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A novel ranking method for intuitionistic fuzzy set based on information fusion and application to threat assessment

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Abstract

A novel ranking method based on multi-time information fusion is proposed for intuitionistic fuzzy sets (IFSs) and applied to the threat assessment problem, a multi-attribute decision making (MADM) one. This method integrates a designed intuitionistic fuzzy entropy (IFE), the closeness degree of technique for order preference by similarity to ideal solution (TOPSIS), the decision maker¡¯s (DM¡¯s) risk attitude and decision information from DMs in multiple times. Firstly, a novel IFE is designed in order to sufficiently express the fuzziness and the unknown of uncertainty conveyed by IFSs. Secondly, considering the distance from the positive ideal solution and the negative ideal solution simultaneously, the closeness degree of TOPSIS is utilized to characterize the amount of information conveyed by the IFS. The amount of information and the uncertainty of information are combined by the introduction of DMs¡¯ risk attitudes and then; a novel single time ranking method for IFSs is structured. Next, to reflect the subjective ranking intention on alternatives, the comprehensive preference model of DMs is established. The optimal attribute weights are derived from the linear programming model which incorporates the subjective ranking intension and the objective ranking result. Finally, the decision information from DMs in multiple times is aggregated to give the final ranking results. A threat assessment example and comparison analysis demonstrates the flexibility and effectiveness of the proposed method.

Keywords: Decision making, threat assessment, intuitionistic fuzzy set, intuitionistic fuzzy entropy, comprehensive preferences.

1 Introduction

The threat assessment plays an important role in the fire allocation and assistant decision making in military operations [13, 14, 16], and thus; is involved in the research in various contexts. The threat assessment problem can be regard as a multi-attribute decision making (MADM) one, where each target carries with multiple attributes and the aim is to give the ranking orders of all the targets. Due to the complexity of battlefield and the restriction of sensors [15, 17, 18] , the obtained target data usually consist of uncertainty [8, 9]. In addition, the decision maker (DM) often shows hesitation during making decisions because of lack of professional knowledge. The intuitionistic fuzzy set (IFS) [28] extends Zadeh¡¯s fuzzy sets [29] by introducing the hesitation degree on the basis of membership degree and non-membership degree, and thus describes the internal fuzzy essence of threat assessment more precisely. Additionally, the vague set [37] has been proved to be the IFS [11], and the IFS has been applied to various problems MADM problems [12, 19].

However, such an order relationship problem is still a critical issue in the MADM under intuitionistic fuzzy environment. The existing achievements on this problem can be roughly divided into two parts. The first part [22, 41] uses the combinatorial calculations on the two or three functions (membership function, non-membership function, and hesitation margin). Such methods utilize traditional tools and ranking results are sometimes counterintuitive or unobtainable. The second one employs the geometric representation to rank IFSs, which is more effective in solving MADM problems, such as entropy-based measures [2, 5, 21, 23, 25, 31, 32, 33, 34, 38] and distance-based measures [3,

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4, 6, 7, 10, 20, 24, 26, 27, 30, 40]. References [2, 5, 21, 23, 31, 33, 34, 38] construct various forms of intuitionistic fuzzy entropies (IFEs) and apply them to the MADM problem. References [25, 32] point out and justify that the uncertainty of an IFS contains the fuzziness and the unknown (lack of knowledge), and then propose new definitions of IFE for each of the two points. As to the distance-based measures, reference [10] adopts the grey relational analysis (GRA) to solve the MADM problem where the information about attribute weights is incompletely known. Reference [3] defines the group consistency and inconsistency indices, and then constructs a linear programming model aiming at minimizing the group consistency index to calculate the intuitionistic fuzzy positive ideal solution (IFPIS) and weights. Reference [20] proposes two new aggregation operators to extend VIKOR method for dynamic intuitionistic fuzzy MADM. Reference [4] utilizes the amount and the reliability of information conveyed by an IFS to construct a new ranking method. The analysis in references [6] and [7] is concerned with an intrinsic relationship between the positive and negative information and a lack of information expressed by the hesitation margin to measure the amount of information conveyed by IFSs. References [26] and [27] further introduce the DM¡¯s risk attitude and propose two attitudinal-based intuitionistic fuzzy decision models on the basis of [4]. Reference [40] combines Atanassov¡¯s order [24] and Szmidt and Kacprzyk¡¯s order [4] to propose a new model, not easily affected by extreme data, for MADM. Reference [30] puts forward two score functions for evaluating the suitability of an alternative and utilizes the DM¡¯s attitude to determine the indeterminacy during the evaluation.

While the existing work has made seminal contributions to the MADM problem in intuitionistic fuzzy environment, there are still some issues to be addressed. Firstly, some IFEs [5, 23, 33, 34, 38] utilize the difference between the membership degree and non-membership degree to characterize the uncertainty conveyed by an IFS. However, such a difference just describes the fuzziness of uncertainty but can¡¯t express the unknown of uncertainty. Though some IFEs [2, 21, 31] do use the hesitation degree to represent the unknown, they sometimes suffer from counterintuitive results. Secondly, the structured IFEs in [2, 5, 21, 23, 25, 31, 32, 33, 34, 38] are not reasonable and direct ranking measures for IFSs, because they can¡¯t distinguish an IFS and its complementary set. Thirdly, papers [3, 4, 7, 26, 27, 40] using the distance to rank IFSs just consider the distance from positive ideal solutions but neglect negative ideal solutions. Fourthly, most existing work only focuses on the current moment but neglects decision information in previous times, easily resulting in partial ranking results. Additionally, different DMs often hold different preferences and different risk attitudes towards alternatives. Therefore, it¡¯s necessary to introduce DMs¡¯ preferences [1, 35] and risk attitudes [1, 26, 27, 30, 35, 36] into the ranking method.

Aiming at the issues above, a novel MADM method is proposed and then is applied to the threat assessment problem. The main contributions of this paper are as follows.

1) A new IFE is designed to overcome the drawbacks in existing IFEs. It not only employs the difference between the membership degree and the non-membership degree to describe the fuzziness of uncertainty conveyed by IFSs, but also exploits the hesitation degree to represent the unknown of uncertainty. As such, the two aspects of uncertainty of information are sufficiently expressed. Moreover, many examples show that the proposed IFE can avoid the counterintuitive ranking results in the existing IFEs.

2) Considering distance from the positive solution and the negative ideal solution simultaneously, the closeness degree of TOPSIS is used to characterize the amount of information conveyed by the IFS.

3) The DM¡¯s risk attitude is introduced to integrate the amount and the uncertainty of information conveyed by IFSs. Thereby, a single time ranking method for IFSs is established.

4) The comprehensive preference model for alternatives of all DMs is established to reflect DMs¡¯ subjective intention. On the condition of incomplete weight information, the linear programming model, aiming at minimizing the distance from the single time evaluation value to the ideal solutions and to the comprehensive preferences to alternatives simultaneously, is built to calculate optimal attribute weights. In the linear programming model, a coefficient is introduced to reflect DMs¡¯ inclination to the subjective ranking intension or the objective ranking result, which brings more flexibility for the decision making.

5) The decision information from DMs in multiple times is fused through formulating the time sequence weights, and a multi-time ranking measure for IFSs is structured. A threat assessment example and comparison analysis shows the reasonability and the practicality of the proposed method.

