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Analyzing the Advantage of Combination of Density Forecasts in Tehran Stock Exchange

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ABSTRACT

Today, stock market plays a key role in the economy of any country and is considered as one of the growth indicators of any economy. Gaining the skills of gathering and analyzing data simultaneously, as well as using this knowledge in economic investigations, make time and precision factors to be the drawcard of any investor in competition with others. Therefore, having a predictive approach with the lowest degree of error will lead to smarter management of resources. Due to the complex and stochastic nature of the stock market, conventional forecasting approaches in this field have usually faced serious challenges, most notably losing the robustness when the data type changed over time. Moreover, by focusing on point forecasting, some useful statistical information about the objective random variable has been ignored inadvertently, undermining the prediction efficiency. The focus of this study is on density forecasting models which, unlike point forecasting, contain a description of uncertainty. Also, to take advantage of the diversity and robustness features of the combination, instead of an individual prediction, a combination of the density forecasting given by different structures of ARMA, ANN, and RBF models is presented. In order to analyze the capabilities of these approaches in Tehran Stock Exchange (TSE), two basic methods of this category have been used to predict the price of MAPNA stock -one of the fifty active companies in this market- in the period 2012 to 2019.

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I. INTRODUCTION

Basically, all managers need efficient predictions for the appropriate management of their resources. Therefore, forecasting plays a key role in their precise decision making. However, in practice, predictions are not usually accurate, so one of the most important objectives of the management systems is to decrease forecasting errors as much as possible.

On the other hand, it is clear that, nowadays, stock market investment is a significant part of every country's economy. Therefore, the national economic indicators are heavily influenced by stock market performance [1]. The multiplicity of effective factors on stock market and the existence of many other unknown factors cause uncertainty in the field of financial time series forecasting; and consequently, it has inappropriate impacts on the investments which are based on these forecasts. Thus, it is necessary to use such appropriate scientific methods in order to make applicable and reliable predictions. In this particular application, due to the nature of prediction, the occurrence of an error in different forecasting models is unavoidable. So, naturally, a model with a lower prediction error rate is preferred over others. Hence, the necessity of a relevant forecasting model that maximizes the efficiency of investment via optimizing resource allocation is extremely sensible.

The prediction of stock return and price is possible by discovering behavioral patterns of the stock price generating process. The ability to detect such behavioral patterns determines the efficiency of the prediction method. This process may be a linear, nonlinear, or stochastic model [2]. The stock market forecasting methods can be classified into three general categories: traditional or classic methods, modern

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methods, and combined methods.

In different forecasting fields, several approaches with various success degrees have been applied; however, it should be noted that each of these approaches has such disadvantages as the inability to model nonlinear relationships, getting stuck in local optimums, over-fitting, and the difficulty of choosing numerous regulation parameters, each directly affecting prediction accuracy. In addition, sometimes the prediction accuracy of these approaches is different for data with various behaviors. In other words, in case of using a particular approach, the robustness is likely to be lost while the data type changes. Therefore, determining which one is the best choice for any given situation is not straightforward and depends on many factors. Indeed, when only one particular approach is used to solve a problem, other structures of this method and even other methods, each with its own advantages and disadvantages, are ignored and actually removed from the solution set.

To solve this problem and, simply put, to take advantage of the different approaches simultaneously, combination mechanisms are proposed, in which a combination of the solution of different approaches for a specific problem is considered. Using these methods will lead to many achievements, such as reducing the uncertainty in parameter setting, decreasing the randomness of the training process, and so on. Therefore, a suitable combination mechanism to achieve the stated goals would be very useful.

One approach to improve the structure of forecast combination mechanisms is to change components that are combined together in order to make the final prediction. In this regard, by substituting the point forecasting models instead of the density forecasting models, in addition to predicting price as the only advantage of the point forecasting, further useful information can be achieved, too.

The focus of this study is on two basic forecast combination method, namely Simple Average (SA) and Bates & Granger (B&G). Although these methods were proposed a long time ago, they are still being widely used. In particular, it may be interesting to know that some researchers are still attempting to improve their forecast combination methods such that the robustness of their mechanisms can be comparable with SA. According to the key role of risk management in investment, helping investors to minimize the risk of their investments is one of this study's goals. So, by using those forecast combination methods and then applying them to combine some density forecast models, the advantages of using these approaches in Tehran Stock Exchange (TSE) will be investigated, in this study.

