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Complex fuzzy ordered weighted distance measures

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Abstract

In this paper, we introduce the complex fuzzy ordered weighted distance (CFOWD) measure. It is a new measure that uses the main characteristics of the the ordered weighted averaging (OWA) operator and uncertain information represented as complex fuzzy values. This measure includes a wide range of distance measures such as the complex fuzzy maximum distance, the complex fuzzy minimum distance, the normalized complex fuzzy distance (NCFD), the complex fuzzy ordered weighted Hamming distance (CFOWHD), complex fuzzy ordered weighted Euclidean distance (CFOWED), complex fuzzy ordered weighted geometric distance (CFOWGD), the normalized complex fuzzy Hamming distance (NCFHD) and the normalized complex fuzzy Euclidean distance (NCFED). We study some of its main properties. Finally, based on CFOWD measures, we give an illustrative example regarding the selected problem.

Keywords: Complex fuzzy sets, OWA operator, distance measures, complex fuzzy ordered weighted distance, decision making.

1 Introduction

Complex fuzzy set [21] (CFS), as an interesting extension of the standard fuzzy sets (FS), has been investigated and successfully used in various fields such as decision making[2, 3, 10, 22], time series prediction [9, 14, 15, 17], signal processing [11, 20, 26] and image restoration [16]. Recently, different concepts and measures of CFS have been studied, these including entropy measures [5], distance measures [1, 7, 12, 26], linguistic variables [1], rotational invariance [8], parallelity and orthogonality relations [4, 11, 13].

The ordered weighted averaging (OWA) operator [24] is a very useful tool in many fields of decision making. Motivated by the idea of the OWA operator, Xu and Chen [23] emphasized the ordered position of each deviation value and developed the ordered weighted distance (OWD) measure which assigns low weights to those unduly high deviation values. After that, the ordered weighted distance measures have been studied by several authors. Zeng and Su [27, 28] developed an intuitionistic fuzzy OWD measures and applied them to decision making. Yager [25] introduced a variety of ordered weighted averaging norms. Merigó and Gil-Lafuente [18, 19] provided the ordered weighted averaging distance (OWAD) operator for decision making.

However, it seems that in the literature there is no investigation on OWD measures between CFSs. In many complicated practical situations, the input data are usually provided with complex numbers, such as in signal processing, image representation and decision making. Thus, it is necessary to extend the OWD measures to accommodate the situation with complex fuzzy information. In order to do so, in this paper, we develop some complex fuzzy ordered distance measures and apply them to solve decision making problems with complex fuzzy information.

Since complex membership grade is given by an amplitude term and a phase term. Many researchers [1, 12, 26] calculate the deviation of CFSs by combining the deviation between the amplitude terms and the deviation between the phase terms. However, in some cases, they cannot represent the deviation between CFSs. For example, the amplitude term and phase term respectively represent the distance and direction of the target in [2, 3]. Let $A \equiv 0.01e^{j1.5\pi}, B \equiv 0.01e^{j0.5\pi}$, they are very close as $\|A-B\| = 0.02$. But by distance measures in [1, 12, 26], we have $d(A,B) \geq 1/2$. This is not consistent with our intuition. So traditional measurement of complex numbers is used in this paper.

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The rest of this paper is organized as follows. In Section 2, we develop the complex fuzzy ordered weighted distance (CFOWD) measure, and study some its various properties. In Section 3, we analyze different types of CFOWD operators. Section 4 briefly describes the decision making process based on CFOWD measures and we illustrate the process in detail with a practical example in Section 5. Conclusions are presented in Section 6.

2 Complex fuzzy ordered weighted distance

Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set, a CFS A on X is given as following:

$$A = \big\{(x, \mu_A(x))\big|x \in X\big\},$$

where the membership function $\mu_A(x)$ has the form $r_A(x) \cdot e^{j\theta_A(x)}$, $j = \sqrt{-1}$, the amplitude term $r_A(x)$ belongs to $[0,1]$ and the phase term $\theta_A(x)$ is real-valued. The complex number $\mu_A(x)$ (or $r_A(x) \cdot e^{j\theta_A(x)}$) is called a complex fuzzy value (CFV) and each CFV can be simply denoted as $a = r_a \cdot e^{j\theta_a}$, where $r_a \in [0,1]$.

