

Two-level supply chain for a deteriorating item with stock and promotional cost dependent demand under shortages

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Abstract

In this research work, a wholesaler-retailer-customer supply chain model for a deteriorating item is considered, where the retailer's warehouse in the market place has a limited capacity. The retailer can rent an additional warehouse (rented warehouse) if needed, with a higher rent compared to the existing warehouse (own warehouse). The customers' demand of the item is linearly influenced by the stock level and in case of shortages the base demand is partially backlogged. Being the leader of the supply chain, the retailer introduces some promotional cost to boost the base demand of the item. To participate in joint marketing decision, the wholesaler shares a compromise part of this promotional cost. Goal of this research work is to maximize the individual profits (when the retailer is the leader and the wholesaler is the follower) as well as the channel profit (when the retailer and the wholesaler jointly make marketing decision) of the system. It is established that if the wholesaler shares a part of the promotional cost, then the channel profit as well as the individual profits increase. The supply chain model is also considered in imprecise environment, where different inventory parameters are fuzzy/rough in nature. In this case, the individual profits as well as the channel profit become fuzzy/rough in nature. As optimization of fuzzy/rough objective is not well defined, following credibility/trust measure of fuzzy/rough event, an approach is followed for comparison of fuzzy/rough objectives and a Particle Swarm Optimization algorithm is implemented to find the marketing decisions. Efficiency of the algorithm in solving the problem is statistically established. The existence of the joint marketing decision is established analytically and numerically (with illustration) in crisp as well as in imprecise environments.

Keywords: Deteriorating inventory, two-warehouse model, promotional cost, credibility measure, trust measure, particle swarm optimization.

1 Introduction

The classical inventory models on deteriorating items are normally developed with the common assumption that the capacity of the retailer's outlet is sufficient, i.e., the outlet has sufficient space to store the order quantity [2, 5, 30, 37, 43, 44]. However, in several real-life problems, this assumption may not appropriate. There are a number of factors which influence the marketing decisions in different ways. Sometimes these factors may force the retailer to buy more than his/her own warehouse (OW) capacity. The retailer may overcome the situation using an additional rented warehouse (RW), having sufficient capacity, normally with higher rent relative to the OW [8, 24].

Influence of displayed inventory level on the demand of any item is a well established phenomenon [12, 11]. Due to this reason, a retailer normally uses a decorated outlet at the market place to attract the customers and uses another storehouse near the outlet to stock the excess order quantity [23, 24, 29, 35]. Also inventory modelings of the deteriorating items draw significant attention by the researchers [2, 3, 5, 21, 26, 30, 41, 44]. During last two decades, several researchers on inventory control problems developed their models incorporating the above mentioned

two important phenomenon, i.e., inventory models of deteriorating items with displayed inventory dependent demand under retailer's two warehouse facility [4, 7, 8, 27, 37, 39].

From the perspective of the customers' buying behavior, it is noticed that besides stock of the products other factors such as promotional cost through advertising, free gift coupon etc., also influence customers' preferences and their purchasing decisions and hence the market demand. Many researchers also have considered promotional effort dependent demand in their research. Wu [40] considered price and promotional effort dependent demand in a SC. Giri and Sharma [10] discussed manufacturer's pricing strategy in a two-level SC with competing retailers and advertising cost dependent demand. Pal et al. [34] have considered two-echelon SC with price and promotional effort sensitive non-linear demand. None of these investigations gives attention to study the joint effect of the stock level and the promotional effort on the demand of a deteriorating item specially in a SC under coordination mechanism.

For some products when a retailer is out of stock, the demand is lost which means that the customer finds the item or a similar one in another store. Yang et al. [43] developed an inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Sarkar and Sarkar [38] proposed an improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. But, none of these investigations on deteriorating items, studied the effects of shortages when stock and promotional efforts jointly influences the demand of the item specially in a SC under promotional cost sharing strategy.

To overcome the above stated lacunas of the SC models on deteriorating items, in this study, a wholesaler-retailer-customer SC model of a deteriorating item is considered where the retailer runs his/her business with a warehouse having limited capacity. Due to limited capacity of own warehouse (OW), the retailer rents another warehouse (named RW) to store the excess order quantity of the item (if required). Units are sold from the OW and are continuously replenished from the RW. The item has a base demand d and another portion of the demand is linearly influenced by the stocks level at OW. Retailer invests some promotional cost to improve the base demand of the item. The product is deteriorated with a constant rate. Shortages are also considered and backlogged partially. As the retailer introduces the promotional cost and there is no bonding of the retailer with the wholesaler, in the first phase of the study, it is assumed that the retailer is the leader and the wholesaler is the follower, i.e., the retailer determines his/her marketing strategy according to his/her interest and accordingly the wholesaler fixes his/her marketing decision. This situation is named as non-coordination scenario (NCS). In the second phase of the study, it is assumed that the wholesaler will to share a compromise portion of the promotional cost spends by the retailer to take part in the joint marketing decision with the retailer and this situation is named as coordination scenario (CS). It is established that the profits of both the parties improves in CS. The crisp (precise) model is solved following GRG method using LINGO 14.0 software. Model is analyzed in imprecise environment also when different inventory costs like set-up cost, holding cost and the promotional cost function are fuzzy/rough in nature. As optimization of fuzzy/rough objective is not well defined, following credibility/trust measure of fuzzy/rough event an approach is followed for the comparison of fuzzy/rough objectives. A Particle Swarm Optimisation (PSO) technique [18] is implemented and used to determine the marketing decisions of the model in imprecise environment. Proper parameter setting of PSO for solving the model is made following Taguchi approach [42]. The crisp models are also solved using PSO and compared the results with those obtained using LINGO. Another heuristic, Artificial Bee Colony (ABC) [19] is also implemented to establish the uses of PSO in solving the models. Models are illustrated with numerical examples and some managerial insights are outlined. The existence of the joint marketing decision is established analytically and numerically in crisp as well as in imprecise environments.

Among various heuristic algorithms, here PSO is considered due to the following reasons:

- The model considered in this paper involves continuous optimisation in crisp and imprecise environment. PSO is well established algorithm for continuous optimisation and it needs only comparison of objectives for finding optimal/near-optimal solution. Due to this property of PSO, using valid fuzzy/rough comparison approach of fuzzy/rough objectives, here it is used to find marketing decisions in imprecise environment.
- PSO can be easily implemented using any computer language. Its implementation involves lesser number of coded lines compared to other existing heuristics in the literature.
- PSO needs relatively small number of function evaluations for finding optimal/ near-optimal solution of continuous optimisation problems.
- GRG method (using LINGO 14.0 software) is not capable of finding solutions of the optimisation problems in imprecise environment. Also no proper guidelines is available in the literature for the optimisation in imprecise environment. Due to this reason, here, using credibility/trust measure of fuzzy/rough event for comparison of fuzzy/rough objectives, PSO is used to find marketing decision of the model in fuzzy/rough environment.

Table 1: Comparison of Previous Research Works

References	No. of warehouses	Deterioration	Stock-dependent Demand	Promotional Effort Dependent Demand	Other Demand	Allow Shortages	Coordination Mechanism	Imprecise Environment
Bhunia and Maiti (1998)	one	✓	x	x	✓	✓	x	x
Mukhopadhyay et al. (2004)	one	✓	x	x	✓	x	x	x
Yang (2006)	two	✓	x	x	✓	✓	x	x
Chung and Huang (2007)	two	✓	x	x	✓	x	x	x
Dye (2007)	one	✓	x	x	✓	✓	x	x
Gayen and Pal (2009)	two	✓	✓	x	x	x	x	x
Lee and Hsu (2009)	two	✓	x	x	✓	x	x	x
Yang et al. (2010)	one	✓	✓	x	x	✓	x	x
Das et al. (2012)	two	✓	✓	x	x	x	x	x
Yu et al. (2012)	one	✓	x	x	✓	x	x	x
Ghiami et al. (2013)	two	✓	✓	x	x	✓	x	x
Sarkar and Sarkar (2013)	one	✓	✓	x	x	✓	x	x
Wu (2013)	one	x	x	✓	✓	x	x	x
Giri and Sharma (2014)	one	x	x	✓	x	x	✓	x
Mishra (2014)	one	✓	x	x	✓	✓	x	x
Prasad and Mukherjee (2014)	one	✓	✓	x	✓	✓	x	x
Pal et al. (2015)	one	x	x	✓	✓	x	✓	x
Mishra and Shaikh (2017)	two	x	✓	x	x	x	✓	x
Pakhira et al. (2017)	one	✓	x	✓	✓	✓	✓	✓
Pramanik et al. (2017)	one	x	x	✓	✓	x	✓	✓
Tiwari et al. (2017)	two	✓	✓	x	x	✓	x	x
Present Paper	two	✓	✓	✓	x	✓	✓	✓

Comparison of previous research works with the proposed model have been presented in Table 1. The new ideas incorporated in this investigation are as follows:

- The joint effect of stock and promotional cost on demand of a deteriorating item is studied in a SC under retailer's two-warehouse facility.
- The SC models with the above mentioned realistic demand pattern are formulated under proper coordination strategy and solved in both crisp and imprecise (fuzzy and rough) environments.
- Uses of PSO in solving such complicated models is established through statistical test, like ANOVA.
- Moreover, a guideline is presented for proper parameter setting of the PSO for solving realistic SC/inventory control problems following Taguchi method [42].

2 Assumptions and notations

The following assumptions and notations are used in this study:

Assumptions:

- It is an infinite time horizon EOQ model for the retailer for a constantly deteriorating item.
- Lead time is zero.
- Demand is stock and promotional effort dependent.
- Shortages are allowed and partially backlogged.
- Rate of replenishment is infinite.
- Two warehouses – OW and RW are considered. Sales are performed from OW and units are transferred from RW to OW by continuous release pattern.
- The promotional cost to boost the demand is shared by both the wholesaler and the retailer.

Notation Meaning

- W the capacity of the OW.
 $I_o(t)/I_r(t)$ the inventory level at the OW/RW at time t .

Notation	Meaning
$S(t)$	the shortage level of the retailer.
$I_W(t)$	the inventory level of the wholesaler.
$I_W^i(t)$	the inventory level of the wholesaler during i -th interval of its inventory period.
α	the deterioration rate at the OW.
β	the deterioration rate at the RW ($\beta > \alpha$).
γ	the deterioration rate at the wholesaler.
δ	the percentage of demand which is backlogged during shortage time.
A_R/A_W	the replenishment fixed cost per order of the retailer/wholesaler.
p_R/p_W	the purchase price of the retailer/wholesaler per unit.
s_R/s_W	the selling price of the retailer/wholesaler per unit ($s_W = p_R$).
h_o/h_r	the unit holding cost per unit time at the OW/RW.
h_W	the unit holding cost per unit time of the wholesaler.
d_R/d_W	the deterioration cost per unit time of the retailer/wholesaler.
c_{sf}	the unit lost sale cost.
c_{sv}	the unit shortage cost per unit time for backlogged demand.
t_o/t_r	the time at which the inventory level at the OW/RW reaches zero.
t_s	the time during which the retailer is out of stock and demand is partially backlogged.
T_R/T_W	the inventory period of the retailer/wholesaler.
Q_R/Q_W	the order quantity of the retailer/wholesaler.
D_R/D_W	the quantity of deterioration of the retailer/wholesaler.
ρ	the retailer promotional effort, $\rho \geq 1$.
F	the fraction of the retailer's promotion cost shared by the wholesaler.
PrC	the promotional effort cost per unit time for the item: $PrC = g(\rho - 1)^2 d^m$; where, g, m are the parameters so chosen to best fit the promotional cost and d is the base demand of the item.
$D(t)$	the demand rate at time t , the demand is assumed to be deterministic, stock-dependent and promotional cost dependent: $D(t) = cI_o(t) + d\rho$; where, c is a parameter so chosen to best fit the demand function and d is the base demand of the item.

