

An integrated model of fuzzy multi-criteria decision making and stochastic programming for the evaluating and ranking of advanced manufacturing technologies

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Abstract

Investment appraisal in advanced manufacturing technologies (AMTs) has been receiving considerable attention over the past three decades. As stated in numerous studies, traditional engineering economic methods cannot adequately justify investments in AMTs. Thus, beside these methods, some other solutions have been proposed in this field. The methods applied in the evaluation of AMTs can be classified into four groups including economic, strategic, analytical and integrated methods. In this paper, an integrated model is presented for the evaluation of AMTs. The model is developed according to a fuzzy multi-criteria decision making procedure. It is assumed that a group of decision makers evaluate different AMT alternatives according to certain strategic and economic criteria. The importance of the criteria and the weights of the alternatives versus the criteria are expressed using linguistic variables set by the decision makers. To compute the economic criterion of each alternative, a linear program with chance constraints is proposed. Finally, using different fuzzy operators, the AMT alternatives are scored and ranked. In this regard, an example is presented to show the performance of the model.

Keywords: Advanced manufacturing technology, linguistic variable, multi-criteria decision making, linear programming, stochastic programming, chance constraint.

1 Introduction

Nowadays, with increasing competition in markets, manufacturing companies have become more and more aware of the necessity of investment in advanced manufacturing technologies (AMTs). Affective selection and application of an appropriate AMT can bring the company several competitive advantages in terms of productivity, profitability, reliability, etc. In competitive arena, application of suitable AMTs is crucial for the survival of a manufacturing company as an inappropriate decision can adversely affect the competitiveness and productivity of the company.

An AMT can be characterized by multiple attributes. Thus selection and evaluation of the AMTs need the skills and expertise that are beyond the ability of a single expert. Thus, in this process, a committee or a group of experts with different expertise is usually involved. Some attributes of the AMTs can be expressed using monetary values or other quantitative measures while there exist the attributes that are intangible or difficult to be presented using a measurable scale. On the other hand, the recent progress in computer science provides different AMT alternatives for the manufacturing companies. All of the aforementioned factors lead to the complexity of the AMT selection and evaluation. To tackle the problem, since 1990, the researchers have developed several decision tools and methods to assist the managers of the manufacturing companies in evaluation and selection of the most appropriate AMT according to their expectations. This methods range from the traditional engineering economic methods like internal rate of return, net present value, and payback periods to the more advanced integrated methods.

An advanced manufacturing technology (AMT) can be defined as any type of advanced technology that is applied in a manufacturing system and exerts major effects on the product, process and information aspects of that system [16]. Recently, the evaluation and selection of AMTs have been receiving considerable attention. In this case, a wide variety of methods and techniques have been developed. Manufacturing firms have been investing in AMTs to improve the manufacturing performance in terms of cost, flexibility, quality and reliability [5]. According to [17] and [16], investment in AMTs aims at (but is not limited to) stand-alone systems, e.g., computer-aided designs (CAD) and robots, intermediate systems, e.g., automated storage and retrieval systems (AS/RS), and integrated systems, e.g., flexible manufacturing systems (FMS).

In comparison with conventional manufacturing technologies, investment in AMTs has several distinct features. The main features include long life and adaptability, need for investment in several years, increase of return over time, and various intangible or system-wide benefits [16]. Because of these features, justification of investment in AMTs through traditional engineering economic methods is difficult [16]. Over time, therefore, other methods and techniques have been developed for this purpose. Generally, the evaluation techniques for AMT selection can be classified into economic, analytical, strategic and integrated techniques [7]. They are different due to the effect of non-monetary and qualitative factors. Table 1 shows the techniques and their advantages and disadvantages. The table is the updated version of the one presented in [10].

Discounted cash flow (DCF), which is developed based on the concept of the time value of money, underlies economic techniques. The net present worth, annual uniform value and internal rate of return are among the parameters used in such techniques. Some other economic techniques address the payback period and do not consider the concept of time value of money. It seems that the economic approaches developed based on the discounted cash flow are the most popular. Because DCF-based techniques do not take into account the effect of non-monetary factors, using them alone is not sufficient for the evaluation of AMTs [16].

Table 1: Classification of the techniques for AMT evaluation

Techniques	Criteria	Advantages	Disadvantages
Economic	<ul style="list-style-type: none"> - Payback - Return investment - Discounted cash flow (NPW, IRR) 	<ul style="list-style-type: none"> - Ease of data collection - Intuitive appeal 	<ul style="list-style-type: none"> - Do not take into account strategic and non-economic benefits - Consider a single objective of cash flows and ignore other benefits such as quality and flexibility
Strategic	<ul style="list-style-type: none"> - Technical importance - Business objectives - Competitive advantage - Research and development 	<ul style="list-style-type: none"> - Requireless technical data - Use the general objectives of the firm 	<ul style="list-style-type: none"> - Necessity to use these techniques with economic or analytic ones because they consider only long-term intangible benefits
Analytic	<ul style="list-style-type: none"> - Scoring models (AHP etc.) - Mathematical programming - Goal programming DEA 	<ul style="list-style-type: none"> - The uncertainty of the future and the multi-objectiveness can be incorporated - Subjective criteria can be introduced in the modelling phase 	<ul style="list-style-type: none"> - Require more data - Usually more complex than the economic technique
Integrated	<ul style="list-style-type: none"> - Combination of two or more of the other techniques 	<ul style="list-style-type: none"> - Benefit from the advantages of other techniques 	<ul style="list-style-type: none"> - Require more data - Usually more complex than the other techniques

