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Original paper

Uncertain network DEA models for evaluating efficiencies of Bi-echelon supply chain with asymmetric power

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Abstract

Nowadays, some burgeoning issues occurring in green and sustainable supply chain face an increasing number of tackles, including uncertain factors and unobserved data, which makes it even more complicated to assess supply chain efficiency. To address this issue, this paper applies uncertainty theory to two-stage network DEA intending to deal with inaccurate data. Moreover, the bi-echelon supply chain generally suffers asymmetric power among involved members. Therefore, this paper proposes attrition rate (D) and fulfillment rate (G) in the first and second stages respectively so as to reveal the bi-echelon supply chain within different leader-follower patterns. The first two models assume the upstream firm is the leader while the last two models regard the downstream firm as the leader. We find out that the results of the evaluation vary under divergent patterns accordingly by running a numerical example. Adopting proposed models can help decision-makers of the multistage supply chain make decisions more effectively and avoid possible mistakes.

Keywords: Uncertainty theory, data envelopment analysis, supply chain management.

1 Introduction

Data envelopment analysis (DEA) has developed and received considerable attention in various fields over the past decades [6, 7, 8, 10, 13, 28]. After the appearance of the most classical DEA model generated by Charnes, Cooper and Rhods [2], numerous DEA models have been put forward to evaluate whether the peer decision-making units (DMUs) are efficient or not (Chang et al. [1]) without considering the internal structure inside it. According to Wei [38], if and only if all the sub-processes attain efficiency, the entire system can reach efficiency. Therefore, it is essential to investigate the inefficiency of subprocess, which can access the root cause of the inefficiency of the whole system. The decision-makers can thereby modify the decisions on the basis of distinguishing the inefficient sub-processes in the system so that the overall performance can be enhanced.

Applying network DEA, many researchers have made a huge number of efforts to obtain outstanding academic achievements in order to transform the "black box" into "glass box" [5]. For example, Färe and Grosskopt [9] proposed a network DEA model that discussed the concept of an intermediate product. Seiford and Zhu [33] originated two-stage DEA models to evaluate sub-processes such as profitability and marketability of 55 top-class banks in the US. Cook et al. [4] presented a general summary of new researches on network DEA models.

One of the most basic and prevailing structures of the network operation is two-stage DEA models because they can be easily generalized into other structures with more than two processes. Generally speaking, most two-stage models decompose the whole procedure process into two stages in which both inputs and outputs are frequently seen in other "black-box" DEA models [38]. Moreover, two-stage DEA models often use intermediate measures that are the outputs of the first sub-process and the inputs of the second sub-process to produce final outputs. Kao and Hwang [13] created

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two-stage models, which include intermediate products. They can connect the two sub-processes so that it is capable of representing the mutual influence between the entire system and its components.

However, when illustrating into some more complex systems such as the supply chain, the previous models are not reasonable enough because the power of members of a supply chain is asymmetric [18]. The unbalance strengthens those who are more powerful in the whole supply chain and can be regarded as focal firms, and we can call them "leader". Otherwise, we call them "follower" [17]. This circumstance prevails in real life. For example, as an upstream firm, Qualcomm is a chip supplier of smart-phone manufacturers. The capacity of many companies relies to a large extent on the supply of the chips bought from it. In this context, Qualcomm has domain advantages, so it can be seen as the core enterprise in the supply chain. On the other hand, as a downstream firm, large chain retailer Carrefour recently becomes increasingly more important for its massive marketing channels, which indicates that Carrefour can be treated as a focal company in the supply chain. For the virtue of the diversity of the two situations, it is infeasible to view the focal firm and non-focal firm as same when evaluating the efficiency of the supply chain. If the sub-process, which is more powerful in the whole system is not taken into consideration, then the improper result that is far away from the real one, may occur. To avoid this kind of potential error, Liang et al. [19] develop two-stage models that have the ability to characterize the efficiency of each and overall system, in a way that defines a leader-follower pattern for the two sub-systems, and the results of overall efficiency under different scenarios quite differ.

