$, *N* individuals, *R(v)*s, are generated randomly.

Step 3: Producing binary quantum $R(v)$ *by observing* $Q(v)$ *states:* a binary chromosome is a solution of length n. Making a binary chromosome is done by observing $Q(v)$ states and we use the following equation:

$$x_{ji}^r = \begin{cases} 0 & U(0,1) < \alpha t_\pi^2 \\ 1 & Otherwise \end{cases} \tag{15}$$

where $U(.,.)$ is a function that produces random numbers uniformly.

Step 4: Cost assignment and Evaluation of $R(v)$*:* The cost is defined as inthe verse of minimum *T* that *T* is obtained by *feasp* function. The function *feasp* solves the following auxiliary convex program[33]:

Minimize *T* subject to $L(x) - J(x) < TI$,

where x is the vector of (scalar) decision variables and T_{min} is the minimized value of *T*. Evaluation process is a little different and divided in two cases.

<u>*Case I*</u>: When $R(v)$ is infeasible, *i.e.*, it does not fulfill Equ. (9): It is not necessary to determine admissible delay. The

value of $\Delta\theta_i$ is assigned a large negative value to increase the probability of state $|0>$. In this case, the cost value corresponds to it will be assigned large enough in order to have a chance to discard this feedback gain.

Case II: When $R(v)$ is feasible, *i.e.*, it does fulfill Equ. (9): The maximum admissible time delay, τ_I, would be computed by the bisection method. The value of $\Delta\theta_i$ is assigned a large positive value to increase the probability of state $|1>$. The cost of $R(v)$is determined according to its cost value. This step is repeated for the number of individuals.

Step 5: Update (v) *:* This updating is done based on rotation Q-gate by finding a probabilistic value of α and β.

Step 6: selecting the best solution among $K(v-1)$ *and* $R(v)$ *and store it in* $K(v)$.

These steps go on until the number of generation is reached. The diagram of the algorithm is shown in Fig.4 in details.

C. co-Evolutionary QGA-LMI with Particle Swarm Optimization

Exploration and exploitation are two challenging objectives of global search procedures. In the lack of prior knowledge, knowledge about the problem is gathered by uniform measurements over the solution domain. Exploration assures that searching to find a consistent estimate of the optimum is done appropriately. Exploitation searches around the best solution. In order to balance between exploration and exploitation, search methods are combined with each other. Particle swarm optimization [34] is one of the population-based algorithms to find global optimum, it employs social interaction. Particle trajectories regulate social rules. At each time step, fitness function demonstrating a quality measure is computed by the position of each particle, as input. Changing velocity explores solution space. Current location of each particle is defined by the best fitness that particle has achieved in its velocity vector. In fact, solutions do not change straight in PSO algorithm. Each solution is changed over time based on its velocity vector. A swarm of particles moves across the problem space with the velocity update rule. Information distribution and merging the advantages of both QGA and PSO can lead to finding better solutions.

The cooperation of QGA and PSO can lead to a hybrid model. In that, each of QGA and PSO may be used as an optimizer of a part of solution space or pre optimizer of another. In the hybrid algorithm used here, the first population is divided into two parts randomly, next half of the population of the next generation is evolved by the PSO and the remainder by QGA. These two parts in the resulting population recombined. In the next generation, the recombined population divided into another two parts randomly and again each part evolved by PSO or QGA. Fig.5 shows the flow diagram of the described algorithm.

V. Numerical Results

The proposed approach is examined for a one-link scara robot that described by the following second order differential equation using Newton's second law:

$$\begin{aligned} &m\ddot{q} + c\dot{q} + gq = u, \\ &c = 5(1 + \Delta_c), \\ &g = 5(1 + \Delta_g) \end{aligned} \tag{16}$$

where Δ represents uncertainty. The stat space representation of system without uncertain parameters is expressed as:

$$\begin{aligned} &A = \begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix} \\ &B = [83 \quad 0] \\ &C = [1 \quad 0] \\ &D = 0 \end{aligned} \tag{17}$$