In order to measure the deviation between any two CFVs $a = r_a \cdot e^{j\theta_a}$ and $b = r_b \cdot e^{j\theta_b}$. We defined the following distance:

Definition 2.1. *Let $a = r_a \cdot e^{j\theta_a}$ and $b = r_b \cdot e^{j\theta_b}$ be two CFVs. Then*

$$d_{CFD}(a, b) = \frac{1}{2} \parallel a - b \parallel . \tag{1}$$

is called a complex fuzzy distance (CFD) between CFVs a and b.

Remark 2.2. *$\parallel x \parallel$ is 2-norm of any complex number x. $\parallel x - y \parallel$ is the Euclidean distance between complex number x, y. Since $\frac{1}{2}$ is used in the above equation, for any two CFVs μ_a and μ_b, we have $d_{CFD}(a, b) \in [0,1]$.*

Obviously, CFD has the following properties: for any CFVs a, b, c,

(1) Non-negativity: $d_{CFD}(a, b) \geq 0$;

(2) Commutativity: $d_{CFD}(a, b) = d_{CFD}(b, a)$;

(3) Reflexivity: $d_{CFD}(a, a) = 0$;

(4) Triangle inequality: $d_{CFD}(a, b) + d_{CFD}(b, c) \geq d_{CFD}(a, c)$.

Definition 2.3. [24] *An ordered weighted averaging (OWA) operator of dimension n is a mapping OWA: $\mathbb{R}^n \to \mathbb{R}$ that has the following form:*

$$OWA(a_1, a_2, ..., a_n) = \sum_{i=1}^{n} w_i a_{\sigma(i)}, \tag{2}$$

where $(\sigma(1), \sigma(2), ..., \sigma(n))$ is any permutation of $(1, 2, ..., n)$ such that $a_{\sigma(i-1)} \geq a_{\sigma(i)}$ for all $i = 1, 2, ..., n$, $w = (w_1, w_2, ..., w_n)$ is the weighting vector associated with the OWA operator, $w_i \in [0,1]$ for all i, and $\sum_{i=1}^{n} w_i = 1$.

Motivated by the idea of the OWA operator, Xu and Chen [23] developed the OWD measure for real values.

Definition 2.4. [23] *Let $A = (a_1, a_2, ..., a_n)$ and $B = (b_1, b_2, ..., b_n)$ be two collections of real numbers, and $\triangle(a_i, b_i) = |a_i - b_i|$ be the distance between a_i and b_i, then*

$$OWD(A, B) = \Big(\sum_{i=1}^{n} w_i \big(\triangle(a_{\sigma(i)}, b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}, \tag{3}$$

is called a complex fuzzy ordered weighted distance (CFOWD) between a and b, where $\lambda > 0$, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is any permutation of $(1, 2, ..., n)$, such that

$$\triangle(a_{\sigma(i-1)}, b_{\sigma(i-1)}) \geq \triangle(a_{\sigma(i)}, b_{\sigma(i)}), j = 2, 3, ..., n, \tag{4}$$

and $w = (w_1, w_2, ..., w_n)$ is the weighting vector of the ordered positions of the distances $\triangle(a_i, b_i)$ $(i = 1, 2, ..., n)$, $w_i \geq 0$, $i = 1, 2, ..., n$, and $\sum_{i=1}^{n} w_i = 1$.

Distance measure of complex fuzzy sets is an important research topic which has been investigated by many authors [26, 12, 1]. However, there is no investigation on ordered weighted distance measures between CFSs, all the above suggested distance measures consider just an arithmetic average or weighted average for deviation between complex fuzzy values. Motivated by the idea of the OWD measure, next we develop an ordered weighted distance to accommodate the situation with complex fuzzy information.

Definition 2.5. *Let $A = (a_1, a_2, ..., a_n)$ and $B = (b_1, b_2, ..., b_n)$ be two collections of CFVs, and $d_{CFD}(a_i, b_i) = \frac{1}{2} \parallel a_i - b_i \parallel$ be the distance between a_i and b_i, then*

$$CFOWD(A, B) = \Big(\sum_{i=1}^{n} w_i\big(d_{CFD}(a_{\sigma(i)}, b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}, \tag{5}$$

is called a complex fuzzy ordered weighted distance (CFOWD) between a and b, where $\lambda > 0$, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is any permutation of $(1, 2, ..., n)$, such that

$$d_{CFD}(a_{\sigma(i-1)}, b_{\sigma(i-1)}) \geq d_{CFD}(a_{\sigma(i)}, b_{\sigma(i)}), j = 2, 3, ..., n, \tag{6}$$

and $w = (w_1, w_2, ..., w_n)$ is the weighting vector of the ordered positions of the distances $d_{CFD}(a_i, b_i)$ $(i = 1, 2, ..., n)$, $w_i \geq 0$, $i = 1, 2, ..., n$, and $\sum\limits_{i=1}^{n} w_i = 1$.

