Iranian Journal of Fuzzy Systems

Volume 17, Number 5, (2020), pp. 23-28

University of Sistan and Baluchestan

Greedy decomposition integrals

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Abstract

In this contribution we define a new class of non-linear integrals based on decomposition integrals. These integrals are motivated by greediness of many real-life situations. Another view on this new class of integrals is that it is a generalization of both the Shilkret and PAN integrals. Moreover, it can be seen as an iterated Shilkret integral. Also, an example in time-series analysis is provided.

Keywords: Decomposition integral, Shilkret integral, PAN integral, greediness.

1 Introduction

Theory of decomposition integrals is prolific and contemporary area of research. These integrals were introduced by Even and Lehrer [1] in 2014 as a framework unifying different non-linear integrals and are studied since [4, 5, 7, 8, 9, 10].

In this article we define a new class of integrals, called greedy decomposition integrals. This definition is motivated by a greedy algorithm trying to solve the Knapsack problem [3]: Imagine you want to pack as many items as you can into a bag that can hold only a certain weight. This is a time-consuming optimization problem. In a real-life situation the time to find an optimum takes longer than the time that is available. Thus, a greedy approach is needed and tolerated.

The rest of this article is organized as follows. In Section 2 we summarize the basic definitions and tools needed later. The third section is the main section of this article, where the definition of a greedy decomposition integral is presented first for collections then for decomposition systems. In Section 4 we show an application of the greedy decomposition integral in time-series modeling. Lastly, in Section 5, some concluding remarks are added.

2 Preliminaries

Throughout this paper we will consider a finite non-empty set X. Without loss of generality we may assume that $X = \{1, 2, \ldots, n\} \subsetneq \mathbb{N}$. By a function we mean any mapping $f\colon X \to [0, \infty[$. A class of all such functions will be denoted by $\mathcal{F}$. We will write $f = (f_1, f_2, \ldots, f_n)$ for a function $f \in \mathcal{F}$ where $f(i) = f_i$ for $i = 1, 2, \ldots, n$. For a set $A \subseteq X$ we denote $\{f_i : i \in A\}$ by $f(A)$.

A set function on 2^X is any mapping $\mu\colon 2^X \to \mathbb{R}$. A capacity is a set function μ that is grounded, i.e., $\mu(\emptyset) = 0$, and non-decreasing with respect to set inclusion, i.e., if $A \subseteq B \subseteq X$ then $\mu(A) \leq \mu(B)$. It is standard in literature that a capacity μ satisfies $\mu(X) = 1$ but we will omit this condition. The class of all capacities on 2^X is denoted by $\mathcal{M}$. To shorten the notation, we will write $\mu_{xy\ldots z}$ instead of $\mu(\{x, y, \ldots, z\})$, i.e., $\mu_{134} = \mu(\{1, 3, 4\})$.

A collection, mostly denoted by $\mathcal{D}$, is any subset of 2^X. It is convenient also to assume that $\mathcal{D}$ is non-empty but due to some technicalities we will assume that $\mathcal{D}$ can also be the empty set. This change has no influence on the results of this paper. A decomposition system is non-empty set of collections.

Let $\mathcal{D}$ be any non-empty collection and let $A \in \mathcal{D}$ be any element of this collection. The set $A\sharp\mathcal{D}$ is an element of $\mathcal{D}$ such that conditions

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Received: July 2019; Revised: November 2019; Accepted: March 2020.

- $A \subseteq A\sharp\mathcal{D}$; and
- if $B \in \mathcal{D}$ and $A\sharp\mathcal{D} \subseteq B$ then $A\sharp\mathcal{D} = B$

hold. In other words, $A\sharp\mathcal{D}$ denotes a maximal element of $\mathcal{D}$ such that A is its subset. Note that this element may not be defined uniquely, e.g., if $\mathcal{D} = \{\{1\},\{1,2\},\{1,3\}\}$ then $\{1\}\sharp\mathcal{D}$ can be either $\{1,2\}$ or $\{1,3\}$. For our purpose it does not matter which of these elements we choose to be $\{1\}\sharp\mathcal{D}$. For a correct definition we can introduce a total order on the collection $\mathcal{D}$ and select the smallest element such that the conditions for $A\sharp\mathcal{D}$ hold.

