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Reliable hub-and-spoke network design problems under uncertainty through multi-objective programming

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Abstract

HLP (hub location problem) tries to find locations of hub facilities and assignment of nodes to extended facilities. Hubs are facilities to collect, arrange, and distribute commodities in telecommunication networks, cargo delivery systems, etc. Hubs are very crucial and their inaccessibility impresses on network whole levels. In this paper, first, total reliability of the network is defined based on considering the reliability values of nubs and arcs. Then, a reliable hub-and-spoke network design problem under uncertainty is introduced through the multi - objective programming method in which the parameters are random fuzzy variables. Indeed, we are making effort to either maximize the average reliability or minimize total cost. Then, the proposed reliable multi - objective hub-and-spoke network design problem under uncertainty is solved by a new method using Zimmermann fuzzy multi - objective programming and random fuzzy chance-constrained programming based on possibility theory. Finally, some benchmark problems are solved as numerical examples to clarify the described method and show its efficiency.

Keywords: Hub-and-Spoke, reliability, multi-objective programming, random fuzzy chance-constrained programming, possibility theory.

1 Introduction

Hub location problem (HLP) is one of the most attractive fields in location problems and relatively new extension of classical facility location problems. HLPs are classical optimization problems that have many practical applications in telecommunication networks, cargo delivery systems, railroad transports systems, airlines, postal networks and other delivery networks that have multiple send and receive. Hubs are used to decrease the number of transportation links between origin and destination nodes. It plays a great role in air trips, posting services, telecommunication industries and computer systems.

Hub location is one of the newest sciences for locating in different fields. In a typical HLP, the following questions are supposed to be answered: A) where to locate hub facilities B) how to allocate demand nodes to hubs for objective optimization. Hub has some important responsibilities in different areas. For instance in customer management, we ought to understand customer requirements in order to allocate work to spokes or in quality, risk and performance management respectively we are supposed to set quality, risk and performance standards for the hub and spokes and monitor performance and eventually report or initiate corrective actions wherever required. Hubs and spokes can include different positions with each other [1].

The problem of deciding which nodes should become hubs and how the flow in the network should be consolidated and redistributed defines the basic setting of a hub location problem. When a non-hub node has to be assigned to exactly one hub, we face a so-called single-allocation hub location problem. However, in multiple allocation a non-hub node can be allocated or connected to more than one hub.

In the last two decades, many researchers have investigated hub location problems; however, reliable hub location under uncertain environment is discussed newly and it is advanced field. In this section at first, the research on classical

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and original hub location problems are reviewed. Then, we review in brief some related works to this paper, specifically those considering uncertainty and reliability. Furthermore, we review the research on reliable hub location problem under uncertainty.

OKelly [24] introduced the first mathematical model in HLP. He presented a quadratic integer programming whose objective is to minimize the total delivery cost between nodes and locating a pre-specified number of hubs. The interested reader could review the papers by Campbell and OKelly [7] and Farahani et al. [10] to read full survey of hub location problems and its subcategories.

Recently, Yang et al. [27] discussed a fuzzy p-hub center problem with generalized value-at-risk criterion. Also, Qin and Gao [25] proposed p-hub location with fixed costs and uncertain flows. An et al. [3] proposed a reliable hub-and-spoke design problem in which the selection of backup hubs and alternative route are taken into account to handle hub disruptions. For these reliable network design problems, they introduced nonlinear mixed integer programming (MIP) models and presented linearize formulae. Lagrangian relaxation and branch-and-bound methods are developed to solve those difficult problems.

The reader is referred to the papers [21, 23, 28] for other recent work of hub location problems under uncertainty and to the works [4, 20] for reliable HLPs under uncertainty.

In most of the classical hub location problems, demand of the nodes or in other words, the flow between any origin - destination (O-D) nodes and transportation cost are considered as a deterministic parameter. However, in the reality because of many environmental aspects such as traffic intensity or climate changes, it is required to assume these deterministic parameters as uncertain parameters. One of the suggested approaches to confront uncertain variables in models is fuzzy stochastic programming. In this research, we study and develop a reliable multi - objective hub-and-spoke network design problem (RMOHSP) under uncertain framework by random fuzzy variables (RFVs).

The remainder of this paper is organized as follows. Section 2 reviews the literature on RMOHSP. Section 3 includes some basic concepts of fuzzy theory and fuzzy stochastic theory. In Section 4, novel RMOHSPs with RFVs are proposed, and new methods are presented to solve them satisfying optimistic and pessimistic decision makers (DMs) and using Zimmermann fuzzy multi - objective programming (MOP). In Section 5, we report our computational results concluded on the popular CAB data set, and some benchmark problems as numerical examples are solved and analyzed to show the efficiency of described methods. The conclusions and suggestions for further research are discussed in the last section.

2 Literature review

Deterministic modelling work began with the first mathematical model formulated in O'Kelly [24] which was named the single allocation p-hub median problem. A few years later O'Kelly introduced the single allocation hub location problem, which is similar to the p-hub median problem, but differing by accounting for some given fixed costs of setting up the hubs at specific nodes. This additional cost was added to the objective function, resulting in a solution suggesting the number of hubs to set up instead of using the predetermined p number of hubs.

Campbell [6] was the first to introduce the use of multiple allocations to the p-hub median problem and a couple of years later Campbell proposed two additional sets of problems, namely, the p-hub center problem and the hub covering problem. The median, including the hub location problem, the center, and the covering problems have become benchmark problems in the literature. All of them have been both improved upon computationally, via reformulation or exact or heuristic algorithm developments, or extended upon to capture more real life system properties, by several different authors. Marianov and Serra [19] developed a heuristic algorithm based on tabu search to solve a hub location problem with chance constraints. They built an M/D/c queuing model, which considered the number of airplanes waiting to land based on an assumed Poisson arrival rate of the aircraft. The result from the M/D/c queuing model was then used in the chance constraint requiring the average number of airplanes, arriving to some hub, to be less than or equal to the found value. They were able to solve networks up to 30 nodes in size.

Later Hult [13] studied the same problem as well presented exact solution methods based on pre-processing and a separation algorithm which were able to solve up to 50 node networks. Yang [26] developed a two - stage model considering stochastic demand and stochastic discount factors on the hub-hub links for an air freight hub location problem. They considered three possible discrete scenarios on demand and conducted a case study of a 10-node air freight network in Taiwan and China.

2.1 Reliable hub location problems with uncertainty

In real world problems, there might be vagueness or ambiguity in the parameters of the model, and in some cases, these parameters have both possibility and probability properties simultaneously. For example, the flow of commodities from one city to another might be uncertain for the DMs and this uncertainty has both fuzzy and random aspects. This is why optimization under uncertainty is discussed. In the literature of HLPs, there is less attention to the uncertainty of problem and most of the models have been formulated in deterministic environment.

A robust optimization model was introduced for multi - objective operation of capacitated p-hub location problems under uncertainty by Makui et al. [18]. Alumur et al. [2] addressed several aspects concerning hub location problems under uncertainty, presented a comprehensive model considering all source of uncertainty and used direct approach for solution.

Ghodratnama et al. [12] proposed a fuzzy possibilistic bi - objective hub covering problem considering production facilities, time horizons and transporter vehicles. They proposed a fuzzy goal programming approach to obtain solution. The interested readers are referred to the recent works [21, 23, 25, 27, 28] about hub location problems under uncertainty.

The reliable facility location problems have attracted the attention of many researchers. However, only several recent studies have considered reliable hub-and-spoke network design problems [15, 16, 30]. A single-allocation hub-and-spoke network was designed by Davari et al. [8] in which the reliability of the network was maximized. For this reliable hub-and-spoke problem, an expected value maximization version of the problem was proposed and a simulation-embedded simulated annealing was presented to solve the problem.

Zarandi et al. [29] designed a reliable single-allocation hub-and-spoke network using an interactive fuzzy goal programming. To model and solve the problem, a fuzzy goal programming approach was developed for design of network in an interactive manner between decision maker and the model. Recently, An et al. [3] proposed a set of reliable hub-and-spoke network design models, where the selection of backup hubs and alternative routes were taken into consideration to proactively handle hub disruptions. To solve these nonlinear mixed integer formulations for reliable network design problems, Lagrangian relaxation and Branch-and-Bound methods were developed to efficiently obtain optimal solutions.

Mohammadi et al. [20] designed a reliable logistics network with hub disruption under uncertainty. A new mixed-integer programming model was proposed to minimize the total sum of the nominal and expected failure costs. This model considered complete and partial disruption at hubs. In addition, they proposed a new hybrid meta-heuristic algorithm based on genetic and imperialist competitive algorithms to solve that presented problem. Azizi et al. [4] investigated the impact of hub failure in hub-and-spoke networks. In this study, they first presented a novel mathematical model that built hub-and-spoke systems under the risk of hub disruption. They adopted a linearization for the model and presented an efficient evolutionary approach with specifically designed operators. Obviously, there has not been much attempt in literature to reliable hub location problems with RFVs. Through maximization of average reliability and minimization of total cost in hub-and-spoke network design problems, we introduce RMOHSPs under random fuzzy framework.