Usually, strategic techniques are qualitative. They evaluate AMTs according to intangible and long-term benefits. Thus, it is better to use them along with economic and analytical techniques. Analytical techniques are based on mathematical processes that usually require considerable data to reach a conclusion. To evaluate investment in AMTs, quantifying some of the potential incomes and advantages may be very difficult if not impossible. Many managers declare that accounting procedures limit the acceptance and use of AMTs. Moreover, they express that many advantages obtained from AMTs cannot be quantified using these procedures. In this regard, different studies have emphasized the inadequacy of traditional financial techniques, e.g., the use of present value and payback period, due to ignoring strategic advantages. Hence, some researchers have combined quantitative and non-quantitative advantages, applied integrated approaches, and developed integrated models. The integrated models developed to evaluate AMTs are appropriate because they benefit from the advantages of different techniques, i.e., economic, analytic and strategic techniques.

To take into consideration the vagueness and uncertainty involved in AMT investments, the fuzzy theory can be effectively used. In the present study, an integrated model is developed on the basis of this theory. The model involves a fuzzy multi-criteria decision making (MCDM) procedure that takes into account the economic and strategic criteria of AMTs. To compute the economic criteria, a linear program with chance constraints is developed. The results of the

programming are incorporated in the fuzzy MCDM procedures. Finally, AMS alternatives are scored according to the strategic and economic criteria.

Regarding the application of the fuzzy theory in the selection and evaluation of AMTs, some studies have been conducted using this concept. One can refer to such studies as Karsak and Tolga[10], Dournal and Agilo [7], Karsak and Kuzkungaia[6], Kulak and Kahraman [11], and Chin Chu [6]. Also, there are some other models in the literature on AMTs that have not applied the fuzzy theory. In this line of research, one can mention [22], [3], Talluri et al.[19], Maldunado et al.[13], [20], Ordoobadi [16], Nath and Sarkar [14], Bai and Sarkis [1], He et al.[8], Wang and Chine[21], and Parameshwaran et al.[18]. For more research regarding the evaluation and selection of AMTs it can be referred to [2],[12] and [15].

The rest of the paper is organized as follows. Section 2 provides the general structure of the proposed integrated model. In Section 3, to compute the economic criteria, a linear program is developed. Section 4 presents the derivation of the chance constraints of the model. In Section 5, a fuzzy multi-criteria decision making framework is developed while the result of Section 3 is incorporated in it. Section 6 presents a numerical example to clarify the performance of the model. Finally, Section 7 concludes the paper.

2 The general structure of the proposed integrated model

It should be noted that the two phrases of AMT and advanced manufacturing system (AMS) will be used interchangeably throughout the paper. In this section, the general structure of the proposed integrated model is discussed. This structure is illustrated in Figure 1.

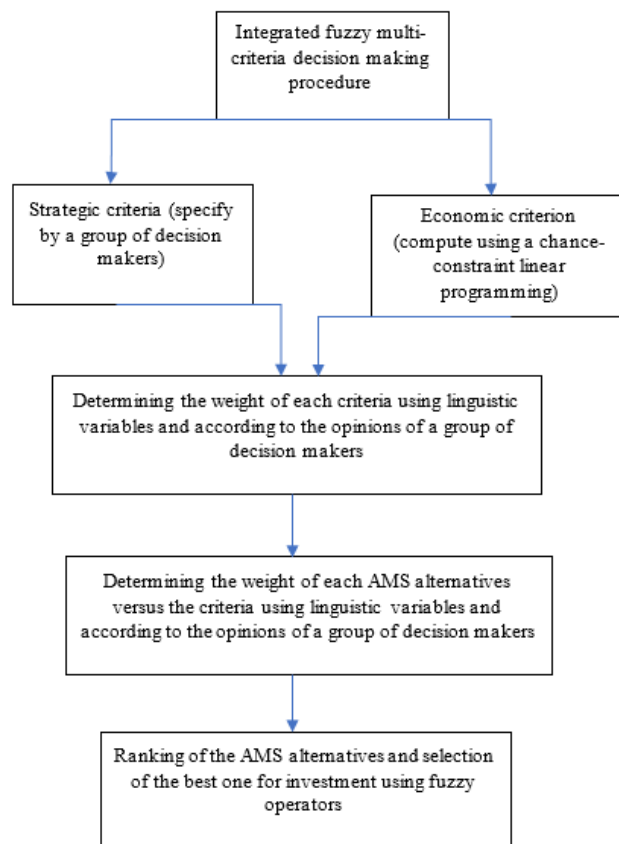


Figure 1. General Structure of the integrated model

The basis of the model is a fuzzy multi-criteria decision making (MCDM) procedure. The model takes into account the economic and strategic criteria of AMTs. Throughout the study, it is assumed that there are a group of n decision makers (D_1, D_2, \dots, D_n) . The group evaluates the suitability of m AMS alternatives $(Al_1, Al_2, \dots, Al_m)$ according to k criteria (C_1, C_2, \dots, C_k) . Criteria C_1, C_2, \dots, C_{k-1} are strategic ones, and criterion C_k is of an economic type. The

value of the net present worth (*NPW*) of each alternative, which is optimized through stochastic programming, is incorporated in the model as an economic criterion. The stochastic programming is a linear type of programming with chance constraints involved. Thus, the value of *NPW* of each AMS alternative is computed using a linear program with chance constraints.