Considering the supply chain of reality under various conditions, it is easy to find a vast number of differences between the perspectives of assessing the efficiency in the past and at present. The later broadens the range of the evaluation. Traditional supply chain attaches great importance to statistics on precise data such as cost and time. In contrast, the emerging supply chain pays increasingly more attention to the environment, customer satisfaction, and social benefits (e.g., emergency and poverty supply chain). For example, for the environment and social performance, in 2011, Chen and Delmas [3] proposed a new way for corporate social performance evaluation using DEA. However, when adopting DEA models to evaluate the efficiency of a new-type supply chain, inputs and outputs are uncertain data because some of them do exist, but there are no exact figures available. For instance, the amount of carbon dioxide engendered by the producing process cannot be measured in an accurate way. As for customer satisfaction, it is a subjective experience that is impossible to access the precise number. Another reason for uncertainty in supply chain evaluation is an unpredictable incident like an emergency (e.g., flood and C9 leakage). Thus, if it is necessary to evaluate the inputs and outputs of DMUs in a supply chain while the data is imprecise, conventional DEA models are not able to figure out the right conclusion.

To cope with uncertainty, many kinds of paradigms have been put forward, such as robust optimization, fuzzy set theory, and probability theory. Robust optimization first occurred in 1973 [35]. Then it was improved and established by Mulvey [30] in 1995. Since robust optimization has many merits, some scholars focusing on dealing with uncertainty are backed by robust optimization, it is a popular method in a supply chain that mainly aims to prevent the negative impact of uncertainty. After that, in 2007, Leung et al. [16] considered and minimized total costs consisting of production cost, labor cost, inventory cost, and workforce changing cost under uncertainty and built a robust optimization model. Then they analyzed both solution robustness and model robustness. Besides, Ouyang and Daganzo [31] brought forward robust analytical conditions so as to diagnose the bullwhip effect and bound its degree in a single-stage supply chain with arbitrary customer demands. In 2011, Mirzapour, Al-e-hashem et al. [29] developed a supply chain model with multiple vendors, multiple suppliers, and multiple consumers, using a new robust multi-objective mixed integer nonlinear programming. With the help of robust optimization, above works have made great contributions to the supply chain management science. Nevertheless, none of them concentrated on efficiency evaluation. In 2008, Sadjadi and Omrani [32] proposed the first robust DEA model. It considered uncertain parameters when assessed the performance of electricity distribution companies. In 2010, Shokouhi et al. [34] came up with an adaptation of the standard DEA under conditions of uncertainty, which was based on a robust optimization model. It surmounted the problem that it was hard to model with fuzzy representations in an uncertain DEA model and made results less complicated.

Though robust optimization is classic, it is not ideally suited to our research. The reason is that in principle, robust optimization always considers the worst-case, but the worst results might not happen at all. The general robust optimization problem can be transformed into a min or max problem. It can be seen that the maximum is always reduced to the minimum (which is often more expensive than the average to the minimum). However, the probability that the actual situation is just the worst-case is very small, so we can say that robust optimization is conservative (because the worst-case optimization is considered). In this paper we focus on discussing how to effectively evaluate the efficiency of a supply chain and recognize the leader when the inputs and outputs are inaccurate. Therefore, using robust optimization may lead to unreasonable results.

Many scholars employed the fuzzy set theory to resolve inaccurate inputs and outputs in DEA models. Kao and Liu [14] came up with fuzzy observations which expressed efficiency measures by membership functions instead of a crisp number. Wanke and Barros [37] assessed the productivity efficiency of some Mozambican banks, using integrated fuzzy

DEA to assess underlying uncertainty. Tavana et al. [36] developed two-stage fuzzy DEA models that decomposed and measured the efficiency of the system. Based on their unique experience, they have designed fuzzy rules and membership functions in their paper. However, we can think in a different way. If we define customer satisfaction as a fuzzy variable, and assign it a membership function. Then we would conclude the following three propositions: (1) the customer satisfaction is "exactly 0.8" with possibility measure 1; (2) the customer satisfaction is "not 0.8" with possibility measure 1; (3) "exactly 0.8" has the same possibility of "not 0.8". This kind of indeterminate quantities cannot be quantified by possibility measure and then they are not fuzzy concepts (Liu [21]).