Remark 2.6. *Since $\mathbb{C}$ is isomorphic to $\mathbb{R}^2$, Eq.(5) also is an OWD when $a_i, b_i \in \mathbb{R}^2$. Moreover, when $a_i, b_i \in \mathbb{R}^2$, let $d_{CFD}(a_i, b_i) = \frac{1}{m} \parallel a_i - b_i \parallel$, Eq.(5) also is an OWD.*

Several properties of CFOWD measure can be proved with the following theorem:

Theorem 2.7. *For any two collections of CFVs $A = (a_1, a_2, ..., a_n)$ and $B = (b_1, b_2, ..., b_n)$, then*

(1) (Reflexivity)

$$CFOWD(A, A) = 0.$$

(2) (Non-negativity)

$$CFOWD(A, B) \geq 0.$$

(3) (Idempotency) if $d_{CFD}(a_i, b_i) = d$ for all $i = 1, 2, ..., n$, then

$$CFOWD(A, B) = d.$$

(4) (Commutativity) If $\big((a'_1, b'_1), (a'_2, b'_2), ..., (a'_n, b'_n)\big)$ is a permutation of $\big((a_1, b_1), (a_2, b_2), ..., (a_n, b_n)\big)$, then

$$CFOWD\big((a'_1, b'_1), (a'_2, b'_2), ..., (a'_n, b'_n)\big) = CFOWD\big((a_1, b_1), (a_2, b_2), ..., (a_n, b_n)\big).$$

(5) (Boundary)

$$\min_i d_{CFD}(a_i, b_i) \leq CFOWD(A, B) \leq \max_i d_{CFD}(a_i, b_i).$$

(6) (Monotonicity) Let $C = (c_1, c_2, ..., c_n)$ be set of CFVs, if $d_{CFD}(a_i, b_i) \geq d_{CFD}(a_i, c_i)$ for all $i = 1, 2, ..., n$, then

$$CFOWD(A, B) \geq CFOWD(A, C).$$

Proof. The proofs of (1) and (2) are straightforward and thus omitted.

(3) Because $d_{CFD}(a_i, b_i) = d$ for all $i = 1, 2, ..., n$, and $\sum\limits_{i=1}^{n} w_i = 1$, then

$$CFOWD(A, B) = \Big(\sum_{i=1}^{n} w_i\big(d_{CFD}(a_{\sigma(i)}, b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda} = \Big(\sum_{i=1}^{n} w_i\big(d\big)^{\lambda}\Big)^{1/\lambda} = d.$$

(4) Let

$$CFOWD\big((a_1, b_1), (a_2, b_2), ..., (a_n, b_n)\big) = \Big(\sum_{i=1}^{n} w_i\big(d_{CFD}(a_{\sigma(i)}, b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}.$$

$$CFOWD\big((a'_1,b'_1),(a'_2,b'_2),...,(a'_n,b'_n)\big)=\Big(\sum_{i=1}^{n}w_i\big(d_{CFD}(a'_{\sigma(i)},b'_{\sigma(j)})\big)^{\lambda}\Big)^{1/\lambda}.$$

Because $\big((a'_1,b'_1),(a'_2,b'_2),...,(a'_n,b'_n)\big)$ is a permutation of $\big((a_1,b_1),(a_2,b_2),...,(a_n,b_n)\big)$, and we have $d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})=d_{CFD}(a'_{\sigma(i)},b'_{\sigma(i)})$ for all $i=1,2,...,n$, then

$$CFOWD\big((a_1,b_1),(a_2,b_2),...,(a_n,b_n)\big)=CFOWD\big((a'_1,b'_1),(a'_2,b'_2),...,(a'_n,b'_n)\big).$$

(5) Let $\max_i d_{CFD}(a_i,b_i)=s$ and $\min_i d_{CFD}(a_i,b_i)=t$, since $\sum_{i=1}^{n}w_i=1$, then we have

$$CFOWD(A,B)=\Big(\sum_{i=1}^{n}w_i\big(d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}\leq\Big(\sum_{i=1}^{n}w_i\big(s\big)^{\lambda}\Big)^{1/\lambda}=s.$$

and

$$CFOWD(A,B)=\Big(\sum_{i=1}^{n}w_i\big(d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}\geq\Big(\sum_{i=1}^{n}w_i\big(t\big)^{\lambda}\Big)^{1/\lambda}=t.$$