The strategic criteria are specified by a group of decision makers. The group also determines the importance, or weight, of each criterion and the suitability of each AMS alternative versus the strategic criteria. These parameters are expressed using linguistic variables. The membership functions of the linguistic variables are assumed as triangular fuzzy numbers. Nevertheless, the fuzzy MCDM proposed in this study can be applied to different types of fuzzy membership functions and fuzzy numbers, e.g., trapezoidal or parabolic membership. The advantages of the proposed approach can be expressed as follows:

- The model has an integrated structure that benefits from the advantages of the economic, strategic and analytical approaches.
- Unlike most of the integrated models developed for assessment of AMT, the economic criterion incorporated in the model is optimized using a linear programming with chance constraints .
- The model can be applied for the different types of fuzzy membership functions and fuzzy numbers, e.g., trapezoidal or parabolic membership.
- Rate of inflation have been taken into account when developing the model.

Notation	Description
Q_{jn}	Lot size of product j at period n (Decision Variable)
TR	Tax rate
d_n	Depreciation at period n
SV	Salvage value
R_n	Revenues of period n
C_n	Costs of period n
I_0	Initial investment
I_{jn}	The inventory level of product j at period n (Decision variable)
B_{jn}	Black-order level of product j at period n (Decision variable)
b_j	Unit back-order cost for product j at period n
h_j	Unit inventory carrying cost for product j at period n
C_{ij}	The amount of resource i required to produce one unit of product j
L	Number of different work station
M	Number of products
N	Number of periods
P_j	Unit selling price of product j
Y_{in}	Allocated budget of recourse i in period n
Y_{Tn}	Total budget at period n
μD_{jn}	Mean of sum of n periods of demands for product j
$\sigma_{D_{jn}}$	The standard deviation of sum of n periods of demand for product j
$\Phi^{-1}(\cdot)$	Unit production time of product j at work station 1
F_n	Number of production shifts at period n
$\Phi^{-1}(\cdot)$	The inverse of the cumulative distribution function of a standard normal distribution
H	Number of hours in each shift
a_j	The allowable probability of inventories for product j can occur
β_j	The allowable probability of back-orders for product j can occur
i_{cm}	Interest rate of period m
f_m	A general inflation rate of income
e_{Rm}	Effective inflation rate of income for period m
e_{cim}	Effective inflation rate of resource i at period m
e_{hm}	Effective inflation rate of carrying inventory at period m
e_{bm}	Effective inflation rate of back-order at period m

3 Computation of the economic criteria through linear programming

In this section, to compute the economic criterion of each AMS alternative, a linear program with chance constraints is developed. Linear programming optimizes the value of the net present worth (*NPW*) for each alternative. It is worth noting that the linear programming presented in this section is an extension of the model devised by Chang and Tsou[3]. The extensions of the current linear programming with respect to the model of Chang and Tsou are as follows:

- Taking inflation into account in developing the model.
- Relaxing the assumption of a fixed interest rate in the whole planning horizon.
- Incorporating the result of optimization the linear programming as an economic criterion into the model.

The following notations are applied to the presentation of linear programming. The decision variables are specified and the remaining notations are the parameters of the model.

The cash flow after tax for an alternative in different time periods can be stated as follows:

$$CFAT_n = \begin{cases} -I_0, & n = 0 \\ (R_n - C_n)(1 - TR) + d_n \times TR, & n = 1, 2, \dots, N - 1 \\ SV + (R_N - C_N)(1 - TR) + d_N \times TR, & n = N \end{cases} \quad (1)$$

The first term of Equation (1) presents *CFAT*, i.e., cash flow after tax, at the start of the project. It means that the project needs I_0 as an initial investment cost. The second term states the *CFAT* from period 1 to $N - 1$. Finally, the last term expresses the *CFAT* at the end of the project life time.

According to Equation (1), the present worth of an alternative can be computed as follows:

$$NPW = -I_0 + SV \prod_{m=1}^N (1 + f_m)(1 + i_{cm})^{-1} + \sum_{n=1}^N [(R_n - C_n)(1 - TR) + d_n \times TR] \times \prod_{m=1}^n (1 + i_{cm})^{-1} \quad (2)$$

Equation (2) can be rewritten as follows:

$$NPW = NPW_1 + NPW_2 \quad (3)$$

While

$$NPW_1 = -I_0 + SV \prod_{m=1}^N (1 + f_m)(1 + i_{cm})^{-1} + \sum_{n=1}^N (d_n \times TR) \prod_{m=1}^n (1 + i_{cm})^{-1} \quad (4)$$

$$NPW_2 = \sum_{n=1}^N (R_n - C_n)(1 - TR) \times \prod_{m=1}^n (1 + i_{cm})^{-1} \quad (5)$$