A collection integral with respect to a collection $\mathcal{D}$ is an operator $\mathcal{I}_\mathcal{D}\colon \mathcal{F}\times\mathcal{M} \to [0,\infty[$ given by

$$\mathcal{I}_\mathcal{D}(f,\mu) = \bigvee \left\{ \sum_{A\in\mathcal{D}} \alpha_A \mu(A)\colon \sum_{A\in\mathcal{D}} \alpha_A 1_A \le f, \alpha_A \ge 0 \right\}$$

if $\mathcal{D}$ is non-empty, and $I_\mathcal{D}(f,\mu) = 0$ if $\mathcal{D} = \emptyset$. A decomposition integral with respect to a decomposition system, see [1], $\mathcal{H}$ is an operator $I_\mathcal{H}\colon \mathcal{F}\times\mathcal{M} \to [0,\infty[$ such that

$$I_\mathcal{H}(f,\mu) = \bigvee_{\mathcal{D}\in\mathcal{H}} \mathcal{I}_\mathcal{D}(f,\mu).$$

3 Greedy decomposition integral

Let us start with a definition of a Greedy decomposition integral for a collection. This will be the basic building block of Greedy decomposition integrals build upon decomposition systems.

Definition 3.1. *A greedy collection integral with respect to a collection $\mathcal{D}$ is an operator $\mathcal{G}_\mathcal{D}\colon \mathcal{F}\times\mathcal{M} \to [0,\infty[$ such that $\mathcal{G}_\mathcal{D}(f,\mu)$ is defined recursively:*

- *if $\mathcal{D} = \emptyset$ then $\mathcal{G}_\mathcal{D}(f,\mu) = 0$;*
- *if $\mathcal{D} \neq \emptyset$ then we find a real number*

$$\alpha = \bigvee_{A\in\mathcal{D}} \mu(A) \min f(A)$$

and we construct a sub-collection $\mathcal{S} \subseteq \mathcal{D}$ given by

$$\mathcal{S} = \{A \in \mathcal{D}\colon \mu(A)\min f(A) = \alpha\}.$$

Then the value of $\mathcal{G}_\mathcal{D}(f,\mu)$ is set to

$$\mathcal{G}_\mathcal{D}(f,\mu) = \alpha + \bigvee_{B\in\mathcal{S}} \mathcal{G}_{\mathcal{D}_B}(f_B,\mu),$$

where $f_B = f - 1_B \cdot \min f(B)$ and $\mathcal{D}_B = \{A \in \mathcal{D}\colon A \subseteq \operatorname{supp} f_B\}$, where the support of f_B is the set $\operatorname{supp} f_B = \{x \in X\colon f_B(x) \neq 0\}$.

Note that every collection $\mathcal{D}$ is of a finite cardinality and that $\mathcal{D}_B \subsetneq \mathcal{D}$ for any $B \in \mathcal{D}$. Thus the recursive computation of $G_\mathcal{D}(f,\mu)$ will end in a finite number of steps. This result is summarized in the following proposition.

Proposition 3.2. *$\mathcal{G}_\mathcal{D}(f,\mu)$ is well-defined for all non-negative functions $f \in \mathcal{F}$ and all capacities $\mu \in \mathcal{M}$.*

Example 3.3. *Let $X = \{1,2,3\}$ and let $\mathcal{D} = \{\{1,2\},\{2\},\{2,3\},\{3\}\}$ be a collection. For a function $f\colon X \to [0,\infty[$, $f = (1,3,4)$, and a capacity $\mu\colon 2^X \to [0,\infty[$ such that $\mu_{12} = 2$, $\mu_2 = 1$, $\mu_{23} = 1$, and $\mu_3 = 1/2$, we obtain that*

$$\mathcal{G}_\mathcal{D}(f,\mu) = 3 + \max\{\mathcal{G}_{\mathcal{D}_1}(g,\mu), \mathcal{G}_{\mathcal{D}_2}(h,\mu)\} = 3 + \max\{2, 1/2\} = 5,$$

where $g = (1,0,4)$, $h = (1,0,1)$, and $\mathcal{D}_1 = \mathcal{D}_2 = \{\{3\}\}$.