The rest of this paper is organized as follows. Some basic concepts about IFSs are reviewed in Section 2. In Section 3, a novel IFE is proposed and utilized to structure the ranking method for IFSs by the combination with TOPSIS after introducing DMs¡¯ risk attitudes. The optimization linear programming model for the calculation of attribute weights is established in Section 4. Section 5 formulates time sequence weights and fuses decision information in multiple times. A threat assessment example and ranking result discussions are given in Section 6. Section 7 concludes the paper.

2 Preliminaries

Definition 2.1. [1] *Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed non-empty universe set, an IFS A in X is defined as $A = \{< x_i, \mu_A(x_i), \nu_A(x_i) > | x_i \in X\}$, which is characterized by a membership function $\mu_A(x_i) \in [0,1]$ and a non-membership function $\nu_A(x_i) \in [0,1]$ with the condition $0 \le \mu_A(x_i) + \nu_A(x_i) \le 1$. In addition, for each IFS A in X, $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$ is denoted the hesitation degree.*

Definition 2.2. [41] *Let $A_i =< \mu_i, \nu_i > (i = 1, 2)$ be two intuitionistic fuzzy values (IFVs, the basic components of an IFS), and k is a real number which satisfies $k > 0$. Then the relations and operations for IFVs are defined as:*
1) $A_1 + A_2 =< \mu_1 + \mu_2 - \mu_1\mu_2, \nu_1\nu_2 >$;
2) $kA_1 =< 1 - (1 - \mu_1)^k, \nu_1^k >$;
3) *The complementary value* $A_1^c =< \nu_1, \mu_1 >$;
4) *The containment* $A_1 \subseteq A_2$ *if and only* $\mu_1\mu_2$ *and* $\nu_1\nu_2$;
5) *The equivalence* $A_1 = A_2$ *if and only* $A_1 \subseteq A_2$ *and* $A_1 \supseteq A_2$.

Definition 2.3. [5] *A mapping E: $IFS(X) \longrightarrow [0,1]$ is called an entropy measure of IFSs, if it satisfies the following axiomatic requirements:*
1) $E(A) = 0$ *if and only if A is a crisp set;*
2) $E(A) = 1$ *if and only if* $\mu_A(x_i) = \nu_A(x_i)$, *for* $\forall x_i \in X$
3) $E(A) = E(A^c)$
4) $E(A) \le E(B)$ *for* $\forall x_i \in X$, $\mu_A(x_i) \le \mu_B(x_i)$ *and* $\nu_A(x_i) \ge \nu_B(x_i)$ *when* $\mu_B(x_i) \le \nu_B(x_i)$; *or* $\mu_A(x_i) \ge \mu_B(x_i)$ *and* $\nu_A(x_i) \le \nu_B(x_i)$ *when* $\mu_B(x_i) \ge \nu_B(x_i)$.

Definition 2.4. [41] *Assume that $A_i =< \mu_i, \nu_i > (i = 1, 2, n)$ is a set of IFVs, the intuitionistic fuzzy weighted average operator (IFWA) is expressed as:*

$$\sum_{i=1}^{n} \omega_i A_i =< 1 - \prod_{i=1}^{n} (1 - \mu_i)^{\omega_i}, \prod_{i=1}^{n} (\nu_i)^{\omega_i} > \tag{1}$$

where ω_i is the weight of $A_i (i = 1, 2, n)$, which satisfies $0 \le \omega_i \le 1$.

3 Ranking method for IFVs based on TOPSIS and IFE

3.1 A novel IFE

At first, a survey is conducted at the drawbacks in the existing IFEs.

1) The IFE presented by Zeng [38]:

$$E_{zeng}(A) = 1 - \frac{1}{n} \sum_{i=1}^{n} |\mu_A(x_i) - \nu_A(x_i)| \tag{2}$$

2) The IFE proposed by Zhang [33]:

$$E_{Zhang}(A) = \frac{1}{n} \sum_{i=1}^{n} [\frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2} \log(\frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2}) + \frac{\nu_A(x_i) + 1 - \mu_A(x_i)}{2} \log(\frac{\nu_A(x_i) + 1 - \mu_A(x_i))}{2}] \tag{3}$$

3) The IFE offered by Ye [23]:

$$E_{Ye}(A) = \frac{1}{n} \sum_{i=1}^{n} [(\sqrt{2} \cos \frac{\mu_A(x_i) - \nu_A(x_i)}{4} \pi - 1) \times \frac{1}{\sqrt{2} - 1}] \tag{4}$$

4) The IFE structured by Verma [34]:

$$\begin{aligned} E_{Verma}(A) = \frac{1}{n(\sqrt{e} - 1)} \sum_{i=1}^{n} [&\frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2} \exp(1 - \frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2}) + \\ &\frac{\nu_A(x_i) + 1 - \mu_A(x_i)}{2} \exp(1 - \frac{\nu_A(x_i) + 1 - \mu_A(x_i)}{2}) - 1] \end{aligned} \tag{5}$$

However, the IFEs above all neglect the influence of the hesitation degree on the uncertainty conveyed by IFSs. Thus, the obtained values of IFEs are equivalent when two IFVs have the same difference between the membership degree and the non-membership degree.

Though some other IFEs do take the influence of the hesitation degree into consideration, there are still some limitations.

5) The IFE put forward by Wang [21]:

$$E_{Wang}(A) = \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{\pi}{4} + \frac{|\mu_A(x_i) - \nu_A(x_i)|}{4(1+\pi_A(x_i))}\pi) \tag{6}$$

The IFE provided by Wei [2]:

$$E_{Wei}(A) = \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{\mu_A(x_i) - \nu_A(x_i)}{2(1+\pi_A(x_i))}\pi) \tag{7}$$

7) The IFE designed by Gao [31]:

$$E_{Gao}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{1 - |\mu_A(x_i) - \nu_A(x_i)|^2 + \pi_A(x_i)^2}{2} \tag{8}$$

Example 3.1. *Consider Four IFVs $A_1 =< 0.2, 0.6 >, A_2 =< 0.13, 0.565 >, A_3 =< 0.5, 0.3 >$ and $A_4 =< 0.5, 0.5 >$. It is apparent that the fuzziness of A_1 and A_2 are different, and so are A_3 and A_4. However, the obtained values by Eq. (6) to Eq. (8) are $E_{Wang}(A_1) = E_{Wang}(A_2) = 0.8458, E_{wei}(A_1) = E_{wei}(A_2) = 0.8660$, and $E_{Gao}(A_3) = E_{Gao}(A_4) = 0.5$. It is not conform to the intuitionistic fact. Therefore, we devise a novel IFE.*

For an arbitrary IFS $A = \{< x_i, \mu_A(x_i), \nu_A(xi) > |x_i \in X, i = 1, 2, ..., n\}$, we define:

$$E(A) = \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{(\mu_A(x_i) - \nu_A(x_i))(1 - \frac{\pi_A(x_i)^2)}{2})}{2}\pi) \tag{9}$$

is an IFE.

