Iranian Journal of Fuzzy Systems

Volume 16, Number 5, (2019), pp. 29-31

A note on decision making in medical investigations using new divergence measures for intuitionistic fuzzy sets

Y. F. Li[1] and X. X. He[2]

[1] *School of Economics Information Engineering, Southwestern University of Finance and Economics, Chengdu 611130, Sichuan, China*

[2] *School of Mathematics, Southwest Jiaotong University, Chengdu 610031, Sichuan, China*

liyf@swufe.edu.cn, x.he@home.swjtu.edu.cn

Abstract

Srivastava and Maheshwari (Iranian Journal of Fuzzy Systems 13(1) (2016) 25-44) introduced a new divergence measure for intuitionistic fuzzy sets (IFSs). The properties of the proposed divergence measure were studied and the efficiency of the proposed divergence measure in the context of medical diagnosis was also demonstrated. In this note, we point out some errors in the proving process of two properties of the proposed divergence measure. Then we give a modification of the proving process.

Keywords: Intuitionistic fuzzy sets, divergence measures, decision making.

1 Introduction

An intuitionistic fuzzy set A in X is defined as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where $\mu_A, \nu_A : X \longrightarrow [0, 1]$ are such that $0 \leq \mu_A(x)+\nu_A(x) \leq 1$ for all $x \in X$. The number $\mu_A(x)$ and $\nu_A(x)$ represent respectively the degree of membership and nonmembership of $x \in X$ in A. The complement set of A is defined as $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$. We denote all IFSs in X by $IFS(X)$. The intuitionistic index (or hesitancy degree)[1] of an element $x \in X$ in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ with $\pi_A(x) \in [0, 1]$. Throughout this paper, $X = \{x_1, x_2, \ldots, x_n\}$ is used to denote the discourse set.

Definition 1.1. [1] *If $A, B \in IFS(X)$ defined by $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\}$ and $B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle : x_i \in X\}$, then*

(1) $A \subseteq B$ if and only if $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for each $x_i \in X$.

(2) $A = B$ if and only if $\mu_A(x_i) = \mu_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$ for each $x_i \in X$.

(3) $A = \emptyset$ if and only if $\mu_A(x_i) = 0$ and $\nu_A(x_i) = 1$ for each $x_i \in X$.

(4) $A \cup B = \{\langle x_i, \max(\mu_A(x_i), \mu_B(x_i)), \min(\nu_A(x_i), \nu_B(x_i))\rangle : x_i \in X\}$.

(5) $A \cap B = \{\langle x_i, \min(\mu_A(x_i), \mu_B(x_i)), \max(\nu_A(x_i), \nu_B(x_i))\rangle : x_i \in X\}$.

2 Discussion

In Srivastava and Maheshwari's work [3], they used the following concept of divergence measure for IFSs.

Definition 2.1. [2] *Let $A, B \in IFS(X)$ be two IFSs in X. A mapping $\mathcal{D} : IFS(X) \times IFS(X) \longrightarrow R$ is a divergence measure for IFS if it fulfils the following axioms:*

Corresponding Author: X. X. He

Received: March 2018; Revised: August 2018; Accepted: February 2019.

($\mathcal{D}$1) $\mathcal{D}(A,B)=\mathcal{D}(B,A)$.

($\mathcal{D}$2) $\mathcal{D}(A,B)=0$ *if and only if* $A=B$.

($\mathcal{D}$3) $\mathcal{D}(A\cap C,B\cap C)\le\mathcal{D}(A,B)$ *for every* $C\in IFS(X)$.

($\mathcal{D}$4) $\mathcal{D}(A\cup C,B\cup C)\le\mathcal{D}(A,B)$ *for every* $C\in IFS(X)$.

Srivastava and Maheshwari [3] proposed the following new divergence measure for IFS.

$$\begin{aligned}\mathcal{D}(A,B) &= 1-\log_2(1+\frac{1}{n}\sum_{i=1}^{n}(\min(\mu_A(x_i),\mu_B(x_i))+\min(\nu_A(x_i),\nu_B(x_i))+\min(\pi_A(x_i),\pi_B(x_i))))\\ &= 1-\log_2(1+\frac{1}{2n}\sum_{i=1}^{n}(2-|\mu_A(x_i)-\mu_B(x_i)|-|\nu_A(x_i)-\nu_B(x_i)|-|\pi_A(x_i)-\pi_B(x_i)|)).\end{aligned}\tag{1}$$

Then they proved that $\mathcal{D}(A,B)$ given by Equation (1) satisfies twelve properties. However, we observe some mistakes in the proof of the following two properties.