As it can be seen, the value of NPW_1 is constant and does not depend on the performance of the production system. On the other hand, the value of NPW_2 depends on the performance of the production system. Thus, linear programming is proposed to optimize the value of NPW_2 . The linear programming with chance constraints is modeled as follows:

$$\begin{aligned} & \text{Max} \sum_{n=1}^N \left\{ \sum_{j=1}^M \left\{ \left[\left(P_j \prod_{m=1}^n (1 + e_{Rm}) - \sum_{i=1}^k c_{ij} \prod_{m=1}^n (1 + e_{cim}) \right) (1 - TR) \right] Q_{jn} \times \prod_{m=1}^n (1 + i_{cm})^{-1} \right\} \right. \\ & - \sum_{n=1}^{N-1} \left\{ \sum_{j=1}^M \left[P_j \left(\prod_{m=1}^n (1 + e_{Rm}) - \frac{\prod_{m=1}^n (1 + e_{Rm})}{(1 + i_{c(n+1)})} \right) (1 - TR) + h_j \prod_{m=1}^n (1 + e_{im}) \right] I_{jn} \times \prod_{m=1}^n (1 + i_{cm})^{-1} \right\} \\ & - \sum_{j=1}^M \left\{ \left(P_j \prod_{m=1}^N (1 + e_{Rm}) + h_j \prod_{m=1}^N (1 + e_{hm}) \right) (1 - TR) I_{jN} \times \prod_{m=1}^n (1 + i_{cm})^{-1} \right\} \\ & \left. - \sum_{n=1}^N \left\{ \sum_{l=1}^M \left(b_j \prod_{m=1}^n (1 + e_{bm}) \right) (1 - TR) B_{jn} \times \prod_{m=1}^n (1 + i_{cm})^{-1} \right\} \right\} \quad (6) \end{aligned}$$

Subject to:

$$\sum_{i=1}^M C_{ij} \prod_{m=1}^n (1 + e_{cim}) Q_{jn} \leq Y_{in}, \quad i = 1, 2, \dots, k - 1, \quad n = 1, 2, \dots, N \quad (7)$$

$$\sum_{j=1}^M \left(h_j \prod_{m=1}^n (1 + e_{hm}) I_{jn} + b_j \prod_{m=1}^n (1 + e_{bm}) B_{jn} \right) \leq Y_{hn}, \quad n = 1, 2, \dots, N \quad (8)$$

$$\sum_{i=1}^k Y_{in} \leq Y_{Tn}, \quad n = 1, 2, \dots, N \quad (9)$$

$$\sum_{m=1}^n Q_{jm} - I_{jn} \leq \phi^{-1}(\alpha_j) \times \sigma_{D_{jn}} + \mu_{D_{jn}}, \quad j = 1, 2, \dots, M, \quad n = 1, 2, \dots, N \quad (10)$$

$$\sum_{m=1}^n Q_{jm} + B_{jn} \geq \phi^{-1}(1 - \beta_j) \times \sigma_{D_{jn}} + \mu_{D_{jn}}, \quad j = 1, 2, \dots, M, \quad n = 1, 2, \dots, N \quad (11)$$

$$\sum_{j=1}^M T_{lj} Q_{jn} \leq E_f F_n H, \quad l = 1, 2, \dots, L \quad (12)$$

Equation (6) is the objective function of the model which maximizes the value of NPW_2 . It consists of four major terms. The first term is the present worth of incomes obtained from the selling of the products. The second term computes the present worth of costs (and sometimes incomes) and originates from carrying an inventory and selling it in the next period. In the last period, if there is an inventory, the cost of holding it and not selling the items has to be incurred. This is computed in the third term of the objective function. Finally, the last term of the objective function computes the present worth of the shortages. In derivation of Equation (6), the inflation and interest rates are considered different during each period of planning horizon. The first constraint, i.e., inequality 7, guarantees that the costs of each resource do not exceed the budget allocated to that resource. This constraint should be satisfied for each period and each resource. Inequality 8 ensures that the cost of the inventory system does not exceed the budget allocated to each period. Inequality 9 ensures that the sum of the cost in each period does not violate the budget allocated to that period. Inequalities 10 and 11 are the chance constraints of the model. Finally, the last constraint ensures that, at each work station, the time of production does not exceed the production capacity. Descriptions of how the chance constraints have been derived are presented in the next section.

The model can be optimized using operational research software programs, e.g., Lingo and GAMS. For each alternative, the model is optimized and the value of NPW_2 is computed. Also, using Equation (4), the value of NPW_1 is obtained for each alternative. Thus, according to Equation (3), when the values of NPW_1 and NPW_2 are added up, the value of NPW is obtained for each alternative. As stated in Section (3), the value of NPW for each AMS alternative is incorporated in the integrated model as an economic criterion.

If decision making is performed solely based on the economic criteria, an alternative with the largest value of NPW is selected. As mentioned before, however, beside the economic criteria, the alternatives are compared with respect to the strategic criteria as well.

4 Derivation of the chance constraints of the linear programming

Derivation of Equation (10) is discussed in this section. Equation (11) is derived in the same manner. Let assume the demand is a random variable that follows a normal distribution. In other words,

$$\sum_{m=1}^n D_{jm} \sim N(\mu_{D_{jn}}, \sigma_{D_{jn}}^2)$$

For item j , the probability that the production level will be greater than demand can be obtained as follows:

$$P\left(\sum_{m=1}^n Q_{jm} - I_{jn} \geq \sum_{m=1}^n D_{jm}\right) \leq \alpha_j, \quad j = 1, 2, \dots, M \quad (13)$$