The remainder of this paper is outlined as follows. In section II, an overview of related work is provided. The structure of the desired combination system is described in section III. Also, two famous linear combination methods are explained in this section. Then, the method of producing the system inputs is presented. Simulation results are provided in the next section and finally the paper is concluded in section V.

II. Literature Review

Since financial time series is dynamic, nonlinear, complex, nonparametric and disordered in nature, it is a generally challenging task to predict the stock market [3]. Many people have made the prediction with different approaches. Before the advent of computers and their being used for forecasting in the financial domain, classic methods, such as technical and fundamental analysis, had been employed to solve these problems. Since the mid-1970s, great efforts have been made to investigate the predictability of stock prices using new mathematical methods based on time series and more advanced tools [4]- [6]. Among the classic methods that have been widely used to predict the stock market, one can refer to Autoregressive (AR), Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) models [7]. Also, a number of these methods that are the practical approach for time series prediction have been introduced in [8] by Boxes and Jenkins. These models and, in particular, the ARIMA model are widely used in many different fields, but they can only perform linear prediction in time series. This feature makes such models inappropriate for predicting time series with nonlinear components [9].

Through the efforts of mathematical and dynamic systems scientists to address the deficiencies of linear models, new methods, called modern methods, have been proposed to predict prices in the stock market [10]. They are able to be applied in various sciences, especially economics. Therefore, nonlinear system practitioners have tried to explain stock price behavior and predict it through advanced nonlinear methods in financial markets, too. Due to the uncertainty of stock market and based on the articles reviewed in the survey article [11], soft computing techniques, belonging to the modern methods, are suitable for considering nonlinear relationships in the stock and, in most cases, outperform classic methods as they have better results for financial systems and have higher prediction accuracy without prior knowledge about the statistical distribution of input data. Machine learning's popularity is still increasing in financial forecasting. The ability of data mining and the flexibility of these algorithms have made them a very proper option for working on complex financial data [12]. There are different machine learning algorithms, each with respect to its own strengths, designed to solve a specific problem. Some of them work like the decision tree, where the user can fully understand the steps of the machine inference, and some act like neural networks similar to a black box. Artificial Neural Network (ANN) is one of the computational tools that is known as the most practical and widely used method for modeling large and complex problems [3], [13]. After an initial study of White in [14], the artificial neural network was first applied in the financial field and was subsequently considered as a suitable model for predicting the stock market. A review of the application of ANN in stock market forecasting is presented in [11], [15]. The results of the studies show that ANN is robust in model specification compared to parametric models, and, for this reason, it has been used frequently to predict stock prices and other financial derivatives [16]. The long history of using ANN in the financial field has led to more advanced versions of it, like the Radial Basis Function (RBF) network, being released in order to improve its performance. Reference [17] is one of the first articles that uses Support Vector Machines (SVMs) for financial forecasting. Then, the performance of SVMs is compared with the performance of other algorithms, such as backpropagation and RBF. Support Vector Regression (SVR) is another nonlinear forecasting algorithm used to predict financial time series. The efficiency of SVR as a predictive method is shown in [18]- [20]. Artificial neural networks and their variants have been widely used in the literature, but with the appearance of kernel-based methods such as SVR and considering their advantages, they fell out of fashion over time. Then, in 2006, Hinton et al. rekindled interest in neural networks by demonstrating an excellent performance of deep neural networks in [21]. Since then, researchers have conducted numbers of studies on using deep learning for financial time series.

With numerous choices in forecasting tools, to address the shortcomings of various approaches, perhaps one obvious way is to combine two or more models and present one prediction. In this way, the strengths of a method can be used to overcome the weaknesses of the others. In this regard, the combination of linear and nonlinear models has been suggested by several studies [22]- [24]. McDonald et al. in [9] showed that among several combined models, the combination of ARIMA models and Self-Organizing Fuzzy Neural Networks (SOFNN) performs better than others. Raoofi et al. in [25] compared several combined methods for forecasting Tehran Stock Exchange Index based on ARIMA, Fuzzy ARIMA (FARIMA), ANN and Adaptive Neuro-Fuzzy Inference System (ANFIS) models. By using some forecast accuracy measures such as Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), they concluded that the combined models of ANFIS and FARIMA have outperformed other models. Also, Fallahpour et al. in [26] showed that the prediction of the combined SVM model based on Genetic Algorithm (GA) for stock price movement trend is far better and more accurate than simple SVM. The application of different types of neural network and their combined models for the financial forecasting is one of the areas of interest to researchers. [27]- [29] can be mentioned as proper examples.