(6) Let

$$CFOWD\big((a_1,b_1),(a_2,b_2),...,(a_n,b_n)\big)=\Big(\sum_{i=1}^{n}w_i\big(d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}.$$

$$CFOWD\big((a_1,c_1),(a_2,c_2),...,(a_n,c_n)\big)=\Big(\sum_{i=1}^{n}w_i\big(d_{CFD}(a_{\sigma(i)},c_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}.$$

Because $d_{CFD}(a_i,b_i)\geq d_{CFD}(a_i,c_i)$ for all $i=1,2,...,n$, then $d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})\geq d_{CFD}(a_{\sigma(i)},c_{\sigma(i)})$, for all $i=1,2,...,n$. Then $CFOWD(A,B)\geq CFOWD(A,C)$. □

3 Families of the CFOWD measures

By using a different weighting vector w and parameter λ, we are able to obtain some particular types of the CFOWD measure. Let two collections of CFVs be $A=(a_1,a_2,...,a_n)$ and $B=(b_1,b_2,...,b_n)$, then

(1) If $w=(1,0,0,...)$, then the CFOWD measure is reduced to the max complex fuzzy distance ($MCFD_1$) measure,

$$MCFD_1(A,B)=\max_i d_{CFD}(a_i,b_i)=\frac{1}{2}\max_i\parallel a_i-b_i\parallel. \tag{7}$$

(2) If $w=(0,...,0,1)$, then the CFOWD measure is reduced to the min complex fuzzy distance ($MCFD_2$) measure,

$$MCFD_2(A,B)=\min_i d_{CFD}(a_i,b_i)=\frac{1}{2}\min_i\parallel a_i-b_i\parallel. \tag{8}$$

(3) If $w=(1/n,1/n,...,1/n)$, then the CFOWD measure is reduced to the normalized complex fuzzy distance (NCFD) measure

$$NCFD(A,B)=\Big(\frac{1}{n}\sum_{i=1}^{n}\big(d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})\big)^{\lambda}\Big)^{1/\lambda}=\Big(\frac{1}{2n}\sum_{i=1}^{n}\big(\parallel a_{\sigma(i)},b_{\sigma(i)}\parallel\big)^{\lambda}\Big)^{1/\lambda}. \tag{9}$$

(4) If $\lambda=1$, then the CFOWD measure is reduced to complex fuzzy ordered weighted Hamming distance (CFOWHD) measure

$$CFOWHD(A,B)=\sum_{i=1}^{n}w_i\big(d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})\big)=\frac{1}{2}\sum_{i=1}^{n}w_i\big(\parallel a_{\sigma(i)},b_{\sigma(i)}\parallel\big). \tag{10}$$

(5) If $\lambda=2$, then the CFOWD measure is reduced to the complex fuzzy ordered weighted Euclidean distance (CFOWED) measure

$$CFOWED(A,B)=\Big(\sum_{i=1}^{n}w_i\big(d_{CFD}(a_{\sigma(i)},b_{\sigma(i)})\big)^{2}\Big)^{1/2}=\Big(\frac{1}{2}\sum_{i=1}^{n}w_i\big(\parallel a_{\sigma(i)},b_{\sigma(i)}\parallel\big)^{2}\Big)^{1/2}. \tag{11}$$

(6) If $\lambda \to 0$, then the CFOWD measure is reduced to the complex fuzzy ordered weighted geometric distance (CFOWGD) measure

$$CFOWGD(A,B) = \prod_{i=1}^{n} \left(d_{CFD}(a_{\sigma(i)}, b_{\sigma(i)})\right)^{w_i} = \frac{1}{2}\prod_{i=1}^{n} \left(\parallel a_{\sigma(i)}, b_{\sigma(i)} \parallel \right)^{w_i}. \tag{12}$$

(7) If $\lambda = 1$ and $w = (1/n, 1/n, ..., 1/n)$, then the CFOWD measure is reduced to the normalized complex fuzzy Hamming distance (NCFHD) measure

$$NCFHD(A,B) = \frac{1}{n}\sum_{i=1}^{n} \left(d_{CFD}(a_{\sigma(i)}, b_{\sigma(i)})\right) = \frac{1}{2n}\sum_{i=1}^{n} \left(\parallel a_{\sigma(i)}, b_{\sigma(i)} \parallel \right). \tag{13}$$

