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Redundancy of mset topologies

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Abstract

In this note, our aim is to present the relation between msets and L-fuzzy sets. We prove that an mset can be viewed as an L-fuzzy set. An mset topology can be viewed as an L-topology. Therefore, it follows that mset topologies are redundant in theoretical sense.

Keywords: L-fuzzy set, L-topology, mset, mset topology.

1 Introduction

The rapid development of science has led to an urgent need for the sets theory development. For example, Zadeh [23] introduced the fuzzy sets theory as an extension of the crisp sets theory. Later, Atanassov [1] extended the fuzzy sets by adding another degree of nonmembership. The new structure given the name intuitionistic fuzzy sets. The attempts to offer generalizations of crisp sets theory did not stop. In 1999, Molodtsov [15] proposed the soft sets theory to deal with the uncertainty in a parametric behavior.

However as proved in [7], intuitionistic fuzzy sets are redundant and represent unnecessarily complicated, strictly special subcases of standard fixed-basis set theory. Analogously it follows that (fuzzy) soft set and (fuzzy) soft topologies are redundant and unnecessarily complicated in theoretical sense (see [16, 17, 18]).

In the real world it is observed that there is enormous repetition. For instance, there are many hydrogen atoms, many water molecules, many strands of DNA, etc.

In classical set theory, a set is a well-defined collection of distinct objects. If repeated occurrences of any object is allowed in a set, then a mathematical structure, that is known as multiset (mset or bag, for short [2, 3, 4, 12, 22]). Thus, a multiset differs from a set in the sense that each element has a multiplicity—a natural number not necessarily one—that indicates how many times it is a member of the multiset. One of the most natural and simplest examples is the multiset of prime factors of a positive integer 'n'. The number 504 has the factorization $504 = 2^3 3^2 7^1$ which gives the multiset {2,2,2,3,3,7}.

In [9], Girish and John presented the construction of topological structures using binary relations and rough sets in the context of multisets. The basic concepts of general topology on classical sets are generalized on multisets. The concept of rough multiset is introduced and the relationship between rough multiset and multiset topology is investigated [5, 6, 9, 10, 13, 14, 19, 20, 21].

In [8], Ghareeb concluded that multiset topology is exactly a special subcase of general topology. Thus there is no benefit from continuing to study the theoretical aspects of multiset topology. Then a natural problem is as follows. Is Ghareeb's conclusion right?

In this note, we shall present the condition under which Ghareeb's results are correct. We prove that an mset can be viewed as an L-fuzzy set. An mset topology can be viewed as an L-topology.

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2 Msets as L-fuzzy sets

The notion of msets was introduced by Yager [22]. The basic definitions and notions of relations and functions in multiset context introduced by Girish and John [9, 10] are presented.

Definition 2.1. [10] *An mset M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \to \mathbb{N}$ where $\mathbb{N}$ represents the set of non-negative integers.*

Let M be an mset from the set $X = \{x_1, x_2, \cdots, x_n\}$ with x_i appearing k_i times in M. It is denoted by $x_i \in^{k_i} M$.

Clearly, a set is a special case of an mset. Let M and N be two msets drawn from a set X. Then, the following are defined:

(i) $M = N$ if $C_M(x) = C_N(x), \forall x \in X$.

(ii) $M \subseteq N$ if $C_M(x) \leq C_N(x), \forall x \in X$.

(iii) $P = M \cup N$ if $C_P(x) = \max\{C_M(x), C_N(x)\}, \forall \in X$.

(iv) $P = M \cap N$ if $C_P(x) = \min\{C_M(x), C_N(x)\}, \forall x \in X$.

(v) $P = M \oplus N$ if $C_P(x) = C_M(x) + C_N(x), \forall x \in X$.

(vi) $P = M \ominus N$ if $C_P(x) = \max\{C_M(x) - C_N(x), 0\}, \forall x \in X$.

The mset space $[X]^m$ is the set of all msets whose elements are in X such that no element in the mset occurs more than m times.

Let $\{M_i \mid i \in \Omega\}$ be a collection of msets drawn from $[X]^m$. Then the following operations are possible.

(i) The union $\bigcup_{i\in\Omega} M_i$ of $\{M_i \mid i \in \Omega\}$ is defined as $C_{\bigcup_{i\in\Omega} M_i}(x) = \max_{i\in\Omega} C_{M_i}(x)$, $x \in X$.

(ii) The intersection $\bigcap_{i\in\Omega} M_i$ of $\{M_i \mid i \in \Omega\}$ is defined as $C_{\bigcap_{i\in\Omega} M_i}(x) = \min_{i\in\Omega} C_{M_i}(x)$, $x \in X$.

(iii) The mset complement M^c of M is defined as $C_{M^c}(x) = m - C_M(x), x \in X$.

The power set of an mset M is the support set of the power mset and is denoted by $P^*(M)$.