Some researchers believe that it is better to forecast the time series with high uncertainty, like financial market time series, by using forecast combination methods instead of using only one forecast model. Timmermann made a thorough review of the forecast combination mechanisms in [30]. He expressed the reasons for using forecast combination methods in order to produce better forecast and also discussed the conditions under which these mechanisms are admissible and useful. Due to the key role of forecast combination in different fields of forecasting, particularly in financial market and economic, after Timmermann's paper in 2006, many researchers are keeping their study on it till now. Most of these literatures ([31]- [36]) are about the structures of the combination and the weight assignment mechanisms. The large number of researcher concord on this point that even the simplest structure of the forecast combination can be helpful when necessary conditions are available. According to [37], [38], even if a simple forecast combination method does not be able to improve the forecast accuracy, it can decrease model selection risk. more recently, Atiya in [39] provides a brief analysis of the reasons for the success of forecast combination. Ultimately, reviewing the literature shows that the forecast combinations generally perform well [40].

As mentioned in economic forecasting and some other similar fields, the inability of point forecasting to describe the uncertainty of the variables leads to the use of density forecasting which has been used recently in forecasting the variables with high degree of uncertainty, like financial forecasting, [41]- [43]. That is, in order to forecast variables with a high degree of uncertainty, a density forecast is a comprehensive form of the point forecast. So, one idea is to use the density forecast combination to take advantage of both combination and density forecast abilities that is addressed in this paper.

According to all superiority of forecast combination and also density forecast combination in forecasting field, it will be very helpful to investigate the advantages of their application in Tehran Stock Exchange, which unfortunately is not done so far. Therefore, more researches still need to be done in this field for helping investors to have accurate decision making in this market.

III. METHODOLOGY

To take advantage of several available prediction models, one can derive the results of each model and combined them by the appropriate approach to achieve a single prediction, rather than using them to present an improved hybrid model. Actually, faced with various forecasts of the same variable, an issue that immediately arises is whether one should identify the best prediction or combine the underlying forecasts. Different conditions must be considered to answer this question, but in general and from a theoretical point of view, if a specific prediction model can be found having smaller prediction error than its competitors, then it justifies relying on a single prediction; otherwise, diversification gains of combination makes this approach more attractive than choosing the best model. Even if the best model can be recognized at any moment, the combination may still be an absorbing diversification strategy, though its success will depend on the structure of the combination and method of determining its weights. In fact, the diversification of a simple portfolio was the initial idea and the reason for using the forecast combinations [44].

Also, if each predictor model has useful and effective input variables, using all of which leads to a comprehensive model, the comprehensive model must be used in order to improve the outcome. Yet, when these variables are not available, the only option is the combination.

The next reason for using forecast combinations that have been mentioned by many researchers, including Stock and Watson in [45], [46], is that individual forecasting models may be subject to misspecification bias of unknown form. It is clear that even the best model at the specific time may not be preferred over other models at all times. Also, the best model may be time-variant, so it can be hard to track based on past forecasting performance. Forecast combinations can be viewed as a way to making the forecast more robust against such misspecification biases and measurement errors in the data sets underlying the individual forecasts [47].

Another argument for combining predictions is that basic prediction models may be based on different loss functions. Given the structure of the desired loss functions, forecast combinations can be helpful in improving the outcome, in these conditions, too.

Therefore, to benefit from the capabilities of combining forecasts, this approach can be applied to the stock market domain or any other domain with a similar data structure. If a combination mechanism, as shown in Fig.1, is considered as a system, the individual forecasts and the final predictions for the desired variable are input and output of this system, respectively. It combines predictions in such a way that, in the worst case, the use of combination theory is justified. In order to improve the performance of each system, one can change various parts of the system such as input, output and main structure of the system.

Fig. 1. Forecast Combination System.

Due to the important role of the combination in the field of prediction and the achievements of its application, several good review articles such as [30], [48], [49] have been published so far, but among them, Timmerman's survey article in 2005 [30] is the most recent and also the most comprehensive, according to numerous subsequent articles, including [50]. After developing the initial idea of the combination, different researchers have worked on its weights with various approaches to improving this idea; in this article, however, the combination is done by two primary and, of course, basic methods.