Symbols $\tilde{\sim}$ and $\underset{\sim}{\sim}$ are used on the top of some of the above notations to indicate fuzzy and rough variable respectively.

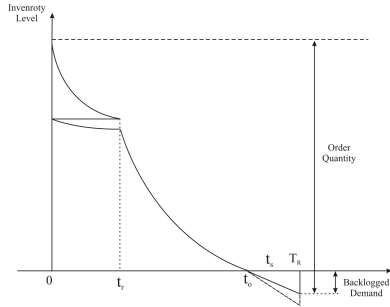


Figure 1: Inventory level at the RW and the OW of the retailer

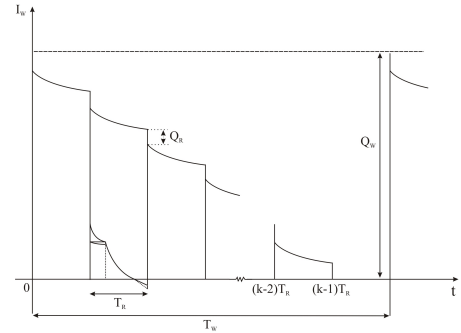


Figure 2: Inventory level of the wholesaler

3 Mathematical formulation

3.1 Inventory level of the retailer (OW and RW)

It is assumed that at the beginning of each cycle the retailer orders an amount of Q_R units of the item to the wholesaler. Among Q_R units, W units are stored at OW and remaining units are stored at RW. The units are sold from OW and are continuously replenished from RW (cf. Figure 1). Let $I_o(t)$ and $I_r(t)$ be the inventory level at the OW and the RW respectively. At the RW, the inventory is depleted by a demand which is connected to the inventory level at the OW. Therefore, the changes of inventory level at the RW between the start of the inventory period and t_r can be presented by the following differential equation [7, 8, 29]:

$$\frac{dI_r(t)}{dt} = -cI_o(t) - d\rho - \beta I_r(t), \text{ for } 0 \leq t \leq t_r \quad (1)$$

While the retailer is using the inventory at the RW to meet the demand, the inventory level at the OW goes down by a constant rate of the inventory level due to deterioration as follows:

$$\frac{dI_o(t)}{dt} = -\alpha I_o(t), \text{ for } 0 \leq t \leq t_r \quad (2)$$

At time t_r , the inventory at the RW is depleted completely and the inventory at the OW is used. The inventory level at the OW decreases due to the demand and deterioration until it reaches zero at t_o . This changes of inventory level at the OW is presented by the following differential equation:

$$\frac{dI_o(t)}{dt} = -cI_o(t) - d\rho - \alpha I_o(t), \text{ for } t_r \leq t \leq t_o \quad (3)$$

From t_o to T_R the system is out of stock and unmet demand is partially backlogged.

$$\frac{dI_o(t)}{dt} = -\delta d\rho, \text{ for } t_o \leq t \leq T_R \quad (4)$$

In order to solve the presented differential equations, the following boundary conditions should be considered:

$$I_o(0) = W, \quad I_o(t_o) = 0, \quad I_r(t_r) = 0$$

By solving the differential equations in (1)-(4), the inventory levels at the OW and RW are obtained:

$$I_r(t) = \frac{cW e^{-\alpha t}}{\beta - \alpha} \{e^{(\beta - \alpha)(t_r - t)} - 1\} + \frac{d\rho}{\beta} \{e^{\beta(t_r - t)} - 1\}, \text{ for } 0 \leq t \leq t_r \quad (5)$$

$$I_o(t) = W e^{-\alpha t}, \text{ for } 0 \leq t \leq t_r \quad (6)$$

$$I_o(t) = \frac{d\rho}{c + \alpha} \{e^{(c + \alpha)(t_o - t)} - 1\}, \text{ for } t_r \leq t \leq t_o \quad (7)$$

$$S(t) = -I_o(t) = \delta d\rho(t - t_o), \text{ for } t_o \leq t \leq T_R \quad (8)$$

Equating the inventory level of OW at $t = t_r$ from (6) and (7), the following result is derived.

$$t_o = t_r + \frac{1}{c + \alpha} \ln \left(1 + \frac{c + \alpha}{d\rho} W e^{-\alpha t_r} \right) \quad (9)$$

which shows that t_o is a function of t_r and ρ .

The order quantity for the retailer is the sum of the initial inventory level at the RW and the OW and the total backlogged demand during one inventory period.

$$Q_R = I_r(0) + I_o(0) + S(T_R) = \frac{cW}{\beta - \alpha} \{e^{(\beta - \alpha)t_r} - 1\} + \frac{d\rho}{\beta} \{e^{\beta t_r} - 1\} + W + \delta d\rho t_s \quad (10)$$

which shows that Q_R is a function of t_r , t_s and ρ .

The length of the inventory period of the retailer is the sum of t_o and t_s .

$$T_R = t_o + t_s = t_r + \frac{1}{c + \alpha} \ln \left(1 + \frac{c + \alpha}{d\rho} W e^{-\alpha t_r} \right) + t_s \quad (11)$$

This shows that T_R is a function of t_r , t_s and ρ .

3.2 Inventory level of the wholesaler

The order quantity and the length of the inventory period of the wholesaler are Q_W and T_W respectively. Here it is assumed that T_W is a multiple of T_R (i.e., $T_W = kT_R$, where k is an integer) [8].

The order quantity of the wholesaler is equal to the inventory needed for k periods of the retailer, plus the amount of deterioration during the wholesaler's inventory cycle. It should be noted that during the k -th interval, there is no inventory at the wholesaler and after receiving Q_W units of the item at the end of T_W , Q_R is again sent to the retailer. Therefore there is no deterioration during this interval at the wholesaler. The order quantity of the wholesaler can be calculated as follows:

$$Q_W = kQ_R + D_W \quad (12)$$

where, D_W is the deterioration during the wholesaler's inventory cycle.

Figure 2 illustrates the inventory level at the wholesaler. Here, one inventory period of the wholesaler consists of k retailer's inventory periods. At the time $(k-2)T_R$ and $(k-1)T_R$, the inventory level of the wholesaler drops by Q_R and a constant rate of the inventory is deteriorated during the interval $[(k-2)T_R, (k-1)T_R]$. The change in inventory level of the wholesaler during this interval can be presented by the following differential equation:

$$\frac{dI_W(t)}{dt} = -\gamma I_W(t) \quad (13)$$

Considering the inventory level of the wholesaler at $(k-1)T_R$ which is Q_R , the inventory level for the specific period will be:

$$I_W(t) = Q_R e^{\gamma[(k-1)T_R - t]} \text{ for } (k-2)T_R \leq t \leq (k-1)T_R \quad (14)$$

In a similar way, the inventory level of the wholesaler can be obtained for the period starts at $(k-3)T_R$ using (13), considering the boundary condition derived from (14) at $t = (k-2)T_R$:

$$I_W(t) = Q_R(1 + e^{\gamma T_R})e^{\gamma[(k-2)T_R - t]} \text{ for } (k-3)T_R \leq t \leq (k-2)T_R \quad (15)$$

In this way, the inventory level of the wholesaler during i -th interval can be calculated as follows:

$$I_W^i(t) = Q_R \left\{ \sum_{m=i+1}^k e^{(k-m)\gamma T_R} \right\} e^{\gamma[iT_R - t]} \text{ for } i = 1, 2, \dots, k-1. \quad (16)$$

3.3 Profit of the retailer

Total deteriorated units (D_R) [2, 14, 21] during the retailer's inventory cycle is the sum of the deteriorated units at the RW (D_{RW}) and at the OW (D_{OW}):

$$D_R = D_{RW} + D_{OW} \quad (17)$$

$$\text{Deteriorated units at the RW} = D_{RW} = \int_0^{t_r} \beta I_r(t) dt = (e^{\beta t_r} - 1) \left\{ \frac{cW e^{-\alpha t_r}}{\beta - \alpha} + \frac{d\rho}{\beta} \right\} + \frac{\beta cW}{\alpha(\beta - \alpha)} (e^{-\alpha t_r} - 1) - d\rho t_r$$

$$\text{Deteriorated units at the OW} = D_{OW} = \int_0^{t_r} \alpha I_o(t) dt + \int_{t_r}^{t_o} \alpha I_o(t) dt = W \left\{ 1 - \frac{ce^{-\alpha t_r}}{c + \alpha} \right\} - \frac{\alpha d\rho}{c + \alpha} (t_o - t_r)$$

Hence, the total selling price per unit time of the retailer:

$$TS_R = \frac{(Q_R - D_R)s_R}{T_R} \quad (18)$$

The retailer has different types of costs: ordering cost (A_R), purchase, carrying, deterioration and shortage costs. The purchase cost of the retailer is

$$PC_R = p_R Q_R \quad (19)$$

Total inventory carrying cost (ICC_R) during the retailer's inventory period is the sum of the carrying cost at the RW (ICC_{RW}) and at the OW (ICC_{OW}):

$$ICC_R = ICC_{RW} + ICC_{OW} \quad (20)$$

$$\text{Carrying Cost at the RW} = ICC_{RW} = h_r \int_0^{t_r} I_r(t) dt = \frac{h_r}{\beta} D_{RW}$$

$$\text{Carrying Cost at the OW} = ICC_{OW} = h_o \int_0^{t_r} I_o(t) dt + h_o \int_{t_r}^{t_o} I_o(t) dt = \frac{h_o}{\alpha} D_{OW}$$

Deterioration cost (DC_R) of the retailer includes both the deterioration at the RW and the OW:

$$DC_R = DC_{RW} + DC_{OW} \quad (21)$$

Deterioration cost at the RW = $DC_{RW} = d_R D_{RW}$

Deterioration cost at the OW = $DC_{OW} = d_R D_{OW}$

During the shortage period, the demand is partially backlogged. There are two different types of shortage cost; one is based on per unit for the lost sale and the second is for the backlogged demand which is per unit per unit of time.

$$SC_R = c_{sf} \int_{t_o}^{T_R} \{(1 - \delta)d\rho\}dt + c_{sv} \int_{t_o}^{T_R} \delta d\rho(t - t_o)dt = c_{sf}(1 - \delta)d\rho t_s + \frac{1}{2}c_{sv}\delta d\rho t_s^2 \quad (22)$$

Hence, the total cost per unit time of the retailer:

$$TC_R = \frac{1}{T_R}(A_R + PC_R + ICC_R + DC_R + SC_R) \quad (23)$$

With the above costs, the retailer spends some promotional cost (PrC) to increase the demand as follows:

$$PrC = g(\rho - 1)^2 d^m \quad (24)$$

where, g and m are the parameters so chosen to best fit the promotional cost.