If we denote $\mathbb{N}\cup\{+\infty\}$ by $\overline{\mathbb{N}}$, then it is easy to see that $\overline{\mathbb{N}}$ is a complete lattice. Thus an mset can be regarded as an $\overline{\mathbb{N}}$-fuzzy set [11]. Further we can easily prove that the operations $\bigcup$ and $\bigcap$ in $[X]^m$ are exactly the operations $\bigvee$ and $\bigwedge$ in $\overline{\mathbb{N}}^X$ for any $m \in \mathbb{N}$. Thus we easily obtain the fact that $([X]^m, \bigvee, \bigwedge)$ is a complete sublattice of $\overline{\mathbb{N}}^X$ for any $m \in \mathbb{N}$.

In [9], Girish and John introduced the notion of mset topology as follows.

Definition 2.2. [10] *Let $M \in [X]^m$ and $\tau \subseteq P^*(M)$. Then τ is called an mset topology if it satisfies the following properties.*

(1) $\emptyset$ and M are in τ.

(2) The union of the elements of any sub-collection of τ is in τ.

(3) The intersection of the elements of any finite sub collection of τ is in τ.

Given an mset M in $[X]^m$, Ghareeb [8] construct an isomorphism $\psi : (P^*(M), \bigcup, \bigcap) \to (\psi(M), \bigcup, \bigcap)$, where $\psi : [X]^m \to X \times \downarrow \mathbb{N}$ is defined as

$$\psi(M) = \bigcup_{x\in M^*} \{x\} \times \downarrow C_M(x), \text{ and } \downarrow C_M(x) = \{n \in \mathbb{N} \mid n \leq C_M(x)\}.$$

And Ghareeb proved the following result.

Proposition 2.3. [8] *(M, τ) is an mset topological space if and only if $(\psi(M), \psi^{\to}(\tau))$ is a topological space, where $\psi^{\to}(\tau) = \{\psi(V) \mid V \in \tau\}$.*

From Proposition 2.3 we know that if we consider only two operations $\bigcup$ and $\bigcap$, then an mset topology defined on M is equivalent to a topology defined on the family of all down sets $\downarrow \psi(M)$.

When the complement operation is not involved, all claims around general topology can be acclimatized to preclude any possible scheme based on mset topology.

But if the complement operation is involved, then the above statement is not correct. In fact, we can easily see that for an mset M, $\psi(M^c) = (\psi(M))^c$ is not true. In this case, not all conclusions in the general topology can be translated into the mset topology.

We must ask a principal question. Is an mset topology mathematically redundant, in which sense does its redundancy occur?

Now we answer the above question. We shall prove that an mset topology can be regarded as a special L-fuzzy topology.

Let $(X, \mathcal{T})$ be an L-topological space and $A \in L^X$. If we let $\mathcal{T}|_A = \{A \wedge T \mid T \in \mathcal{T}\}$, then the following conditions are true.

(1) $\emptyset, A \in \mathcal{T}|_A$;

(2) If $U, V \in \mathcal{T}|_A$, then $U \wedge V \in \mathcal{T}|_A$;

(3) If $\{U_i\}_{i\in I}$ is a family of L-fuzzy sets in $\mathcal{T}|_A$, then $\bigvee\limits_{i\in I} U_i \in \mathcal{T}|_A$.

We shall say that $(A, \mathcal{T}|_A)$ is a quasi L-topological space.

The following theorem present the relation between mset topological spaces and quasi L-topological spaces.

Theorem 2.4. *Let $M \in [X]^m$ and let τ be an mset topology on M. Then (M, τ) can be regarded as a quasi $[0,1]$-topological space.*

Proof. As we mentioned, $M \in [X]^m$ can be regarded as an L-fuzzy set in which $L = \{0, 1, 2, \cdots, m\}$ is a complete lattice and a clain. L can be embedded into $[0,1]$ in the following form. For any $n \in \{0, 1, 2, \cdots, m\}$, define $\phi(n) = \frac{n}{m}$. Thus an mset M can be mapped to a $[0,1]$-fuzzy set $\varphi(M)$ such that for all $x \in X$, $\varphi(M)(x) = \phi(C_M(x)) = \frac{C_M(x)}{m}$.

Obviously $\varphi : [X]^m \to [0,1]^X$ is a one to one mapping. It is easy to see that

$$\varphi(\emptyset)(x) = \phi(C_\emptyset(x)) = \frac{0}{m} = 0, \forall x \in X.$$

Given a family of msets $\{M_i \mid i \in \Omega\}$, We have the following equality $\forall x \in X$,

$$\varphi\left(\bigcup_{i\in\Omega} M_i\right)(x) = \frac{\max\limits_{i\in\Omega} C_{D_i}(x)}{m} = \max_{i\in\Omega} \frac{C_{D_i}(x)}{m} = \bigvee_{i\in\Omega} \varphi(M_i)(x).$$

Analogously we have the following.