A. Simple Average

For the reasons stated, the researchers decided to use a combination of several predictions instead of the results of a single prediction model, in which the new problem was how to combine these results, appropriately. In this regard, SA based on Equal Weights (EW) is considered as the simplest and most primitive way that comes to mind. It is defined as:

$$f^{EW}_{t+h|t} = \frac{1}{m}\sum_{i=1}^{m} f_{i,t+h|t} \tag{1}$$

where

h	forecast horizon;
y_{t+h}	target variable of forecast;
$f_{i,t+h|t}$	output of i-th forecast model;
m	number of forecast models;
$f^{EW}_{t+h|t}$	output of SA forecast combination method.

This simple method of combination, as people in different fields asserted, has been able to improve the prediction results [30] and even performs better than other approaches presented later.

B. Linear Combination with Optimal Weights

After developing the SA method, in order to improve its results, the researchers decided to obtain the optimal value of the combination weights to minimize the prediction error instead of the equal weights. This idea was first proposed in 1969 by Bates and Granger in [44] as Optimal Weights (OW). The weights are determined by the Least Square (LS) estimation method using previous prediction error data. They call the weights optimal so that the variance of the in-sample error is minimized. The result of their work is:

$$\begin{gathered} f^{BG}_{t+h|t} = \widehat{W}'_{OLS} F_{t+h|t} = \left(\left(\iota' \hat{\Sigma}_e^{-1} \iota\right)^{-1} \hat{\Sigma}_e^{-1} \iota \right)' F_{t+h|t} \, , \\ \hat{\Sigma}_e = (t-h)^{-1} \sum_{\tau=1}^{t-h} \boldsymbol{e}_{\tau+h|\tau} \boldsymbol{e}'_{\tau+h|\tau} \end{gathered} \tag{2}$$

where

ι	column vector of ones;
$\boldsymbol{e}_{\tau+h|\tau}$	prediction error vector;
$\hat{\Sigma}_e$	estimation of covariance matrix;
$\widehat{W}_{OLS}$	weight vector;
$f^{BG}_{t+h|t}$	output of B&G forecast combination method.

It should be noted that in the optimization process, the weights are considered to be positive and the sum of them equal to one. In this study, this method is known as B&G.

It is notable that if all the predictions in the combination have the same error variance and the correlation between them is equal, the optimal weights will have the same value, too. In other words, under these conditions, the optimal weights are EW [30].

In this combination approach, the role of estimation error in the efficiency of the method is very prominent, so researchers have made many efforts to reduce this error and its consequences.

C. Density Forecast Combinations

So far, the focus was on the combination of point forecasts, which is influenced by the fact that a large proportion of academic studies are focused on these types of forecasts. Nevertheless, the tendency of researchers to study density forecasts is also growing, and published articles in the field of economics employ combination methods for these categories of forecasts. In addition, similar approaches have also been used for forecasting in other fields, such as meteorology.

One reason for the tendency of researchers to density forecasts is that a density forecast is an estimate of the probability distribution of the probable values of a random variable at some future time. Therefore, in contrast to a point forecast, which by itself has no description of the prediction uncertainty, a density forecast provides a full description of the associated uncertainty.

Today in many contexts of finance and economics, it is helpful and even in some cases necessary to provide density forecasts and evaluate them. Therefore, the next cause of people's shift from point forecasting to density forecasting is its valuable application in important financial fields, such as risk management. Many peoples have investigated density forecasts and their applications in macroeconomics and finance. Among them [43], [51]– [53] can be noted. Besides, [54] has addressed how to evaluate density forecasts. In this article, the combination of density forecasts is computed as follow:

$$p(y_{t+h}|D) = \sum_{i=1}^{m} w_i p(y_{t+h}|M_i,D) \tag{3}$$

where

D	data and observations;
M_i	i-th forecast model;
w_i	i-th weight determined by density forecast combination methods.

D. Input Generation Process

As mentioned above, inputs of the system shown in Fig. 1, have been generated using the predictions of several models. What the forecast models are and how they are selected can be a separate research topic, but this article focuses on a number of points for these choices. For example, both linear and nonlinear methods exist in the models and, from each category, select methods that are widely used for forecasting in the financial field. Therefore, in the first step, different structures of selected models including ARMA, ANN and RBF are used to predict the target random variable.