Using (18), (23) and (24), the total profit per unit time of the retailer (TP_R) is

$$TP_R = TS_R - TC_R - PrC \quad (25)$$

3.4 Profit of the wholesaler

Based on (16), the amount of the deterioration in each interval can be calculated as follows [8]:

Time Period	Deterioration
$[(k - 1)T_R, kT_R]$	0
$[(k - 2)T_R, (k - 1)T_R]$	$Q_R(e^{\gamma T_R} - 1)$
$[(k - 3)T_R, (k - 2)T_R]$	$Q_R(e^{\gamma T_R} - 1)(1 + e^{\gamma T_R})$
$[(k - 4)T_R, (k - 3)T_R]$	$Q_R(e^{\gamma T_R} - 1)(1 + e^{\gamma T_R} + e^{2\gamma T_R})$
...	...
$[(i - 1)T_R, iT_R]$ (i -th interval)	$Q_R(e^{\gamma T_R} - 1) \sum_{m=i+1}^k e^{(k-m)\gamma T_R}$

Therefore, the total deteriorated units of the wholesaler (D_W) during T_W can be obtained as:

$$D_W = Q_R(e^{\gamma T_R} - 1) \sum_{i=1}^{k-1} \sum_{m=i+1}^k e^{(k-m)\gamma T_R} = Q_R \left(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1} - k \right) \quad (26)$$

Using (12) and (26), the wholesaler's order quantity can be calculated as follows:

$$Q_W = kQ_R + Q_R \left(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1} - k \right) = Q_R \left(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1} \right) \quad (27)$$

Hence, the total selling price per unit time of the wholesaler:

$$TS_W = \frac{(Q_W - D_W)s_W}{T_W} \quad (28)$$

The wholesaler has the following costs: ordering cost (A_W), purchase, carrying and deterioration costs. The purchase cost of the wholesaler is

$$PC_W = p_W Q_W \quad (29)$$

Inventory carrying cost of the wholesaler during the i -th interval is

$$\int_{(i-1)T_R}^{iT_R} h_W I_W^i(t) dt = \frac{h_W Q_R}{\gamma} (e^{\gamma T_R} - 1) \sum_{m=i+1}^k e^{(k-m)\gamma T_R}$$

Hence, the inventory carrying cost of the wholesaler (ICC_W) during one inventory period (consider that there is no carrying cost during k -th interval) is

$$ICC_W = \frac{h_W Q_R}{\gamma} (e^{\gamma T_R} - 1) \sum_{i=1}^{k-1} \sum_{m=i+1}^k e^{(k-m)\gamma T_R} = \frac{h_W Q_R}{\gamma} \left(\frac{e^{k\gamma T_R} - 1}{e^{\gamma T_R} - 1} - k \right) = \frac{h_W}{\gamma} D_W \quad (30)$$

The total deterioration cost of the wholesaler (DC_W) during T_W is

$$DC_W = d_W D_W \quad (31)$$

Hence, the total cost per unit time of the wholesaler:

$$TC_W = \frac{1}{T_W} (A_W + PC_W + ICC_W + DC_W) \quad (32)$$

Using (28) and (32), the total profit per unit time of the wholesaler (TP_W) is

$$TP_W = TS_W - TC_W \quad (33)$$

According to the above discussion, two cases may arise:

- The wholesaler does not share any part of the promotional cost. In this case, retailer is the primary decision maker. So inventory decisions are made by the retailer first, i.e., the retailer will fix his/her marketing decision to maximise his/her profit only. Depending upon the retailer's decision, the wholesaler will fix his/her marketing decision, i.e., here the retailer is the leader and the wholesaler is the follower.
- The wholesaler shares a compromise part F of the promotional cost; i.e., the wholesaler pays $Fg(\rho - 1)^2 d^m$ and the retailer pays the remaining part $(1 - F)g(\rho - 1)^2 d^m$ of the promotional cost. In this case, inventory decisions are made jointly by the retailer and the wholesaler, i.e., joint profit of the retailer and the wholesaler is maximized to find marketing decision.

These phenomena are termed as *non-coordination scenario* and *coordination scenario* respectively. These two scenarios are discussed separately.

3.5 Non-coordination scenario (NCS)

In this scenario the retailer spends the total amount of the promotional cost and hence he/she is the primary decision maker [31, 34]. Goal of the retailer is to maximize the profit function TP_R , which is a function of t_r , t_s and ρ . So the problem of the retailer mathematically takes the following form:

$$\left. \begin{array}{l} \text{Determine } t_r, t_s \text{ and } \rho \\ \text{to maximize } TP_R(t_r, t_s, \rho) \\ \text{subject to } t_r, t_s \geq 0; \rho \geq 1 \end{array} \right\} \quad (34)$$

Depending upon the decision of the retailer, the wholesaler tries to improve his/her profit. So in this scenario the problem of the wholesaler mathematically takes the following form:

$$\left. \begin{array}{l} \text{Determine } k \\ \text{to maximize } TP_W(t_r, t_s, \rho, k) \\ \text{where, } t_r, t_s \text{ and } \rho \text{ are determined by the retailer and } k > 0 \text{ is an integer.} \end{array} \right\} \quad (35)$$

To solve these single objective optimisation problems, here a heuristic algorithm PSO is implemented and tested (cf. §3.9). Problems are solved using this PSO. To check the validity of the results obtained using PSO, the problems are also solved following generalised reduced gradient (GRG) approach using LINGO 14.0 software.

3.6 Coordination scenario (CS)

In this scenario, the wholesaler likes to take part in the joint marketing decision with the retailer to improve his/her profit [15, 34, 36]. For this purpose he/she agrees to pay a compromise part (F) of the promotional cost spend by the retailer. For this contribution of the promotional cost by the wholesaler, the contribution of the retailer towards the promotional cost reduces with the same amount. Then the retailer's profit becomes

$$TP_R^F = TS_R - TC_R - (1 - F)g(\rho - 1)^2 d^m \quad (36)$$

The wholesaler's profit reduces to

$$TP_W^F = TS_W - TC_W - Fg(\rho - 1)^2 d^m \quad (37)$$

As the retailer and the wholesaler both have the same power to take part in the joint marketing decision, here the joint profit of the retailer and the wholesaler (TP) is to be maximized and TP is a function of t_r , t_s , ρ , k and is given by

$$TP = TP_R^F + TP_W^F \quad (38)$$

The joint optimal decision will be acceptable to the retailer as well as the wholesaler if the decision improves their individual profits, i.e., the retailer's and the wholesaler's profits under the NCS are viewed as the lower bounds for the model under the coordination scenario. Let $TP_{R'}$ and $TP_{W'}$ be the profits of the retailer and the wholesaler respectively in NCS. So the profit of the retailer in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned} & (\text{Retailer's profit in CS}) - (\text{Retailer's profit in NCS}) \geq 0 \\ \text{i.e., if } & TP_R^F(t_r, t_s, \rho) - TP_{R'} \geq 0 \\ \text{i.e., if } & \{TS_R - TC_R - (1 - F)PrC\} - TP_{R'} \geq 0 \\ \text{i.e., if } & F.PrC \geq TP_{R'} - (TS_R - TC_R - PrC) \\ \text{i.e., if } & F \geq [TP_{R'} - (TS_R - TC_R - PrC)]/PrC = F_{min} \text{ (say)} \end{aligned}$$

Again, the profit of the wholesaler in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned} & (\text{Wholesaler's profit in CS}) - (\text{Wholesaler's profit in NCS}) \geq 0 \\ \text{i.e., if } & TP_W^F(t_r, t_s, \rho, k) - TP_{W'} \geq 0 \\ \text{i.e., if } & \{TS_W - TC_W - FPrC\} - TP_{W'} \geq 0 \\ \text{i.e., if } & F \leq [(TS_W - TC_W) - TP_{W'}]/PrC = F_{max} \text{ (say)} \end{aligned}$$

From the above discussion, the following proposition follows.

Proposition 3.1. *If $F_{min} < F < F_{max}$, then the profits for both the parties (the retailer and the wholesaler) increase in the CS than the NCS, where*

$$\begin{aligned} F_{min} &= [TP_{R'} - (TS_R - TC_R - PrC)]/PrC \\ \text{and } F_{max} &= [(TS_W - TC_W) - TP_{W'}]/PrC \end{aligned}$$

So for a compromise value of $F \in (F_{min}, F_{max})$ the problem in this scenario mathematically takes the following form:

$$\left. \begin{aligned} & \text{Determine } t_r, t_s, \rho \text{ and } k \\ & \text{to maximize } TP(t_r, t_s, \rho, k) \\ & \text{subject to } t_r, t_s \geq 0; \rho \geq 1; k \text{ is an integer.} \end{aligned} \right\} \quad (39)$$

3.7 Fuzzy model

It is discussed in the introduction section that in real life most of the inventory parameters are imprecise in nature and can be represented by fuzzy numbers. When some of the inventory parameters are fuzzy in nature, the model reduces to a fuzzy model [9, 11, 13, 17, 20, 23]. Normally set up cost, holding cost, etc., are imprecise in nature. In this model, the set up costs A_{R_2} , A_W , holding costs h_r , h_o , h_W and the constant g of the promotional cost are considered as fuzzy numbers \tilde{A}_R , \tilde{A}_W , \tilde{h}_r , \tilde{h}_o , \tilde{h}_W , \tilde{g} respectively, and hence the profits in both the scenarios become imprecise in nature and are presented below.

3.7.1 Fuzzy model in non-coordination scenario

According to the above assumptions in this case individual profits of the retailer, the wholesaler and their joint profit are transformed to the fuzzy numbers $\widetilde{TP}_R, \widetilde{TP}_W, \widetilde{TP}$ respectively and are represented by [31, 33, 32]

$$\begin{aligned}\widetilde{TP}_R &= TS_R - \widetilde{TC}_R - \widetilde{PrC} \\ \widetilde{TP}_W &= TS_W - \widetilde{TC}_W \\ \widetilde{TP} &= \widetilde{TP}_R + \widetilde{TP}_W\end{aligned}$$

$$\begin{aligned}\text{where, } \widetilde{TC}_R &= \frac{1}{T_R}(\widetilde{A}_R + PC_R + \widetilde{ICC}_R + DC_R + SC_R) \\ \widetilde{PrC} &= \tilde{g}(\rho - 1)^2 d^m \\ \widetilde{TC}_W &= \frac{1}{T_W}(\widetilde{A}_W + PC_W + \widetilde{ICC}_W + DC_W) \\ \text{where, } \widetilde{ICC}_R &= \frac{\tilde{h}_r}{\beta} D_{RW} + \frac{\tilde{h}_o}{\alpha} D_{OW} \\ \widetilde{ICC}_W &= \frac{\tilde{h}_W}{\gamma} D_W\end{aligned}$$

Considering the fuzzy numbers $\widetilde{A}_R, \widetilde{A}_W, \tilde{h}_r, \tilde{h}_o, \tilde{h}_W, \tilde{g}$ as triangular fuzzy numbers (TFNs) $(A_{R1}, A_{R2}, A_{R3}), (A_{W1}, A_{W2}, A_{W3}), (h_{r1}, h_{r2}, h_{r3}), (h_{o1}, h_{o2}, h_{o3}), (h_{W1}, h_{W2}, h_{W3}), (g_1, g_2, g_3)$ respectively, the fuzzy numbers $\widetilde{TP}_R, \widetilde{TP}_W, \widetilde{TP}$ becomes $(TP_{R1}, TP_{R2}, TP_{R3}), (TP_{W1}, TP_{W2}, TP_{W3}), (TP_1, TP_2, TP_3)$ respectively, where for $i = 1, 2, 3$

$$\begin{aligned}TP_{Ri} &= TS_R - TC_{R(4-i)} - PrC_{4-i} \\ TP_{Wi} &= TS_W - TC_{W(4-i)} \\ TP_i &= TP_{Ri} + TP_{Wi}\end{aligned}$$

$$\begin{aligned}\text{where, } TC_{Ri} &= \frac{1}{T_R}(A_{Ri} + PC_R + ICC_{Ri} + DC_R + SC_R) \\ PrC_i &= g_i(\rho - 1)^2 d^m \\ TC_{Wi} &= \frac{1}{T_W}(A_{Wi} + PC_W + ICC_{Wi} + DC_W) \\ \text{where, } ICC_{Ri} &= \frac{h_{ri}}{\beta} D_{RW} + \frac{h_{oi}}{\alpha} D_{OW} \\ ICC_{Wi} &= \frac{h_{Wi}}{\gamma} D_W\end{aligned}$$