$$\varphi\left(\bigcap_{i\in\Omega} M_i\right)(x) = \frac{\min\limits_{i\in\Omega} C_{D_i}(x)}{m} = \min_{i\in\Omega} \frac{C_{D_i}(x)}{m} = \bigwedge_{i\in\Omega} \varphi(M_i)(x).$$

In particular given two msets M_1 and M_2 in M. We have $\varphi(M_1 \cap M_2) = \varphi(M_1) \wedge \varphi(M_2)$. This shows that $(\varphi(M), \varphi(\tau))$ is a quasi $[0,1]$-topological space, where $\varphi(\tau) = \{\varphi(D) \mid D \in \tau\}$. □

Given two msets D and E in M with $D \leq E$. The complement $P = E - D$ of D in E is defined as

$$C_P(x) = C_E(x) - C_D(x).$$

If we define the complement $\varphi(E) - \varphi(D)$ of two fuzzy sets $\varphi(D)$ and $\varphi(E)$ as

$$(\varphi(E) - \varphi(D))(x) = \varphi(E)(x) - \varphi(D)(x),$$

then we have

$$(\varphi(E) - \varphi(D))(x) = \frac{C_E(x) - C_D(x)}{m} = \varphi(E - D)(x),$$

which implies φ preserves complement operation.

Now, we can say that an mset topology can be regarded as a $[0,1]$-fuzzy set topology. Therefore, it follows that mset topologies are redundant in theoretical sense.

3 Conclusion

In this note, we proved that an mset can be viewed as an L-fuzzy set and an mset topology can be viewed as an L-topology. Therefore, it follows that mset topologies are redundant in theoretical sense.

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References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87-96.

[2] W. D. Blizard, *Multiset theory*, Notre Dame Journal of Formal Logic, **30** (1988), 36-66.

[3] J. Casasnovas, G. Mayor, *Discrete t-norms and operations on extended multisets*, Fuzzy Sets and Systems, **159** (2008), 1165-1177.

[4] K. Chakrabarty, *Bags with interval counts*, Foundations of Computing and Decision Sciences, **25** (2000), 23-36.

[5] S. A. El-Sheikh, R. A. K. Omar, M. Raafat, *γ-operation in m-topological space*, General Mathematics Notes, **7** (2015), 40-54.

[6] S. A. El-Sheikh, R. A. K. Omar, M. Raafat, *Separation axioms on multiset topological space*, Journal of New Theory, **7** (2015), 11-21.

[7] J. G. Garcia, S. E. Rodabaugh, *Order-theoretic, topological, categorical redundancies of interval-valued sets, grey sets, vague sets, interval-valued intuitionistic sets, intuitionistic fuzzy sets and topologies*, Fuzzy Sets and Systems, **156** (2005), 445-484.

[8] A. Ghareeb, *Redundancy of multiset topological spaces*, Iranian Journal of Fuzzy Systems, **14** (2017), 163-168.

[9] K. P. Girish, S. J. John, *Multiset topologies induced by multiset relations*, Information Sciences, **188** (2012), 298-313.

[10] K. P. Girish, J. S. Jacob, *On multiset topologies*, Theory and Applications of Mathematics and Computer Science, **2** (2012), 37-52.

[11] J. A. Goguen, *L-fuzzy sets*, Journal of Mathematical Analysis and Applications, **18** (1967), 145-174.

[12] S. P. Jena, S. K. Ghosh, B. K. Tripathy, *On the theory of bags and lists*, Information Sciences, **132** (2001), 241-254.

[13] A. Kandil, O. Tantawy, S. El-Sheikh, A. Zakaria, *Multiset proximity spaces*, Journal of the Egyptian Mathematical Society, **24** (2016), 562-567.

[14] P. M. Mahalakshmi, P. Thangavelu, *m-connectedness in m-topology*, International Journal of Pure and Applied Mathematics, **8** (2016), 21-25.

[15] D. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications, **37** (1999), 19-31.

[16] F. G. Shi, C. Z. Fan, *Fuzzy soft sets as L-fuzzy sets*, Journal of Intelligent and Fuzzy Systems, **37** (2019), 5061-5066.

[17] F. G. Shi, B. Pang, *Redundancy of fuzzy soft topological spaces*, Journal of Intelligent and Fuzzy Systems, **27** (2014), 1757-1760.

[18] F. G. Shi, B. Pang, *A note on soft topological spaces*, Iranian Journal of Fuzzy Systems, **5** (2015), 149-155.

[19] K. Shraven, B. C. Tripathy, *Generalised closed sets in multiset topological spaces*, Proyecciones, **37** (2018), 223-237.

[20] K. Shraven, B. C. Tripathy, *Multiset ideal topological spaces and local functions*, Proyecciones, **37** (2018), 699-711.

[21] K. Shraven, B. C. Tripathy, *Multiset mixed topological space*, Soft Computing, **23** (2019), 9801-9805.

[22] R. R. Yager, *On the theory of bags*, International Journal of General Systems, **13** (1986), 23-37.

[23] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (1965), 338-353.