In the density forecasting literature, the density function is often assumed to be a normal or Gaussian distribution, whereas in practice, many financial time series exhibit asymmetric features or patterns [43]. In this study, the density forecasts are estimated by a non-parametric method, named kernel smoothing algorithm. As a result, no specific distribution is considered for densities.

IV. Results and Discussion

To demonstrate the advantages of density forecasting methods and their combinations in Tehran Stock Exchange, they are used to forecast the price of MAPNA company as one of the 50 active companies in this market. Then, daily price of this stock, from 1 January 2012 to 31 August 2019 (1447 data), is considered. Fig.2 shows the real values of MAPNA stock price over this period. For each type of prediction models, they are first trained with specific n-observation (75% of total data) input-output patterns. Then, the models are employed to forecast the remaining observations (25% of total data) and the performances are evaluated in terms of Mean Squared Error (MSE). All data are normalized by Min_Max method as follows:

$$D_{Normal} = \frac{D_{Real} - min(D_{Real})}{max(D_{Real}) - min(D_{Real})} \tag{4}$$

where

D_{Real}	real value of data;
D_{Normal}	normalized data.

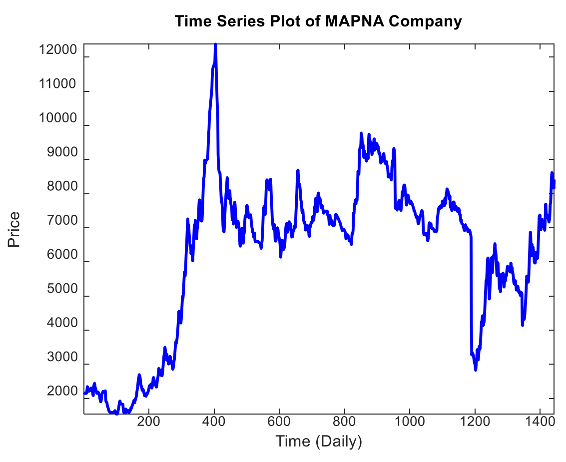

Fig. 2. Real Values of MAPNA Stock Price from Jan. 2012 to Aug. 2019.

Due to the structures, considered for ARMA, ANN, and RBF models, finally 8 prediction models have been used and their characteristics are presented in Table I. The parameter settings and the model structure selections are done such that both high-performance and low-performance models are included in the combination. Given these assumptions, the key role of combination in decreasing the sensitivity of model selection becomes evident, that is one of the purposes of this study. In this regard, no attempt was made to provide the best structures of the prediction models.

Normalized data are given to all forecast models as input, and each of them predicts a value of price for h-day ahead (where 'h' is the prediction horizon and it is considered 3 in this study, arbitrarily).

TABLE I

STRUCTURE OF PREDICTION MODELS

Model	Structure				
ARMA	**Model Order**		**Estimation Method**		
	$AR = 2, MA = 0$		RLS		
	$AR = 2, MA = 1$		ELS		
ANN	**Learning Method**	**No. of Hidden Layers**	**No. of Neurons**	**Epochs**	**Learning Parameters**
	Gradient Descent	$l_h = 1$	$n_h = 3$	500	$\eta = 0.001$
	Gradient Descent	$l_h = 1$	$n_h = 5$	500	$\eta = 0.001$
	Levenberg Marquardt	$l_h = 1$	$n_h = 3$	60	$\alpha = 5$
	Levenberg Marquardt	$l_h = 1$	$n_h = 5$	60	$\alpha = 5$
RBF	**Learning Method**	**Radial Function**	**No. of Neurons**	**Epochs**	**Learning Parameters**
	Levenberg Marquardt	$l_h = 1$	$n_h = 30$	100	$\alpha = 1$
	Levenberg Marquardt	$l_h = 1$	$n_h = 50$	100	$\alpha = 1$

To performance comparison, MSE values of all models have been computed. For brevity, Table II shows this value for only three selected models.