Thus, the following equations are obtained:

$$P\left(Z \leq \frac{\sum_{m=1}^n Q_{jm} - I_{jn} - \mu_{D_{jn}}}{\sigma_{D_{jn}}}\right) \leq \alpha_j, \quad j = 1, 2, \dots, M \quad (14)$$

$$\frac{\sum_{m=1}^n Q_{jm} - I_{jn} - \mu_{D_{jn}}}{\sigma_{D_{jn}}} \leq \phi^{-1}(\alpha_j), \quad j = 1, 2, \dots, M \tag{15}$$

$$\sum_{m=1}^n Q_{jm} - I_{jn} \leq \phi^{-1}(\alpha_j) \times \sigma_{D_{jn}} + \mu_{D_{jn}}, \quad j = 1, 2, \dots, M \tag{16}$$

5 Fuzzy multi-criteria decision making

Following the computation and discussion of the economic criteria, an integrated model is developed in this section. This development occurs through determining the importance or weight of the strategic criteria, evaluating the AMS alternatives with respect to the criteria, and ranking those alternatives.

5.1 Determination of the importance of individual strategic criteria

The decision makers specify the strategic criteria and their importance. To determine the importance of each strategic criterion, linguistic variables are applied. These variables are used in conditions where it is difficult to assign a certain clear-cut value to a criterion. Linguistic variables are those whose value is expressed using words or phrases. They are usually presented by such words as good and very good. To quantify the value of a linguistic variable, the fuzzy set theory can be applied. On this basis, the present study employs triangular fuzzy numbers. The fuzzy MCDM proposed in this study, however, can be applied to different types of fuzzy membership functions and fuzzy numbers, e.g., trapezoidal or parabolic membership.

Every member of the group of decision makers expresses his/her opinion regarding the importance of each criterion using linguistic variables. The variables used to weight the criteria are defined by the set $W = \{VL, L, M, H, VH\}$, where VL, L, M, H and VH denote very low, low, medium, high and very high respectively. Table 2 presents the members of set W along with the equivalent fuzzy numbers.

Table 2: Equivalent triangular fuzzy number for the linguistic variables of set W

Linguistic variable	Description	Fuzzy number
VL	<i>verylow</i>	(0,0,0.3)
L	<i>low</i>	(0, 0.3,0.5)
M	<i>Medium</i>	(0.2,0.5,0.8)
H	<i>High</i>	(0.5,0.7,1)
VH	<i>VeryHigh</i>	(0.7,1,1)

The equivalent fuzzy value of a linguistic variable that is specified by the j^{th} decision maker for the importance of the t^{th} criterion is denoted by W_{tj} . Thus, the following equation holds true:

$$W_{ti} = (a_{ti}, b_{ti}, c_{ti}), \quad t = 1, \dots, k, \quad j = 1, \dots, n \tag{17}$$

Finally, the weight of each criteria is computed as follows:

$$W_t = \left(\frac{1}{n}\right) \otimes (W_{t1} \oplus W_{t2} \oplus \dots \oplus W_{tn}), \quad t = 1, \dots, k \tag{18}$$

$$W_t = (a_t, b_t, c_t), \quad t = 1, \dots, k$$

$$a_t = \sum_{j=1}^n \left(\frac{a_{tj}}{n}\right), \quad b_t = \sum_{j=1}^n \left(\frac{b_{tj}}{n}\right), \quad c_t = \sum_{j=1}^n \left(\frac{c_{tj}}{n}\right)$$

Where \otimes and \oplus denote the multiplication and addition algebraic operators for the fuzzy numbers respectively.

5.2 Determination of the weight of each alternative with respect to the criteria

In this stage, the importance of each AMS alternative versus the criteria is specified by the group of decision makers. They apply linguistic variables for this purpose, specifically the following set $A = \{VP, P, F, G, VG\}$ is applied. In this set, VP, P, F, G , and VG denote very poor, poor, fair, good, and very good respectively. Table 3 presents the members of set A along with the corresponding triangular fuzzy numbers.

Table 3: Equivalent triangular fuzzy number for the linguistic variables of set A

Linguistic variable	Description	Fuzzy number
VP	<i>Verypoor</i>	(0,0,0.2)
P	<i>Poor</i>	(0, 0.2,0.4)
F	<i>Fair</i>	(0.3,0.5,0.7)
G	<i>Good</i>	(0.6,0.8,1)
VG	<i>Verygood</i>	(0.8,1,1)

The equivalent fuzzy number of each linguistic variable specified by the j^{th} decision maker for the i^{th} alternative versus the t^{th} criterion is denoted by A_{ijt} . Thus, the following equation holds true:

$$A_{itj} = (o_{itj}, p_{itj}, q_{itj}), \quad i = 1, \dots, m, \quad t = 1, \dots, k - 1, \quad j = 1, \dots, n \tag{19}$$

It is worth restating that, among the k criteria, $k - 1$ criteria are strategic and one criterion is economic. Thus, decision-makers evaluate the alternatives with respect to the strategic criteria. To evaluate the alternatives versus the economic criteria, the value of the net present worth is applied.