3 Basic concepts

In this section, we review some technical terms presented by Dubois and Prade [9] and Katagiri et al.[14]. In the following definitions, we assume that $(\Omega, \Im, \Pr)$ is a probability space where Ω is a set of all outcomes of a random experiment, $\Im$ is called a σ-algebra and Pr is referred to as a probability measure, and $(\Theta, P(\Theta), Pos)$ is a possibility space where Θ is universe, $P(\Theta)$ is the power set of Θ and Pos is a possibility measure defined on fuzzy sets. A fuzzy set on $\Re$ is called a fuzzy number if it is normal, convex and upper semi - continuous and its support set is compact. LR fuzzy number [9] is a special fuzzy number used frequently.

Definition 3.1. *If X is a fuzzy number, then α-cut $X_\alpha = \{t \in \Re | \mu_X(t) \geq \alpha\} = [X_\alpha^-, X_\alpha^+]$ is a real interval for every $\alpha \in (0, 1]$. Let $C_\alpha(X) = \frac{X_\alpha^- + X_\alpha^+}{2}$ and $M_\alpha(X) = \frac{C_1(X) + C_\alpha(X)}{2}$. We define the mean value of fuzzy number X, denoted by $M(X)$, as follows:*

$$M(X) = \int_0^1 M_\alpha(X) d\alpha.$$

RFVs introduced by Liu [17] are used instead of fuzzy random variables when the mean of a random variable is expressed as a fuzzy number due to lake of information.

Definition 3.2. [14] *Let Γ be a collection of random variables. Then, a RFV $\bar{\tilde{A}}$ on the possibility space $(\Theta, P(\Theta), Pos)$ is defined by its membership function $\mu_{\bar{\tilde{A}}} : \Gamma \to [0, 1]$.*

We assume that the mean value of $\bar{\tilde{A}}$ is an LR fuzzy number $\tilde{M} = (m^0, m^1, \alpha, \beta)_{LR}$ defined by the following membership function:

$$\mu_{\tilde{M}}(t) = \begin{cases} L(\frac{m^0 - t}{\alpha}) & \text{if } m^0 - \alpha \leq t \leq m^0 \\ 1 & \text{if } m^0 \leq t \leq m^1 \\ R(\frac{t - m^1}{\beta}) & \text{if } m^1 \leq t \leq m^1 + \beta, \end{cases}$$

where $[m^0, m^1]$ denotes the peak of fuzzy number $\tilde{M}$ and α, β represent the left and right spread respectively; $L, R : [0,1] \to [0,1]$ with $L(0) = R(0) = 1$ and $L(1) = R(1) = 0$ are strictly decreasing, continuous functions. Indeed, a RFV is a function from the possibility space $(\Theta, P(\Theta), Pos)$ to the set of random variables.

Definition 3.3. *Let X be a RFV. The mean value of X is a fuzzy number defined by $E(X)$. We define the scalar expected value (SEV) of X, denoted by $SE(X)$ such that $SE(X) = M(E(X))$.*

4 Model formulation

In this section, the reliability is supposed to be represented completely. We are going to inspect the reliability as a crucial criterion for specified objective function, which will be demonstrated in the following part.

Kim and O'Kelly [16] presented a procedure to compute the reliability of a route based on single reliabilities of its arcs. However, it can be claimed that in some situations, it is impossible to represent the reliability appropriately.

It is known that in a hub-and-spoke network, to reach a destination, the flow must pass hubs and arcs. Therefore, the reliability of each hub is of high importance, along with the reliability of arcs. This paper adds the reliabilities of hubs to the procedure of reliability calculation by presenting a procedure to consecutive multiplication of single reliabilities of hubs and arcs.

Our problem assumes infinite capacity in hub nodes and a complete and symmetric hub network. Hub location problems include single allocation and multiple allocation. In single allocation models, a non-hub node could be connected to only one unique hub. However, in multiple allocation a non-hub node can be allocated or connected to more than one hub. Let $G = (N, E)$ be an undirected complete graph with node set $N = \{1, 2, ..., n\}$ and arc set E. Each arc (i, j) has a cost (time, flow, distance, etc.) C_{ij} where $C_{ij} = C_{ji}$ and satisfies triangular inequality ($C_{ij} \leq C_{im} + C_{mj}$ $\forall i, j, m$). Each origin - destination pair $i - j$ should be connected through hub nodes and it is assumed that there is a pre-defined reduction factor (α such that $0 \leq \alpha \leq 1$) between hub nodes so the cost between pairs is reduced, comparing when they connected directly.

Indeed, for inter-hub connections, a discount factor α reflecting the economy of scale associated with increased traffic between hubs is considered. It is known that in classical HLPs, to reach a destination, the flow must pass through at least one and at most two hubs (and the link between these two hubs). Therefore, the cost of routing one unit of flow from origin into destination j via hubs k and m can be calculated as:

$$C_{ijkm} = C_{ik} + \alpha C_{km} + C_{mj}. \tag{1}$$

Furthermore, it is supposed that same as the arcs, there should be a discount factor associated with hubs which will be shown as β in the consequence. It should be noted that this value could be completely different from the arcs discount factor. The reliabilities of these two sorts of flows would be shown in Figure 1 [11].

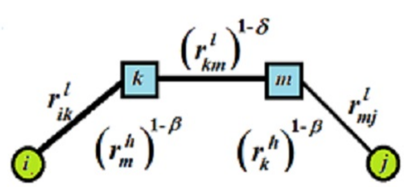

Figure 1: Components of reliability in a two-stop O/D route

The reliability of this route can be found as follows:

$$R_{ijkm} = {r_{ik}}^l ({r_k}^h)^{1-\beta} ({r_{km}}^l)^{1-\delta} ({r_m}^h)^{1-\beta} r^l_{mj}, \forall i, j, k, m, \tag{2}$$

where δ, $0 < \delta < 1$, is the discount factor for the reliability of hub-to-hub links and β, $0<\beta<1$, is the discount factor for the reliability hubs. Note that if the O/D (origin/destination) from node i to node j (W_{ij}), passes through only one single hub, then we will have $k = m$ and the R_{ijkk} will be computed as follows:

$$R_{ijkk} = {r_{ik}}^{l}({r_k}^{h})^{1-\beta}{r_{kj}}^{l}. \tag{3}$$

Also, in case both the origin and destination of the O/D pair are hubs, we will have $i = k$ and $j = m$, hence the R_{kmkm} will be computed as follows:

$$R_{kmkm} = (r_k^h)^{1-\beta}(r_{km}^l)^{1-\delta}(r_m^h)^{1-\beta}. \tag{4}$$

Now, we will discuss a RMOHSP in single (RMOSAHSP) and multiple (RMOMAHSP) states by making effort to either maximize the average reliability or minimize total cost.

4.1 RMOHSP in multiple stat

In multiple allocation, each none-hub node could be allocated to more than one hub node. In the following, the sets, parameters and decision variables of the RMOHSP are defined:

i, j:	Non-hub node index
k, m:	Hub node index
F_k:	The fixed cost for hub k
C_{ijkm}:	The cost per unit of the flow between nodes i and j routed via hubs k and m
W_{ij}:	The flow between nodes i and j
R_{ijkm}:	The reliability of the flow between nodes i and j which is routed via hubs k and m
X_{ijkm}:	The fraction of O/D flow W_{ij} that is routed via the hubs k and m
Z_K:	Binary variable which is equal to 1 if a hub is located at node k

The reliable multi - objective MIP for the RMOHSP in multiple allocation is introduced as follows:
Problem 1:

$$\max\ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{5}$$

$$\min\ Z = \sum_k F_k Z_k + \sum_i \sum_j \sum_k \sum_m w_{ij} C_{ijkm} X_{ijkm} \tag{6}$$

$$\text{s.t.} \sum_i \sum_j \sum_m W_{ij} X_{ijkm} \leq \Gamma_k Z_k, \quad \forall k, \tag{7}$$

$$\sum_k \sum_m X_{ijkm} = 1, \quad \forall i, j, \tag{8}$$

$$\sum_{m(m \neq k)} X_{ijkm} + \sum_m X_{ijmk} \leq Z_k, \quad \forall i, j, k, \tag{9}$$

$$X_{ijkm},\ \overline{R} \geq 0,\ \forall i, j, k, m, \tag{10}$$

$$z_k \in \{0, 1\},\ \forall k. \tag{11}$$

Objective function (5) maximizes the mean reliability of the network which is calculated as the sum of reliabilities for all O/D pairs divided by the total number of O/D pairs. Objective function (6) minimizes the amount of money spent on establishment of hubs plus transportations costs. Constraint (7) is the capacity constraint that limits the inbound plus outbound flows to and from each hub by the hub capacity values. Constraints (8) guarantee that there is a single path connecting the origin and destination nodes of every O/D flow. Constraints (9) prohibit flows from being routed via a non-hub node. Finally, constraints (10) and (11) are the standard non-negativity and integrality constraints.