(8) If $\lambda = 2$ and $w = (1/n, 1/n, ..., 1/n)$, then the CFOWD measure is reduced to the normalized complex fuzzy Euclidean distance (NCFED) measure

$$NCFED(A,B) = \left(\frac{1}{n}\sum_{i=1}^{n} \left(d_{CFD}(a_{\sigma(i)}, b_{\sigma(i)})\right)^2\right)^{1/2} = \left(\frac{1}{2n}\sum_{i=1}^{n} \left(\parallel a_{\sigma(i)}, b_{\sigma(i)} \parallel \right)^2\right)^{1/2}. \tag{14}$$

In order to understand these measures,we give a simple numerical example in the following.

Example 3.1. *Let two collections of CFVs are given as*

$$A = \{0.5 + 0.5j, 0.4 + 0.3j, -0.4 + 0.5j, 0.7 - 0.2j\},$$

$$B = \{0.2 + 0.1j, 0.4 - 0.3j, -0.4 - 0.3j, 0.6 - 0.2j\}.$$

Then

$$d_{CFD}(a_1, b_1) = \frac{1}{2} \parallel (0.5 + 0.5j) - (0.2 + 0.1j) \parallel = 0.25.$$

Similarly, we have

$$d_{CFD}(a_2, b_2) = 0.3, d_{CFD}(a_3, b_3) = 0.4, d_{CFD}(a_4, b_4) = 0.05.$$

and thus

$$d_{CFD}(a_{\sigma(1)}, b_{\sigma(1)}) = d_{CFD}(a_3, b_3) = 0.4, \quad d_{CFD}(a_{\sigma(2)}, b_{\sigma(2)}) = d_{CFD}(a_2, b_2) = 0.3,$$

$$d_{CFD}(a_{\sigma(3)}, b_{\sigma(3)}) = d_{CFD}(a_1, b_1) = 0.25, \quad d_{CFD}(a_{\sigma(2)}, b_{\sigma(4)}) = d_{CFD}(a_4, b_4) = 0.05.$$

Suppose that $w = (1, 0, 0, 0)$ is the weight vector of the distance $d_{CFD}(a_i, b_i)(i = 1, 2, ..., 4)$, then we have

$$MCFD_1(A,B) = \max\{0.25, 0.3, 0.4, 0.05\} = 0.4.$$

Suppose that $w = (0, 0, 0, 1)$ is the weight vector of the distance $d_{CFD}(a_i, b_i)(i = 1, 2, ..., 4)$, then we have

$$MCFD_2(A,B) = \min\{0.25, 0.3, 0.4, 0.05\} = 0.05.$$

Suppose that $w = (0.1, 0.2, 0.3, 0.4)$ is the weight vector of the distance $d_{CFD}(a_i, b_i)(i = 1, 2, ..., 4)$, and, let $\lambda = 1$, then we have

$$CFOWHD(A,B) = (0.4 \times 0.4 + 0.3 \times 0.3 + 0.2 \times 0.25 + 0.1 \times 0.05) = 0.305,$$

let $\lambda = 2$, then we have

$$CFOWED(A,B) = (0.4 \times 0.4^2 + 0.3 \times 0.3^2 + 0.2 \times 0.25^2 + 0.1 \times 0.05^2)^{1/2} = 0.3221,$$

let $\lambda \to 0$, then we have

$$CFOWGD(A,B) = (0.4^{0.4} \times 0.3^{0.3} \times 0.25^{0.2} \times 0.05^{0.1}) \approx 0.2713.$$

Suppose that $w = (0.25, 0.25, 0.25, 0.25)$ is the weight vector of the distance $d_{CFD}(a_i, b_i)(i = 1, 2, ..., 4)$, and, let $\lambda = 1$, then we have

$$NCFHD(A,B) = (0.25 \times 0.4 + 0.25 \times 0.3 + 0.25 \times 0.25 + 0.25 \times 0.05) = 0.25,$$

let $\lambda = 2$, then we have

$$NCFED(A,B) = (0.25 \times 0.4^2 + 0.25 \times 0.3^2 + 0.25 \times 0.25^2 + 0.25 \times 0.05^2)^{1/2} \approx 0.2806.$$

4 Illustrative example

We give an illustrative example in the selection problem using CFOWD measures. The positions of three alternatives A_1, A_2, A_3 and decision makers B are estimated in Table 1 by three experts E_1, E_2, E_3, where r and θ represent the distance (in km) and direction, respectively. Then which is the nearest alternative?