In this scenario, the retailer spends the total amount of promotional cost and hence he/she is the primary decision maker. Goal of the retailer is to maximize the profit function \widetilde{TP}_R , which is a function of t_r, t_s and ρ . So the problem of the retailer mathematically takes the following form:

$$\left. \begin{aligned} &\text{Determine } t_r, t_s \text{ and } \rho \\ &\text{to maximize } \widetilde{TP}_R = (TP_{R1}, TP_{R2}, TP_{R3}) \\ &\text{subject to } t_r, t_s \geq 0; \rho \geq 1 \end{aligned} \right\} \quad (40)$$

Depending upon the decision of the retailer, the wholesaler likes to improve his/her profit. So in this scenario, the problem of the wholesaler mathematically takes the following form:

$$\left. \begin{aligned} &\text{Determine } k \\ &\text{to maximize } \widetilde{TP}_W = (TP_{W1}, TP_{W2}, TP_{W3}) \\ &\text{where, } t_r, t_s \text{ and } \rho \text{ are determined by the retailer and } k > 0 \text{ is an integer.} \end{aligned} \right\} \quad (41)$$

The problems are solved using proposed PSO (cf. §3.9) where comparisons of the objectives are made by the credibility measure approach of fuzzy events. Let $\widetilde{TP}_{Ra}, \widetilde{TP}_{Rb}$ be the two objectives corresponding to two solutions X_a, X_b respectively. Then the credibility measure approach of comparison of two solutions is given below:

- According to this approach X_a dominates X_b if the credibility measure (Cr) of the fuzzy event $(\widetilde{TP}_{Ra} > \widetilde{TP}_{Rb})$ is greater than 0.5, i.e., if $Cr(\widetilde{TP}_{Ra} > \widetilde{TP}_{Rb}) > 0.5$ [31]. In this approach, no crisp equivalent of the fuzzy numbers are used to find marketing decisions. This is a valid fuzzy comparison operation as $Cr(\widetilde{A} > \widetilde{B}) + Cr(\widetilde{A} \leq \widetilde{B}) = 1$ [22].

3.7.2 Fuzzy model in coordination scenario

For the coordination scenario, the individual profits and the total profit as fuzzy numbers are represented by [31, 33, 32]

$$\begin{aligned}\widetilde{TP}_R^F &= TS_R - \widetilde{TC}_R - (1 - F) \cdot \widetilde{PrC} \\ \widetilde{TP}_W^F &= TS_W - \widetilde{TC}_W - F \cdot \widetilde{PrC} \\ \widetilde{TP} &= \widetilde{TP}_R^F + \widetilde{TP}_W^F\end{aligned}$$

As discussed in the CS of crisp model, the wholesaler bears a compromise part (F) of the promotional cost spend by the retailer to take joint marketing decision with the retailer. The joint optimal decision will be acceptable to the retailer as well as the wholesaler if the decision improves their individual profits, i.e., the retailer's and the wholesaler's profits under the NCS are viewed as the lower bounds for the model under the coordination scenario. Let $\widetilde{TP}_{R'}$ and $\widetilde{TP}_{W'}$ be the profits of the retailer and the wholesaler respectively in NCS. If a proper value of F is chosen which improves the profits of both the parties, then for that chosen value of F , they will take joint marketing decision for the benefit of them. Now according to Liu [22] and Pakhira et al. [33], the profit of the retailer in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned}Cr(\text{Retailer's Profit in CS} \geq \text{Retailer's Profit in NCS}) &> 0.5 \\ \text{i.e., if } Cr(\widetilde{TP}_R^F(t_r, t_s, \rho) \geq \widetilde{TP}_{R'}) &> 0.5 \\ \text{where, } \widetilde{TP}_R^F &= (TP_{R1}^F, TP_{R2}^F, TP_{R3}^F) \text{ and } \widetilde{TP}_{R'} = (TP_{R'1}, TP_{R'2}, TP_{R'3}) \\ \text{i.e., if } TP_{R2}^F &\geq TP_{R'2} \text{ [cf. Lemma 6.3]} \\ \text{i.e., if } \{TS_R - TC_{R2} - (1 - F)PrC_2\} - TP_{R'2} &\geq 0 \\ \text{i.e., if } F &\geq [TP_{R'2} - \{TS_R - TC_{R2} - PrC_2\}] / PrC_2 = F_{min} \text{ (say)}\end{aligned}$$

Also, the profit of the wholesaler in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned}Cr(\text{Wholesaler's Profit in CS} \geq \text{Wholesaler's Profit in NCS}) &> 0.5 \\ \text{i.e., if } Cr(\widetilde{TP}_W^F(t_r, t_s, \rho, k) \geq \widetilde{TP}_{W'}) &> 0.5 \\ \text{where, } \widetilde{TP}_W^F &= (TP_{W1}^F, TP_{W2}^F, TP_{W3}^F) \text{ and } \widetilde{TP}_{W'} = (TP_{W'1}, TP_{W'2}, TP_{W'3}) \\ \text{i.e., if } TP_{W2}^F &\geq TP_{W'2} \text{ [cf. Lemma 6.3]} \\ \text{i.e., if } \{TS_W - TC_{W2} - FPrC_2\} - TP_{W'2} &\geq 0 \\ \text{i.e., if } F &\leq [\{TS_W - TC_{W2}\} - TP_{W'2}] / PrC_2 = F_{max} \text{ (say)}\end{aligned}$$

From the above discussion, the following proposition follows.

Proposition 3.2. *In fuzzy environment, if $F_{min} < F < F_{max}$, then the profits for both the parties (the retailer and the wholesaler) increase in the CS than the NCS, where*

$$\begin{aligned}F_{min} &= [TP_{R'2} - (TS_R - TC_{R2} - PrC_2)] / PrC_2 \\ \text{and } F_{max} &= [(TS_W - TC_{W2}) - TP_{W'2}] / PrC_2\end{aligned}$$

When the assumed fuzzy parameters reduces to the crisp parameters then $TP_{R1} = TP_{R2} = TP_{R3} = TP_R$, $TP_{W1} = TP_{W2} = TP_{W3} = TP_W$, $TP_{R'1} = TP_{R'2} = TP_{R'3} = TP_{R'}$, $TP_{W'1} = TP_{W'2} = TP_{W'3} = TP_{W'}$, and then clearly the Proposition 3.2 reduces to the Proposition 3.1, i.e., Proposition 3.1 is a special case of Proposition 3.2.

So for a compromise value of $F \in (F_{min}, F_{max})$, the problem in this scenario mathematically takes the following form:

$$\left. \begin{aligned} &\text{Determine } t_r, t_s, \rho \text{ and } k \\ &\text{to maximize } \widetilde{TP} = (TP_1, TP_2, TP_3) \\ &\text{subject to } t_r, t_s \geq 0; \rho \geq 1; k \text{ is an integer} \end{aligned} \right\} \quad (42)$$

The problem is solved using proposed PSO (cf. §3.9), where comparisons of the objectives are made using Credibility Measure approach, which is discussed earlier.

3.8 Rough model

It is discussed in the introduction section that another tool to represent the imprecise parameters is rough set [28, 32, 36]. In this model, set up costs A_R, A_W , holding costs h_r, h_o, h_W and the constant g of the promotional cost are considered as rough variables $\check{A}_R, \check{A}_W, \check{h}_r, \check{h}_o, \check{h}_W, \check{g}$ respectively. Then profits in both the scenario become rough in nature.

3.8.1 Rough model in non-coordination scenario

According to the above assumptions in this case, the individual profits and the total profit of the retailer and the wholesaler are reduces to rough variables and are represented by [32, 36]

$$\begin{aligned} \check{TP}_R &= TS_R - \check{TC}_R - Pr\check{C} \\ \check{TP}_W &= TS_W - \check{TC}_W \\ \check{TP} &= \check{TP}_R + \check{TP}_W \end{aligned}$$

$$\begin{aligned} \text{where, } \check{TC}_R &= \frac{1}{T_R}(\check{A}_R + PC_R + I\check{C}C_R + DC_R + SC_R) \\ Pr\check{C} &= \check{g}(\rho - 1)^2 d^m \\ \check{TC}_W &= \frac{1}{T_W}(\check{A}_W + PC_W + I\check{C}C_W + DC_W) \\ \text{where, } I\check{C}C_R &= \frac{\check{h}_r}{\beta} D_{RW} + \frac{\check{h}_o}{\alpha} D_{OW} \\ I\check{C}C_W &= \frac{\check{h}_W}{\gamma} D_W \end{aligned}$$

Considering the rough variables $\check{A}_R, \check{A}_W, \check{h}_r, \check{h}_o, \check{h}_W, \check{g}$ as $([A_{R1}, A_{R2}], [A_{R3}, A_{R4}])$, where $A_{R3} \leq A_{R1} \leq A_{R2} \leq A_{R4}$; $([A_{W1}, A_{W2}], [A_{W3}, A_{W4}])$, where $A_{W3} \leq A_{W1} \leq A_{W2} \leq A_{W4}$; $([h_{r1}, h_{r2}], [h_{r3}, h_{r4}])$, where $h_{r3} \leq h_{r1} \leq h_{r2} \leq h_{r4}$; $([h_{o1}, h_{o2}], [h_{o3}, h_{o4}])$, where $h_{o3} \leq h_{o1} \leq h_{o2} \leq h_{o4}$; $([h_{W1}, h_{W2}], [h_{W3}, h_{W4}])$, where $h_{W3} \leq h_{W1} \leq h_{W2} \leq h_{W4}$; $([g_1, g_2], [g_3, g_4])$, where $g_3 \leq g_1 \leq g_2 \leq g_4$ respectively, the rough variables $\check{TP}_R, \check{TP}_W, \check{TP}$ becomes $([TP_{R1}, TP_{R2}], [TP_{R3}, TP_{R4}])$, $([TP_{W1}, TP_{W2}], [TP_{W3}, TP_{W4}])$, $([TP_1, TP_2], [TP_3, TP_4])$ respectively, where

$$\begin{aligned} \text{For } i = 1, 2 \quad TP_{Ri} &= TS_R - TC_{R(3-i)} - PrC_{3-i} \\ TP_{Wi} &= TS_W - TC_{W(3-i)} \\ TP_i &= TP_{Ri} + TP_{Wi} \\ \text{For } i = 3, 4 \quad TP_{Ri} &= TS_R - TC_{R(7-i)} - PrC_{7-i} \\ TP_{Wi} &= TS_W - TC_{W(7-i)} \\ TP_i &= TP_{Ri} + TP_{Wi} \end{aligned}$$

and for $i = 1, 2, 3, 4$ the following relations hold

$$\begin{aligned} TC_{Ri} &= \frac{1}{T_R}(A_{Ri} + PC_R + ICC_{Ri} + DC_R + SC_R) \\ PrC_i &= g_i(\rho - 1)^2 d^m \\ TC_{Wi} &= \frac{1}{T_W}(A_{Wi} + PC_W + ICC_{Wi} + DC_W) \\ \text{where, } ICC_{Ri} &= \frac{h_{ri}}{\beta} D_{RW} + \frac{h_{oi}}{\alpha} D_{OW} \\ ICC_{Wi} &= \frac{h_{Wi}}{\gamma} D_W \end{aligned}$$