($\mathcal{D}$3) $\mathcal{D}(A\cap C,B\cap C)\le\mathcal{D}(A,B)$ for every $C\in IFS(X)$.

($\mathcal{D}$11) $\mathcal{D}(A,B)\le\mathcal{D}(A,C)$ for $A\subseteq B\subseteq C$.

The mistakes associated with the results of Srivastava and Maheshwari are listed as follows.

(1): On page number 39 of [3], Srivastava and Maheshwari proved that $\mathcal{D}$ satisfies the property $\mathcal{D}3$ through the following inequality

$$\begin{aligned}&|\min(\mu_A(x_i),\mu_C(x_i))-\min(\mu_B(x_i),\mu_C(x_i))|+|\max(\nu_A(x_i),\nu_C(x_i))-\max(\nu_B(x_i),\\ &\nu_C(x_i))|+|1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i))-(1-\min(\mu_B(x_i),\mu_C(x_i))\\ &-\max(\nu_B(x_i),\nu_C(x_i)))|\le|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|1-\mu_A(x_i)-\nu_A(x_i)\\ &-(1-\mu_B(x_i)-\nu_B(x_i))|.\end{aligned}\tag{2}$$

Actually, Inequality (2) is valid. However, the proof of Inequality (2) in [3] is not true since the authors used an inequality

$$\begin{aligned}&|1-\min(\mu_A(x_i),\mu_C(x_i))-\max(\nu_A(x_i),\nu_C(x_i))-(1-\min(\mu_B(x_i),\mu_C(x_i))-\\ &\max(\nu_B(x_i),\nu_C(x_i)))|\le|1-\mu_A(x_i)-\nu_A(x_i)-(1-\mu_B(x_i)-\nu_B(x_i))|\end{aligned}\tag{3}$$

in the proof. It is shown from the following numerical example that Inequality (3) is invalid.

Example 2.2. *Let* $A=\{\langle x,0.3,0.2\rangle\},B=\{\langle x,0.1,0.4\rangle\}$ *and* $C=\{\langle x,0.2,0.2\rangle\}$ *be three IFSs in* $X=\{x\}$*. Then* $|1-\min(\mu_A(x),\mu_C(x))-\max(\nu_A(x),\nu_C(x))-(1-\min(\mu_B(x),\mu_C(x))-\max(\nu_B(x),\nu_C(x)))|=0.1$ *and* $|1-\mu_A(x)-\nu_A(x)-(1-\mu_B(x)-\nu_B(x))|=0$*. In this case, we have* $|1-\min(\mu_A(x),\mu_C(x))-\max(\nu_A(x),\nu_C(x))-(1-\min(\mu_B(x),\mu_C(x))-\max(\nu_B(x),\nu_C(x)))|>|1-\mu_A(x)-\nu_A(x)-(1-\mu_B(x)-\nu_B(x))|$.

(2): On page number 41 of [3], Srivastava and Maheshwari proved that $\mathcal{D}$ satisfies the property $\mathcal{D}11$ through the following inequality

$$\begin{aligned}&2(1-|\mu_A(x_i)-\mu_B(x_i)|-|\nu_A(x_i)-\nu_B(x_i)|-|\pi_A(x_i)-\pi_B(x_i)|)\\ &\ge 2(1-|\mu_A(x_i)-\mu_C(x_i)|-|\nu_A(x_i)-\nu_C(x_i)|-|\pi_A(x_i)-\pi_C(x_i)|).\end{aligned}\tag{4}$$

Actually, Inequality (4) is valid. However, the proof of Inequality (4) in [3] is not true since the authors used an inequality

$$|\pi_A(x_i)-\pi_B(x_i)|\le|\pi_A(x_i)-\pi_C(x_i)|\tag{5}$$

in the proof. It is shown from the following numerical example that Inequality (5) is invalid.