Theorem 3.2. *The definition of $E_(A)$ by Eq. (9) is an IFE.*

Proof. To demonstrate $E(A)$ is an IFE, $E(A)$ should meet the four requirements in Definition 2.3. It is obvious that $E(A)$ always satisfies $0 \leq E(A) \leq 1$.

For requirement 1) in Definition 2.3: when A is a crisp set, then we have $\mu_A(x_i) = 0, \nu_A(x_i) = 1$ or $\mu_A(x_i) = 1, \nu_A(x_i) = 0$. Thus, $E(A) = 0$ for $\forall x_i \in X$. Otherwise, if $E(A) = 0$, then for $\forall x_i \in X$ satisfies $\cos((\mu_A(x_i) - \nu_A(x_i))\frac{(1-\frac{\pi_A(x_i)^2}{2})}{2}\pi) = 0$. Only $\mu_A(x_i) = 0, \nu_A(x_i) = 1$ or $\mu_A(x_i) = 1, \nu_A(x_i) = 0$ can be obtained. Thus, A is a crisp set.

For requirement 2) in Definition 2.3: When $\mu_A(x_i) = \nu_A(x_i)$ for $\forall x_i \in X$, we have $E(A) = 1$. Otherwise, suppose $E(A) = 1$, for all $x_i \in X$, $\cos((\mu_A(x_i) - \nu_A(x_i))\frac{(1-\frac{\pi_A(x_i)^2}{2})}{2}\pi) = 1$, we obtain $\mu_A(x_i) = \nu_A(x_i), \forall x_i \in X$.

For requirement 3) in Definition 2.3: From $A^c = \{< x_i, \nu_A(x_i), \mu_A(x_i) > |x_i \in X\}$, we easily obtain:

$$\begin{aligned} E(A^c) = \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{(\nu_A(x_i) - \mu_A(x_i))(1 - \frac{\pi_A(x_i)^2)}{2})}{2}\pi) = \\ \frac{1}{n}\sum_{i=1}^{n}\cos(\frac{(\mu_A(x_i) - \nu_A(x_i))(1 - \frac{\pi_A(x_i)^2)}{2})}{2}\pi) = E(A) \end{aligned} \tag{10}$$

For requirement 4) in Definition 2.3: We construct the function: $f(x, y) = \cos[\pi(x-y)(1-(1-x-y)^2/2)/2]$, where $x, y \in [0, 1]$ and $x + y \leq 1$. Then, the partial derivative of $f(x, y)$ to x and y are:

$$\begin{aligned} \frac{\partial f(x,y)}{\partial x} &= -\frac{\pi}{2}\sin[\frac{\pi}{2}(x-y)(1-\frac{(1-x-y)^2}{2})][(1-\frac{(1-x-y)^2}{2}) + (x-y)(1-x-y)] \\ &= -\frac{\pi}{2}\sin[\frac{\pi}{2}(x-y)(1-\frac{(1-x-y)^2}{2})][1-2(x-\frac{1}{2})^2 + \frac{1}{2}(y-x)^2] \end{aligned} \tag{11}$$

$$\frac{\partial f(x,y)}{\partial y}=\frac{\pi}{2}\sin[\frac{\pi}{2}(x-y)(1-\frac{(1-x-y)^2}{2})][(1-\frac{(1-x-y)^2}{2})-(x-y)(1-x-y)] \quad (12)$$

When $x\le y, x-y\le 0, 1/2\le 1-\frac{(1-x-y)^2}{2}\le 1$. Assume $g(x)=1-2(x-1/2)^2$ in Eq. (11), and $g(x)$ achieves the minimum $1/2\ge 0$ when $x=0$ or $x=1$, then we obtain $\partial f(x,y)/\partial x\ge 0$. Similarly, we can obtain $\partial f(x,y)/\partial y\le 0$. Therefore, $f(x,y)$ is a monotonic increasing function to x, whereas a monotonic decreasing function to y. Thus, we obtain $f(\mu_A(x_i),\mu_B(x_i))\le f(\mu_B(x_i),\mu_A(x_i))$ when $\mu_B(x_i)\le\nu_B(x_i), \mu_A(x_i)\le\mu_B(x_i)$ and $\nu_A(x_i)\ge\nu_B(x_i)$. Namely, $E(A)\le E(B)$. Otherwise, when $x\ge y$, $\partial f(x,y)/\partial x\le 0$ and $\partial f(x,y)/\partial y\ge 0$. Therefore, we obtain $f(\mu_A(x_i),\mu_B(x_i))\le f(\mu_B(x_i),\mu_A(x_i))$ when $\mu_B(x_i)\ge\nu_B(x_i), \mu_A(x_i)\ge\mu_B(x_i)$ and $\nu_A(x_i)\le\nu_B(x_i)$. Namely, $E(A)\le E(B)$, which completes the proof. □

Remark 3.3. *The proposed IFE not only considers the influence of the difference between the membership degree and the non-membership degree:* $\mu_A(xi)-\nu_A(xi)$*, but also introduces the hesitation degree. Thus, the fuzziness and unknown of uncertainty conveyed by IFSs are well expressed. Consider the examples above where the IFEs in [2, 21, 31] can¡¯t distinguish, the results are as follows when applying the proposed IFE:* $E(A_1)=0.8163, E(A_2)=0.7952, E(A_3)=0.9530, E(A_4)=1$. *Therefore, the proposed IFE has better distinguish ability for IFVs.*

3.2 Ranking function based on TOPSIS and IFE

From the aspect of information, an IFS contains two contents: the amount of information and the reliability of information. As such, references [4, 7, 26, 27, 40] utilize the distance from the positive ideal point to represent the amount of information, and adopt the hesitation degree to represent the reliability of information. However, they all neglect the distance to the negative point. Additionally, the hesitation degree can¡¯t sufficiently express the reliability of information. At present, TOPSIS has been widely applied in similarity measures of IFSs, where a closer distance from the point to the positive ideal point while a farther distance from the point to the negative ideal point means a better one. As such, TOPSIS is a reasonable measure for the amount of information [36]. Furthermore, the IFE precisely depicts the uncertainty of IFSs and thus expresses the reliability of information in another aspect. Therefore, we combine TOPSIS with IFE and propose a novel ranking method for IFSs.

In the threat assessment problem, the following attributes are concerned: the target type (T_Y), the target jamming ability (J_A), the straight distance from the target to the radar (S_D), the target velocity (V), the target heading angle (H_A), and the target height (H). The first two attributes are qualitative ones, and the other four attributes are quantitative ones. The target jamming ability and the target velocity are benefit attributes, where the higher value means the higher threat level. The other four attributes are cost ones, where the lower value means the lower threat level. For benefit attributes, the positive ideal point μ_j^+ is adopted as $<1,0>$, and the negative ideal point ν_j^- is adopted as $<0,1>$. However, for cost attributes, the positive ideal point μ_j^+ is $<0,1>$, and the negative ideal point ν_j^- is $<1,0>$. Additionally, we transform cost attributes $<c_{ij},d_{ij}>$ into benefit attributes $<a_{ij},b_{ij}>$ by Eq. (13):

$$a_{ij}=d_{ij}, b_{ij}=c_{ij} \quad (13)$$

Then, the Hamming distance from the jth attribute of ith target in time t_k to the positive ideal point and to the negative ideal point are separately expressed as:

$$D_{ij}^+(t_k)=\frac{1}{2}[|\mu_{ij}(t_k)-\mu_j(t_k)^+|+|\nu_{ij}(t_k)-\nu_j(t_k)^+|+|\pi_{ij}(t_k)-\pi_j(t_k)^+|]=1-\mu_{ij}(t_k) \quad (14)$$

$$D_{ij}^-(t_k)=\frac{1}{2}[|\mu_{ij}(t_k)-\mu_j(t_k)^-|+|\nu_{ij}(t_k)-\nu_j(t_k)^-|+|\pi_{ij}(t_k)-\pi_j(t_k)^-|]=1-\nu_{ij}(t_k) \quad (15)$$