Table 1: Positions of the alternatives and decision maker given by experts.

	A_1	A_2	A_3	B
E_1	$0.5 \cdot e^{j1.2\pi}$	$0.8 \cdot e^{j1.7\pi}$	$0.9 \cdot e^{j0.5\pi}$	$0.3 \cdot e^{j0.2\pi}$
E_2	$0.5 \cdot e^{j1.3\pi}$	$0.8 \cdot e^{j1.8\pi}$	$0.9 \cdot e^{j0.5\pi}$	$0.4 \cdot e^{j0.1\pi}$
E_3	$0.6 \cdot e^{j1.2\pi}$	$0.7 \cdot e^{j1.7\pi}$	$0.9 \cdot e^{j0.6\pi}$	$0.4 \cdot e^{j0.2\pi}$

Table 2: Distance results for experts.

	$d_{CFD}(B, A_1)$	$d_{CFD}(B, A_2)$	$d_{CFD}(B, A_3)$
E_1	0.5	0.4272	0.3817
E_2	0.4283	0.3255	0.4323
E_3	0.4	0.4031	0.4323

Table 3: Distance results.

	$MCFD_1$	$MCFD_2$	NCFHD	CFOWHD	CFOWED	CFOWGD
$d(B, A_1)$	0.5	0.4	0.4428	0.4513	0.4532	0.4495
$d(B, A_2)$	0.4247	0.3255	0.3853	0.3972	0.3990	0.3953
$d(B, A_3)$	0.4323	0.3817	0.4154	0.4222	0.4227	0.4217

Table 4: Ordering of the alternatives.

	Ordering
$MCFD_1$	$d(B, A_1) \succ d(B, A_3) \succ d(B, A_2)$
$MCFD_2$	$d(B, A_1) \succ d(B, A_3) \succ d(B, A_2)$
NCFHD	$d(B, A_1) \succ d(B, A_3) \succ d(B, A_2)$
CFOWHD	$d(B, A_1) \succ d(B, A_3) \succ d(B, A_2)$
CFOWED	$d(B, A_1) \succ d(B, A_3) \succ d(B, A_2)$
CFOWGD	$d(B, A_1) \succ d(B, A_3) \succ d(B, A_2)$

For each expert, the distance results are shown in Table 2. By using the CFOWD measure with the weighting vector $w = (0.4, 0.4, 0.2)$, the results are shown in Table 3. As we can see in Table 4, A_2 is the nearest alternative for all cases.

5 Conclusions

In this paper, we have presented a complex fuzzy ordered weighted distance (CFOWD) measure, which is very useful to deal with the decision information represented in complex fuzzy values under uncertain situations. This CFOWD measure includes a wide range of complex fuzzy distance measures such as the complex fuzzy maximum distance, the complex fuzzy minimum distance, CFOWHD, CFOWED, CFOWGD, NCFHD and NCFED measures. We gave an illustrative example regarding the selection problem by using the proposed CFOWD measures.

As we know, complex values are frequently encountered with many different applications, such as engineering, economics and medicine. In future research, we expect to develop these extensions and apply them.

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References

[1] A. U. M. Alkouri, A. R. Salleh, *Linguistic variables, hedges and several distances on complex fuzzy sets*, Journal of Intelligent and Fuzzy Systems, **26** (2014), 2527-2535.

[2] L. Bi, S. Dai, B. Hu, *Complex fuzzy geometric aggregation operators*, Symmetry, **10** (2018), 251.

[3] L. Bi, S. Dai, B. Hu, S. Li, *Complex fuzzy arithmetic aggregation operators*, Journal of Intelligent and Fuzzy Systems, **26** (2019), 2765-2771.

[4] L. Bi, B. Hu, S. Li, S. Dai, *The parallelity of complex fuzzy sets and parallelity preserving operators*, Journal of Intelligent and Fuzzy Systems, **34** (2018), 4173-4180.

[5] L. Bi, Z. Zeng, B. Hu, S. Dai, *Two classes of entropy measures for complex fuzzy sets*, Mathematics, **7** (2019), 96.