Example 3.4. *A choice of $\mathcal{D} = 2^X$, i.e., a maximal collection on X, leads to an iterated Shilkret integral. In the first iteration we obtain a value of the classical Shilkret integral and then, in other iterations, the Shilkret integral is computed for the residual function and the values are summed up. Consider, for example, $\mathcal{D} = 2^X$, where $X = \{1,2,3\}$, a function $f = (1,3,4)$, and a capacity $\mu\colon 2^X \to \mathbb{R}$ given by $\mu_1 = \mu_2 = \mu_{23} = 1$, $\mu_3 = 1/2$, $\mu_{12} = \mu_{13} = 2$ and $\mu_{123} = 3$. The value of the iterated Shilkret integral of f with respect to μ is then equal to $13/2 = 6.5$. Observe that considering the same function f and the same capacity μ but a smaller collection $\mathcal{D}$ from Example 1 we obtain 5.*

Example 3.5. *If we consider a normed counting measure* μ*, i.e.,* $\mu(A) = \operatorname{card} A/\operatorname{card} X$*, and a collection* $\mathcal{D}$ *such that*

$$\{\{x\}: x \in X\} \subseteq \mathcal{D}$$

then $\mathcal{G}_{\mathcal{D}}(f,\mu)$ *is the arithmetic mean of values of* f*, i.e.,*

$$\mathcal{G}_{\mathcal{D}}(f,\mu) = \frac{1}{n}\sum_{i=1}^{n} f_i.$$

Due the greediness, these integrals lack many properties, but some are preserved. It can easily be seen that these integrals are positively homogeneous. Unfortunately, these integrals are not monotone in either argument, in general, see following two examples.

Example 3.6. *Let* $\mathcal{D} = \{\{1,2,3\},\{2,3\},\{3\}\}$*, a capacity* μ *be such that* $\mu_{123} = 2$*,* $\mu_{23} = 1$*, and* $\mu_3 = 1/4$*. Last, consider functions* $f = (1, 1.9, 3)$ *and* $g = (1, 2.2, 3)$*. Then* $\mathcal{G}_{\mathcal{D}}(f,\mu) = 3.175$*, and* $\mathcal{G}_{\mathcal{D}}(g,\mu) = 2.4$*. Thus* $\mathcal{G}_{\mathcal{D}}$ *is not monotone in the first argument, i.e.,* $f \le g$ *does not imply* $\mathcal{G}_{\mathcal{D}}(f,\mu) \le \mathcal{G}_{\mathcal{D}}(g,\mu)$*.*

Example 3.7. *Consider* $\mathcal{D} = \{\{1,2,3\},\{2,3\},\{3\}\}$*, a function* $f = (1,2,3)$*, and two capacities* μ,ν*, given by* $\mu_{123} = \nu_{123} = 2$*,* $\mu_{23} = 0.9$*,* $\nu_{23} = 1.1$*, and* $\mu_3 = \nu_3 = 1/4$*. Then, trivially,* $\mu \le \nu$*, but* $\mathcal{G}_{\mathcal{D}}(f,\mu) = 3.15 \not\le 2.45 = \mathcal{G}_{\mathcal{D}}(f,\nu)$*.*

Previous examples and discussion lead to the following proposition.