The closeness degree is:

$$D_{ij}(t_k)=\frac{D_{ij}^-(t_k)}{D_{ij}^+(t_k)+D_{ij}^-(t_k)}=\frac{1-\nu_{ij}(t_k)}{1+\pi_{ij}(t_k)} \quad (16)$$

The IFE of the attribute is calculated as follows according to Eq. (9):

$$E_{ij}(t_k)=\cos(\frac{(\mu_{ij}(t_k)-\nu_{ij}(t_k))(1-\frac{\pi_{ij}(t_k)^2}{2})}{2}\pi) \quad (17)$$

Since the closeness degree reflects the amount of information conveyed by IFSs, and the IFE describes the uncertainty of information conveyed by IFSs, a novel ranking function for IFSs is put forward by introducing the DM¡¯s risk attitude:

$$P_{ij}(t_k) = (1-\alpha)D_{ij}(t_k) + \alpha E_{ij}(t_k) \tag{18}$$

where bigger P_{ij} means a higher threat level. α is the risk factor of the DM, which satisfies:

$$\alpha = \int_0^1 Q(s)ds \tag{19}$$

where $Q(s)$ is an unit-interval monotonic function. When $0.5 < \alpha < 1$, the DM prefers the uncertainty of information: E_{ij}. When $0 < \alpha < 0.5$, the DM prefers the amount of information: D_{ij}. When $\alpha = 0.5$, the DM is neutral towards the amount of information and the uncertainty of information. In application, α is able to be adjusted according to different $Q(s)$ functions. For example, if $Q(s) = s^t (t > 0)$, Eq. (18) is:

$$P_{ij}(t_k) = \frac{t}{1+t} D_{ij}(t_k) + \frac{1}{1+t} E_{ij}(t_k) \tag{20}$$

when $Q(s) = (\sin(\pi s/2))^t (t > 0)$, Eq. (18) is:

$$P_{ij}(t_k) = \begin{cases} E_{ij}(t_k), & t \to 0 \\ \frac{\pi-2}{\pi} E_{ij}(t_k) + \frac{2}{\pi} D_{ij}(t_k), & t = 1 \\ \frac{1}{2} E_{ij}(t_k) + \frac{1}{2} D_{ij}(t_k), & t = 2 \\ D_{ij}(t_k), & t \to \infty \end{cases} \tag{21}$$

Therefore, the DM can choose the appropriate $Q(s)$ and α according to his own risk attitude, which brings more flexibility to the MADM.

Remark 3.4. *The proposed ranking function is particular as follows.*

1) Many existing ranking measures [3, 4, 7, 26, 27, 40] *only take the distance from the IFV to the positive ideal point into account but neglect the distance from the negative ideal point. However, the proposed method combines the distance from the positive ideal point with the distance from the negative ideal point by the closeness degree of TOPSIS.*

2) The methods in [36] *and* [39] *are similar with the proposed ranking function. However,* [39] *neglect the DM¡¯s risk attitude. Though* [36] *utilizes the geometric measurement* $(1-\pi^2/2)$ *to express the reliability of IFS, it overlooks the other aspect contained in the uncertainty: the fuzziness. (The uncertainty conveyed by an IFS contains two aspects: the fuzziness and the unknown* [25, 32], and $\pi^2/2$ only describes the unknown.) However, the proposed method utilizes the structured IFE in Section 3 and expresses the two points of uncertainty more sufficiently.

4 Synthesis of the objective ranking intension and the subjective ranking result under incomplete weight information

4.1 Attribute preference model

It is often the case that multiple decision makers (DMs) will join in the MADM. Since different DMs have different knowledge and experiences, they have different evaluations on the attribute importance. Considering DMs¡¯ recognition on the attribute importance and DMs¡¯ authority, the DM¡¯s preference to jth attribute of ith alternative is defined as:

$$p_{ij} = \sigma r_{ij} w_j \tag{22}$$

where σ is the measure of DM¡¯s authority, r_{ij} is the original evaluation value of jth attribute of ith alternative, which satisfies $r_{ij} =< \mu_{ij}, \nu_{ij} >$. w_j is jth attribute weight obtained from the DM. Assume that there are g DMs and qth DM¡¯s authority is σ_q, the comprehensive preference of jth attribute of ith alternative from all DMs is:

$$p_{ij} = \sum_{q=1}^{g} \sigma_q r_{ij} w_j \tag{23}$$

where $p_{ij} =< \mu_{ij}^p, \nu_{ij}^p >$.

4.2 Optimization model of attribute weight

In most cases, the information about attribute weights is incomplete. In order to calculate optimal attribute weights, the evaluation values of all alternatives should have the shortest distance from the positive ideal solutions, at the same time; have the farthest distance from the negative ideal solutions. However, such a calculation just reflects the objective evaluation results. In the threat assessment, the subjective intention from DMs should also be taken into account to guide the final ranking results. That means the distance from the final evaluation results of all alternatives to the subjective preferences of DMs and to the objective evaluation values should achieve the minimum simultaneously to decide the weight ω_j of jth attribute. Thus, the following linear programming optimization model is established as:

$$\begin{cases} minZ(\omega) = \beta \sum_{i=1}^{m} [d(r_i, D_i^+) - d(r_i, D_i^-)] + \gamma \sum_{i=1}^{m} d(r_i, p_i) \\ s.t. \Lambda = \omega | \sum_{j=1}^{n} \omega_j = 1, \omega_j > \varepsilon \end{cases} \tag{24}$$

where β and γ are the weight of the objective and the subjective programming models respectively, which satisfy $\beta \in [0, 1], \gamma \in [0, 1]$ and $\beta + \gamma = 1$. $\omega_j > \varepsilon$ ensures each weight ω_j to be bigger than a positive valid number. $d(.)$ is the weighted Hamming distance, which satisfies:

$$d(r_i, D_i^+) = \sum_{j=1}^{n} \omega_j (1 - \mu_{ij}) \tag{25}$$

$$d(r_i, D_i^-) = \sum_{j=1}^{n} \omega_j (1 - \nu_{ij}) \tag{26}$$

$$d(r_i, p_i) = \frac{1}{2} \sum_{j=1}^{n} \omega_j (|\mu_{ij} - \mu_{ij}^p| + |\nu_{ij} - \nu_{ij}^p| + |\pi_{ij} - \pi_{ij}^p|) \tag{27}$$

where r_i is the final evaluation result of alternative i. D_i^+ and D_i^- are distances from the final evaluation value of ith alternative to positive ideal solutions and to negative ideal solutions respectively. p_i is DMs¡¯ comprehensive preference to ith alternative. When $\beta = 1$ and $\gamma = 0$, Eq. (24) will be changed into the calculation method for attribute weights that only considering the smallest deviation from the ideal solutions, namely, the objective assignment method. Conversely, Eq. (24) will be changed into the calculation method only for the smallest deviation from the preferences of DMs when $\beta = 0$ and $\gamma = 1$, namely, the subjective assignment method. It is usually adopted as $\gamma \in (0, 1)$ and $\beta \in (0, 1)$ in order to reflect both of the objective ranking result and the subjective preferences of DMs.