Example 2.3. *Let $A = \{\langle x, 0.1, 0.5\rangle\}, B = \{\langle x, 0.4, 0.4\rangle\}$ and $C = \{\langle x, 0.5, 0.1\rangle\}$ be three IFSs in $X = \{x\}$ satisfying $A \subseteq B \subseteq C$. Then we have $|\pi_A(x)-\pi_B(x)| = 0.2$ and $|\pi_A(x)-\pi_C(x)| = 0$ and thus $|\pi_A(x)-\pi_B(x)| > |\pi_A(x)-\pi_C(x)|$.*

Since Inequalities (2) and (4) are indispensable in the proof of properties $\mathcal{D}3$ and $\mathcal{D}11$, we give a modification of their proving process in the following two propositions.

Proposition 2.4. *Let $A, B, C \in IFS(X)$ be three IFSs in X. Then we obtain that Inequality (2) is valid for all $x_i \in X$.*

Proof. Let us denote $|\min(\mu_A(x_i), \mu_C(x_i)) - \min(\mu_B(x_i), \mu_C(x_i))| + |\max(\nu_A(x_i), \nu_C(x_i)) - \max(\nu_B(x_i), \nu_C(x_i))| + |1 - \min(\mu_A(x_i), \mu_C(x_i)) - \max(\nu_A(x_i), \nu_C(x_i)) - (1 - \min(\mu_B(x_i), \mu_C(x_i)) - \max(\nu_B(x_i), \nu_C(x_i)))|$ and $|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |1 - \mu_A(x_i) - \nu_A(x_i) - (1 - \mu_B(x_i) - \nu_B(x_i))|$ by Δ_1 and Δ_2, respectively. It is aimed to prove that $\Delta_1 \leq \Delta_2$ for all $x_i \in X$. According to the values of $\mu_A(x_i), \mu_B(x_i), \nu_A(x_i)$ and $\nu_B(x_i)$, four cases should be considered.

(a): If $\mu_A(x_i) \geq \mu_B(x_i), \nu_A(x_i) \geq \nu_B(x_i)$, then $\Delta_1 = 2(\min(\mu_A(x_i), \mu_C(x_i)) - \min(\mu_B(x_i), \mu_C(x_i)) + \max(\nu_A(x_i), \nu_C(x_i)) - \max(\nu_B(x_i), \nu_C(x_i)))$, $\Delta_2 = 2(\mu_A(x_i) - \mu_B(x_i) + \nu_A(x_i) - \nu_B(x_i))$. Since $\min(\mu_A(x_i), \mu_C(x_i)) - \min(\mu_B(x_i), \mu_C(x_i)) \leq \mu_A(x_i) - \mu_B(x_i)$ and $\max(\nu_A(x_i), \nu_C(x_i)) - \max(\nu_B(x_i), \nu_C(x_i)) \leq \nu_A(x_i) - \nu_B(x_i)$, we have $\Delta_1 \leq \Delta_2$.

(b): If $\mu_A(x_i) \geq \mu_B(x_i), \nu_A(x_i) \leq \nu_B(x_i)$, then $\Delta_1 = 2\max((\min(\mu_A(x_i), \mu_C(x_i)) - \min(\mu_B(x_i), \mu_C(x_i))), (\max(\nu_B(x_i), \nu_C(x_i)) - \max(\nu_A(x_i), \nu_C(x_i))))$ and $\Delta_2 = 2\max((\mu_A(x_i) - \mu_B(x_i)), (\nu_B(x_i) - \nu_A(x_i)))$. Since $\min(\mu_A(x_i), \mu_C(x_i)) - \min(\mu_B(x_i), \mu_C(x_i)) \leq \mu_A(x_i) - \mu_B(x_i)$ and $\max(\nu_B(x_i), \nu_C(x_i)) - \max(\nu_A(x_i), \nu_C(x_i)) \leq \nu_B(x_i) - \nu_A(x_i)$, we have $\Delta_1 \leq \Delta_2$.