Proposition 3.8. *The operator* $\mathcal{G}_{\mathcal{D}}$ *is positively homogeneous, in general, but not monotone in either argument.*

Example 3.9. *One of the properties of the collection integral is collection monotonicity, i.e., if* $\mathcal{D}_1$ *and* $\mathcal{D}_2$ *are two collections such that* $\mathcal{D}_1 \subseteq \mathcal{D}_2$ *then* $\mathcal{I}_{\mathcal{D}_1} \le \mathcal{I}_{\mathcal{D}_2}$*. Unfortunately, the same does not hold for greedy collection integrals. Let us consider a space* $X = \{1,2,3\}$*, collections* $\mathcal{D}_1 = \{\{1,2\},\{2,3\}\}$ *and* $\mathcal{D}_2 = \{\{1,2\},\{1,2,3\},\{2,3\}\}$*, a capacity* μ *such that* $\mu_{12} = \mu_{23} = 2$ *and* $\mu_{123} = 3$*, and a function* $f = (1,2,1)$*. Then* $\mathcal{G}_{\mathcal{D}_1}(f,\mu) = 4$ *and* $\mathcal{G}_{\mathcal{D}_2}(f,\mu) = 3$*, i.e., the greedy collection integral is not monotone with respect to collection monotonicity, in general.*

Now we can define a greedy decomposition integral.

Definition 3.10. *A greedy decomposition integral with respect to a decomposition system* $\mathcal{H}$ *is an operator* $G_{\mathcal{H}}\colon \mathcal{F} \times \mathcal{M} \to [0,\infty[$ *such that*

$$G_{\mathcal{H}}(f,\mu) = \bigvee_{\mathcal{D}\in\mathcal{H}} \mathcal{G}_{\mathcal{D}}(f,\mu).$$

Due to Definition 1 and 2, the algorithms for processing Greedy integrals can be obtained straight-forwardly, what is not the case of decomposition (collection) integrals.

Based on the previous discussion we obtain the following theorem:

Theorem 3.11. $G_{\mathcal{H}}$ *is a positively homogeneous operator.*

It is also very easy to see that

Theorem 3.12. $G_{\mathcal{H}} \le I_{\mathcal{H}}$ *for all decomposition systems* $\mathcal{H}$*.*

Note that, we can not obtain more general properties due the involved greediness. The definition of a greedy decomposition integral might show potential in applications involving greediness and time-optimization, i.e., in modeling of economical time series, see Section 4.

Example 3.13. *Setting* $\mathcal{H} = \{\{A\}: A \in 2^X\setminus\{\emptyset\}\}$ *leads to the Shilkret integral, and setting* $\mathcal{H} = \{\mathcal{D}: \mathcal{D}$ *is a partition of* $X\}$ *leads to the PAN integral. Hence, for particular collections* $\mathcal{D}$*, as well as for particular decomposition systems* $\mathcal{H}$*,* $\mathcal{G}_{\mathcal{D}}$ *and* $G_{\mathcal{H}}$ *may be monotone both for functions and for capacities.*

Remark 3.14. *(i) Recall that for* $\mathcal{D}^* = 2^X$ *and* $\mathcal{H}^* = \{\mathcal{D}^*\}$*,* $\mathcal{I}_{\mathcal{D}^*} = I_{\mathcal{H}^*}$ *is the concave integral introduced by Lehrer [2].* $\mathcal{H}$ *is the greatest collection (decomposition) integral and it extends the standard Lebesgue integral. While the second property is valid also for greedy integrals, i.e.,* $\mathcal{G}_{\mathcal{D}^*} = G_{\mathcal{H}^*}$ *extends the Lebesgue integral, the first one need not be valid in the greedy integrals case. So, for example, let* $X = \{1,2\}$ *and* $f = 1_X$*,* $\mathcal{G}_{\mathcal{D}^*}(f,\mu) = \mu_{12}$*. However, if* $\mathcal{D} = \{\{1\},\{2\}\}$*, then* $\mathcal{G}_{\mathcal{D}}(f,\mu) = \mu_1 + \mu_2$*, i.e., if* $\mu_{12} < \mu_1 + \mu_2$*, then* $\mathcal{G}_{\mathcal{D}^*}(f,\mu) < \mathcal{G}_{\mathcal{D}}(f,\mu)$*.*

(ii) When applying Definition 1 to compute $\mathcal{G}_{\mathcal{D}^*}(f,\mu)$*, the first step corresponds to the evaluation of the related Shilkret integral,* $\alpha = \operatorname{Sh}(f,\mu)$*. Then, in the second step, we evaluate the Shilkret integral* $\operatorname{Sh}(f_B,\mu)$*, etc. Therefore,* $\mathcal{G}_{\mathcal{D}^*}$ *can be also called as an iterated Shilkret integral.*