The above problem is a multi - objective linear programming (MOLP) problem. Now, Zimmermann fuzzy approach [31] will be explained to solve this multi - objective optimization problem. In this approach, Bellman and Zadeh's max-min operator [5] has been used. Let $R^{(1)}$ be upper bound of $\overline{R}$, and $R^{(1)} - P^{(1)}$ be its initial value and $Z^{(1)}$ be lower bound of Z and $Z^{(1)} + q^{(1)}$ be its initial values. The objective functions of problem1 have the following membership functions:

$$\mu_1(\overline{R}) = \begin{cases} 0 & \text{if } \overline{R} \leq R^{(1)} - P^{(1)} \\ \frac{\overline{R} - (R^{(1)} - P^{(1)})}{P^{(1)}} & \text{if } R^{(1)} - P^{(1)} \leq \overline{R} \leq R^{(1)} \\ 1 & \text{if } \overline{R} \geq R^{(1)}, \end{cases} \tag{12}$$

$$\mu_2(Z)=\begin{cases} 1 & \text{if } Z\le Z^{(1)} \\ \frac{-Z+(Z^{(1)}+q^{(1)})}{q^{(1)}} & \text{if } Z^{(1)}\le Z\le Z^{(1)}+q^{(1)} \\ 0 & \text{if } Z\ge Z^{(1)}+q^{(1)}. \end{cases} \tag{13}$$

By applying these membership functions and Bellman and Zadeh's max-min operator, above multi objective problem is converted to the following problem:
Problem 2:

$$\max\ \lambda \tag{14}$$

$$s.t.\ \frac{\sum_i\sum_j\sum_k\sum_m R_{ijkm}X_{ijkm}}{|N|^2}\ge R^{(1)}-(1-\lambda)P^{(1)}, \tag{15}$$

$$\sum_k F_kZ_k+\sum_i\sum_j\sum_k\sum_m w_{ij}C_{ijkm}X_{ijkm}\le Z^{(1)}+(1-\lambda)q^{(1)}\ , \tag{16}$$

Constraints 7-11 from *Problem 1.*

The above problem is a mixed integer-programming problem which has been easily solved with one of the MIP solvers like GAMS software. Furthermore, all MOP problems obtained later will be solved by Zimmermann fuzzy approach.

4.2 RMOHSP in single stat

First, the RMOSAHSP is introduced which is a MOP problem. Then, the proposed problem is converted to a MIP problem by applying Zimmermann fuzzy approach. The sets, parameters and decision variables are de?ned as follows:

- i,j: Non-hub node index
- k,m: Hub node index
- α: Discount factor reflecting the economy of scale associated with increased traffic between hubs
- F_k: The fixed cost for hub k
- C_{ijkm}: The cost per unit of the flow between nodes i and j routed via hubs k and m
- W_{ij}: The flow between nodes i and j
- R_{ijkm}: The reliability of the flow between nodes i and $j$which is routed via hubs k and m
- X_{ijkm}: The fraction of O/D flow W_{ij} that is routed via the hubs k and m
- z_{ik}: Binary variable which is equal to 1 if node i is connected to node k
- z_{kk}: Binary variable which is equal to 1 if a hub is located at node k (if and only if k is a hub node).

We introduce the multi - objective MIP model for single allocation of the RMOHSP as follows:
Problem 3:

$$\max\ \overline{R}=\frac{\sum_i\sum_j\sum_k\sum_m R_{ijkm}X_{ijkm}}{|N|^2} \tag{17}$$

$$\min\ Z=\sum_k F_kZ_{kk}+\sum_i\sum_j\sum_k\sum_m w_{ij}C_{ijkm}X_{ijkm} \tag{18}$$

$$\text{s.t.}\ \sum_i\sum_j\sum_m W_{ij}X_{ijkm}\le \Gamma_kZ_{kk},\quad \forall k, \tag{19}$$

$$\sum_k z_{ik}=1,\quad \forall i, \tag{20}$$

$$z_{ik}\le z_{kk},\quad \forall i,k, \tag{21}$$

$$\sum_m X_{ijkm}=z_{ik},\quad \forall i,j,k, \tag{22}$$

$$\sum_k X_{ijkm}=z_{jm},\quad \forall i,j,m, \tag{23}$$

$$X_{ijkm},\ \overline{R}\ge 0,\quad \forall i,j,k,m, \tag{24}$$

$$z_{ik}\in\{0,1\}\ ,\ \forall i,k. \tag{25}$$

Most real decision making issues include parameters that are unknown, and in some cases DMs face with fuzzy and probability uncertainty simultaneously. As a new approach, it is useful to consider the fuzziness and randomness of the parameters as RFVs when the mean and the variance of a random variable are estimated as fuzzy numbers due to a lack of information. Therefore, a RMOHSP with RFVs will be discussed.

4.3 RMOMAHSP with RFVs

The multi - objective MIP model for multiple allocation of the RMOHSP with RFVs are introduced as follows:
Problem 4:

$$\max \ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{26}$$

$$\min \ \bar{\tilde{Z}} = \sum_k \bar{\tilde{F}}_k Z_k + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \tag{27}$$

$$\text{s.t.} \ \sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \leq \bar{\tilde{\Gamma}}_k Z_k, \quad \forall k, \tag{28}$$

Constraints 8-11 from *Problem 1.*

In the above model, $\bar{\tilde{F}}_k$, $\bar{\tilde{w}}_{ij}$ and $\bar{\tilde{\Gamma}}_k$ are RFVs. In this paper, we supposed that, Γ is a collection of all possible random variables $\bar{\gamma} = \gamma^0 + \gamma^1 \bar{t}$ in which $\bar{t}$ is a random variable.

The $\bar{\tilde{F}}_k$, $\bar{\tilde{w}}_{ij}$ and $\bar{\tilde{\Gamma}}_k$ are expressed as RFVs with the following membership functions:

$$\mu_{\bar{\tilde{F}}_k}(\bar{\gamma}_k) = \{\mu_{\tilde{M}_k}(\gamma_k^0) | \bar{\gamma}_k = \gamma_k^0 + \gamma_k^1 \bar{t}\}, \ \forall \bar{\gamma}_k \in \Gamma, \tag{29}$$

$$\mu_{\bar{\tilde{W}}_{ij}}(\bar{\gamma}_{ij}) = \{\mu_{\tilde{M}_{ij}}(\gamma_{ij}^0) | \bar{\gamma}_{ij} = \gamma_{ij}^0 + \gamma_{ij}^1 \bar{t}\}, \ \forall \bar{\gamma}_{ij} \in \Gamma, \tag{30}$$

$$\mu_{\bar{\tilde{\Gamma}}_k}(\bar{\rho}_k) = \{\mu_{\tilde{N}_k}(\rho_k^0) | \bar{\rho}_k = \upsilon_k^0 + \rho_k^1 \bar{t}\}, \ \forall \bar{\rho}_k \in \Gamma, \tag{31}$$

where $\tilde{M}_k = (m_k^0, m_k^1, \alpha_k, \beta_k)$, $\tilde{M}_{ij} = (m_{ij}^0, m_{ij}^1, \alpha_{ij}, \beta_{ij})$and $\tilde{N}_k = (n_k^0, n_k^1, \delta_k, \varphi_k)$ represent the mean value of $\bar{\tilde{F}}_k$, $\bar{\tilde{w}}_{ij}$ and $\bar{\tilde{\Gamma}}_k$ respectively and $\bar{t}$ is a random variable with the cumulative distribution function T.

In what follows, the SEV model of Problem 4 will be proposed by using the SEV of RFVs.

4.3.1 Scalar expected value model

The SEV model of Problem 4, represented by SE- RMOMAHSP with RFVs, is defined as follows:
Problem 5:

$$\max \ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{32}$$

$$\min \ SE(\bar{\tilde{Z}}) = \sum_k SE(\bar{\tilde{F}}_k) Z_k + \sum_i \sum_j \sum_k \sum_m SE(\bar{\tilde{w}}_{ij}) C_{ijkm} X_{ijkm} \tag{33}$$

$$\text{s.t.} \ \sum_i \sum_j \sum_m SE(\bar{\tilde{W}}_{ij}) X_{ijkm} \leq SE(\bar{\tilde{\Gamma}}_k) Z_k, \quad \forall k, \tag{34}$$

Constraints 8-11 from *Problem 1.*

By this model whose all parameters are real, solutions with optimal scalar expected return subject to constraints will be obtained. We solve the obtained deterministic MIP problem by one of the MIP solvers. The obtained optimal solution is called SE-optimal solution of the original problem.

4.3.2 Random fuzzy chance-constrained programming

Let $\bar{\tilde{Z}} = \sum_k \bar{\tilde{F}}_k Z_k + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm}$. From extension principle of Zadeh, the membership functions of

$\bar{\tilde{Z}}$ and $\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm}$ are given as

$$\mu_{\bar{\tilde{Z}}}(\bar{u}) = \sup_{\bar{\gamma}_k, \bar{\gamma}_{ij}} \left\{ \min\left\{ \min_k \mu_{\bar{\tilde{F}}_k}(\bar{\gamma}_k), \min_{i,j} \mu_{\bar{\tilde{W}}_{ij}}(\bar{\gamma}_{ij}) \right\} | \bar{u} = \sum_k \bar{\gamma}_k Z_k + \sum_i \sum_j \sum_k \sum_m \bar{\gamma}_{ij} C_{ijkm} X_{ijkm} \right\} = \sup_{\gamma_k^0, \gamma_{ij}^0} \left\{ \min\left\{ \min_k \mu_{\tilde{M}_k}(\gamma_k^0), \min_{i,j} \mu_{\tilde{M}_{ij}}(\gamma_{ij}^0) \right\} | \bar{u} = \sum_k (\gamma_k^0 + \gamma_k^1 \bar{t}) Z_k + \sum_i \sum_j \sum_k \sum_m (\gamma_{ij}^0 + \gamma_{ij}^1 \bar{t}) C_{ijkm} X_{ijkm} \right\}, \tag{35}$$

$$\mu_{\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm}}(\bar{u}_k) = \sup_{\bar{\gamma}_{ij}} \left\{ \min_{i,j} \mu_{\bar{\tilde{W}}_{ij}}(\bar{\gamma}_{ij}) | \bar{u}_k = \sum_i \sum_j \sum_m \bar{\gamma}_{ij} X_{ijkm} \right\} = \sup_{\gamma_{ij}^0} \left\{ \min_{i,j} \mu_{\tilde{M}_{ij}}(\gamma_{ij}^0) | \bar{u}_k = \sum_i \sum_j \sum_m (\gamma_{ij}^0 + \gamma_{ij}^1 \bar{t}) X_{ijkm} \right\}. \tag{36}$$