The rank of the alternative i^{th} versus the strategic criterion t^{th} is denoted by A_{it} . To compute A_{it} , the mean operator is applied as follows:

$$A_{it} = \begin{cases} (o_{it}, p_{it}, q_{it}), & t = k, \quad i = 1, \dots, m \\ \left(\frac{1}{n}\right) \otimes (A_{it1} \oplus A_{it2} \oplus \dots \oplus A_{itn}) = (o_{it}, p_{it}, q_{it}), & t = 1, \dots, k, \quad i = 1, \dots, m \end{cases} \tag{20}$$

$$o_{it} = \sum_{j=1}^n \left(\frac{o_{ij}}{n}\right), \quad p_{it} = \sum_{j=1}^n \left(\frac{p_{ij}}{n}\right), \quad q_{it} = \sum_{j=1}^n \left(\frac{q_{ij}}{n}\right)$$

It should be noted that there exist several methods of ranking criteria in multi-criteria decision making problems, e.g., AHP, VICOR, and TOPSIS. In this study, however, the mean operator is used as in other studies on the assessment of AMTs, such as [10] and [9]. As Equation 20 shows, to compute the rank of the alternative i^{th} versus the economic criterion A_{ik} , the present worth of each AMS alternative is directly applied. Because the value of A_{it} is dimensionless for $t = 1, 2, \dots, k - 1$, the present worth of each AMS alternative should be dimensionless as well. If NPW_i denotes the present worth of alternative i , then the dimensionless value of NPW_i , which is denoted by NPW'_i , can be obtained as follows:

$$NPW'_i = NPW_i / (NPW_1 + NPW_2 + \dots + NPW_M) \tag{21}$$

Because the value of NPW'_i is not a fuzzy number, to incorporate it in the model, it can be considered as a fuzzy number as follows:

$$A_{ik} = (O_{ik}, P_{ik}, Q_{ik}) = (NPW'_i, NPW'_i, NPW'_i) \text{ and } A_{ik} = (o_{ik}, p_{ik}, q_{ik}), \quad t = 1, 2, \dots, k - 1$$

Finally, the following equation is obtained:

$$A_{itj} = (o_{itj}, p_{itj}, q_{itj}), \quad i = 1, \dots, m, \quad t = 1, \dots, k \tag{22}$$

5.3 Ranking of the AMS alternatives

In this subsection, using the weighted mean operator, the rank of each alternative is computed with respect to all the criteria. The following equation is applied for this purpose:

$$FA_i = \left(\frac{1}{k}\right) \otimes [(A_{i1} \otimes W_1) \oplus (A_{i2} \otimes W_2) \oplus \dots \oplus (A_{ik} \otimes W_k)], \quad i = 1, \dots, m \tag{23}$$

where FA_i is a fuzzy number with the following membership function [10]

$$f_{FA_i}(x) = \begin{cases} -H_{i1} + \left[H_{i1}^2 + \frac{(x-Y_i)}{T_{i1}}\right]^{\frac{1}{2}} & Y_i \leq x \leq Q_i \\ -H_{i2} + \left[H_{i2}^2 + \frac{(x-Z_i)}{U_{i1}}\right]^{\frac{1}{2}} & Q_i \leq x \leq Z_i, \quad i = 1, \dots, m \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

FA_i is not a fuzzy triangular number, and can be represented as follows:

$$FA_i = (Y_i, Q_i, Z_i; H_{i1}, T_{i1}, H_{i2}, U_{i1}), \quad i = 1, \dots, m \tag{25}$$

The following equation can be used for estimation of FA_i as a triangular fuzzy number [10]:

$$FA_i \cong (Y_i, Q_i, Z_i), \quad i = 1, \dots, m \tag{26}$$

The values of Z_i, Q_i and Y_i are computed as follows:

$$Y_i = \sum_{t=1}^k \frac{o_{it} a_t}{k}, \quad Q_i = \sum_{t=1}^k \frac{p_{it} b_t}{k}, \quad Z_i = \sum_{t=1}^k \frac{q_{it} c_t}{k}$$

So far, the rank of each alternative has been specified using triangular fuzzy numbers. It is now the triangular fuzzy numbers to rank. To this end, different methods have been proposed. In this paper, the centroid method is applied to rank the fuzzy numbers. Accordingly, the corresponding crisp value of $A = (a, b, c)$ is obtained as follows:

$$D(A) = \frac{1}{3}(a + b + c) \tag{27}$$

Using this method, the value of FA_i , which is a triangular fuzzy number, is transformed to a real number as $D(FA_i)$. Now, the score of each alternative can be obtained as follows [4]:

$$\text{Score}(Alt_i) = \frac{D(FA_i) - \min_{i=1,2,\dots,m} \{D(FA_i)\}}{\max_{i=1,2,\dots,m} \{D(FA_i)\} - \min_{i=1,2,\dots,m} \{D(FA_i)\}} \tag{28}$$

Considering the scores obtained in Equation (28), the rank of each alternative is specified. Finally, based on the rank, the best AMS alternative is selected. Finally, regarding the proposed integrated model the following note is worth mentioning. To compute the fuzzy weights for every criterion the authors used the mean operator to aggregate the triangular fuzzy numbers given by DMs. It should be noted that although there exist several traditional and fuzzy MCDM methods, e.g., AHP, TOPSIS, to decrease the complexity of the proposed integrated model, it is preferred to use the mean operator instead of the more complex MCDM methods. Also, As stated by [20], in comparison with traditional MCDM techniques, the fuzzy versions of MCDM techniques have more complexities especially regarding the mathematical calculations. This, to some extent, has led to the reluctance of the managers of manufacturing systems for applications of this techniques during their decision making.