[6] S. Dai, *A generalization of rotational invariance for complex fuzzy operations*, IEEE Transactions on Fuzzy Systems, (2020), doi:10.1109/TFUZZ.2020.2969874.

[7] S. Dai, L. Bi, B. Hu, *Distance measures between the interval-valued complex fuzzy sets*, Mathematics, **7** (2019), 549.

[8] S. Dick, *Towards complex fuzzy logic*, IEEE Transactions on Fuzzy Systems, **13** (2005), 405-414.

[9] Z. Chen, S. Aghakhani, J. Man, S. Dick, *ANCFIS: A neuro-fuzzy architecture employing complex fuzzy sets*, IEEE Transactions Fuzzy Systems, **19** (2011), 305-322.

[10] H. Garg, D. Rani, *Some generalized complex intuitionistic fuzzy aggregation operators and their application to multicriteria decision-making process*, Arabian Journal for Science and Engineering, **44**(3) (2019), 2679-2698.

[11] B. Hu, L. Bi, S. Dai, *The orthogonality between complex fuzzy sets and its application to signal detection*, Symmetry, **9** (2017), 175.

[12] B. Hu, L. Bi, S. Dai, S. Li, *Distances of complex fuzzy sets and continuity of complex fuzzy operations*, Journal of Intelligent and Fuzzy Systems, **35** (2018), 2247-2255.

[13] B. Hu, L. Bi, S. Dai, S. Li,*The approximate parallelity of complex fuzzy sets*, Journal of Intelligent and Fuzzy Systems, **35** (2018), 6343-6351.

[14] C. Li, *Complex neuro-fuzzy ARIMA forecasting. A new approach using complex fuzzy sets*, IEEE Transactions on Fuzzy Systems, **21** (2013), 567-584.

[15] C. Li, T. W. Chiang, L. C. Yeh, *A novel self-organizing complex neuro-fuzzy approach to the problem of time series forecasting*, Neurocomputing, **99** (2013), 467-476.

[16] C. Li, T. Wu, F. T. Chan, *Self-learning complex neuro-fuzzy system with complex fuzzy sets and its application to adaptive image noise canceling*, Neurocomputing, **94** (2012), 121-139.

[17] J. Ma, G. Zhang, J. Lu, *A method for multiple periodic factor prediction problems using complex fuzzy sets*, IEEE Transactions on Fuzzy Systems, **20** (2012), 32-45.

[18] J. M. Merigó, A. M. Gil-Lafuente, *The ordered weighted averaging distance operator*, Lectures on Modelling and Simulation, **8** (2007), 1-11.

[19] J. M. Merigó, A. M. Gil-Lafuente, *Using the OWA operator in the Minkowski distance*, International Journal of Electrical and Computer Engineering, **3** (2008), 149-157.

[20] D. Ramot, M. Friedman, G. Langholz, A. Kandel, *Complex fuzzy logic*, IEEE Transactions on Fuzzy Systems, **11** (2003), 450-461.

[21] D. Ramot, R. Milo, M. Friedman, A. Kandel, *Complex fuzzy sets*, IEEE Transactions on Fuzzy Systems, **10** (2002), 171-186.

[22] D. Rani, H. Garg, *Complex intuitionistic fuzzy power aggregation operators and their applications in multicriteria decision-making*, Expert Systems, (2018), e12325.

[23] Z. S. Xu, J. Chen, *Ordered weighted distance measure*, Journal of Systems Science and Systems Engineering, **17** (2008), 432-445.

[24] R. R. Yager, *On ordered weighted averaging aggregation operators in multi-criteria decision making*, IEEE Trans Systems Man Cybernet B, **18**(1) (1988), 183-190.

[25] R. R. Yager, *Norms induced from OWA operators*, IEEE Transactions on Fuzzy Systems, **18** (2010), 57-66.

[26] G. Zhang, T. S. Dillon, K. Y. Cai, J. Ma, J. Lu, *Operation properties and delta-equalities of complex fuzzy sets*, International Journal of Approximate Reasoning, **50** (2009), 1227-1249.

[27] S. Zeng, W. Su, *Intuitionistic fuzzy ordered weighted distance operator*, Knowledge-Based Systems, **24** (2011), 1224-1232.

[28] S. Zeng, W. Su, A. Le, *Fuzzy generalized ordered weighted averaging distance operator and its application to decision making*, International Journal of Fuzzy Systems, **14**(3) (2012), 402-412.