The probability that the objective function $\bar{\tilde{Z}}$ is smaller than or equal to upper bound variable Z is introduced as follows:

$$\tilde{P} = \Pr\left\{ \omega | \tilde{Z}(\omega) = \sum_k \tilde{F}_k(\omega) Z_k + \sum_i \sum_j \sum_k \sum_m \tilde{w}_{ij}(\omega) C_{ijkm} X_{ijkm} \le Z \right\}, \tag{37}$$

where $\tilde{P}$ is a fuzzy set defined by the following membership function:

$$\mu_{\tilde{P}}(p) = \sup_{\bar{u}} \left\{ \mu_{\bar{\tilde{Z}}}(\bar{u}) | p = \Pr\{\omega | u(\omega) \le Z\} \right\}. \tag{38}$$

Now, $\bar{\tilde{Z}} \le Z$ is a random fuzzy constraint, whose possibility with a permissible probability level θ is defined as

$$Pos_\theta(\bar{\tilde{Z}} \le Z) = \sup_p \left\{ \mu_{\tilde{P}}(p) | p \ge \theta \right\}, \tag{39}$$

and whose necessity with a permissible probability level θ is

$$Nec_\theta(\bar{\tilde{Z}} \le Z) = \inf_p \left\{ 1 - \mu_{\tilde{P}}(p) | p \ge \theta \right\}. \tag{40}$$

Furthermore, the degree of possibility $Pos_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_k)$ that the random fuzzy value $\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm}$ is smaller than or equal to the random fuzzy value $\bar{\tilde{\Gamma}}_k Z_k$ with a permissible probability level θ_k is defined as

$$Pos_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_k) = \sup_{p_k} \left\{ \mu_{\tilde{P}_k}(p_k) | p_k \ge \theta_k \right\}, \qquad \forall k \tag{41}$$

where $\tilde{P}_k = \Pr\left\{ \omega | \sum_i \sum_j \sum_m \tilde{W}_{ij}(\omega) X_{ijkm} \le \tilde{\Gamma}_k(\omega) Z_k \right\}$ is a fuzzy set with the following membership function:

$$\mu_{\tilde{P}_k}(p_k) = \sup_{\bar{u}_{k1}, \bar{u}_{k2}} \left\{ \min\{ \mu_{\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm}}(\bar{u}_{k1}), \mu_{\bar{\tilde{\Gamma}}_k}(\bar{u}_{k2}) \} | p_k = \Pr\{\omega | u_{k1}(\omega) \le u_{k2}(\omega) Z_k\} \right\}. \tag{42}$$

The necessity of $\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_k$ with a permissible probability level θ_k is defined as

$$Nec_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_k) = \inf_{p_k} \left\{ 1 - \mu_{\tilde{P}_k}(p_k) | p_k \ge \theta_k \right\}, \quad \forall k. \tag{43}$$

For optimistic DMs who wish to prefer a risk, an optimistic model of the original RMOMAHSP with RFVs problem is proposed as follows by applying the RFCCP based on possibility measures:

Problem 6:

$$\max\ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{44}$$

$$\min\ Z \tag{45}$$

$$\text{s.t.}\ \ Pos_\theta(\sum_k \bar{\tilde{F}}_k Z_k + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \le Z) \ge \eta, \tag{46}$$

$$Pos_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_k) \ge \lambda_k, \quad \forall k, \tag{47}$$

Constraints 8-11 from *Problem 1.*

where η and λ_k are permissible possibility levels defined by the decision makers. In what follows, we try to transform (46) and (47) into deterministic constraints. We obtain the following theorem:

Theorem 4.1.

$$\begin{aligned}&Pos_{\theta}(\sum_k \bar{\tilde{F}}_k Z_k + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \leq Z) \geq \eta \\ \Leftrightarrow \quad & \sum_k (m_k^0 + T^*(\theta)\gamma_k^1 - L^*(\eta)\alpha_k) Z_k + \sum_i \sum_j \sum_k \sum_m (m_{ij}^0 + T^*(\theta)\gamma_{ij}^1 - L^*(\eta)\alpha_{ij}) C_{ijkm} X_{ijkm} \leq Z,\end{aligned} \tag{48}$$

$$\begin{aligned}&Pos_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \leq \bar{\tilde{\Gamma}}_k Z_k) \geq \lambda_k \\ \Leftrightarrow \quad & \sum_i \sum_j \sum_m (m_{ij}^0 + T^*(1-\theta)\gamma_{ij}^1 - L^*(\lambda_k)\alpha_{ij}) X_{ijkm} \leq (n_k^1 + T^*(1-\theta)\rho_k^1 + \varphi_k R^*(\lambda_k)) Z_k,\end{aligned} \tag{49}$$

where L^* and T^* are pseudo inverse function and defined as $L^*(\lambda) = \sup\{t | L(t) \geq \lambda\},\ T^*(\lambda) = \inf\{t | T(t) \geq \lambda\}$.

For the proof of this theorem and Theorem 4.2, the reader can be referred to the recent study [22] about fuzzy stochastic programming approach.

As a direct effect of Theorem 4.1, problem 6 is equivalently transformed into the following problem:

Problem 7:

$$\max\ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{50}$$

$$\min\ Z \tag{51}$$

$$\text{s.t.} \sum_k (m_k^0 + T^*(\theta)\gamma_k^1 - L^*(\eta)\alpha_k) Z_k + \sum_i \sum_j \sum_k \sum_m (m_{ij}^0 + T^*(\theta)\gamma_{ij}^1 - L^*(\eta)\alpha_{ij}) C_{ijkm} X_{ijkm} \leq Z, \tag{52}$$

$$\sum_i \sum_j \sum_m (m_{ij}^0 + T^*(1-\theta)\gamma_{ij}^1 - L^*(\lambda_k)\alpha_{ij}) X_{ijkm} \leq (n_k^1 + T^*(1-\theta)\rho_k^1 + \varphi_k R^*(\lambda_k)) Z_k, \quad \forall k, \tag{53}$$

Constraints 8-11 from *problem 1.*

The above problem is a deterministic multi - objective MIP problem. The optimal solution of problem 7 is called P-optimal solution of original RMOMAHSP with RFVs.

For pessimistic and risk - averse DMs, we introduce the necessity-based model of original problem with RFVs as follows by applying the RFCCP:

Problem 8:

$$\max\ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{54}$$

$$\min\ Z \tag{55}$$

$$\text{s.t. } Nec_{\theta}(\sum_k \bar{\tilde{F}}_k Z_k + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \leq Z) \geq \eta, \tag{56}$$

$$Nec_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \leq \bar{\tilde{\Gamma}}_k Z_k) \geq \lambda_k, \quad \forall k, \tag{57}$$

Constraints 8-11 from *problem 1.*

Now, we obtain the following theorem for converting relations (55) and (56) to deterministic constraints.

Theorem 4.2.

$$\begin{aligned}&Nec_{\theta}(\sum_k \bar{\tilde{F}}_k Z_k + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \leq Z) \geq \eta \\ \Leftrightarrow \quad & \sum_k (m_k^1 + T^*(\theta)\gamma_k^1 + R^*(1-\eta)\beta_k) Z_k + \sum_i \sum_j \sum_k \sum_m (m_{ij}^1 + T^*(\theta)\gamma_{ij}^1 + R^*(1-\eta)\beta_{ij}) C_{ijkm} X_{ijkm} \leq Z,\end{aligned} \tag{58}$$

$$\begin{aligned}&Nec_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \leq \bar{\tilde{\Gamma}}_k Z_k) \geq \lambda_k \\ \Leftrightarrow \quad & \sum_i \sum_j \sum_m (m_{ij}^1 + T^*(1-\theta)\gamma_{ij}^1 + R^*(1-\lambda_k)\beta_{ij}) X_{ijkm} \leq (n_k^0 + T^*(1-\theta)\rho_k^1 - \delta_k R^*(1-\lambda_k)) Z_k,\end{aligned} \tag{59}$$

where R^* is pseudo inverse function defined as $R^*(\lambda) = \sup\{t | R(t) \geq \lambda\}$.

From Theorem 4.2, Problem 8 is equivalently rewritten as follows which is deterministic multi - objective MIP problem:

Problem 9:

$$\max\ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{60}$$

$$\min\ Z \tag{61}$$

$$\text{s.t. } \sum_k (m_k^1 + T^*(\theta)\gamma_k^1 + R^*(1-\eta)\beta_k) Z_k + \sum_i \sum_j \sum_k \sum_m (m_{ij}^1 + T^*(\theta)\gamma_{ij}^1 + R^*(1-\eta)\beta_{ij}) C_{ijkm} X_{ijkm} \leq Z, \tag{62}$$

$$\sum_i \sum_j \sum_m (m_{ij}^1 + T^*(1-\theta)\gamma_{ij}^1 + R^*(1-\lambda_k)\beta_{ij}) X_{ijkm} \leq (n_k^0 + T^*(1-\theta)\rho_k^1 - \delta_k R^*(1-\lambda_k)) Z_k, \quad \forall k, \tag{63}$$

Constraints 8-11 from *Problem 1.*

The optimal solution of Problem 9 is called N-optimal solution of original RMOMAHSP with RFVs. The obtained parametric MIP problems are solved for different permissible levels by one of the MIP solvers.