(c): If $\mu_A(x_i) \leq \mu_B(x_i), \nu_A(x_i) \leq \nu_B(x_i)$, then $\Delta_1 = 2(\min(\mu_B(x_i), \mu_C(x_i)) - \min(\mu_A(x_i), \mu_C(x_i)) + \max(\nu_B(x_i), \nu_C(x_i)) - \max(\nu_A(x_i), \nu_C(x_i)))$, $\Delta_2 = 2(\mu_B(x_i) - \mu_A(x_i) + \nu_B(x_i) - \nu_A(x_i))$. Since $\min(\mu_B(x_i), \mu_C(x_i)) - \min(\mu_A(x_i), \mu_C(x_i)) \leq \mu_B(x_i) - \mu_A(x_i)$ and $\max(\nu_B(x_i), \nu_C(x_i)) - \max(\nu_A(x_i), \nu_C(x_i)) \leq \nu_B(x_i) - \nu_A(x_i)$, we have $\Delta_1 \leq \Delta_2$.

(d): If $\mu_A(x_i) \leq \mu_B(x_i), \nu_A(x_i) \geq \nu_B(x_i)$, then $\Delta_1 = 2\max((\min(\mu_B(x_i), \mu_C(x_i)) - \min(\mu_A(x_i), \mu_C(x_i))), (\max(\nu_A(x_i), \nu_C(x_i)) - \max(\nu_B(x_i), \nu_C(x_i))))$ and $\Delta_2 = 2\max((\mu_B(x_i) - \mu_A(x_i)), (\nu_A(x_i) - \nu_B(x_i)))$. Since $\min(\mu_B(x_i), \mu_C(x_i)) - \min(\mu_A(x_i), \mu_C(x_i)) \leq \mu_B(x_i) - \mu_A(x_i)$ and $\max(\nu_A(x_i), \nu_C(x_i)) - \max(\nu_B(x_i), \nu_C(x_i)) \leq \nu_A(x_i) - \nu_B(x_i)$, we have $\Delta_1 \leq \Delta_2$. □

Proposition 2.5. *Let $A, B, C \in IFS(X)$ be three IFSs in X satisfying $A \subseteq B \subseteq C$. Then we obtain that Inequality (4) is valid for all $x_i \in X$.*

Proof. Since $A \subseteq B \subseteq C$, we have $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i) \geq \nu_C(x_i)$ for each $x_i \in X$. Therefore, $|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)| = 2\max((\mu_B(x_i) - \mu_A(x_i)), (\nu_A(x_i) - \nu_B(x_i))) \leq 2\max((\mu_C(x_i) - \mu_A(x_i)), (\nu_A(x_i) - \nu_C(x_i))) = |\mu_A(x_i) - \mu_C(x_i)| + |\nu_A(x_i) - \nu_C(x_i)| + |\pi_A(x_i) - \pi_C(x_i)|$. Thus we obtain that $2(1 - |\mu_A(x_i) - \mu_B(x_i)| - |\nu_A(x_i) - \nu_B(x_i)| - |\pi_A(x_i) - \pi_B(x_i)|) \geq 2(1 - |\mu_A(x_i) - \mu_C(x_i)| - |\nu_A(x_i) - \nu_C(x_i)| - |\pi_A(x_i) - \pi_C(x_i)|)$ for all $x_i \in X$. □

3 Conclusions

In this note, we pointed out some errors in the proving process of two properties of the proposed divergence measure in Srivastava and Maheshwaris work (Iranian Journal of Fuzzy Systems 13(1) (2016) 25-44). Then we gave a modification of the proving process.

Acknowledgement

Authors would like to express their sincere thanks to the valuable comments and suggestions from editors and reviewers. This research is supported by the National Natural Science Foundation of China (Grant No. 61603307) and the Grant from MOE (Ministry of Education in China) Project of Humanities and Social Sciences (Grant No. 19YJCZH048).

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87–96.

[2] I. Montes, N. R. Pal, V. Janiš, S. Montes, *Divergence measures for intuitionistic fuzzy sets*, IEEE Transactions on Fuzzy Systems, **23**(2) (2015), 444–456.

[3] A. Srivastava, S. Maheshwari, *Decision making in medical investigations using new divergence measures for intuitionistic fuzzy sets*, Iranian Journal of Fuzzy Systems, **13**(1) (2016), 25–44.