6 Numerical example

In this section, a numerical example is presented to clarify the performance of the model. A company wants to produce five new products, i.e., p_1, p_2, p_3, p_4, p_5 . There exist three AMS alternatives, i.e., alt1, alt2, alt3, with which to produce the products. According to the performance of the integrated model, linear programming is done first to compute the net present worth of each alternative. Then, in the second stage of the model, fuzzy multi-criteria decision making is carried out.

6.1 Economic analysis of the alternatives

The specifications of each AMS alternative are presented in Tables 4-9. The interest rate is considered 20% for the whole planning horizon. It is assumed that the general inflation rate is 22%. The production efficiency of the alternatives is 75%, 80% and 85% respectively. Also, each shift lasts for 7.33 hours. To optimize the stochastic linear programming, different operational research software programs can be used, e.g., Lingo and Gams. In this paper, we use Lingo 12. The details of the Lingo code applied to optimize the linear program can be provided by the authors. The results of the model optimization are shown in Tables 11-13. The values of NPW for the alternatives are presented in Table 13. As mentioned in Section 3, the value of NPW for each alternative is the sum of NPW_1 and NPW_2 . While the value of NPW_1 does not depend on the performance of the production system, the value of NPW_2 depends on it. The value of NPW_2 is the value of the objective function in the stochastic programming. If the assessment is solely performed according to the economic criteria, alternative 2 is selected because it has the largest value of NPW .

Table 4: Initial cost of the alternatives.

Alternative	Cost
Al_1	1000,000,000
Al_2	1000,000,000
Al_3	1000,000,000

Table 5: Normal probability distribution of the demand during the planning horizon

Period \ Product	P_1	P_2	P_3	P_4	P_5
1	N (83450,3900)	N (71340,2700)	N (63560,2900)	N (20940,700)	N (42370, 1300)
2	N (83450,3900)	N (71340,2700)	N (63560,2900)	N (20940,700)	N (42370, 1300)
3	N (79400,3800)	N (70200,2650)	N (62000,2800)	N (24500, 800)	N (43200, 1350)
4	N (78500,3750)	N (69500,2650)	N (61500,2750)	N (25600,900)	N (44300, 1400)
5	N (78500,3750)	N (68450,2600)	N (61500,275)	N (26800,900)	N (45800, 1450)

Table 6: Unit production time of each product.

Production times			
Product	Al_1	Al_2	Al_3
P_1	0.268	0.237	0.191
P_2	0.215	0.175	0.115
P_3	0.283	0.193	0.121
P_4	0.909	0.618	0.40
P_5	0.341	0.165	0.112

Table 7: Budget allocated for each period

Period	Budget	Number of shifts
1	4,000,000,000	282
2	4,000,000,000	280
3	4,000,000,000	281
4	4,000,000,000	285
5	4,000,000,000	276

Table 8: The value of α and β for each product

	P1	P2	P3	P4	P5
α	0.2	0.2	0.2	0.1	0.2
β	0.1	0.1	0.1	0.5	0.1

Table 9: Unit production costs and selling prices.

	Raw material	Labor	Tool	Software	Set up	Maintenance	Failure	Manufacturing cost	Inventory	Shortage	Selling price
Al_1	P_1	7995	107	90	0	89	36	8353	6	12	11300
	P_2	789	86	80	0	71	29	1079	6	8	1500
	P_3	736	113	80	0	94	38	1102	6	6	1800
	P_4	36455	363	150	0	303	121	37458	7	22	42000
	P_5	12152	136	100	0	113	45	12583	6	16	15000
Al_2	P_1	7995	76	0	15	10	17	8198	6	12	11300
	P_2	789	41	0	10	15	8	916	6	8	1500
	P_3	669	48	0	9	13	13	832	6	8	1800
	P_4	3345	160	0	7	30	27	33788	7	22	42000
	P_5	12152	45	0	17	13	12	12327	6	16	15000
Al_2	P_1	7589	80	0	6	13	11	7837	6	12	11300
	P_2	769	42	0	11	7	6	903	6	8	1500
	P_3	669	42	0	14	8	7	823	6	8	1800
	P_4	3189	145	0	3	23	21	32406	7	22	42000
	P_5	12000	40	0	9	7	8	12134	6	16	15000
Inflation rate	12%	21.1%	15%	21%	21%	20%	16%	--	10%	15%	22%

Table 10: The inventory level of the items for the alternatives

	Period					
		1	2	3	4	5
Al_1	P_1	0	0	0	0	0
	P_2	0	0	0	0	0
	P_3	45778	0	0	0	0
	P_4	0	0	0	41710	0
	P_5	0	0	0	0	0
Al_2	P_1	0	0	0	0	0
	P_2	0	0	0	0	0
	P_3	0	0	0	0	0
	P_4	0	0	0	0	0
	P_5	0	0	0	0	0
Al_3	P_1	0	0	0	0	0
	P_2	0	0	0	0	0
	P_3	0	0	0	0	0
	P_4	0	0	0	0	0
	P_5	0	0	0	0	0

Table 11: Shortage level of each item for each alternative

	Period					
		1	2	3	4	5
Al_1	P_1	133	187	228	263	293
	P_2	110	156	190	219	245
	P_3	0	162	197	6271	252
	P_4	20983	2289	344	0	185
	P_5	76	108	133	155	175
Al_2	P_1	133	187	228	263	293
	P_2	110	156	190	219	245
	P_3	114	162	197	226	252
	P_4	20983	109	137	163	185
	P_5	42416	6524	133	155	175
Al_3	P_1	132	187	228	263	293
	P_2	110	156	190	219	244
	P_3	114	161	196	226	252
	P_4	77	109	137	163	185
	P_5	42614	108	133	155	175