4.4 RMOSAHSP with RFVs

We introduce the multi - objective MIP model for single allocation of the RMOHSP with RFVs as follows:
Problem 10:

$$\max \ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{64}$$

$$\min \ \bar{\tilde{Z}} = \sum_k \bar{\tilde{F}}_k Z_{kk} + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \tag{65}$$

$$\text{s.t.} \sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_{kk}, \quad \forall k, \tag{66}$$

Constraints 20-25 from *Problem 3.*

Similar to the methodology used in section 4.3, we proposed the scalar expected value model and the random fuzzy chance-constrained programming approach to the RMOSAHSP with RFVs.

4.4.1 Scalar expected value model

The SEV model of Problem 10, represented by SE- RMOSAHSP with RFVs, is defined as follows:
Problem 11:

$$\max \ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{67}$$

$$\min \ SE(\bar{\tilde{Z}}) = \sum_k SE(\bar{\tilde{F}}_k) Z_{kk} + \sum_i \sum_j \sum_k \sum_m SE(\bar{\tilde{w}}_{ij}) C_{ijkm} X_{ijkm} \tag{68}$$

$$\text{s.t.} \sum_i \sum_j \sum_m SE(\bar{\tilde{W}}_{ij}) X_{ijkm} \le SE(\bar{\tilde{\Gamma}}_k) Z_{kk}, \quad \forall k, \tag{69}$$

Constraints 20-25 from *Problem 3.*

4.4.2 Random fuzzy chance-constrained programming

By applying the RFCCP based on possibility measures, an optimistic model of the original RMOSAHSP with RFVs is proposed as follows:
Problem 12:

$$\max \ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{70}$$

$$\min \ Z \tag{71}$$

$$\text{s.t.} \ Pos_\theta(\sum_k \bar{\tilde{F}}_k Z_{kk} + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \le Z) \ge \eta, \tag{72}$$

$$Pos_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_{kk}) \ge \lambda_k, \quad \forall k, \tag{73}$$

Constraints 20-25 from *Problem 3.*

By considering Theorem 4.1, the optimistic model of the original RMOSAHSP with RFVs is equivalently transformed into the following problem:
Problem 13:

$$\max \ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{74}$$

$$\min \ Z \tag{75}$$

$$\text{s.t.} \sum_k (m_k^0 + T^*(\theta)\gamma_k^1 - L^*(\eta)\alpha_k) Z_{kk} + \sum_i \sum_j \sum_k \sum_m (m_{ij}^0 + T^*(\theta)\gamma_{ij}^1 - L^*(\eta)\alpha_{ij}) C_{ijkm} X_{ijkm} \le Z, \tag{76}$$

$$\sum_i \sum_j \sum_m (m_{ij}^0 + T^*(1-\theta)\gamma_{ij}^1 - L^*(\lambda_k)\alpha_{ij}) X_{ijkm} \le (n_k^1 + T^*(1-\theta)\rho_k^1 + \varphi_k R^*(\lambda_k)) Z_{kk}, \quad \forall k, \tag{77}$$

Constraints 20-25 from *Problem 3.*

Furthermore, we propose a pessimistic model of the original RMOSAHSP with RFVs as follows by using necessity measures:
Problem 14:

$$\max \ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \tag{78}$$

$$\min \ Z \tag{79}$$

$$\text{s.t.} \ Nec_\theta(\sum_k \bar{\tilde{F}}_k Z_{kk} + \sum_i \sum_j \sum_k \sum_m \bar{\tilde{w}}_{ij} C_{ijkm} X_{ijkm} \le Z) \ge \eta, \tag{80}$$

$$Nec_{\theta_k}(\sum_i \sum_j \sum_m \bar{\tilde{W}}_{ij} X_{ijkm} \le \bar{\tilde{\Gamma}}_k Z_{kk}) \ge \lambda_k, \quad \forall k, \tag{81}$$

Constraints 20-25 from *Problem 3.*

Finally, this pessimistic model is equivalently transformed into the following deterministic programming problem:

Problem 15:

$$\max\ \overline{R} = \frac{\sum_i \sum_j \sum_k \sum_m R_{ijkm} X_{ijkm}}{|N|^2} \quad (82)$$

$$\min\ Z \quad (83)$$

$$\text{s.t. } \sum_k (m_k^1 + T^*(\theta)\gamma_k^1 + R^*(1-\eta)\beta_k) Z_{kk} + \sum_i \sum_j \sum_k \sum_m (m_{ij}^1 + T^*(\theta)\gamma_{ij}^1 + R^*(1-\eta)\beta_{ij}) C_{ijkm} X_{ijkm} \leq Z, \quad (84)$$

$$\sum_i \sum_j \sum_m (m_{ij}^1 + T^*(1-\theta)\gamma_{ij}^1 + R^*(1-\lambda_k)\beta_{ij}) X_{ijkm} \leq (n_k^0 + T^*(1-\theta)\rho_k^1 - \delta_k R^*(1-\lambda_k)) Z_{kk}, \quad \forall k, \quad (85)$$

Constraints 20-25 from *Problem 3.*

5 Computational results

To show the applicability and efficiency of the methods discussed in this study, numerical examples of popular CAB data set [24] have been considered and solved. O'Kelly [24] represented the CAB data set for hub location problems based on Civil Aeronautics Board in 1970, which is generated by airline passengers flow between 25 cities in the United States.

The proposed deterministic problems obtained from both RMOMAHSP and RMOSAHSP in this paper (in both deterministic and random fuzzy parameters) have been coded in GAMS v24.1.2 and run on a PC equipped with a 2.9 GHz Intel Pentium(R)CPU, 4GB of RAM and Windows 7 operating system.

In our experiments, the hub reliability parameters (r^h) are selected randomly from interval [0.9, 1]. The reliability value of each arc (r^l) is considered to be a value within interval [0.5, 1] based on its length. Other parameters selected for testing the described methods are reported in Table 1.

Table1. Parameters of test problems

r^h	r^l	α	δ	β
U[0.9, 1]	[0.5, 1]	{0.6, 0.8}	{0.95,0.9}	{0.9,0.8}

First, we propose numerical results of RMOMAHSP and RMOSAHSP, and then sensitivity analysis of the proposed numerical results are given when the parameters of the problems such as α, δ, β, etc. are changed.

The obtained results of RMOMAHSP and RMOSAHSP for |N|=10, 15, 20, 25 are specified in Tables 2 and 3 respectively. For each test problem, different values are set. In this table, discount factor (α) is displayed in the first column, reliability discount factor values for inter-hub links (δ) and for the hubs (β) are shown in the second and third columns, respectively. The columns labelled "Z" and "$\overline{R}$" give optimal value of the objective functions (total cost and average reliability respectively) for each test problem. The next column labelled as "Hubs" shows the hubs in the optimal solution of the corresponding problem instances. Furthermore, the computer running times are reported in last column entitled as "CPU".

Table 2. Computational results of RMOMAHSP for CAB data set

a) |N|=25

α	δ	β	Z	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1385.567	0.871	2,4,12,13,24	0:12:01.313
		0.85	1383.044	0.869	2,4,12,13,24	0:12:02.135
	0.90	0.9	1381.912	0.865	2,4,12,13,24	0:12:11.192
		0.85	1379.418	0.862	2,4,12,13,24	0:11:22.146
0.8	0.95	0.9	1470.266	0.867	2,4,7,12,24	0:11:53.055
		0.85	1468.451	0.863	2,4,12,13,24	0:08:48.229
	0.90	0.9	1466.572	0.860	2,4,7,12,24	0:08:15.578
		0.85	1464.118	0.857	2,4,8,12,13	0:08:45.186

b) |N|=20

α	δ	β	Z	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1309.124	0.877	4,13,14,17,19	0:02:24.976
		0.85	1307.946	0.873	4,13,14,17,19	0:02:28.330
	0.90	0.9	1306.086	0.871	4,13,14,17,19	0:02:35.373
		0.85	1304.712	0.867	4,13,14,17,19	0:02:31.085
0.8	0.95	0.9	1383.049	0.874	1,4,7,18,19	0:01:56.500
		0.85	1382.548	0.870	4,13,14,17,19	0:02:26.508
	0.90	0.9	1378.796	0.869	1,4,7,18,19	0:01:55.964
		0.85	1378.672	0.865	4,13,14,17,19	0:02:12.158

c) |N|=15

α	δ	β	Z	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1364.421	0.884	4,6,7,8,12,14	0:00:09.66
		0.85	1356.225	0.886	4,6,7,8,12,14	0:00:15.436
	0.90	0.9	1361.026	0.878	4,6,7,8,12,14	0:00:12.725
		0.85	1361.05	0.874	4,6,7,8,12,14	0:00:10.771
0.8	0.95	0.9	1451.876	0.874	1,6,7,11,12	0:00:20.695
		0.85	1451.287	0.870	2,4,8,13,14	0:00:25.109
	0.90	0.9	1446.862	0.869	1,2,4,7,8	0:00:24.777
		0.85	1446.073	0.866	2,4,8,13,14	0:00:21.216

d) |N|=10

α	δ	β	Z	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	997.245	0.855	3,4,6,7,8	0:00:03.510
		0.85	997.146	0.850	3,4,6,7,8	0:00:03.516
	0.90	0.9	993.694	0.848	4,6,7,8	0:00:03.398
		0.85	994.098	0.844	2,4,7,8	0:00:03.322
0.8	0.95	0.9	1030.188	0.852	4,6,7,8	0:00:03.204
		0.85	1029.919	0.848	4,6,7,8	0:00:03.330
	0.90	0.9	1026.270	0.847	4,6,7,8	0:00:03.270
		0.85	1026.024	0.842	4,6,7,8	0:00:03.141