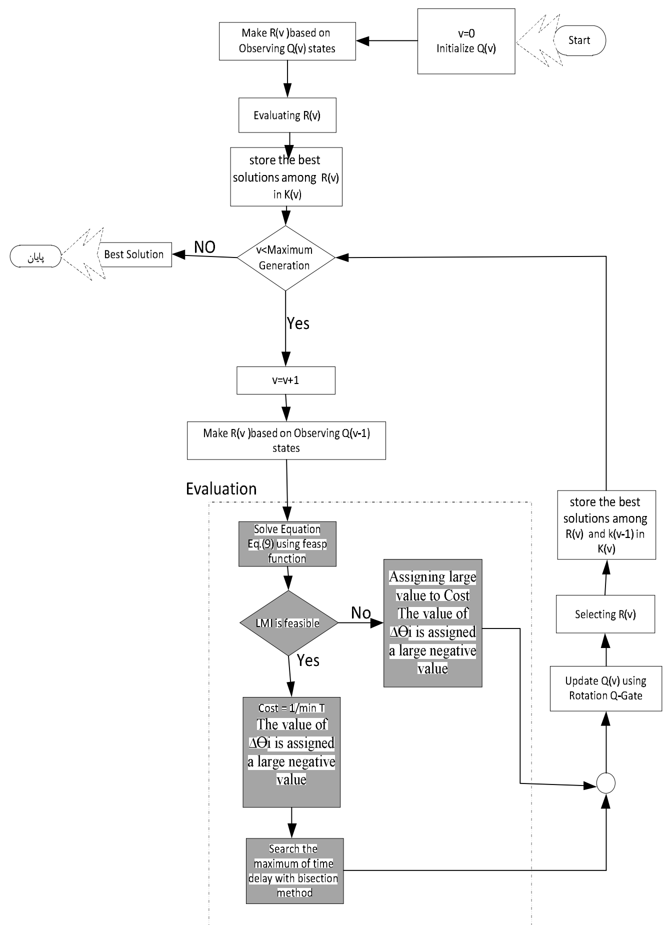

Fig. 4. Flowchart of QGA-LMI algorithm

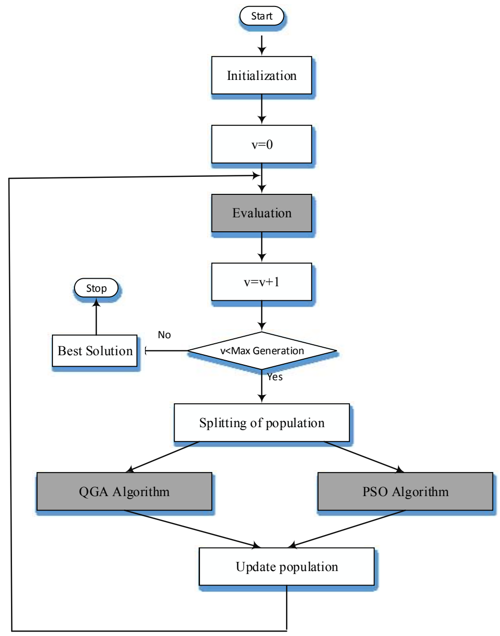

Fig. 5. Flowchart of QGA-PSO algorithm

The uncertain behavior of the original system can be described by an upper linear fractional transformation (LTF):

$$y = F(G,\Delta)u \qquad (18)$$

The views of nominal open and closed loop behavior of the system without considering time delay and frequency behavior of its controller are shown in Figures 6 and 7, respectively.

We define, H_∞ performance, as $\gamma=0.1$. The feedback gain is obtained by QGA as $[-78.94, -47.72]$ in the presence of time delay input. It should be noted that the controller is feasible for the maximum time delay τ_1 means that the designed controller can stabilize the system with H_∞ index, $\gamma=0.1$, for any delay belonging to interval $[0\ \tau 1]$. Rate of convergence with and without time delay is depicted in Fig. 8. The rate represents a normalized mean of the reduction ratio of the fitness difference per generation. As seen, in the presence of delay the system is still stable.