As the retailer is the leader decision maker and the supplier is the follower in NCS, so in this case the problem reduces to:

$$\left. \begin{aligned} &\text{Determine } t_r, t_s \text{ and } \rho \\ &\text{to maximize } \check{TP}_R = ([TP_{R1}, TP_{R2}], [TP_{R3}, TP_{R4}]) \\ &\text{subject to } t_r, t_s \geq 0; \rho \geq 1 \end{aligned} \right\} \quad (43)$$

Depending upon the decision of the retailer, the wholesaler likes to improve his/her profit. So in this scenario, the problem of the wholesaler mathematically takes the following form:

$$\left. \begin{array}{l} \text{Determine } k \\ \text{to maximize } \check{T}P_W = ([TP_{W1}, TP_{W2}], [TP_{W3}, TP_{W4}]) \\ \text{where, } t_r, t_s \text{ and } \rho \text{ are determined by the retailer and } k > 0 \text{ is an integer.} \end{array} \right\} \quad (44)$$

The problems are solved using proposed PSO (cf. §3.9), where comparisons of the objectives are made by the trust measure approach of rough events. Let $\check{T}P_{Ra}$, $\check{T}P_{Rb}$ be the two objectives corresponding to two solutions X_a , X_b respectively. Then the approach of comparison of two solutions using trust measure approach is given below:

- According to this approach X_a dominates X_b , if the trust measure (Tr) of the rough event ($\check{T}P_{Ra} > \check{T}P_{Rb}$) greater than 0.5, i.e., if $Tr(\check{T}P_{Ra} > \check{T}P_{Rb}) > 0.5$ [36, ?]. In this approach, no crisp equivalent of rough variables are used to find the marketing decisions. This is a valid rough comparison operation as $Tr(\check{A} > \check{B}) + Tr(\check{A} \leq \check{B}) = 1$ [22].

3.8.2 Rough model in coordination scenario

For the coordination scenario, the individual profits and the total profit as rough variables are represented by [32, 36]

$$\begin{aligned} \check{T}P_R^F &= TS_R - \check{T}C_R - (1 - F) \cdot PrC \\ \check{T}P_W^F &= TS_W - \check{T}C_W - F \cdot PrC \\ \check{T}P &= \check{T}P_R^F + \check{T}P_W^F \end{aligned}$$

In the crisp and fuzzy model, it is established that a compromise value of F can be chosen which improves the gain of both the parties in the CS, i.e., using joint decision. In this case also, the following proposition ensures the existence of a feasible region of F for which profit of both the parties improves in CS than NCS.

Let $\check{T}P_{R'}$ and $\check{T}P_{W'}$ be the profits of the retailer and the wholesaler respectively in NCS. Now according to Liu [22] and Pramanik et al. [36] the profit of the retailer in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned} &Tr(\text{Retailer's Profit in CS} \geq \text{Retailer's Profit in NCS}) > 0.5 \\ \text{i.e., if } &Tr(\check{T}P_R^F(t_r, t_s, \rho) \geq \check{T}P_{R'}) > 0.5 \\ &\text{where, } \check{T}P_R^F = ([TP_{R1}^F, TP_{R2}^F][TP_{R3}^F, TP_{R4}^F]) \text{ and } \check{T}P_{R'} = ([TP_{R'1}, TP_{R'2}][TP_{R'3}, TP_{R'4}]) \\ \text{i.e., if } &TP_{Ri}^F \geq TP_{R'i} \text{ for } i = 1, 2, 3, 4 \text{ [cf. Lemma 6.4]} \\ \text{i.e., if } &\{TS_R - TC_{R(m-i)} - (1 - F)PrC_{(m-i)}\} - TP_{R'i} \geq 0 \\ &\text{where, } m = 3 \text{ for } i = 1, 2 \text{ and } m = 7 \text{ for } i = 3, 4 \\ \text{i.e., if } &F \geq [TP_{R'i} - \{TS_R - TC_{R(m-i)} - PrC_{(m-i)}\}] / PrC_{(m-i)} = F_{Ri} \text{ for } i = 1, 2, 3, 4 \\ \text{i.e., if } &F \geq \text{Max}\{F_{R1}, F_{R2}, F_{R3}, F_{R4}\} = F_{min} \text{ (say)} \end{aligned}$$

Also, the profit of the wholesaler in this scenario, i.e., in CS, will be better than the NCS, if

$$\begin{aligned} &Tr(\text{Wholesaler's Profit in CS} \geq \text{Wholesaler's Profit in NCS}) > 0.5 \\ \text{i.e., if } &Tr(\check{T}P_W^F(t_r, t_s, \rho, k) \geq \check{T}P_{W'}) > 0.5 \\ &\text{where, } \check{T}P_W^F = ([TP_{W1}^F, TP_{W2}^F][TP_{W3}^F, TP_{W4}^F]) \text{ and } \check{T}P_{W'} = ([TP_{W'1}, TP_{W'2}][TP_{W'3}, TP_{W'4}]) \\ \text{i.e., if } &TP_{Wi}^F \geq TP_{W'i} \text{ for } i = 1, 2, 3, 4 \text{ [cf. Lemma 6.4]} \\ \text{i.e., if } &\{TS_W - TC_{W(m-i)} - FPrC_{(m-i)}\} - TP_{W'i} \geq 0 \\ &\text{where, } m = 3 \text{ for } i = 1, 2 \text{ and } m = 7 \text{ for } i = 3, 4 \\ \text{i.e., if } &F \leq [\{TS_W - TC_{W(m-i)}\} - TP_{W'i}] / PrC_{(m-i)} = F_{Wi} \text{ for } i = 1, 2, 3, 4 \\ \text{i.e., if } &F \leq \text{Min}\{F_{W1}, F_{W2}, F_{W3}, F_{W4}\} = F_{max} \text{ (say)} \end{aligned}$$

From the above discussion, the following proposition follows.

Proposition 3.3. *In rough environment, if $F_{min} < F < F_{max}$, then the profits for both the parties (the retailer and the wholesaler) increase in the CS than the NCS, where*

$$\begin{aligned} F_{min} &= \text{Max}\{F_{R1}, F_{R2}, F_{R3}, F_{R4}\} \\ \text{and } F_{max} &= \text{Min}\{F_{W1}, F_{W2}, F_{W3}, F_{W4}\} \\ \text{where, } F_{Ri} &= [TP_{R'i} - \{TS_R - TC_{R(m-i)} - PrC_{(m-i)}\}] / PrC_{(4-i)} \\ \text{and } F_{Wi} &= [\{TS_W - TC_{W(m-i)}\} - TP_{W'i}] / PrC_{(m-i)} \\ &\text{where, } m = 3 \text{ for } i = 1, 2 \text{ and } m = 7 \text{ for } i = 3, 4 \end{aligned}$$

When the assumed rough parameters reduces to the crisp parameters, then $TP_{R1} = TP_{R2} = TP_{R3} = TP_{R4} = TP_R$, $TP_{W1} = TP_{W2} = TP_{W3} = TP_{W4} = TP_W$, $TP_{R'1} = TP_{R'2} = TP_{R'3} = TP_{R'4} = TP_{R'}$, $TP_{W'1} = TP_{W'2} = TP_{W'3} = TP_{W'4} = TP_{W'}$, and then clearly the Proposition 3.3 reduces to the Proposition 3.1, i.e., Proposition 3.1 is a special case of Proposition 3.3.

So for a compromise value of $F \in (F_{min}, F_{max})$, the problem in this scenario mathematically takes the following form:

$$\left. \begin{aligned} &\text{Determine } t_r, t_s, \rho \text{ and } k \\ &\text{to maximize } TP = ([TP_1, TP_2], [TP_3, TP_4]) \\ &\text{subject to } t_r, t_s \geq 0; \rho \geq 1; k \text{ is an integer} \end{aligned} \right\} \quad (45)$$

The problem is solved using proposed PSO (cf. §3.9), where comparisons of the objectives are made using Trust Measure approach, which is discussed earlier.

3.9 Particle swarm optimization (PSO) technique

Particle Swarm Optimization (PSO) is a heuristic search algorithm was developed by Kennedy and Eberhart [18] in mid-nineties of the last century by mimicking the natural behaviour of a folk of birds searching for their food sources. A folk of birds normally search for their food sources depending upon their own experiences and the best experience among the birds. Same phenomenon is mimicked to create a PSO. A PSO normally starts with a set of solutions (called swarm) of the decision making problem under consideration. Individual solutions are called particles and food is analogous to the optimal solution. In simple terms, the particles are flown through a multi-dimensional search space, where the position of each particle is adjusted according to its own experience and that of its neighbors. The particle i has a position vector ($X_i(t)$), velocity vector ($V_i(t)$), the position at which the best fitness ($X_{pbesti}(t)$) encountered by the particle so far and the best position of all the particles ($X_{gbest}(t)$) in current generation t . In generation $(t + 1)$, the position and velocity of the particle are changed to $X_i(t + 1)$ and $V_i(t + 1)$ using following rules:

$$V_i(t + 1) = wV_i(t) + \mu_1 r_1 (X_{pbesti}(t) - X_i(t)) + \mu_2 r_2 (X_{gbest}(t) - X_i(t)) \quad (46)$$

$$X_i(t + 1) = X_i(t) + V_i(t + 1) \quad (47)$$

The parameters μ_1 and μ_2 are set to constant values, which are normally taken as 2, r_1 and r_2 are two random values uniformly distributed in $[0, 1]$, $w(0 < w < 1)$ is inertia weight which controls the influence of previous velocity on the new velocity. Here $X_{pbesti}(t)$ and $X_{gbest}(t)$ are normally determined by comparison of objectives due to different solutions. So for optimization problem involving crisp objective the algorithm works well. The comparison of fuzzy/rough objectives is made following the technique as stated at the end of the section §3.7.1/§3.8.1. If the objective value due to the solution X_i dominates the objective value due to the solution X_j , we say that X_i dominates X_j . Using this dominance property, PSO can be used to optimize crisp as well as fuzzy/rough optimization problem. In the algorithm V_{max} represent maximum velocity of a particle, $B_{il}(t)$ and $B_{iu}(t)$ represent lower boundary and upper boundary of i -th variable respectively. `check_constraint` ($X_i(t)$) function check whether solution $X_i(t)$ satisfies the constraints of the problem or not. It returns 1 if the solution $X_i(t)$ satisfies the constraints of the problem otherwise it returns 0.

Proposed PSO:

1. Initialize N , V_{max} , μ_1 , μ_2 , w , $Maxgen$.
2. Set iteration counter $t = 0$ and randomly generate initial swarm $P(t)$ of N particles (solutions).
3. Determine objective value of each solution $X_i(t)$ and find $X_{gbest}(t)$ using dominance property.