5 Multiple times decision information fusion

5.1 Time sequence weights

In modern air defense wars, the threat level of the target varies from time to time. To evaluate the intention of the target in accuracy and obtain more reasonable and comprehensive threat ranking results, the time sequence weights are necessary. Meanwhile, it should meet the following requirements. 1) The weight sequence is discrete and can be adjusted by the number of times. 2) The closer to the current time, the more important the information is and the bigger the weight is. 3) The sum of weights is 1. Therefore, the target information in the current time p and the former $(p - 1)$ times are comprehensively evaluated and the inverse pattern of Poisson distribution is utilized to calculate the time sequence weights:

$$\eta_k = \frac{k!/\phi^k}{\sum_{k=1}^{p} (k!/\phi^k)} \tag{28}$$

where η_k is the weight of time t_k, which satisfies $\eta_k \geq 0$ and $\sum_{k=1}^{p} \eta_k = 1$. ϕ is the adjustment parameter satisfying $0 < \phi < 2$.

Table 1: Target information in time $t_1 \sim t_3$

Time	Target index	Type(T_Y)	Straight distance(S_D)	Height(H)
t_1	1	$< 0.3, 0.5 >$	$< 0.76, 0.10 >$	$< 0.68, 0.15 >$
	2	$< 0.7, 0.2 >$	$< 0.80, 0.15 >$	$< 0.70, 0.15 >$
	3	$< 0.6, 0.3 >$	$< 0.78, 0.15 >$	$< 0.72, 0.13 >$
	4	$< 0.6, 0.2 >$	$< 0.65, 0.28 >$	$< 0.74, 0.20 >$
	5	$< 0.4, 0.4 >$	$< 0.68, 0.21 >$	$< 0.75, 0.18 >$
t_2	1	$< 0.4, 0.5 >$	$< 0.78, 0.10 >$	$< 0.65, 0.20 >$
	2	$< 0.6, 0.3 >$	$< 0.83, 0.10 >$	$< 0.65, 0.20 >$
	3	$< 0.5, 0.4 >$	$< 0.75, 0.20 >$	$< 0.80, 0.15 >$
	4	$< 0.5, 0.1 >$	$< 0.70, 0.20 >$	$< 0.64, 0.21 >$
	5	$< 0.5, 0.4 >$	$< 0.68, 0.05 >$	$< 0.70, 0.12 >$
t_3	1	$< 0.5, 0.4 >$	$< 0.76, 0.13 >$	$< 0.68, 0.15 >$
	2	$< 0.6, 0.3 >$	$< 0.80, 0.10 >$	$< 0.75, 0.15 >$
	3	$< 0.6, 0.3 >$	$< 0.75, 0.20 >$	$< 0.70, 0.18 >$
	4	$< 0.6, 0.2 >$	$< 0.75, 0.15 >$	$< 0.68, 0.20 >$
	5	$< 0.5, 0.3 >$	$< 0.65, 0.05 >$	$< 0.70, 0.25 >$

Time	Target index	Type(T_Y)	Velocity(V)	Jamming ability(J_A)
t_1	1	$< 0.75, 0.15 >$	$< 0.80, 0.15 >$	$< 0.5, 0.4 >$
	2	$< 0.60, 0.20 >$	$< 0.70, 0.10 >$	$< 0.6, 0.3 >$
	3	$< 0.65, 0.15 >$	$< 0.75, 0.15 >$	$< 0.5, 0.2 >$
	4	$< 0.55, 0.30 >$	$< 0.80, 0.11 >$	$< 0.7, 0.2 >$
	5	$< 0.69, 0.20 >$	$< 0.63, 0.10 >$	$< 0.2, 0.4 >$
t_2	1	$< 0.70, 0.20 >$	$< 0.80, 0.11 >$	$< 0.4, 0.3 >$
	2	$< 0.70, 0.15 >$	$< 0.75, 0.10 >$	$< 0.6, 0.2 >$
	3	$< 0.72, 0.18 >$	$< 0.73, 0.19 >$	$< 0.5, 0.3 >$
	4	$< 0.76, 0.15 >$	$< 0.75, 0.15 >$	$< 0.6, 0.1 >$
	5	$< 0.68, 0.10 >$	$< 0.78, 0.20 >$	$< 0.3, 0.4 >$
t_3	1	$< 0.60, 0.25 >$	$< 0.75, 0.18 >$	$< 0.5, 0.2 >$
	2	$< 0.75, 0.10 >$	$< 0.85, 0.05 >$	$< 0.7, 0.2 >$
	3	$< 0.80, 0.15 >$	$< 0.78, 0.20 >$	$< 0.5, 0.4 >$
	4	$< 0.70, 0.18 >$	$< 0.60, 0.20 >$	$< 0.6, 0.2 >$
	5	$< 0.65, 0.05 >$	$< 0.80, 0.15 >$	$< 0.4, 0.4 >$

5.2 Procedure of the proposed method

Assume that the evaluation matrix on targets in time t_k is $\mathbf{R}(t_k) = (r_{ij}(t_k))_{m\times n}$, where $r_{ij}(t_k) =< \mu_{ij}(t_k), \nu_{ij}(t_k) >$. i is the alternative set ($i = 1, 2, ..., m$) and j is the attribute set ($j = 1, 2, ..., n$). The procedure of the proposed method is summarized as follows.

Step 1 Exchange cost attributes into benefit attributes according to Eq. (13) and obtain the intuitionistic fuzzy evaluation matrix in time t_k: $(r_{ij}(t_k))_{m\times n}$.

Step 2 Calculate the comprehensive preferences for attributes from DMs on the basis of Eq. (22) and Eq. (23).

Step 3 Consider the constraint information for attribute weights and structure the optimization model for calculating attribute weights. (Eq. (24) to Eq. (27))

Step 4 Aggregate $r_{ij}(t_k)$ by Eq. (1) and obtain the weighted intuitionistic fuzzy evaluation matrix on targets: $(f_i(t_k))_{m\times 1}$.

Step 5 Calculate the ranking function values of targets in time t_k: $P_i(t_k)$. (Eq. (14) to Eq. (18))

Step 6 Form the weighted decision matrix $\mathbf{H} = (h_{ik})_{m\times p}$ from time t_1 to time t_p, where $h_{ik} = \eta_k P_i(t_k)$. (Eq. (28))

Step 7 Obtain the final threat ranking results of targets by TOPSIS.

6 A threat assessment example and analysis

6.1 A threat assessment example

Suppose that there are five enemy targets in an air defense war, and the target information obtained from sensors in three continuous times ($t_1 \sim t_3$) are shown in Table 1.