Table 12: Production values of the items

	Period					
		1	2	3	4	5
Al_1	P_1	83397	83428	79383	74848	74888
	P_2	71296	71322	73186	79488	68440
	P_3	112293	14762	61986	55442	67535
	P_4	0	19651	24461	29966	22620
	P_5	42339	42357	43190	44291	45792
Al_2	P_1	83397	83428	79384	78486	78488
	P_2	71296	71132	70186	69488	68440
	P_3	63514	63541	61986	61488	61489
	P_4	0	41832	24487	25589	26790
	P_5	0	78280	49606	49291	45792
Al_3	P_1	83397	83428	79384	78486	78488
	P_2	71296	71322	70186	69488	68440
	P_3	61541	61986	61986	61488	61489
	P_4	20906	20926	24488	25589	26709
	P_5	0	84698	43190	44291	45792

Table 13: The value of NPW for each AMS alternative

Alternative	NPW_1	NPW_2	$NPW = NPW_1 + NPW_2$
Al_1	-815221866	2135171000	1319949134
Al_2	-965860558	2410532000	1444671442
Al_3	-1448790836	2534022000	1085231164

6.2 Fuzzy multi-criteria decision making

A group of three decision makers specify four strategic criteria obtained from the AMS alternatives. Then, the group determines the importance of each criterion. Table 14 shows the computed weight of each criterion. Also, the weights of the alternatives versus the first strategic criterion are shown in Table 15. For the sake of brevity, the details of the weights of the other strategic criteria are not presented. As Table 14 suggests, from the viewpoint of decision maker 2, for example, the importance of strategic criterion 1 is high, which is a linguistic variable from set W. Also, the data in Table 15 shows that, for example, decision maker 1 expresses the importance of alternative 2 versus strategic criterion 1 using the linguistic variable “good”, which is a member of set A. The values of NPW and the corresponding dimensionless values are provided in Table 16. Finally, the scores of the alternatives are given in Table 17. According to the data presented, alternative 3 has the highest score and is selected as the most suitable AMS alternative.

Table 14: Linguistic variables and the corresponding fuzzy numbers assigned by the decision makers to the criteria (DM: decision maker, SC: Strategic criterion, EC: Economic criterion)

	Linguistic variable			Equivalent fuzzy number			Weight of criteria
	DM ₁	DM ₂	DM ₃	DM ₁	DM ₂	DM ₃	W_t
SC ₁	VH	H	VH	(0.7,1,1)	(0.5,0.7,1)	(0.7,1,1)	(0.633, 0.9, 1)
SC ₂	H	H	M	(0.5,0.7,1)	(0.5,0.7,1)	(0.2,0.5,0.8)	(0.4, 0.633, 0.933)
SC ₃	M	M	H	(0.2,0.5,0.8)	(0.2,0.5,0.8)	(0.5,0.7,1)	(0.3, 0.567, 0.867)
SC ₄	VH	VH	VH	(0.7,1,1)	(0.7,1,1)	(0.7,1,1)	(0.7,1,1)
EC	M	H	H	(0.2,0.5,0.8)	(0.5,0.7,1)	(0.5,0.7,1)	(0.4, 0.633, 0.933)

Table 15: Evaluation of the alternatives versus the first strategic criteria

	DM ₁	DM ₂	DM ₃	DM ₁	DM ₂	DM ₃	A_{it}
Al_1	F	F	P	(0.3,0.5,0.7)	(0.3,0.5,0.7)	(0, 0.2,0.4)	(0.2, 0.4,0.6)
Al_2	G	F	VG	(0.6,0.8,1)	(0.3,0.5,0.7)	(0.8,1,1)	(0.567, 0.767,0.9)
Al_3	VG	V	VG	(0.8,1,1)	(0.6,0.8,1)	(0.8,1,1)	(0.733, 0.933,1)

Table 16: The values of NPW for each alternative along with the corresponding dimensionless values

	NPW_i	NPW'_i	A_{ik}
Al_1	1319949134	0.343	(0.343,0.343,0.343)
Al_2	1444671442	0.375	(0.375,0.375,0.375)
Al_3	1085231164	0.282	(0.282,0.282,0.282)

Table 17: The final scores of the AMS alternatives

	FA_i	$D(FA_i)$	$score(AL_{ti})$	RANK
Al_1	(0.11,0.29,0.52)	0.3083	0	3
Al_2	(0.28,0.56,0.77)	0.538	0.81	2
Al_3	(0.33,0.63,0.81)	0.5909	1	1

7 Conclusions

To evaluate AMTs, an integrated model is presented based on a fuzzy multi-criteria decision making procedure. The model takes into account economic and strategic criteria. Through linear programming with chance constraints, the value of the net present worth of each AMS alternative is optimized and incorporated in the integrated model as an economic criterion. A group of decision makers use linguistic variables to evaluate the AMT alternatives versus the strategic and economic criteria. They also express the importance of each criterion using those variables. Finally, the AMT alternatives are scored and ranked by means of fuzzy operators. An example is presented to clarify the performance of the model. For future research on models such as the one developed in this study, fuzzy cash flows and other methods of multi-criteria decision making can be taken into consideration.

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