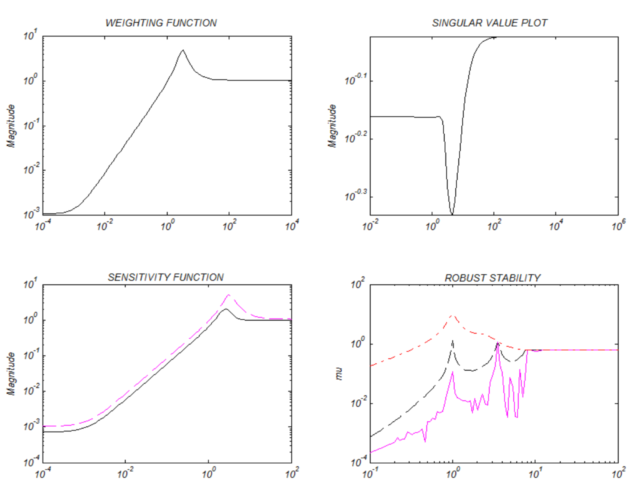

Fig.6. Closed –open loop behavior of the system.

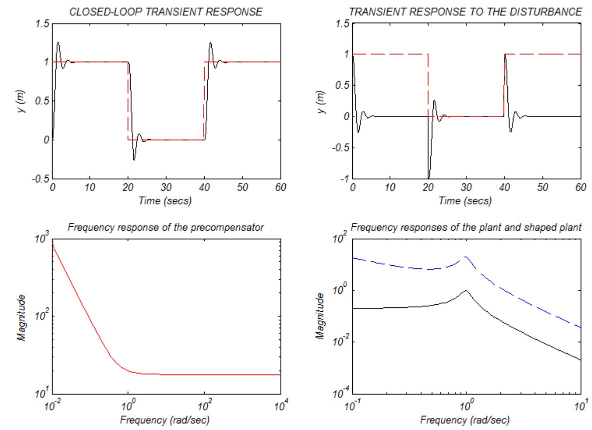

Fig.6.(**Continued**) Closed –open loop behavior of the system.

The peak control input ratio, i.e. $u_{max}(\tau)/u_{max}(0)$ is depicted in Fig. 9. As seen, the more the ratio increases, the more the amount of time delay is. The system is stable and have a reasonable step output when the delay τ is about 137 while the maximum estimated delay obtained 90.7562. From these results, the applicability of the proposed procedure can be concluded such that the procedure validity may remain out of the range found for delay time. Fig. 10 shows the convergence rate of cost function with the number of generation in QGA-PSO.

Table I illustrates the results of the proposed methods based on maximum delay, time-consuming, gain found, tracking error, etc. The fifth row makes a comparison of the four algorithms in term of required time to implement the algorithm. The first row indicates the worst cost of performance. These results show that QGA-PSO algorithm finds a higher maximum delay amount with less worst cost in comparison. Time-consuming for GA-PSO is better than QGA while QGA's better than of GA. However, the complexity of QGA is less than that of GA-PSO. Results obtained by GA-PSO show that GA-PSO is better than those of GA and QGA but still the best results obtained by QGA-PSO.

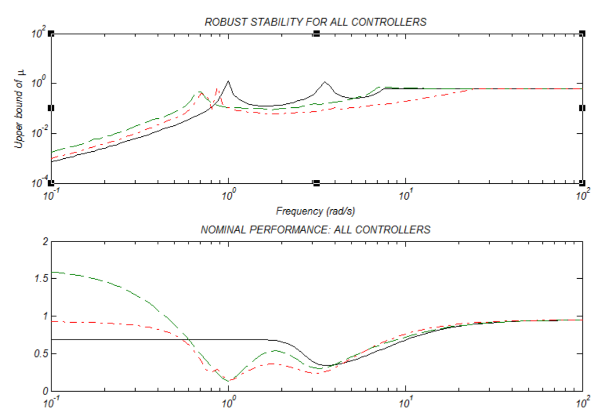

Fig.7. Frequency behavior of the controller.

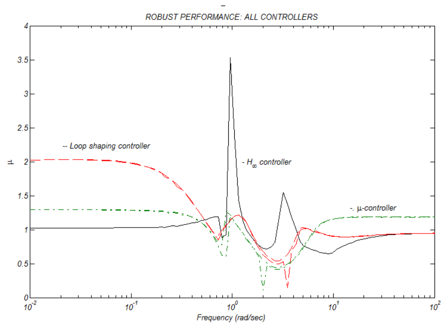

Fig.7.(Continued) Frequency behavior of the controller.