4. Set initial velocity $V_i(t)$, $\forall X_i(t) \in P(t)$ and set $X_{pbest_i}(t) = X_i(t)$, $\forall X_i(t) \in P(t)$.
5. While ($t < Maxgen$) do
6. For $i = 1 : N$ do
7. $V_i(t+1) = wV_i(t) + \mu_1 r_1 (X_{pbest_i}(t) - X_i(t)) + \mu_2 r_2 (X_{gbest}(t) - X_i(t))$.
8. If ($V_i(t+1) > V_{max}$) then set $V_i(t+1) = V_{max}$.
9. If ($V_i(t+1) < -V_{max}$) then set $V_i(t+1) = -V_{max}$.
10. $X_i(t+1) = X_i(t) + V_i(t+1)$.
11. If ($X_i(t+1) > B_{iu}(t)$) then set $X_i(t+1) = B_{iu}(t)$.
12. If ($X_i(t+1) < B_{il}(t)$) then set $X_i(t+1) = B_{il}(t)$.
13. If check_constraint ($X_i(t+1)$) = 0.
14. Set $X_i(t+1) = X_i(t)$, $V_i(t+1) = V_i(t)$.
15. Else
16. If $X_i(t+1)$ dominates $X_{pbest_i}(t)$ then set $X_{pbest_i}(t+1) = X_i(t+1)$.
17. If $X_i(t+1)$ dominates $X_{gbest}(t)$ then set $X_{gbest}(t+1) = X_i(t+1)$.
18. End If.
19. End For
20. Set $t = t + 1$.
21. End While.
22. Output: $X_{gbest}(t)$.
23. End Algorithm.

Parameter setting and implementation: With the above function and values, the algorithm is implemented using C-programming language. Different parametric values of the algorithm used to solve the model are set using Taguchi approach [42] (cf. § 4.2) and Engelbrecht [6] as follows: $N = 20$, $V_{max} = 0.15$, $\mu_1 = 1.49618$, $\mu_2 = 1.49618$, $w = 0.7298$, $Maxgen = 500$.

For these set of parametric values, the performance of the proposed PSO is tested against a benchmark test problems available in the literature and are listed in Appendix-B. The results of the test functions are presented in Table 2. From Table 2, it is clear that the implemented PSO is efficient enough to solve continuous optimisation problems.

Table 2: Results of test functions following PSO approach

TF	Results Obtained	% of Success for 50 Runs	Average Error
TF-1	$SH(x^*) = -186.7309$	96	0.022040
TF-2	$MZ(2.2029, 1.5708) = -1.8013$	68	0.256416
TF-3	$F2(2.048, -0.0002) = -205.8480$	100	0
TF-4	$DJ(0, 0, 0) = 0$	100	0
TF-5	$F(0.5, 0.25) = 0.25$	100	0

For TF-1, 18 global minima (obtained by PSO approach) are

$$x^* = [(-0.0835, 4.8581), (4.8581, 5.4828), (-7.7083, -0.8003), (5.4829, -7.7083), (-7.0835, -1.4251), (-1.4251, -7.0835), (-0.8004, -1.4251), (4.8580, -0.8003), (-0.8004, 4.8580), (5.4829, 4.8581), (5.4829, -1.4251), (-7.0835, -7.7083), (4.8581, -7.0835), (-1.4251, 5.4829), (-0.8003, -7.7084), (-1.4251, -0.8003), (-7.7083, -7.0835), (-7.7084, 5.4829)]$$

4 Numerical illustration and discussion

Real-life Problem-1: Nowadays, in the developing countries, like India, Bangladesh etc., number of middle income group is increasing very fast. Instead of traditional raw spices, they prefer ready-made crashed spices (in packets) for preparation of food as it saves labour and time for the family. In Kharagpur, West Bengal, India, there is a small wholesaler (may be considered as a retailer) who sales cookme crashed spices only in the market. The retailer purchases from big wholesaler in Kolkata, capital of West Bengal. In the market, there are a lot of competitors of cookme such as 'Sunrise', 'Data', 'Rupa', 'Patanjali', etc. Thus, to capture the demand of Kharagpur town, the retailer gives advertisement in electronic media, hoardings in important places etc. From his last few years' experiences, he observed that the demand increases with the promotional effort and amount of displayed units. The retailer fixes his/her marketing decision to increase the annual profit. The big wholesaler also desires to share a part of the promotional cost to make joint marketing decision with the retailer to increase his/her profit. Now, the retailer is in a dilemma whether to share the promotional cost with the wholesaler or not, though it is the fact that the profit increases with the promotional effort. On the part of the wholesaler also, whether it will be profitable to take part in the promotional effort from the business point of view. This model answers to these questions. The following example consists of the data set in appropriate units for one of the such spice and is used to illustrate the model.

Experiment-1: The data sets for the model in different environments are presented below:

Crisp model: $c = 0.2, d = 200, g = 1.1, m = 1.2, W = 200, \alpha = 0.05, \beta = 0.08, \gamma = 0.03, \delta = 0.85, A_R = 1500, A_W = 2500, p_W = 5, s_W = p_R = 8, s_R = 14, h_r = 1.2, h_o = 0.8, h_W = 0.3, d_R = 1, d_W = 1, c_{sf} = 15, c_{sv} = 0.8.$

Fuzzy model: $(A_{R1}, A_{R2}, A_{R3}) = (1400, 1500, 1600), (A_{W1}, A_{W2}, A_{W3}) = (2400, 2500, 2600), (h_{r1}, h_{r2}, h_{r3}) = (1.1, 1.2, 1.3), (h_{o1}, h_{o2}, h_{o3}) = (0.7, 0.8, 0.9), (h_{W1}, h_{W2}, h_{W3}) = (0.25, 0.3, 0.35), (g_1, g_2, g_3) = (1.0, 1.1, 1.2).$ All other parametric values are same as the crisp model.

Rough model: $([A_{R1}, A_{R2}], [A_{R3}, A_{R4}]) = ([1400, 1500], [1350, 1600]), ([A_{W1}, A_{W2}], [A_{W3}, A_{W4}]) = ([2500, 2600], [2400, 2650]), ([h_{r1}, h_{r2}], [h_{r3}, h_{r4}]) = ([1.1, 1.2], [1.05, 1.25]), ([h_{o1}, h_{o2}], [h_{o3}, h_{o4}]) = ([0.8, 0.9], [0.75, 0.95]), ([h_{W1}, h_{W2}], [h_{W3}, h_{W4}]) = ([0.25, 0.3], [0.2, 0.35]), ([g_1, g_2], [g_3, g_4]) = ([1.05, 1.15], [1.0, 1.2]).$ All other parametric values are same as the crisp model.

For the above set of parametric values, for crisp model, in NCS, initially TP_R is optimized to find optimum decision for the retailer and the optimum values of t_r, t_s, ρ are determined. For these values of $t_r, t_s, \rho; TP_W$ is optimized to find optimum k for the wholesaler. Again in CS, the optimum results are obtained by optimizing TP . The value of F is taken as $F = \frac{1}{2}(F_{max} + F_{min})$. Results are obtained using both LINGO 14.0 software and PSO algorithm developed for this purpose and almost same the results are found as in Table 3. In PSO, the parametric study is made on k to optimize TP_W for NCS and to optimize TP for CS and these results are presented in Table 4. It is observed from Table 3 that the results of PSO are at least as good as the results of LINGO software. But LINGO software is not capable of solving fuzzy/rough model. Moreover, efficiency of the implemented proposed PSO in solving continuous optimisation problems is rigorously tested (cf. §3.9). Due to this reason, in further study, PSO is only used to find results in different cases.

Table 3: Optimum results for experiment-1

Technique	Scenario	TP_R	TP_W	TP	t_r	t_s	ρ	k	T_R	T_W	F_{min}	F_{max}	F
GRG (LINGO)	NCS	568.8537	348.2102	917.0639	1.49	0.54	1.59	2	2.58	5.15	-	-	-
	CS	628.0926	407.4526	1035.5452	2.53	1.94	1.90	1	4.91	4.91	0.475	0.705	0.590
PSO	NCS	568.8541	348.1272	916.9813	1.49	0.54	1.59	2	2.58	5.15	-	-	-
	CS	628.0778	407.4680	1035.5459	2.53	1.95	1.90	1	4.91	4.91	0.475	0.705	0.590
ABC	NCS	568.8541	348.1582	917.0123	1.49	0.54	1.59	2	2.57	5.15	-	-	-
	CS	628.1179	407.4279	1035.5459	2.52	1.94	1.90	1	4.91	4.91	0.475	0.705	0.590

For Experiment-1, in crisp model, the total profit is optimized in CS due to sharing of different portion (F) of promotional cost between F_{min} and F_{max} by the wholesaler and the profits of both the parties are tabulated in Table 5. For NCS, $TP_R = 568.85$ and $TP_W = 348.13$. Here it is observed that if $F = 0.47 < F_{min}$, where $F_{min} = 0.475$ then the retailer's profit in the CS (566.30) is less than that in the NCS (568.85). Also if $F = 0.71 > F_{max}$, where $F_{max} = 0.705$ then the wholesaler's profit in the CS (345.81) is less than that in the NCS (348.13). So it is found that for $F_{min} < F < F_{max}$ profit of both the parties increase to some extent. All these calculations are done using PSO technique and the value of k is determined by parametric study on k .

For Experiment-1, in crisp model, a sensitivity analysis of c and d are presented in Table 6 following PSO technique.

Table 4: Parametric study of k for Experiment-1 using PSO technique

Scenario	k	TP_R	TP_W	TP	t_r	t_s	ρ	T_R	T_W
NCS	1	568.85	89.70	658.55	1.49	0.54	1.59	2.57	2.57
	2	568.85	348.13	916.98	1.49	0.54	1.59	2.57	5.15
	3	568.85	270.69	839.55	1.49	0.54	1.59	2.57	7.72
	4	568.85	99.44	668.30	1.49	0.54	1.59	2.57	10.30
	5	568.85	-117.85	451.00	1.49	0.54	1.59	2.57	12.87
CS	1	-	-	1035.55	2.52	1.94	1.90	4.91	4.91
	2	-	-	1034.21	1.84	0.05	2.00	2.32	4.64
	3	-	-	983.30	1.46	0	2.00	1.90	5.71
	4	-	-	883.15	1.23	0	1.99	1.67	6.69
	5	-	-	767.30	1.06	0	1.97	1.51	7.57

Table 5: Values of TP_R, TP_W due to different F in CS for Experiment-1 using PSO technique

F	TP_R	TP_W	TP
0.47	566.30	469.25	1035.55
0.48	571.51	464.03	1035.55
0.54	602.42	433.12	1035.55
0.59	628.08	407.47	1035.55
0.64	653.91	381.63	1035.55
0.70	684.70	350.85	1035.55
0.71	689.73	345.81	1035.55

Here it is noticed that if c or d increases then in both scenarios (NCS and CS) the individual profits as well as the total profit of the retailer and the wholesaler increase. All these observations agrees with reality. A graphical representation of the results obtained in these parametric studies are also presented in Figure 3.