Note that in [6], a property (M) of capacities ensuring the coincidence of PAN and Choquet integrals on finite spaces was given. Hence, if a capacity μ *satisfies the (M)-property then the greedy integral* $\mathcal{G}_{\mathcal{D}}(\cdot,\mu)$ *is just the Choquet integral* $\operatorname{Ch}(\cdot,\mu)$*.*

Lemma 3.15. *Let $\mathcal{D}$ be a collection consisting of disjoint sets. Then $\mathcal{G}_{\mathcal{D}}$ is monotone in both arguments.*

Proof. It is easy to see that

$$\mathcal{G}_{\mathcal{D}}(f,\mu) = \sum_{A\in\mathcal{D}} \mu(A)\min f(A) = \mathcal{I}_{\mathcal{D}}(f,\mu)$$

and thus $\mathcal{G}_{\mathcal{D}}$ is monotone in both arguments. □

Example 3.16. *Let $X = \{1,2,3\}$, $\mathcal{D} = \{\{1\},\{2,3\}\}$. Then*

$$\mathcal{G}_{\mathcal{D}}(f,\mu) = f_1\mu_1 + \min\{f_2,f_3\}\mu_{23}.$$

On this example it is easily seen that $\mathcal{G}_{\mathcal{D}}$ is monotone in both arguments.

Lemma 3.17. *Let $\mathcal{D}$ be a collection such that there are two sets $A,B\in\mathcal{D}$, $A\neq B$, and $A\cap B\neq\emptyset$. Then there exist functions $f,g\in\mathcal{F}$, $f\le g$, $f\neq g$, and a capacity $\mu\in\mathcal{M}$ such that $\mathcal{G}_{\mathcal{D}}(f,\mu)\not\le\mathcal{G}_{\mathcal{D}}(g,\mu)$.*

Proof. We will construct the functions f,g and a capacity μ as an extension of Example 3.6. Let $A,B\in\mathcal{D}$ be two sets such that $A\neq B$ and $A\cap B\neq\emptyset$. Set

$$A^* = A\sharp\mathcal{D} \quad \text{and} \quad B^* = B\sharp(\mathcal{D}\setminus\{A^*\}).$$

Then $A^*\neq B^*$ and $A^*\cap B^*\neq\emptyset$. Let us choose a capacity $\mu\in\mathcal{M}$ given by

$$\mu(S) = \begin{cases} 2, & \text{if } A^*\subseteq S,\\ 1, & \text{if } B^*\subseteq S \text{ and } A^*\not\subseteq S,\\ 0, & \text{otherwise}\end{cases}$$

and functions $f,g\in\mathcal{F}$ given by

$$f(x) = \begin{cases} 1, & \text{if } x\in A^*,\\ 1.9, & \text{if } x\in B^*\setminus A^*,\\ 0, & \text{otherwise}\end{cases} \quad \text{and} \quad g(x) = \begin{cases} 1, & \text{if } x\in A^*,\\ 2.1, & \text{if } x\in B^*\setminus A^*,\\ 0, & \text{otherwise.}\end{cases}$$

It is easy to see that in this setting it is sufficient to consider only the sets A^* and B^* in decomposition in either f and g. Now, it is also trivial to see, that

$$\mathcal{G}_{\mathcal{D}}(f,\mu) = 1\cdot\mu_{A^*} + 0.9\cdot\mu_{B^*} = 2.9$$

and

$$\mathcal{G}_{\mathcal{D}}(g,\mu) = 2\cdot\mu_{B^*} = 2,$$

implying that $\mathcal{G}_{\mathcal{D}}(f,\mu)\not\le\mathcal{G}_{\mathcal{D}}(g,\mu)$ which proves the lemma. □