Table 2: Comprehensive preferences to targets from DMs in time $t_1 \sim t_3$

	$< 0.2140, 0.6310 >$	$< 0.0663, 0.8874 >$	$< 0.0635, 0.8918 >$
	$< 0.0689, 0.9029 >$	$< 0.0543, 0.9306 >$	$< 0.0588, 0.9034 >$
$\mathbf{P}(t_1)$	$< 0.0971, 0.8706 >$	$< 0.0602, 0.9179 >$	$< 0.0543, 0.9089 >$
	$< 0.0971, 0.8326 >$	$< 0.1021, 0.8727 >$	$< 0.0499, 0.9365 >$
	$< 0.1674, 0.7248 >$	$< 0.0919, 0.8689 >$	$< 0.0477, 0.9365 >$
	$< 0.1674, 0.7860 >$	$< 0.0602, 0.9013 >$	$< 0.0706, 0.8888 >$
	$< 0.0971, 0.8706 >$	$< 0.0455, 0.9337 >$	$< 0.0706, 0.8888 >$
$\mathbf{P}(t_2)$	$< 0.1294, 0.8326 >$	$< 0.0694, 0.9147 >$	$< 0.0372, 0.9523 >$
	$< 0.1294, 0.6310 >$	$< 0.0853, 0.8801 >$	$< 0.0731, 0.8858 >$
	$< 0.1294, 0.8326 >$	$< 0.0919, 0.8002 >$	$< 0.0588, 0.8948 >$
	$< 0.1294, 0.8326 >$	$< 0.0663, 0.8979 >$	$< 0.0635, 0.8918 >$
	$< 0.0971, 0.8706 >$	$< 0.0543, 0.9147 >$	$< 0.0477, 0.9249 >$
$\mathbf{P}(t_3)$	$< 0.0971, 0.8706 >$	$< 0.0694, 0.9147 >$	$< 0.0588, 0.9116 >$
	$< 0.0971, 0.8326 >$	$< 0.0694, 0.8979 >$	$< 0.0635, 0.9061 >$
	$< 0.1294, 0.7860 >$	$< 0.1021, 0.7692 >$	$< 0.0588, 0.9294 >$

	$< 0.0395, 0.9415 >$	$< 0.1888, 0.7814 >$	$< 0.0734, 0.9041 >$
	$< 0.0690, 0.8796 >$	$< 0.2587, 0.7413 >$	$< 0.1240, 0.8760 >$
$\mathbf{P}(t_1)$	$< 0.0585, 0.8942 >$	$< 0.1649, 0.7814 >$	$< 0.0734, 0.8377 >$
	$< 0.0803, 0.8796 >$	$< 0.1888, 0.7506 >$	$< 0.1240, 0.8377 >$
	$< 0.0506, 0.9266 >$	$< 0.1212, 0.7413 >$	$< 0.0242, 0.9041 >$
	$< 0.0487, 0.9310 >$	$< 0.1888, 0.7506 >$	$< 0.0546, 0.8760 >$
	$< 0.0487, 0.9197 >$	$< 0.1649, 0.7413 >$	$< 0.0959, 0.8377 >$
$\mathbf{P}(t_2)$	$< 0.0449, 0.9353 >$	$< 0.1565, 0.8058 >$	$< 0.0734, 0.8760 >$
	$< 0.0377, 0.9455 >$	$< 0.1649, 0.7814 >$	$< 0.0959, 0.7762 >$
	$< 0.0526, 0.8970 >$	$< 0.1787, 0.8112 >$	$< 0.0385, 0.9041 >$
	$< 0.0690, 0.8942 >$	$< 0.1649, 0.8002 >$	$< 0.0734, 0.8377 >$
	$< 0.0395, 0.9310 >$	$< 0.2186, 0.6774 >$	$< 0.1240, 0.8377 >$
$\mathbf{P}(t_3)$	$< 0.0308, 0.9605 >$	$< 0.1787, 0.8112 >$	$< 0.0734, 0.9041 >$
	$< 0.0487, 0.9266 >$	$< 0.1123, 0.8112 >$	$< 0.0959, 0.8377 >$
	$< 0.0585, 0.8633 >$	$< 0.1888, 0.7814 >$	$< 0.0546, 0.9041 >$

Table 3: Weights of attributes and ranking function values of targets under different β and γ in time t_1

β and γ	Weights of attributes in time t_1	Ranking function values of targets in time t_1
$\beta = 0, \gamma = 1$	[0.2500,0.3500,0.2500,0.0500,0.0500,0.0500]	[1.0000,0.8071,0.8566,0.9461,0.9244]
$\beta = 0.25, \gamma = 0.75$	[0.3500,0.3500,0.1500,0.0500,0.0500,0.0500]	[1.0000,0.7745,0.8392,0.9226,0.9324]
$\beta = 0.5, \gamma = 0.5$	[0.3500,0.2500,0.0500,0.0500,0.2500,0.0500]	[1.0000,0.8728,0.9243,0.9701,0.9380]
$\beta = 0.75, \gamma = 0.25$	[0.3000,0.2000,0.0500,0.0500,0.2000,0.2000]	[1.0000,0.9399,0.9614,0.9950,0.9341]
$\beta = 1, \gamma = 0$	[0.3000,0.2000,0.0500,0.0500,0.2000,0.2000]	[1.0000,0.9399,0.9614,0.9950,0.9341]

According to the information above, we can obtain the intuitionistic fuzzy evaluation values after exchanging cost attributes into benefit attributes. There are three DMs in the threat assessment process and their authority matrix is: $\sigma = [0.3387, 0.3185, 0.3427]$. The subjective weights of attributes from DMs are $w = [0.20, 0.25, 0.17, 0.14, 0.13, 0.11]$. Thereby, we can obtain the comprehensive preferences to targets from DMs in time $t_1 \sim t_3$, which is shown in Table 2. The constraint information of attribute weights is: $0.10 < \omega_1 < 0.35$, $0.10 < \omega_2 < 0.35$, $0.05 < \omega_3 < 0.25$, $0.05 < \omega_4 < 0.20$, $0.05 < \omega_5 < 0.25$, $0.05 < \omega_6 < 0.20$, $\omega_1 > \omega_5$, $\omega_2 > \omega_3$, $\omega_2 > \omega_5 > \omega_4$, $\omega_2 > \omega_5 > \omega_6$. Additionally, risk factors of the three DMs are $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$. Then, the optimal weights of attributes and ranking function values of targets by Eq. (20) under different β and γ in time $t_1 \sim t_3$ are shown in Table 3 $\sim$ Table 5 respectively.

As to the time sequence weights, $\phi = 1.5$. Therefore, $\eta = [0.2000, 0.2667, 0.5333]$. After forming the weighted decision matrix, final evaluation values of targets and ranking results are shown in Table 6.