TABLE II

PERFORMANCE COMPARISON OF SELECTED PREDICTION MODELS

Forecast Model	MSE	
	In-Sample	**Out-of-Sample**
ARMA	58663	202246
ANN_LM_3	101784	209364
RBF_LM_50	567971	381534

For more clarity of figures and to make them understandable, Figs. 3 and 4 show the true and forecasted value of MAPNA stock price, obtained by only selected models for part of in-sample (from sample 800 to 1000) and out-of-sample (from sample 100 to 290) data, respectively. It has been attempted to select a range of samples in which high volatility of price has occurred to illustrate the difference between the performance of selected methods in the face of this event. According to these diagrams, it is obvious that different models, selected to be in the combination system, have various performance and accuracy over time. But it should be noted that, as it can be seen by the comparison of the efficiency of the RBF_LM_50 model for in-sample and its performance for out-of-sample data, the performance of these forecast models may be changed in any other period or for any other data.

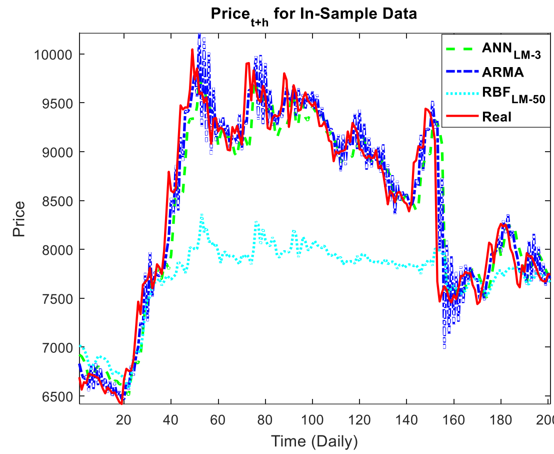

Fig. 3. Real vs. Predicted Values of MAPNA Stock Price for the Part of In-Sample Data (Related to Three Selected Forecast Models).

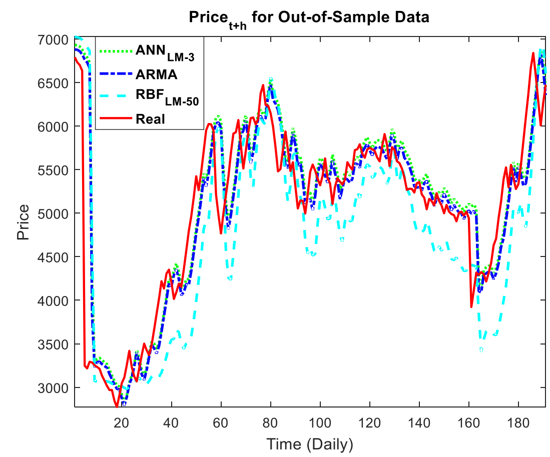

Fig. 4. Real vs. Predicted Values of MAPNA Stock Price for the Part of Out-of-Sample Data (Related to Three Selected Forecast Models).

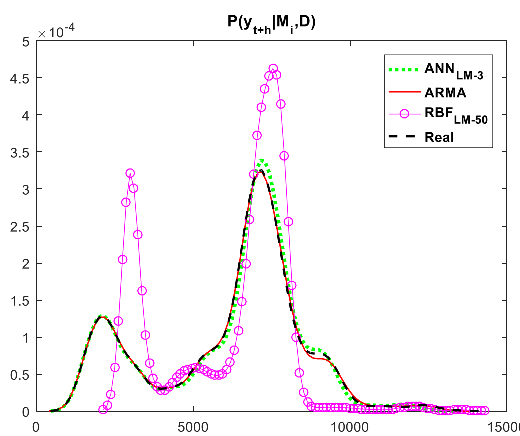

Fig. 5. Real vs. Predicted Density of MAPNA Stock Price (Related to Three Selected Forecast Models).

So far, all models are used to generate some point forecasts while the inputs of the combination system must be in the form of density forecasts. Then in the next step, the probability density of each prediction model output is computed by using a kernel smoothing algorithm. The comparison between Figs. 4 and 5 shows that if one model has a good or poor relative performance in point forecasting, it has a similar performance in density forecasting, too; however, the density forecasting has additional useful information that makes it more valuable. For instance, in contrast to the point forecasts, the density prediction has a description of the stock price uncertainty. In addition, based on the information included in probability density, an investor can be able to have better risk management.