Lemma 3.18. *Let $\mathcal{D}$ be a collection such that there are two sets $A,B\in\mathcal{D}$, $A\neq B$, and $A\cap B\neq\emptyset$. Then there exist a function $f\in\mathcal{F}$ and capacities $\mu,\nu\in\mathcal{M}$, $\mu\le\nu$, $\mu\neq\nu$, such that $\mathcal{G}_{\mathcal{D}}(f,\mu)\not\le\mathcal{G}_{\mathcal{D}}(f,\nu)$.*

Proof. The proof follows the proof of Lemma 2 and extends Example 3.7 with

$$\mu(S) = \begin{cases} 2, & \text{if } A^*\subseteq S,\\ 0.9, & \text{if } B^*\subseteq S \text{ and } A^*\not\subseteq S,\\ 0, & \text{otherwise}\end{cases} \quad \nu(S) = \begin{cases} 2, & \text{if } A^*\subseteq S,\\ 1.1, & \text{if } B^*\subseteq S \text{ and } A^*\not\subseteq S,\\ 0, & \text{otherwise}\end{cases}$$

and

$$f(x) = \begin{cases} 1, & \text{if } x\in A^*,\\ 2, & \text{if } x\in B^*\setminus A^*,\\ 0, & \text{otherwise.}\end{cases}$$

Then,

$$\mathcal{G}_{\mathcal{D}}(f,\mu) = 1\cdot\mu_{A^*} + 1\cdot\mu_{B^*} = 2.9$$

and

$$\mathcal{G}_{\mathcal{D}}(f,\nu) = 2\cdot\mu_{B^*} = 2.2,$$

and thus $\mathcal{G}_{\mathcal{D}}(f,\mu)\not\le\mathcal{G}_{\mathcal{D}}(g,\mu)$. □

Following Lemma 3.15, Lemma 3.17, and Lemma 3.18, we obtain following corollaries.

Corollary 3.19. *$\mathcal{G}_{\mathcal{D}}$ is monotone in either argument if and only if for all $A, B \in \mathcal{D}$, $A \neq B$, we have $A \cap B = \emptyset$, i.e., the collection $\mathcal{D}$ is a system of disjoint sets.*

Corollary 3.20. *$\mathcal{G}_{\mathcal{D}} = \mathcal{I}_{\mathcal{D}}$ if and only if $A \cap B = \emptyset$ for all $A, B \in \mathcal{D}$ such that $A \neq B$.*

As we have seen in Example 3.5, if $\mathcal{D}$ is not a system of disjoint sets, the greedy integral $\mathcal{G}_{\mathcal{D}}$ is not monotone, in general, but it can be monotone for some particular choices of μ and f. So, if $\mathcal{D} = 2^X$ and μ is additive then $\mathcal{G}_{\mathcal{D}}$ is just the Lebesgue integral (in fact, a weighted sum) and hence monotone in functions and in additive measures. Also, for some particular measures, the monotonicity in functions or in collections can be guaranteed. For example, consider $\mu(A) = 0$ whenever $A \neq X$ and $\mu(X) = 1$. Then

$$\mathcal{G}_{\mathcal{D}}(f, \mu) = \begin{cases} \min f(X), & \text{if } X \in \mathcal{D}, \\ 0, & \text{otherwise}, \end{cases}$$

clearly satisfies both properties.

4 Example of possible application: Time-series analysis

As an example for the use of greedy decomposition integrals we present a time-series model build upon them. This model is a generalization of AR(n) model. Let $\{x_1, x_2, \ldots, x_N\}$ be the considered time-series, let $\mathcal{H}$ be a decomposition system, let μ be a capacity, and let $n \geq 1$, $n < N$, be a fixed integer. For $t = n+1, n+2, \ldots, N$ define a function $f_t = (x_{t-1}, x_{t-2}, \ldots, x_{t-n})$. The greedy autoregressive model of order n, denoted by gAR(n), is given by

$$x_t = G_{\mathcal{H}}(f_t, \mu) + \varepsilon_t.$$

Note that this model has $2^n - 1$ parameters representing values of the capacity μ. Nevertheless, only small values of n are considered and thus we have executable number of parameters. Also note that the parameter estimation is a non-linear problem. In the following, we use particle-swarm optimization algorithm to find the parameter estimates.