Table 4: Weights of attributes and ranking function values of targets under different β and γ in time t_2

β and γ	Weights of attributes in time t_2	Ranking function values of targets in time t_2
$\beta = 0, \gamma = 1$	[0.2500,0.3500,0.2500,0.0500,0.0500,0.0500]	[0.9859,0.8955,0.8969,1.0000,0.9496]
$\beta = 0.25, \gamma = 0.75$	[0.3500,0.3500,0.0500,0.0500,0.1500,0.0500]	[1.0000,0.9110,0.9455,0.9870,0.9728]
$\beta = 0.5, \gamma = 0.5$	[0.3500,0.2500,0.0500,0.0500,0.2500,0.0500]	[1.0000,0.9403,0.9560,0.9827,0.9779]
$\beta = 0.75, \gamma = 0.25$	[0.3500,0.1833,0.0500,0.0500,0.1833,0.1833]	[0.9968,0.9757,0.9723,1.0000,0.9668]
$\beta = 1, \gamma = 0$	[0.3500,0.1833,0.0500,0.0500,0.1833,0.1833]	[0.9968,0.9757,0.9723,1.0000,0.9668]

Table 5: Weights of attributes and ranking function values of targets under different β and γ in time t_3

β and γ	Weights of attributes in time t_3	Ranking function values of targets in time t_3
$\beta = 0, \gamma = 1$	[0.2500,0.3500,0.2500,0.0500,0.0500,0.0500]	[0.9775,0.9326,0.9039,0.9195,1.0000]
$\beta = 0.25, \gamma = 0.75$	[0.3500,0.3500,0.1500,0.0500,0.0500,0.0500]	[0.9777,0.9301,0.8971,0.9087,1.0000]
$\beta = 0.5, \gamma = 0.5$	[0.2333,0.2333,0.0500,0.0500,0.2333,0.2000]	[0.9800,1.0000,0.9624,0.9527,0.9805]
$\beta = 0.75, \gamma = 0.25$	[0.2333,0.2333,0.0500,0.0500,0.2333,0.2000]	[0.9800,1.0000,0.9624,0.9527,0.9805]
$\beta = 1, \gamma = 0$	[0.2333,0.2333,0.0500,0.0500,0.2333,0.2000]	[0.9800,1.0000,0.9624,0.9527,0.9805]

6.2 Ranking result discussions and analysis

6.2.1 Rationality of DMs¡¯ subjective intension

Table 3 $\sim$ Table 6 show that β and γ values count in ranking results, and different β and γ will lead to different ranking results. For example, Table 6 shows that the ranking order of target 2 rises as parameter β increases. It is because parameter β indicates the proportion of the objective ranking result which has been considered in solving the optimal attribute weights. Bigger parameter β suggests that DMs are more inclined to the objective ranking method whereas smaller parameter β suggests that DMs are more inclined to the ranking method based on the subjective preference. Therefore, ranking results can be guided by DMs and can be adjusted by parameter β, which indicates the practicality and flexibility of the proposed method.

6.2.2 Rationality of multi-time information fusion

We compare the method that only utilizing the current time information to obtain ranking results and the proposed method.

If the decision information in time t_3 is only utilized, the final ranking results are as follows (as shown in Table 5).$5 > 1 > 2 > 4 > 3$ when $\beta = 0$ and $\gamma = 1$. $5 > 1 > 2 > 4 > 3$ when $\beta = 0.25$ and $\gamma = 0.75$. $2 > 5 > 1 > 3 > 4$ when $\beta = 0.5$ and $\gamma = 0.5$, $\beta = 0.75$ and $\gamma = 0.25$, and $\beta = 1$ and $\gamma = 0$. Obviously, the ranking results are quite different from that of the proposed method (as shown in Table 6) which fuses decision information in multiple times. It is because the proposed method considers target information and decision information from DMs in time $t_1 \sim t_3$ comprehensively. However, the former method only utilizes target information in time t_3 to give ranking results. For example, when $\beta = 0$ and $\gamma = 1$, the ranking result of the proposed method is $1 > 5 > 4 > 2 > 3$. By contrast, the ranking result by only utilizing the information in time t_3 is $5 > 1 > 2 > 4 > 3$. However, Table 3 and Table 4 show that evaluation values of target 1 are higher than target 5 in time t_1 and time t_2. In addition, evaluation values of target 4 are much higher than target 2 in time t_1 and time t_2. From Table 5, we can see that the evaluation value of target 1 is close to target 5 in time t_3, and so are evaluation values of target 2 and target 4 in time t_3. Therefore, the ranking result of the proposed method, which fuses information in multiple times, is more reasonable. Additionally, other examples with different β and γ values also can demonstrate the reasonability of the proposed method.

Table 6: Final evaluation values of targets and ranking results

β and γ	Final evaluation values of targets	Ranking results
$\beta = 0, \gamma = 1$	[1.0000,0.9272,0.9163,0.9604,0.9952]	$1 > 5 > 4 > 2 > 3$
$\beta = 0.25, \gamma = 0.75$	[1.0000,0.9217,0.9172,0.9447,0.9981]	$1 > 5 > 4 > 2 > 3$
$\beta = 0.5, \gamma = 0.5$	[1.0000,0.9819,0.9695,0.9757,0.9869]	$1 > 5 > 2 > 4 > 3$
$\beta = 0.75, \gamma = 0.25$	[1.0000,0.9990,0.9788,0.9837,0.9847]	$1 > 2 > 5 > 4 > 3$
$\beta = 1, \gamma = 0$	[1.0000,0.9990,0.9788,0.9837,0.9847]	$1 > 2 > 5 > 4 > 3$

6.2.3 Rationality of DMs¡¯ risk attitude

Since DMs¡¯ risk attitudes play important roles in the ranking function for IFVs, we discuss the change of final ranking results by Eq. (20) with different DMs¡¯ risk attitudes, which is shown in Table 7. To be convenience, we think the three DMs hold the same risk attitude. The change of final ranking results by Eq. (21) with different DMs¡¯ risk attitudes is shown in Table 8. To be convenience, we think the three DMs hold the same risk attitude. Table 7 and Table 8 show that final ranking results differ a lot when adopting different risk attitudes in the same $Q(s)$. For example, under the condition of $\beta = 0.5, \gamma = 0.5$ and Eq. (20), the ranking result is $4 > 3 > 5 > 1 > 2$ when $t = 0.5$. However, the ranking result is $1 > 5 > 4 > 3 > 2$ when $t = 1$ and the ranking result is $1 > 5 > 2 > 4 > 3$ when $t = 2$. Moreover, final ranking results are distinct when adopting different $Q(s)$ in the same risk attitude. For example, under the condition of $\beta = 0.5, \gamma = 0.5$ and $t = 1$, the ranking result is $1 > 5 > 4 > 3 > 2$ by Eq. (20). By contrast, the ranking result is $1 > 5 > 2 > 4 > 3$ by Eq. (21) under the same condition. Therefore, both of the $Q(s)$ function and DM¡¯s risk attitude (parameter t) are significant in final ranking results and considering different DM¡¯s risk attitudes are necessary and reasonable in the final decision making.