Table 3. Computational results of RMOSAHSP for CAB data set

a) |N|=25

α	δ	β	Z	$\overline{R}$	hubs	CPU (h:m:s)
0.6	0.95	0.9	1399.632	0.793	2, 12, 13	3:47:12.793
		0.85	1399.632	0.792	2, 12, 13	19:01:25.600
	0.90	0.9	1399.632	0.787	2, 12, 13	2:43:08.639
		0.85	1341.873	0.778	2, 12, 21	0:55:05.856
0.8	0.95	0.9	1538.013	0.793	2, 12, 13	2:45:08.639
		0.85	1517.345	0.788	12,13,20	2:09:06.600
	0.90	0.9	1538.013	0.788	12,13,20	2:25:05.450
		0.85	1539.541	0.784	12,13,25	1:13:14.497

b) |N|=20

α	δ	β	Z	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1368.009	0.787	13,19,20	0:30:54.446
		0.85	1368.009	0.785	13,19,20	0:23:47.781
	0.90	0.9	1361.688	0.781	13,19,20	0:21:40.531
		0.85	1361.688	0.779	13,19,20	0:19:20.835
0.8	0.95	0.9	1474.412	0.787	13,19,20	0:29:18.977
		0.85	1474.412	0.785	13,19,20	0:27:48.096
	0.90	0.9	1474.412	0.782	13,19,20	0:29:33.697
		0.85	1474.412	0.780	13,19,20	0:23:25.394

c) |N|=15

α	δ	β	Z	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1373.524	0.771	5,7	0:2:08.049
		0.85	1386.440	0.768	5,11	0:1:30.354
	0.90	0.9	1386.440	0.768	5,11	0:1:52.617
		0.85	1386.44	0.766	5,11	0:1:44.346
0.8	0.95	0.9	1481.860	0.771	6,11	0:1:47.563
		0.85	1481.860	0.768	6,11	0:1:49.727
	0.90	0.9	1432.086	0.766	5,11	0:1:46.266
		0.85	1432.086	0.766	5,11	0:2:01.301

d) |N|=10

α	δ	β	Z	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	944.661	0.755	6,7	0:0:4.805
		0.85	944.661	0.751	6,7	0:0:5.187
	0.90	0.9	944.661	0.751	6,7	0:0:4.921
		0.85	944.661	0.747	6,7	0:0:5.032
0.8	0.95	0.9	1004.578	0.755	6,7	0:0:5.445
		0.85	944.661	0.751	6,7	0:0:5.445
	0.90	0.9	1004.578	0.751	6,7	0:0:5.651
		0.85	1004.578	0.747	6,7	0:0:5.578

As it is specified in Tables 2 and 3, by increasing cost discount factor ($\alpha = 0.6$ and $\alpha = 0.8$), the total costs are increased in both multiple and single states and the average reliability are being lessened in multiple state and has not reasonable difference in single allocation stat. In addition, it can be delivered from above table that increasing the discount factorsδ and βincreases the average reliability in network. For more obviousness of current effect, all the obtained results of Tables 2 and 3 are shown in Figures 2-7 to investigate relationship between optimal average reliability and total cost of different amount of discount factor α.

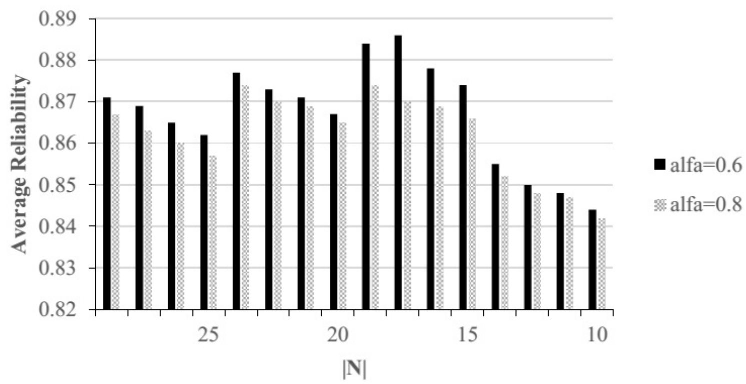

Figure 2: Average reliability of RMOMAHSP for CAB data under different discount factors

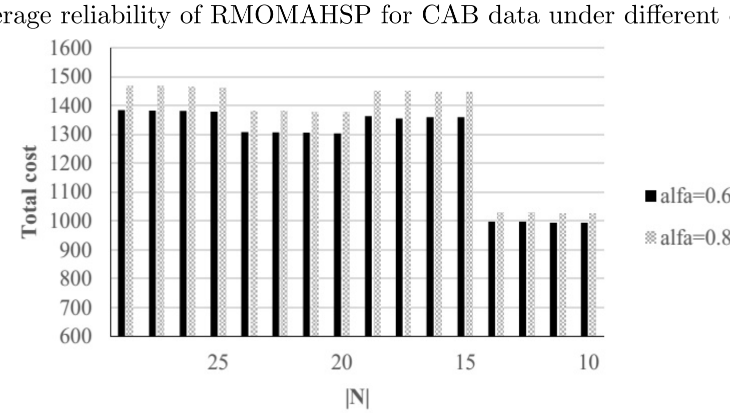

Figure 3: Total cost of RMOMAHSP for CAB data under different discount factors

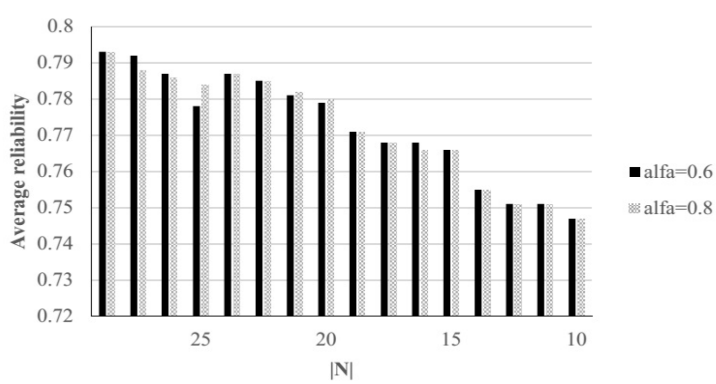

Figure 4: Average reliability of RMOSAHSP for CAB data under different discount factors

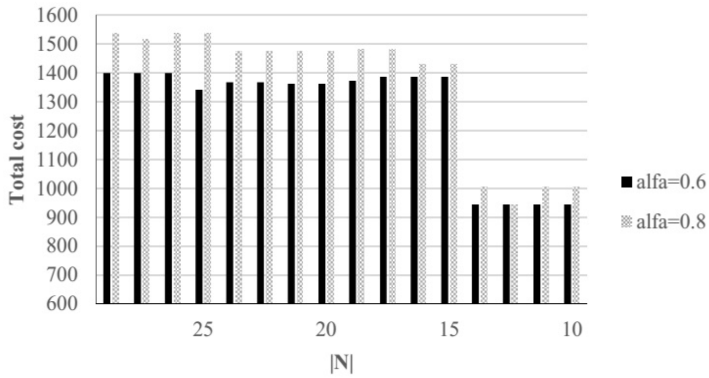

Figure 5: Total cost of RMOSAHSP for CAB data under different discount factors

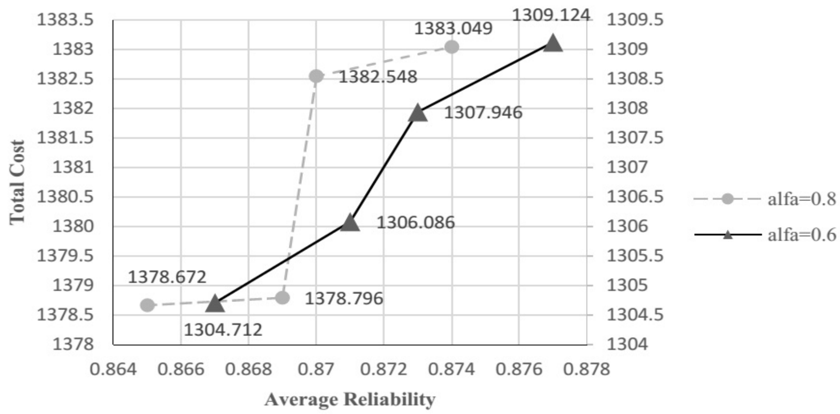

Figure 6: Total cost vs. network reliability in RMOMAHSP under different discount factors for |N|=20