Table 6: Sensitivity analysis of c and d for experiment-1 using PSO technique

	Sensitivity Analysis of c										Sensitivity Analysis of d									
	c	TP_R	TP_W	TP	t_r	t_s	ρ	k	F^*	d	TP_R	TP_W	TP	t_r	t_s	ρ	k	F^*		
NCS	0.16	546.99	333.95	880.94	1.49	0.65	1.58	2	-	190	523.05	314.81	837.86	1.51	0.64	1.58	2	-		
CS		607.74	400.61	1008.35	2.52	2.06	1.89	1	0.59		575.64	382.64	958.28	2.56	2.07	1.90	1	0.59		
NCS	0.17	552.36	337.31	889.67	1.49	0.62	1.58	2	-	195	545.83	331.45	877.27	1.50	0.59	1.59	2	-		
CS		612.83	402.26	1015.09	2.52	2.03	1.90	1	0.59		601.76	395.04	996.80	2.54	2.00	1.90	1	0.59		
NCS	0.18	557.80	341.01	898.80	1.49	0.60	1.58	2	-	200	568.85	348.13	916.98	1.49	0.54	1.59	2	-		
CS		617.99	403.88	1021.87	2.52	2.00	1.90	1	0.59		628.08	407.47	1035.55	2.53	1.94	1.90	1	0.59		
NCS	0.19	563.29	344.53	907.82	1.49	0.57	1.59	2	-	205	592.14	365.08	957.21	1.48	0.49	1.59	2	-		
CS		622.98	405.70	1028.69	2.52	1.97	1.90	1	0.59		655.69	423.48	1079.17	1.83	0.01	2.01	2	0.30		
NCS	0.20	568.85	348.13	916.98	1.49	0.54	1.59	2	-	210	615.69	382.04	997.72	1.47	0.44	1.60	2	-		
CS		628.08	407.47	1035.55	2.53	1.94	1.90	1	0.59		683.42	441.11	1124.53	1.82	0	2.01	2	0.30		
NCS	0.21	574.48	351.86	926.34	1.49	0.51	1.59	2	-	215	639.50	398.89	1038.39	1.47	0.39	1.60	2	-		
CS		635.41	409.98	1045.39	1.84	0.02	2.01	2	0.30		712.05	458.02	1170.08	1.80	0	2.01	2	0.30		
NCS	0.22	580.18	355.54	935.72	1.49	0.48	1.60	2	-	220	663.59	416.16	1079.74	1.46	0.34	1.60	2	-		
CS		641.56	415.17	1056.73	1.84	0	2.01	2	0.30		740.66	475.14	1215.80	1.79	0	2.01	2	0.30		

* Values of F are supplied

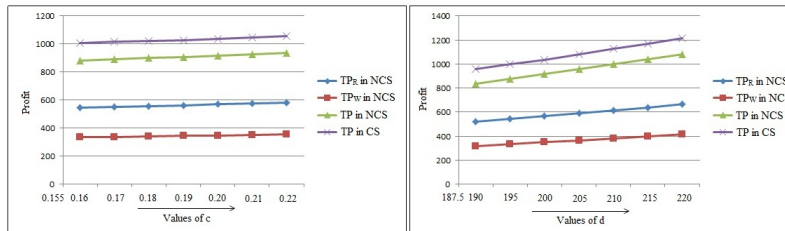


Figure 3: Graphical representation of the results of Experiment-1 of the parametric studies on c and d

For fuzzy and rough models, the Experiment-1 is made using PSO technique following credibility measure and trust measure approach respectively and the results are presented in Table 7. In coordination scenario, the value of F is taken as 0.59 and in both the scenarios optimum k are obtained by parametric studies on k .

Real-life Problem-2: In Haldia, West Bengal, India, there is a retailer (Sampa Fish Center) who sells fresh fishes by collecting fish from a supplier (Digha Fish Supplier), who supplies fish from Shankarpur fishing harbour, Digha, West Bengal, India. As availabilities of different types of fishes vary in different seasons, to keep the demand of his fish high, the retailer gives advertisements in different ways in a regular interval in the local area. The following example consists of the data set in appropriate units for one type of such fish and is used to illustrate the model.

Experiment-2: The data sets for the model in different environments are presented below:

Crisp model: $c = 0.18, d = 200, g = 1.27, m = 1.2, W = 180, \alpha = 0.05, \beta = 0.07, \gamma = 0.02, \delta = 0.83, A_R = 1500, A_W = 4000, p_W = 6, s_W = p_R = 10, s_R = 18, h_r = 1.2, h_o = 1.0, h_W = 0.3, d_R = 0.9, d_W = 0.9, c_{sf} = 6, c_{sv} = 1.4.$

Table 7: Results of fuzzy and rough model following PSO for experiment-1

Fuzzy			Rough		
	NCS	CS		NCS	CS
\widetilde{TP}_R	(480.80, 568.85, 656.91)	(549.10, 628.08, 707.06)	\widetilde{TP}_R	([544.25, 636.32], [477.96, 682.35])	([602.66, 683.31], [552.15, 723.63])
\widetilde{TP}_W	(305.05, 348.13, 391.21)	(359.52, 407.47, 455.41)	\widetilde{TP}_W	([331.87, 375.19], [298.69, 418.50])	([377.92, 426.28], [353.74, 460.65])
\widetilde{TP}	(785.85, 916.98, 1048.12)	(908.63, 1035.55, 1162.46)	\widetilde{TP}	([876.12, 1011.50], [776.65, 1100.85])	([980.58, 1109.59], [905.88, 1184.28])
t_r	1.49	2.53	t_r	1.49	2.57
t_s	0.54	1.94	t_s	0.44	1.90
ρ	1.59	1.90	ρ	1.60	1.91
k	2	1	k	2	1
F	-	0.59	F	-	0.59

Fuzzy model: $(A_{R1}, A_{R2}, A_{R3}) = (1400, 1500, 1600)$, $(A_{W1}, A_{W2}, A_{W3}) = (3900, 4000, 4100)$, $(h_{r1}, h_{r2}, h_{r3}) = (1.1, 1.2, 1.3)$, $(h_{o1}, h_{o2}, h_{o3}) = (0.9, 1.0, 1.1)$, $(h_{W1}, h_{W2}, h_{W3}) = (0.25, 0.3, 0.35)$, $(g_1, g_2, g_3) = (1.25, 1.27, 1.3)$. All other parametric values are same as crisp model.

Rough model: $([A_{R1}, A_{R2}], [A_{R3}, A_{R4}]) = ([1400, 1500], [1350, 1600])$, $([A_{W1}, A_{W2}], [A_{W3}, A_{W4}]) = ([4000, 4100], [3900, 4150])$, $([h_{r1}, h_{r2}], [h_{r3}, h_{r4}]) = ([1.1, 1.2], [1.05, 1.25])$, $([h_{o1}, h_{o2}], [h_{o3}, h_{o4}]) = ([1.0, 1.1], [0.95, 1.15])$, $([h_{W1}, h_{W2}], [h_{W3}, h_{W4}]) = ([0.25, 0.3], [0.2, 0.35])$, $([g_1, g_2], [g_3, g_4]) = ([1.27, 1.3], [1.25, 1.32])$. All other parametric values are same as crisp model.

For the above set of assumed parametric values, TP_R and TP are optimized for NCS and CS respectively and the optimum results obtained using LINGO 14.0 software and PSO technique are presented in Table 8. It is found that the results obtained following both the techniques are almost same. All other results of Experiment-2 are almost the same as Experiment-1. Results of fuzzy and rough model are computed following PSO technique for Experiment-2 and are presented in Table 9.

Table 8: Optimum results for experiment-2

Technique	Scenario	TP_R	TP_W	TP	t_r	t_s	ρ	k	T_R	T_W	F_{min}	F_{max}	F
GRG (LINGO)	NCS	1228.2540	548.7954	1777.0494	1.42	0.78	1.80	3	2.64	7.91	-	-	-
	CS	1319.4982	640.0478	1959.5460	1.96	0.66	2.23	2	2.97	5.95	0.174	0.340	0.257
PSO	NCS	1228.2546	548.6689	1776.9236	1.42	0.78	1.80	3	2.64	7.91	-	-	-
	CS	1319.5983	639.9483	1959.5466	1.96	0.66	2.23	2	2.97	5.94	0.174	0.340	0.257
ABC	NCS	1228.2546	548.7019	1776.9565	1.42	0.78	1.80	3	2.64	7.91	-	-	-
	CS	1319.5483	639.9984	1959.5468	1.97	0.66	2.23	2	2.98	5.95	0.174	0.340	0.257

Table 9: Results of fuzzy and rough model following PSO for experiment-2

Fuzzy			Rough		
	NCS	CS		NCS	CS
\widetilde{TP}_R	(1153.13, 1228.25, 1299.68)	(1225.31, 1322.81, 1413.89)	\widetilde{TP}_R	([1206.16, 1282.24], [1147.04, 1322.12])	([1283.74, 1381.82], [1214.89, 1434.02])
\widetilde{TP}_W	(484.00, 548.67, 613.34)	(576.45, 636.74, 694.77)	\widetilde{TP}_W	([537.57, 601.75], [479.81, 665.93])	([620.69, 681.14], [570.80, 739.35])
\widetilde{TP}	(1637.13, 1776.92, 1913.01)	(1801.75, 1959.55, 2108.67)	\widetilde{TP}	([1743.73, 1883.99], [1626.85, 1988.04])	([1904.43, 2062.95], [1785.69, 2173.38])
t_r	1.42	1.96	t_r	1.42	2.01
t_s	0.78	0.66	t_s	0.74	0.64
ρ	1.80	2.23	ρ	1.80	2.22
k	3	2	k	3	2
F	-	0.26	F	-	0.26

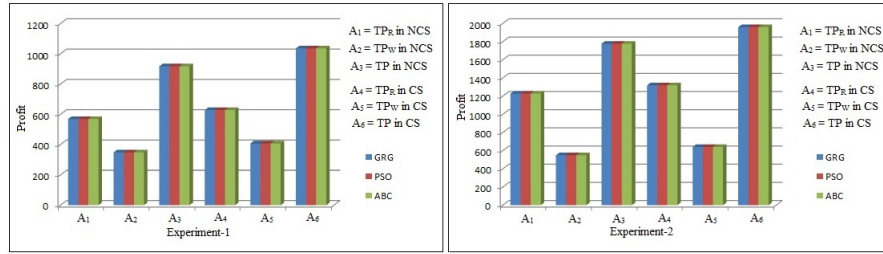


Figure 4: Comparison of the results obtained following different approaches

4.1 ANOVA test [16]

To check the efficiency of the PSO algorithm for solving the model, here along with LINGO software, another heuristic algorithm, ABC is also used to solve the crisp models for both the examples. Results of different models using ABC are presented in Table 3 and Table 8 for Experiment-1 and Experiment-2 respectively. A pictorial representation of the results of both the examples following three different approaches (GRG, PSO, ABC) are presented in Figure 4. From Figure 4 it is clear that the performances of three approaches are almost the same for solving the model. A statistical test ANOVA is also performed on the obtained results following the three approaches. To perform this test, here, the results obtained following these three approaches for six models $M1, M2, M3, M4, M5, M6$ are considered, where $M1 =$ Values of TP_R in NCS for Experiment-1, $M2 =$ Values of TP_W in NCS for Experiment-1, $M3 =$ Values of TP in CS for Experiment-1, $M4 =$ Values of TP_R in NCS for Experiment-2, $M5 =$ Values of TP_W in NCS for Experiment-2, $M6 =$ Values of TP in CS for Experiment-2 respectively, which are presented in Table 10. These three sets of results are considered as three samples ($J = 3$). Clearly size of each sample is $I = 6$. Critical value of the F -ratio is $F(J - 1, J(I - 1)) = F(2, 15) = 3.68$, for significance level 0.05. As three samples are almost same, calculated value of the F -ratio is 0 which is less than the critical value $F(2, 15) = 3.68$. So there is no significant difference between these samples, i.e., performances of these approaches are almost same for solving the proposed model. But here PSO is used to solve the models as it takes less computational time as well as less function evaluations to find the marketing decision (cf. Table 11) relative to ABC.