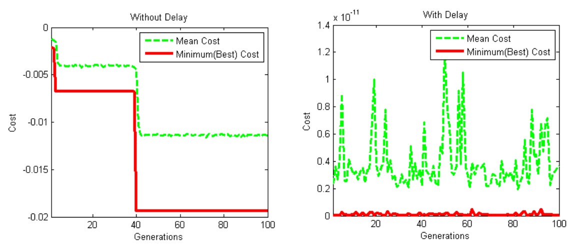

Fig.8. Rate of convergence of QGA.

It should be noted that since the control law is stemmed from the output, more time is required for the model to converge to the desired output, so tracking error exists. The limited error shows the convergence and stability of the whole system. The last row in Table I shows the comparison of different algorithms in error tracking term. These results confirm the priority of QGA-PSO's performance.

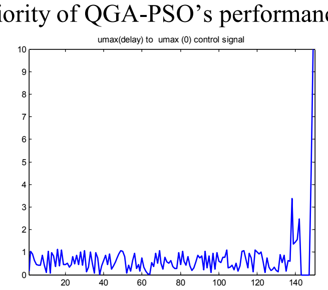

Fig.9. Ratio of control signal vs. time delay

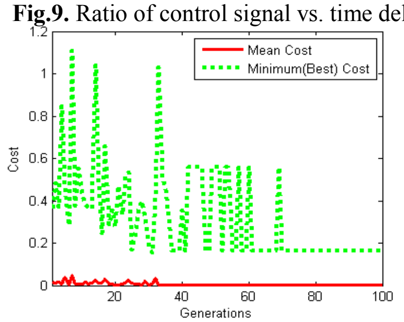

Fig. 10. Convergence rate of the cost function in QGA-PSO (Vertical axis multiplied by 10e-11).

Moreover, the optimal solution, performance, and robust stability depend on the searching range of output feedback gain. Therefore, we can use a larger solution space for the first execution of the algorithm and after we get the early solution, change the range around the values obtained in the first execution. Table II provides the results within initial rang of $[-100, 100]$. As seen, not only the results is improved especially the amount of running time in the second times of execution algorithms.

TABLE I
Comparison of Results

Property	QGA [a] (NO DELAY)	QGA (DELAY)	GA (DELAY)	GA-PSO (DELAY)	QGA-PSO (DELAY)
Worst Cost (10^{-11})	-0.0017	1.35	1.38	1.00	**1.27**
Best Cost (10^{-13})	-0.0009	1.25	1.15	*1.00*	***0.871***
Gain	78.2 ,72.7	-78.94,-47.72	-85.08,-67.52	*-58.2,-85*	***48.68,-39.8***
Maximum Delay	_	90.7562	82.5362	*86.2354*	***90.3351***
Time Consuming (s)	120	130	234	117	**102**
Best Value of $T_{min}(10^{-11})$	-0.0016	0.0091	2.5110	2.1010	**0.098**
Iteration Times	9	15	12	10	**8**
Overshoot (cm)	0.006	0.05	0.025	0.078	**0.02**
Settling Time (ms)	12	35	37	28	**18**
Tracking Error	0.0032	0.029	0.123	0.031	**0.012**

VI. CONCLUSIONS

Little prior knowledge for modeling system when the time delay is main concern leads to a poor controller. Evolutionary algorithms can help in handling this issue. In this paper, an approach is proposed that employs quantum genetic algorithm (QGA) as a searching tool to seek feedback gain and render controller capable of non-convex optimization using LMI solver. Simulation results verify the practicality of the proposed methodology in comparison to alternative approaches. The proposed approach provides larger admissible input time delay without occurring instability. Besides, in this paper, to accelerate convergence speed without damaging the global optimum, the combination of QGA with PSO is suggested. Results show that QGA-PSO gives better solution in different terms comparing to QGA, GA, and GA-PSO. Furthermore, based on the results the effective meta-heuristic algorithm of QGA-PSO has provided an appropriate balance between two competing of exploration and exploitation at the same time. The results of this work can also be readily extended to the state feedback case.

TABLE II
Results related to Shorten Range

Algorithm	Worst Cost (10^{-11})	Best Cost (10^{-14})	Tracking Error	Time(s)
GA	1.2	6.22	0.121	184
QGA	0.09	4.15	0.027	170
GA-PSO	0.324	4.07	0.030	78
QGA-PSO	0.024	2.56	0.01	67

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