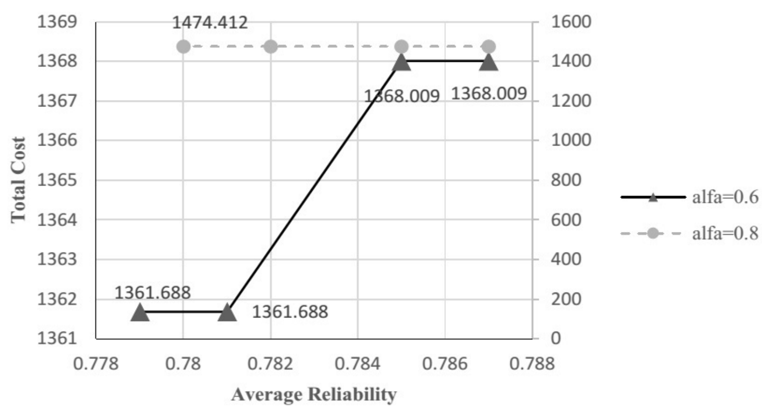

Figure 7: Total cost vs. network reliability in RMOSAHSP under different discount factors for |N|=20

The CAB data set represented by O'Kelly (1987) for hub location problems is generated to new data set denoted by RFCAB data set. The parameters of generated data set are RFVs and their values are distinguished by 5-dimentional vectors $(m^0, m^1, \beta, \gamma, \gamma^1)$ where $m^0 = m^1$ denotes the centre (or mode) of the mean value of RFV which is a fuzzy number, $\beta,\ \gamma$ represent the left and right spread respectively and γ^1 indicates standard deviation of RFV. The centre (or mode) of fuzzy number is assumed as the original number in CAB data set. Assume that φ is the original number in CAB data set, to generate right and left spread and standard deviation, the following relations are used respectively: $(r)\varphi$ and $p\varphi$, where $0 \leq r \leq 1$ and $0.001 \leq p \leq 0.1$. The values of r and p depend on the level of uncertainty and could be changed by the DMs opinion. We assumed $r = 0.2$ and $p = 0.1$ so the right and left values are obtained by 0.2φ and $\gamma^1 = 0.1\varphi$. Furthermore, $R^*(h) = L^*(h) = 1 - h$ and $\bar{t}$ is a normal random variable whose mean 0 and variance 1.

For solving reliable multi - objective hub location problem with RFVs in both single and multiple allocations, the numerical tests are divided into several sub problems by considering different assumption for model features as follows:

1. Size of problem: four problem sizes are obtained by taking the top 10, 15, 20 and 25 nodes from RFCAB data

set.

2. Discount factors:$\alpha \in \{0.6, 0.8\}$, $\delta \in \{0.95, 0.9\}$ and$\beta \in \{0.9, 0.8\}$.

3. Optimistic or pessimistic models.

4. Probability and possibility levels: $\theta, \eta, \lambda = 0.9$.

At first we propose the numerical results for RMOMAHSP and RMOSAHSP with RFVs and then sensitivity analysis of the proposed numerical results are given when the parameters of the problems such as α, p, θ, η, etc. are changed. Now, apply RFCCP method to these problems with RFVs and obtain optimistic optimal solutions for permissible levels $\theta, \eta, \lambda = 0.9$ reported in Tables 4 and 5. The columns labelled "Z (optimistic)", "SEV" and "$\overline{R}$" give optimal value of the objective functions (objective function value (OFV) of optimistic model for total cost, SEV of $\tilde{\bar{Z}}^*$ and average reliability respectively) for each test problem. Furthermore, the optimal OFVs of described solution methods for the problems can be easily compared in Tables 4 and 5.

Table 4. Computational results of RMOMAHSP with RFVs for RFCAB data set: optimistic model

a) |N|=25

α	δ	β	Z(optimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1535.685	1385.372	0.871	2,4,12,13,24	0:09:29.900
		0.85	1533.323	1383.242	0.869	2,4,12,13,24	0:08:84.201
	0.90	0.9	1532.400	1382.408	0.865	2,4,12,13,24	0:09:48.077
		0.85	1529.086	1379.418	0.862	2,4,12,13,24	0:09:43.095
0.8	0.95	0.9	1629.595	1470.090	0.867	2,4,7,12,24	0:09:29.336
		0.85	1627.941	1468.598	0.863	2,4,12,13,24	0:08:42.447
	0.90	0.9	1626.188	1467.017	0.860	2,4,7,12,24	0:10:25.919
		0.85	1622.793	1463.954	0.857	2,4,8,12,13	0:10:38.353

b) |N|=20

α	δ	β	Z(optimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1451.277	1309.226	0.877	4, 13,14, 17, 19	0:02:48.507
		0.85	1449.971	1308.048	0.873	4, 13,14, 17, 19	0:01:51.733
	0.90	0.9	1447.446	1305.770	0.871	4, 13,14, 17, 19	0:02:10.458
		0.85	1446.042	1304.503	0.867	4, 13,14, 17, 19	0:01:10.058
0.8	0.95	0.9	1533.219	1383.147	0.874	1,4,7,18,19	0:02:11.298
		0.85	1532.661	1382.644	0.870	4, 13,14, 17, 19	0:02:16.262
	0.90	0.9	1528.080	1378.512	0.869	1,4,7,18,19	0:03:06.894
		0.85	1527.935	1378.381	0.865	4, 13,14, 17, 19	0:02:39.544

c) |N|=15

α	δ	β	Z(optimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1512.552	1364.503	0.884	4, 6, 7, 8, 12, 14	0:00:13.164
		0.85	1512.428	1364.391	0.879	4, 6, 7, 8, 12, 14	0:00:14.975
	0.90	0.9	1508.920	1361.227	0.878	4, 6, 7, 8, 12, 14	0:00:15.407
		0.85	1508.724	1361.050	0.874	4, 6, 7, 8, 12, 14	0:00:10.898
0.8	0.95	0.9	1609.506	1451.967	0.874	1,6,7,11,12	0:00:33.600
		0.85	1608.644	1451.190	0.870	2, 4, 8, 13, 14	0:00:25.335
	0.90	0.9	1604.232	1447.210	0.869	1, 2, 4, 7, 8	0:00:22.335
		0.85	1602.972	1446.073	0.866	2, 4, 8, 13, 14	0:00:23.668

d) |N|=10

α	δ	β	Z(optimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1105.446	997.245	0.855	4, 6, 7, 8	0:00:04.091
		0.85	1105.249	997.067	0.850	3, 4, 6, 7, 8	0:00:03.840
	0.90	0.9	1101.510	993.694	0.848	4, 6, 7, 8	0:00:03.784
		0.85	1101.786	993.943	0.844	2,4,7,8	0:00:04.218
0.8	0.95	0.9	1141.963	1030.187	0.852	4, 6, 7, 8	0:00:03.479
		0.85	1141.336	1029.622	0.848	4, 6, 7, 8	0:00:03.959
	0.90	0.9	1137.864	1026.490	0.847	4, 6, 7, 8	0:00:03.545
		0.85	1137.184	1025.876	0.842	4, 6, 7, 8	0:00:03.858

Table 5. Computational results of RMOSAHSP with RFVs for RFCAB data set: optimistic model

a) |N|=20

α	δ	β	Z(optimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1516.438	1368.009	0.787	13,19,20	0:25:15.165
		0.85	1516.438	1368.009	0.785	13,19, 20	0:28:33.357
	0.90	0.9	1516.438	1368.009	0.782	13,19, 20	0:24:55.377
		0.85	1516.438	1368.009	0.780	13,19, 20	0:28:10.058
0.8	0.95	0.9	1634.386	1474.412	0.787	13,19, 20	0:31:08.012
		0.85	1634.386	1474.412	0.785	13,19, 20	0:21:18.003
	0.90	0.9	1634.386	1474.412	0.782	13,19, 20	0:24:55.037
		0.85	1634.386	1474.412	0.780	13,19, 20	0:27:47.219

b) |N|=15

α	δ	β	Z(optimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1522.551	1373.524	0.771	5, 7	0:01:19.238
		0.85	1536.869	1386.440	0.768	5, 11	0:01:58.202
	0.90	0.9	1536.869	1386.440	0.768	5, 11	0:01:55.377
		0.85	1536.869	1386.440	0.766	5, 11	0:01:24.018
0.8	0.95	0.9	1642.641	1481.860	0.771	6, 11	0:01:19.129
		0.85	1642.641	1481.860	0.768	6, 11	0:01:28.045
	0.90	0.9	1587.467	1432.086	0.768	5, 11	0:01:39.025
		0.85	1587.467	1432.086	0.766	5, 11	0:01:13.856

c) |N|=10

α	δ	β	Z(optimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1047.157	944.661	0.755	6, 7	0:00:06.214
		0.85	1047.157	944.661	0.751	6, 7	0:00:07.202
	0.90	0.9	1047.157	944.661	0.751	6, 7	0:00:06.556
		0.85	1047.157	944.661	0.747	6, 7	0:00:05.945
0.8	0.95	0.9	1113.574	1004.578	0.755	6, 7	0:00:07.140
		0.85	1113.574	1004.578	0.751	6, 7	0:01:06.439
	0.90	0.9	1113.574	1004.578	0.751	6, 7	0:00:09.134
		0.85	1113.574	1004.578	0.747	6, 7	0:00:06.268

Numerical results based on RFCAB data set for RMOMAHSP and RMOSAHSP with RFVs are given in Tables 4 and 5 respectively. The OFV of the objective functions of the problems for the different size of problems are reported in the first, second, third and fourth parts of Tables 4 and 5. Because of some restrictions and similarity of described solving methods, we solve three problem sizes ($|N| = 10, 15, 20$) for RMOSAHSP with RFVs distinguished in Table 5.