Table 10: Values for ANOVA test

Approach	Obtained optimum value					
	$M1$	$M2$	$M3$	$M4$	$M5$	$M6$
PSO	568.8541	348.1272	1035.5459	1228.2546	548.6689	1959.5466
Lingo	568.8537	348.2102	1035.5452	1228.2540	548.7954	1959.5460
ABC	568.8541	348.1582	1035.5459	1228.2546	548.7019	1959.5468

Table 11: Computational time and number of function evaluation in different approaches

Approach	[Computational Time (in second), No of function evolution]			
	To find $M1, M2$	To find $M3$	To find $M4, M5$	To find $M6$
PSO	[3.261, 1440]	[3.314, 1290]	[3.494, 1140]	[3.352, 1530]
Lingo	[< 1, -]	[< 1, -]	[< 1, -]	[< 1, -]
ABC	[3.709, 1660]	[3.626, 1340]	[4.013, 2140]	[3.702, 1940]

4.2 Tuning of PSO parameter using Taguchi method

To solve the problems of the proposed model, PSO technique is used. In PSO, some parameters are used which are swarm size (N), velocity (V_{max}) for the movement of the solution, some constants (μ_1, μ_2, w). If the values of these parameters are changed, then the number of function evaluations vary. So, there is need to set (tuning) the parameters so that the number of function evaluations is minimized. To tuning these parameters, Taguchi method [42] is used.

In Taguchi method, there is need to set the levels of the parameters of PSO at first, which are shown in Table 12. A three-level PSO parameter counts for two degrees of freedom. Therefore, there are total eight degrees of freedom for the four PSO parameters. The degrees of freedom for the orthogonal array should be greater than or at least equal to those for the design parameters. So, in the present study, an L_9 orthogonal array (cf. Table 13) is selected. This array has four columns and nine

rows. The degrees of freedom of this array is eight. Each PSO parameter is assigned to a column and nine combinations of the parameters are required for L_9 orthogonal array.

Table 12: PSO parameters and their levels

Symbol	PSO parameter	Level-1	Level-2	Level-3
A	swarm size (N)	20	30	40
B	velocity (V_{max})	0.05	0.10	0.15
C	constant ($\mu_1=\mu_2$)	1.45	1.49618	1.55
D	constant (w)	0.71	0.7298	0.75

Table 13: L_9 orthogonal array

Run	PSO parameter			
	A	B	C	D
	swarm size (N)	velocity (V_{max})	constant ($\mu_1=\mu_2$)	constant (w)
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	3	2
5	2	2	1	3
6	2	3	2	1
7	3	1	2	3
8	3	2	3	1
9	3	3	1	2

For the above combinations of the parameters, the number of function evaluations to find optimal decision in different seeds of the random number generator are tabulated in Table 14 and also these numbers are transformed into a signal-to-noise (S/N) ratio by the following formula.

$$\eta = -10\log(M.S.D.) \quad (48)$$

where, M.S.D. is the mean square deviation for the output characteristic.

Table 14: Required number of function evaluation in different seeds and S/N ratio

Run	Symbol				Function Evaluation			S/N ratio
	A	B	C	D	Seed-1	Seed-2	Seed-3	
1	20	0.05	1.45	0.71	1260	1320	1700	-63.17
2	20	0.10	1.49618	0.7298	640	900	1080	-59.01
3	20	0.15	1.55	0.75	940	1000	920	-59.59
4	30	0.05	1.55	0.7298	1380	1200	2310	-64.61
5	30	0.10	1.45	0.75	1080	1200	1290	-61.53
6	30	0.15	1.49618	0.71	1230	960	1110	-60.87
7	40	0.05	1.49618	0.75	1800	2040	2280	-66.23
8	40	0.10	1.55	0.71	2080	1520	1840	-65.24
9	40	0.15	1.45	0.7298	880	1000	1600	-61.60

There are three types of quality characteristic: the-lower-the-better, the-higher-the-better and the-nominal-the-better [42]. Here, the number of function evaluation is the-lower-the-better quality characteristic. The M.S.D. for this characteristic is calculated by the following formula.

$$M.S.D. = \frac{1}{m} \sum_{i=1}^m S_i^2 \quad (49)$$

where, S_i is the value of the number of function evaluation for the i -th test.

The mean S/N ratio for each level of each parameter is summarized and called the S/N response table for the number of function evaluation (cf. Table 15). For example, the mean S/N ratio for the parameter swarm size (N) at levels 1, 2 and 3 can be calculated by averaging the S/N ratios for the run 1-3, 4-6 and 7-9 respectively and so on for the other parameters.

Finally, the ANOVA test is done to investigate which design parameters significantly affect the quality characteristic. From Table 16, it is observed that the values of F-ratio of each parameter is less than the value of $F(2, 6) = 5.14$. So, the change of the parameters are insignificant on the quality characteristic. Based on the S/N and ANOVA analyses, the optimal PSO parameters for the number of function evaluation are the swarm size (N) at level-1, the velocity (V_{max}) at level-3, the constants ($\mu_1 = \mu_2$) at level-2 and the constant (w) at level-2. Due to this reason, this parametric set is used to find marketing decision of the models.

Table 15: S/N response table for function evaluation

Symbol	PSO parameter	Mean S/N ratio			
		Level-1	Level-2	Level-3	(Max - Min)
A	swarm size (N)	-60.59	-62.34	-64.36	3.77
B	velocity (V_{max})	-64.67	-61.93	-60.69	3.98
C	constant ($\mu_1=\mu_2$)	-63.09	-61.74	-62.45	1.35
D	constant (w)	-62.10	-62.04	-63.15	1.11

Table 16: Results of the ANOVA for function evaluation

Symbol	PSO parameter	df	SS_B	MS_B	SS_W	MS_W	F -ratio
A	swarm size (N)	2	21.36	10.68	30.01	5.00	2.13
B	velocity (V_{max})	2	24.88	12.44	26.40	4.40	2.83
C	constant ($\mu_1=\mu_2$)	2	2.74	1.37	48.58	8.10	0.17
D	constant (w)	2	2.34	1.17	49.00	8.17	0.14

5 Conclusions

A coordinated SC sharing of the promotional cost among the wholesaler and the retailer with stock and promotional effort influenced demand is formulated and solved. For the first time, with the above mentioned demand of a deteriorating item, sharing of promotional cost between SC partners is determined for the benefit of the individual profits as well as channel profit. Here, a conventional PSO algorithm is implemented, tested and tacitly used to solve the above problem in crisp and imprecise environments. Its performance is compared with the LINGO 14.0 software and ABC algorithm. From the different tables of the obtained results of the models in different scenarios following marketing decisions can be outlined:

- It is found in all experiments that promotional effort (ρ) of the item is grater than 1. So promotional effort has a positive effect in a SC of deteriorating item.
- It is also found that profits for both the parties (i.e., the wholesaler and the retailer) increase in the coordination scenario than the non-coordination scenario for a compromise value of $F \in (F_{min}, F_{max})$, i.e., if the wholesaler bears a compromise portion of promotional cost, then it is beneficial for both the parties. So theoretical expected result agrees with the numerical findings.
- In all the studies it is observed that $t_r > 0$, i.e., two warehouse strategy is beneficial for the retailer with limited outlet capacity.
- In all the studies it is observed that $t_s > 0$, i.e., backlogging is beneficial for the proposed SC model.
- Efficiency of the PSO of solving such real life complex decision making problem with respect to accuracy and computational time is well established by this study.
- Moreover, for the first time, Taguchi method is used for the parameter setting of a heuristic algorithm to solve any SC/inventory model.

The present model can be extended to three or multi level SC, including trade credit policy (one or more levels), price discount policy, variable deterioration, etc. Here, instead of continuous release pattern, bulk release system also can be used between OW and RW.

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Appendix-A

Lemma 6.1. If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be TFNs then according to Liu [22], Pakhira et al. [31]

$$Cr(\tilde{A} \geq \tilde{B}) = \begin{cases} 1 & \text{if } a_1 \geq b_3 \\ \frac{b_3 + 2(a_2 - b_2) - a_1}{2(b_3 - b_2 + a_2 - a_1)} & \text{if } b_2 \leq a_2, a_1 \leq b_3 \\ \frac{(a_3 - b_1)}{2(a_3 - a_2 + b_2 - b_1)} & \text{if } a_2 \leq b_2, a_3 \geq b_1 \\ 0 & \text{otherwise} \end{cases} \quad (50)$$

Lemma 6.2. If $\tilde{A} = ([a_1, a_2][a_3, a_4])$ and $\tilde{B} = ([b_1, b_2][b_3, b_4])$ be rough variables, then according to Liu [22], Pramanik et al. [36]

$$Tr\{\tilde{A} \geq \tilde{B}\} = \begin{cases} 0 & \text{if } a_4 \leq b_3 \\ \frac{a_4 - b_3}{2(a_4 - a_3 + b_4 - b_3)} & \text{if } a_2 \leq b_1, b_3 \leq a_4 \\ \frac{1}{2} \left[\frac{a_4 - b_3}{a_4 - a_3 + b_4 - b_3} + \frac{a_2 - b_1}{a_2 - a_1 + b_2 - b_1} \right] & \text{if } a_1 \leq b_2, b_1 \leq a_2 \\ \frac{1}{2} \left[\frac{a_4 - b_3}{a_4 - a_3 + b_4 - b_3} + 1 \right] & \text{if } a_3 \leq b_4, b_2 \leq a_1 \\ 1 & \text{if } b_4 \leq a_3 \end{cases} \quad (51)$$

Lemma 6.3. If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are TFNs, then $Cr(\tilde{A} > \tilde{B}) > 0.5$, iff $a_2 > b_2$.

Proof. It follows from Lemma 6.1. □

Lemma 6.4. For any two rough variables $\tilde{A} = ([a_1, a_2][a_3, a_4])$ and $\tilde{B} = ([b_1, b_2][b_3, b_4])$, if $a_1 > b_1, a_2 > b_2, a_3 > b_3, a_4 > b_4$ holds, then $Tr(\tilde{A} > \tilde{B}) > 0.5$.

Proof. It follows from Lemma 6.2. □

Appendix-B List of Test Functions (TF)

TF-1: (Taken from [1]):

$$SH(x_1, x_2) = \sum_{j=1}^5 j \times \cos[(j+1) \times x_1 + j] \times \sum_{j=1}^5 j \times \cos[(j+1) \times x_2 + j],$$

$-10 \leq x_1, x_2 \leq 10$. This problem has 760 local minima and 18 global minima. At global minima (x_1, x_2) , $SH(x_1, x_2) = -186.7309$.

TF-2: (Taken from [1]):

$$MZ(x_1, x_2, \dots, x_n) = - \sum_{i=1}^n \sin(x_i) \cdot [\sin(i \cdot (x_i)^2 / \pi)]^{2m}, \quad -\pi \leq x_1, x_2, \dots, x_n \leq \pi, \text{ where } m = 10. \text{ For } n = 2, \text{ it has one global minima at } (x_1, x_2) = (2.25, 1.57) \text{ and } MZ(2.25, 1.57) = -1.80.$$

TF-3: (Taken from [1]):

$$F2(x_1, x_2) = 100 \times (x_2^2 - x_1) + (1 - x_1), \quad -2.048 \leq x_1, x_2 \leq 2.048. \text{ It has one minima at } (x_1, x_2) = (2.048, 0) \text{ and } F2(2.048, 0) = -205.8480.$$

TF-4: (Taken from [1]):

$$DJ(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2, \quad -5.12 \leq x_1, x_2, x_3 \leq 5.12.$$

It has one global minima at $(x_1, x_2, x_3) = (0, 0, 0)$ and $DJ(0, 0, 0) = 0$.

TF-5: (Taken from [25]):

$$F(x_1, x_2) = 100(x_2 - x_1^2)^2 + (x_1 - 1)^2, \text{ such that } x_1 + x_2^2 \geq 0, x_1^2 + x_2 \geq 0, -0.5 \leq x_1 \leq 0.5, -1.0 \leq x_2 \leq 1.0.$$

It has one global minima at $(x_1, x_2) = (0.5, 0.25)$ and $F(0.5, 0.25) = 0.25$.

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