Table 7: Ranking results by Eq. (20) with different DMs¡¯ risk attitudes

β and γ	Ranking results ($t = 0$)	Ranking results ($t = 0.5$)	Ranking results ($t = 1$)	Ranking results ($t = 2$)	Ranking results ($t = +\infty$)
$\beta = 0, \gamma = 1$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$
$\beta = 0.25, \gamma = 0.75$	$5 > 1 > 4 > 2 > 3$	$5 > 1 > 4 > 2 > 3$	$5 > 1 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$
$\beta = 0.5, \gamma = 0.5$	$3 > 4 > 5 > 1 > 2$	$4 > 3 > 5 > 1 > 2$	$1 > 5 > 4 > 3 > 2$	$1 > 5 > 2 > 4 > 3$	$1 > 2 > 5 > 4 > 3$
$\beta = 0.75, \gamma = 0.25$	$3 > 5 > 4 > 1 > 2$	$3 > 5 > 4 > 1 > 2$	$1 > 5 > 3 > 4 > 2$	$1 > 2 > 5 > 4 > 3$	$2 > 1 > 4 > 5 > 3$
$\beta = 1, \gamma = 0$	$3 > 5 > 4 > 1 > 2$	$3 > 5 > 4 > 1 > 2$	$1 > 5 > 3 > 4 > 2$	$1 > 2 > 5 > 4 > 3$	$2 > 1 > 4 > 5 > 3$

Table 8: Ranking results by Eq. (21) with different DMs¡¯ risk attitudes

β and γ	Ranking results ($t = 0$)	Ranking results ($t = 1$)	Ranking results ($t = 2$)	Ranking results ($t = +\infty$)
$\beta = 0, \gamma = 1$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$
$\beta = 0.25, \gamma = 0.75$	$5 > 1 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$	$5 > 1 > 4 > 2 > 3$	$1 > 5 > 4 > 2 > 3$
$\beta = 0.5, \gamma = 0.5$	$3 > 4 > 5 > 1 > 2$	$1 > 5 > 2 > 4 > 3$	$1 > 5 > 4 > 3 > 2$	$1 > 2 > 5 > 4 > 3$
$\beta = 0.75, \gamma = 0.25$	$3 > 5 > 4 > 1 > 2$	$1 > 2 > 5 > 4 > 3$	$1 > 5 > 3 > 4 > 2$	$2 > 1 > 4 > 5 > 3$
$\beta = 1, \gamma = 0$	$3 > 5 > 4 > 1 > 2$	$1 > 2 > 5 > 4 > 3$	$1 > 5 > 3 > 4 > 2$	$2 > 1 > 4 > 5 > 3$

6.2.4 Comparison with the method in [36]

We compare the method in [36] to show the superiority of the proposed method. Since that method also considers DMs¡¯ risk attitude and it has been shown to be more effective compared with the methods in [10] and [30]. The ranking function values and ranking results by Eq. (17) and by Eq. (19) in [36] are shown in Table 9 and Table 10 respectively. Note that Eq. (17) in [36] has the same $Q(s)$ with Eq. (20) in this paper, and Eq. (19) in [36] has the same $Q(s)$ with Eq. (21) in this paper.

Table 9: Ranking function values and ranking results by Eq. (17) in [36]

t	Ranking function values	Ranking results
$t = 0$	[0.9942,0.9965,0.9995,0.9920,1.0000]	$5 > 3 > 2 > 1 > 4$
$t = 0.5$	[0.9779,1.0000,0.9701,0.9594,0.9817]	$2 > 5 > 1 > 3 > 4$
$t = 1$	[0.9657,1.0000,0.9499,0.9373,0.9683]	$2 > 5 > 1 > 3 > 4$
$t = 2$	[0.9516,1.0000,0.9265,0.9116,0.9528]	$2 > 5 > 1 > 3 > 4$
$t = +\infty$	[0.9152,1.0000,0.8660,0.8454,0.9128]	$2 > 1 > 5 > 3 > 4$

It is obvious that the ranking results by the method in [36] are different from the proposed method. The reasons and advantages of the proposed method are as follows.

Firstly, in the calculation of optimal attribute weights, DMs¡¯ subjective preferences to alternatives/targets is modeled and combined with the objective distance from the positive and the negative solutions in the proposed method. Thus, DMs¡¯ intention can give guidance to the final ranking results. In addition, the integration of the subjective ranking intension and the objective ranking method can be adjusted by parameter ¦Â so that more flexibility is brought for DMs. By contrast, the method in [36] only utilizes the objective ranking method and fails to take DMs¡¯ preferences into consideration. Thus, the results shown in Table 9 and Table 10 are monotonous when DMs¡¯ risk attitude is fixed.

Table 10: Ranking function values and ranking results by Eq. (19) in [36]

t	Ranking function values	Ranking results
$t = 0$	[0.9942,0.9965,0.9995,0.9920,1.0000]	$5 > 3 > 2 > 1 > 4$
$t = 1$	[0.9543,1.0000,0.9310,0.9166,0.9558]	$2 > 5 > 1 > 3 > 4$
$t = 2$	[0.9657,1.0000,0.9499,0.9373,0.9683]	$2 > 5 > 1 > 3 > 4$
$t = +\infty$	[0.9152,1.0000,0.8660,0.8454,0.9128]	$2 > 1 > 5 > 3 > 4$

Secondly, in the ranking function for IFVs, though both of the methods in [36] and this paper take DMs¡¯ risk attitudes into account and utilize the closeness degree of TOPSIS to represent the amount of information conveyed by IFSs, the following difference should be highlighted. The method in [36] adopts $1 - \pi^2/2$ to represent the reliability of information conveyed by IFSs. However, it neglects that the uncertainty of information conveyed by IFSs contains two aspects: the fuzziness and the unknown, and $\pi^2/2$ can only express the unknown. By contrast, the proposed method constructs a novel IFE, which not only employs the hesitation degree to represent the unknown, but also exploits the difference between the membership degree and the non-membership degree to indicate the fuzziness (as shown in section 3.1). Thereby, the two aspects of uncertainty are well expressed.

Thirdly, decision information in multiple times is fused through formulating time sequence weights in the proposed method so that the final ranking result is more comprehensive and reasonable. However, the method in [36] neglects decision information in previous times and easily leads to partial ranking results.

7 Conclusions

A novel ranking method for IFSs is proposed based on intuitionistic fuzzy information fusion in multiple times. It is also applied to the threat assessment problem. The main contributions and advantages of the proposed method are as follows. Firstly, a novel IFE is devised after analyzing the drawbacks in the existing IFEs. The devised IFE not only sufficiently expresses the fuzziness and unknown of uncertainty of information conveyed by IFSs, but also has better distinguish ability in IFVs. Secondly, the closeness degree of TOPSIS is employed to represent the amount of information conveyed by IFS. It takes the distance from the IFV to the positive ideal point and the negative ideal point into consideration jointly. Thirdly, the closeness degree and the devised IFE are integrated by introducing the DM¡¯s risk attitude and a novel ranking method for IFVs is established. Fourthly, the comprehensive preference model to attributes from DMs is established to reflect the subjective ranking intension. Since the distance from the IFS to the positive ideal solutions and to the negative ideal solutions reflects the objective ranking result, a linear programming model, which aims at achieving minimum of the distance from the final ranking results to the subjective ranking intension and to the objective ranking result, is structured to calculate optimal attribute weights. Last but not least, the formulation of time sequence weights is structured and it is also used for fusing decision making information in multiple times. The simulation results and discussions indicate that the proposed method has properties of applicability and flexibility. Furthermore, it can provide reliable and comprehensive assessment results. However, there is still some work to do in the future including the discussion about more $Q(s)$ functions, considering the group consistency in the proposed method and extend our method to the interval intuitionistic fuzzy environment.

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