Example 4.1. *Let us consider a time-series of $N = 251$ values representing the stock price of IBM in 2018. We considered three information criteria,* AIC*,* AICc*, and* BIC*. From the class of* ARMA *models with orders $p, q = 0, 1, \ldots, 7$, based on these information criteria, the best model is* ARMA(1, 1)*. For this model, we obtained the following:*

$$\sigma^2 = 5.058\,71, \quad \text{AIC} = 410.899, \quad \text{AICc} = 410.923, \quad \text{BIC} = 417.95.$$

For gAR *model we considered a decomposition system $\mathcal{H} = \{2^X \setminus \{\emptyset\}\}$ and we estimated parameters of* gAR(2) *and* gAR(3)*. For* gAR(2) *we obtained that*

$$\sigma^2 = 4.933\,73, \quad \text{AIC} = 406.620, \quad \text{AICc} = 406.669, \quad \text{BIC} = 417.196,$$

and for gAR(3) *we get*

$$\sigma^2 = 4.826\,60, \quad \text{AIC} = 409.110, \quad \text{AICc} = 409.340, \quad \text{BIC} = 433.788.$$

We thus found out that gAR(2)*, based on the considered information criteria, is better than any* ARMA *model for this time-series. The estimated capacity is given by*

$$\mu_1 = 0.841\,747, \quad \mu_2 = 0, \quad \mu_{12} = 0.999\,48.$$

If we restrict ourselves to only autoregressive models than the optimal model would be AR(2) model. Because autoregressive models are sub-class of gAR models then we can compare AR(2) and gAR(2) model using statistical tests, more specifically, using F-test. This test rejects the null hypothesis that the gAR(2) model does not provide a significantly better model than AR(2) model (with false-rejection probability $\alpha = 0.05$), i.e., gAR(2) model is statistically significantly better model. The same is obtained for gAR(3).

To compare the results with, e.g., decomposition integrals, more precisely the concave integral (setting $\mathcal{H} = \{2^X \setminus \{\emptyset\}\}$) taking the functions $f_t = (x_{t-1}, x_{t-2})$, we obtain almost identical results as gAR(2) model. Unfortunately we were not able to compute parameters of model taking the functions $f_t = (x_{t-1}, x_{t-2}, x_{t-3})$ because the time complexity of computing decomposition integrals is much longer (solving Nk linear programming problems, where k is the number of iterations of optimizing method) than the time complexity of computing greedy decomposition integrals.

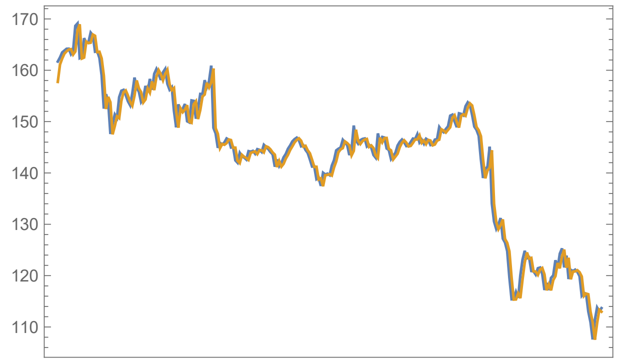

Figure 1: IBM stock prices from January 1 to December 31, 2018 (blue) and gAR(2) estimation of given time-series (orange). Data were obtained using Mathematica software [11].

5 Concluding remarks

In this paper we introduced a modification of the Shilkret integral which covers PAN integrals, among others. These integrals, called greedy decomposition integrals, are motivated by greediness (i.e., local optimization) of many processes in nature and it is observable in behaviour of people. These integrals are first introduced for collections and later for decomposition systems. Some properties of them are also proved, i.e., the positive homogenity and conditions for monotonicity. Lastly, an application of the greedy decomposition integrals in time series analysis is provided.

Acknowledgement

The author was supported by the Slovak Research and Development Agency under the contracts no. APVV-17-0066 and no. APVV-18-0052. Also the support of the grant VEGA 1/0006/19 is kindly announced.

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