As it was shown in Tables 4 and 5, by increasing cost discount factor, the total costs are increased in both multiple and single states and the average reliability are being lessened in multiple state and has not reasonable difference in single allocation stat. Another interesting observation is the effect of increasing the value of δ and β on the average reliability. It can be delivered from above tables that increasing the discount factors δ and βincreases the average reliability in network. For more elaborate presentation of current effect, we have plotted OFVs versus network average reliability for two value of discount factor ($\alpha = 0.6$ and $\alpha = 0.8$) for instances with sizes of 20 and all the obtained results are shown in Figures 6 and 7 respectively.

Table 6. Computational results of RMOMAHSP with RFVs for RFCAB data set and |N|=20: pessimistic model

α	δ	β	Z(pessimistic)	SEV	$\overline{R}$	hubs	CPU(h:m:s)
0.6	0.95	0.9	1713.124	1309.227	0.877	4, 13, 14, 17, 19	0:03:49.267
		0.85	1711.582	1308.049	0.873	4, 13, 14, 17, 19	0:02:15.743
	0.90	0.9	1708.601	1305.771	0.871	4, 13, 14, 17, 19	0:02:19.729
		0.85	1706.943	1304.504	0.867	4, 13, 14, 17, 19	0:02:23.920
0.8	0.95	0.9	1809.847	1383.147	0.874	1, 4, 7, 18, 19	0:02:22.120
		0.85	1809.189	1382.644	0.870	4, 13, 14, 17, 19	0:02:18.331
	0.90	0.9	1803.782	1378.511	0.869	1, 4, 7, 18, 19	0:02:04.732
		0.85	1803.610	1378.380	0.865	4, 13, 14, 17, 19	0:02:18.365

Because of similarity of described solving methods, we will not mention to SEV method, and for pessimistic DMs we will only solve$|N| = 20$ for RMOMAHSP and RMOSAHSP with RFVs and compare with the results of the optimistic method. According to the results of Tables 6 and 7 and Figures 8 and 9, (for RMOMAHSP and RMOSAHSP with RFVs) with the increase of cost discount factor, total cost increases in both optimistic and pessimistic models and the average reliability decreases in both model. However, total costs of optimistic model are smaller than those of pessimistic model. As it was shown in Figures 8 and 9, the average reliability in each described methods are almost not changed with a given amount of discount factor for RMOMAHSP and RMOSAHSP with RFVs.

Table 7. Computational results of RMOSAHSP with RFVs for RFCAB data set and |N|=20: pessimistic model

α	δ	β	Z(pessimistic)	SEV	$\overline{R}$	hubs	CPU (h:m:s)
0.6	0.95	0.9	1790.039	1368.009	0.787	13, 19, 20	0:21:58.263
		0.85	1790.039	1368.009	0.785	13, 19, 20	0:21:44.312
	0.90	0.9	1781.769	1361.688	0.781	13, 19, 20	0:20:08.062
		0.85	1781.769	1361.688	0.779	13, 19, 20	0:28:01.413
0.8	0.95	0.9	1929.269	1474.412	0.787	13, 19, 20	0:35:03.729
		0.85	1929.269	1474.412	0.785	13, 19, 20	0:29:48.537
	0.90	0.9	1929.269	1474.412	0.782	13, 19, 20	0:23:56.367
		0.85	1929.269	1474.412	0.780	13, 19, 20	0:26:08.390

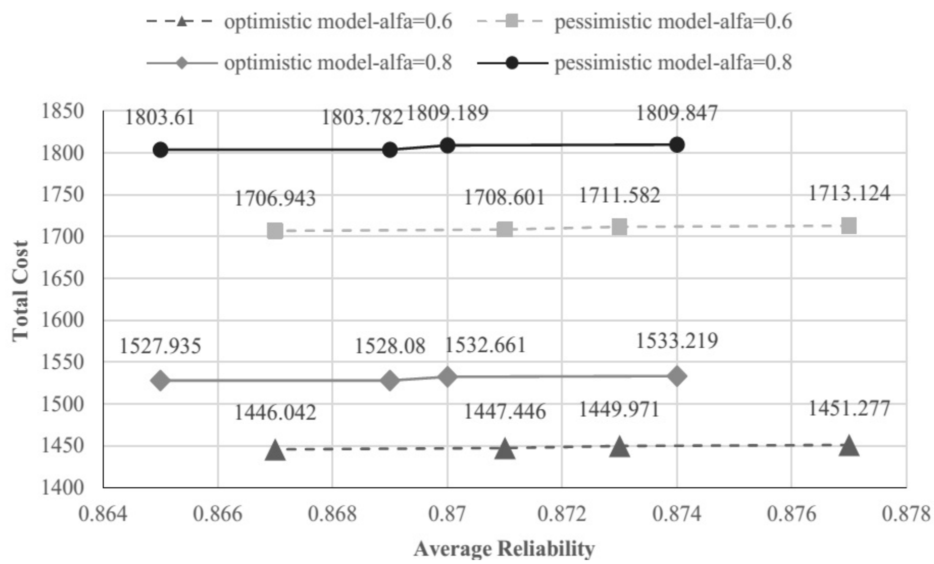

Figure 8: Total cost vs. network reliability in RMOMAHSP with RFVs under different discount factors for |N|=20

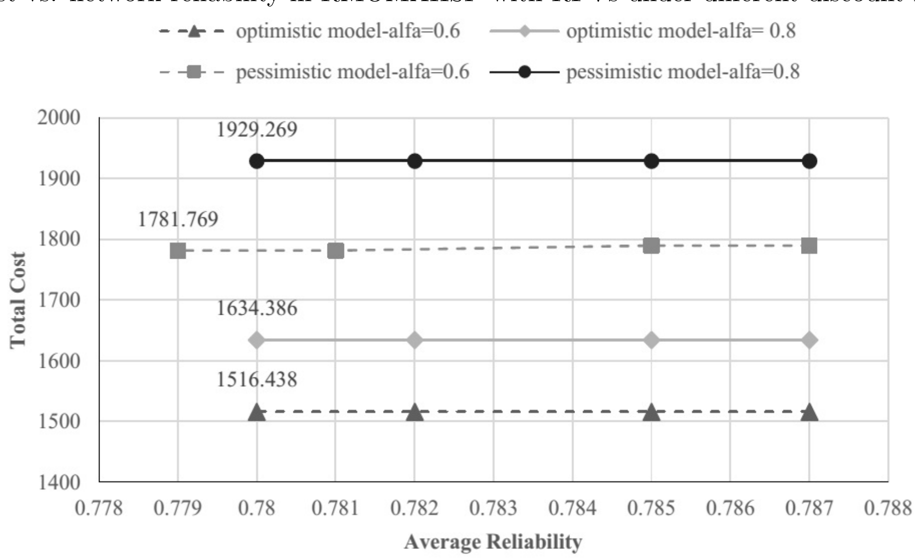

Figure 9: Total cost vs. network reliability in RMOSAHSP with RFVs under different discount factors for |N|=20

Finally in this section, we treated the reliable multi - objective hub location problems in both deterministic and uncertain frameworks. The numerical testes were divided into several sub problems by considering different assumptions for model features. However, there is no limitation for the DM to choose only them to find the optimal solutions. One can choose any other assumption for model features to find his/her optimal solution.

By applying reliability of all origin- destination routes, MOP with average reliability maximization and total cost minimization, and RFVs for parameters, reliable formulations are obtained to these problems compared with the traditional hub location problems because of considering disruption probability of all O/D routes, MOP and uncertain properties of parameters.

6 Conclusion

In this paper, we mention the RMOHSP in that there are two crucial objectives: first goal is average reliability maximization and the second one is total cost minimization. Then we indicate fuzzy approach to explain how to solve multi - objective optimization programming. We present generic models capturing these different sources of uncertainty for the single and the multiple allocation versions of the problems. The flows are described by uncertain variables and an uncertain programming approach is implemented to model the problems. These models are transformed into crisp forms when uncertainty distributions of flows were provided.

First, new reliable multi - objective hub location problems are introduced based on average reliability maximization and total cost minimization. Furthermore, a reliable multi - objective hub location problem with RFVs in single and multiple states is discussed. Then, the proposed hub location problems under uncertainty are solved by new methods using random fuzzy chance-constrained programming based on possibility theory. In the next section, we have collected all numerical consequences to assess all impacts of input factors on optimal solution of the model. We also mentioned all above sentences for single allocation situations.

Finally, for the numerical experiments, we perform extensive computational analysis with more than 350 instances (sub problems) on the RFCAB data set by considering the reliability for all origin - destination nodes reported in Tables 1-4. Furthermore, sensitivity analysis of numerical results have been shown in Figures 2-9 when the parameters of the problems such as number of hubs, Discount factor, possibility level, etc. are changed

There are great opportunities to extend all of these accomplishments in large computations by trying all the priorities in vast fields, besides new constraints can be used for future research and since there is no end of the knowledge for the model developed in this paper, improvement situations can be considered such as multi - objective modelling with objectives of fixed charge costs of the hubs, the central hubs and linking among them or imposing multi allocation situations along with single allocation ones.

For future research suggestions, the proposed approach based on considering the disruption probability of potential hubs under random fuzzy uncertainty framework could be implemented on other hub location problems such as multi - objective hub location problems, hub location under competition and other capacitated hub location problems. Another future research suggestion is providing other solution methods like exact heuristic algorithms to solve more realistic, large-scale instances of this random fuzzy p-hub location problem efficiently.

As another suggestion for future work, using the Australian Post (AP) or Turkish datasets, we will investigate this problem in large-scale instances and use the meta- heuristic methods to solve them.

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