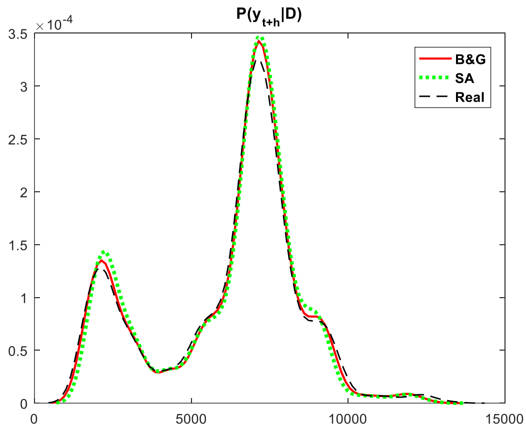

Fig. 6. Density Forecast Combinations for MAPNA Stock Price Using SA and B&G methods.

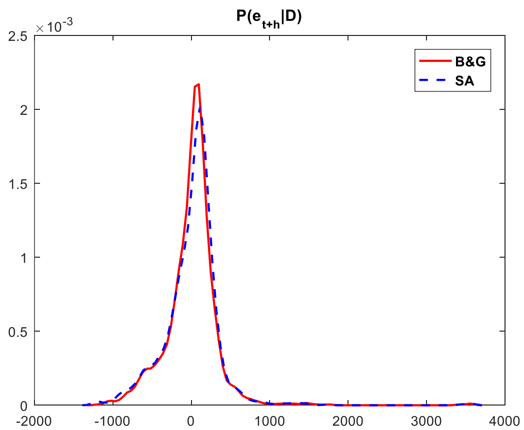

Fig. 7. Probability Density of Error for Two Forecast Combination Mechanisms.

Finally, for two different combination methods, SA and B&G, forecast combination system output is shown in Fig. 6. It demonstrates that both of them have been able to cover the poor performance of some models by relying on the results of the superior models and, ultimately, they have predicted the probability density of the MAPNA stock price with reasonable accuracy.

For a better illustration of the accuracy of density forecasts, probability densities of their errors are shown in Fig. 7. The similarity of these error densities to white noise and this fact that they are relatively symmetric around zero shows that these methods have been able to use the effective information, approximately.

As mentioned above, the efficiency of the same model may be changed for different data or in various periods. Now, assume that the RBF_LM_50 model was an efficient prediction model to forecast the MAPNA stock price in all the times before the period considered in this study. Consequently, according to the good past performance of the RBF_LM_50 model, it is selected as the best model to forecast this specific data. However, according to Fig. 3 can be found that, if so the investor will suffer heavy losses. But his losses were decreased seriously if he considered the RBF_LM_50 model as one of the selected models in forecast combination (see Fig. 6).

Besides the mentioned advantage, the investor can estimate Value at Risk (VaR) and Conditional VaR (CVaR) values of this stock by using density forecast combination. In other words, by using the density forecast, the investor can forecast the risk value of MAPNA or any other stock. Fig. 5 shows that if the investor has only the density forecast obtained by the RBF_LM_50 model, he cannot estimate the CVaR value of MAPNA stock with a reasonable error; and hereby the important advantage of using the density forecast combination is determined. It should be noted that one of the reasons for selecting MAPNA, among the 50 active companies in Tehran Stock Exchange, is that the density of this stock price is such that the selected methods can forecast it with significantly different performances. This property, in particular at the left tail of the probability density is very useful to demonstrate the advantage of density forecast combinations in computing VaR and CVaR.

V. CONCLUSIONS

This study was undertaken to investigate the advantages of using a density forecast combination vs. individual point forecast in Tehran Stock Exchange. The simulation results show that the sensitivity of forecast model selection is decreased by applying combination mechanisms such that, although some individual models have poor efficiency, the precision of forecast combination models are adequate. In addition, because of its diversity property of these mechanisms, the robustness of these prediction methods against the model misspecification biases of the individual forecasts increases. On the other hand, the simulation results confirm that density forecasting implicitly includes point forecasting and also provides more helpful information. For example, investors can apply this probability density function (PDF) to recognize maximum and minimum of any stock price in a specific period of time (from past to future), which helps them to make appropriate decisions. Moreover, they can use these densities to compute risk based on some risk assessments, like VaR and

CVaR. These findings could be exploited in any situation where the prediction of time series with structures similar to stock price is needed. Future work will involve the application of these densities to improve portfolio management in order to have maximum return and minimum risk. Another one will be proposing a proper nonlinear combination mechanism that is able to upgrade the result subject to the nature of these density functions.

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