According to Liu [21], probability theory can only be used when the distribution function is close enough to the real frequency and if not, uncertainty theory should be adopted. However, most of the time the inputs and outputs in an emerging supply chain are impossible to fit in with the condition above. For example, it is obvious that as a variable in DEA models, customer satisfaction cannot be affirmed as a set of numbers whose distribution function is approximately similar to the frequency. To help address this issue, we employ uncertain DEA theory.

Lots of uncertain DEA models have been established. Wen et al. [39] put forward the first uncertain DEA model and then developed it. After that, Lio and Liu [20] constructed an uncertain DEA model that can evaluate the efficiency of DMUs. To supplement it, Jiang et al. [11] presented an uncertain DEA model to identify the scale efficiencies which are more relevant to the private sector in addition to splitting the technical and scale efficiency of decision-making units with the imprecise data. Moreover, Jiang et al. [12] proposed two uncertain DEA models to identify the specific status of each DMU. However, none of the previous researches is specialized in bi-echelon supply chain efficiency evaluation.

Therefore, this paper focuses on raising the research gap by means of applying uncertainty theory to two-stage DEA models. For one thing, bi-echelon DEA models proposed in the current paper can recognize the leader; for another thing, uncertainty theory can help deal with the inaccurate data that occurred in the models [23, 24, 25, 27, 40]. Combined the two convincing tools, the uncertain DEA models for bi-echelon supply chain efficiency evaluation will the help assess the efficiency of the supply chain, especially emerging supply chain in a more effective way. Recognizing the leader in a supply chain, we can propose some constructive suggestions for decision-makers to improve the efficiency like how to integrate and coordinate the members to resist all kinds of crisis in an era of emerging supply chain playing remarkable role under the background of current economic where uncertainty is everywhere.

The rest part of this article is formed as follows. Next section will import some preparatory instructions about uncertainty theory while highlighting the uncertainty DEA models that have been proposed by former studies. After that, the uncertain DEA models for bi-echelon supply chain efficiency evaluation will be presented in the third section. Section 4 will offer a numerical example and discusses the results and explorations of the research. The last section will give a summary of the article and the future study direction.

2 Preliminaries

Uncertainty theory, as a powerful mathematical tool for uncertain variable assessing, is a thriving research area. The characteristic features of uncertainty theory were initially introduced and founded by Liu [21] in 2007. Over the past decade, this theory has been deeply rooted in the study of uncertainty. This part will offer a brief but broad view of its knowledge structure for the reason that the fundamental principles and axioms are helpful for the remainder of the study. The uncertain measure $\mathcal{M}$was defined as a set function on a σ-algebra $\mathcal{L}$ over a nonempty set Γ by the following three axioms (Liu [21]):

Axiom 1. (Normality Axiom) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2. (Duality Axiom) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3. (Subadditivity Axiom) For every countable sequence of events Λ_1, Λ_2, $\cdots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

The fourth axiom (product axiom) was proposed by Liu [22] in 2009 as follows:

Axiom 4. (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for k=1, 2, ... The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\},$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \cdots$, respectively.

Definition 2.1. [21] *The uncertainty distribution Φ of an uncertain variable ξ is defined as $\Phi(x) = \mathcal{M}\{\xi \leq x\}$ for any real number x.*

We will introduce some common uncertainty distributions based on the definitions mentioned above. First, we will introduce the most common one called linear uncertainty distribution, which is

$$\Phi(x)=\begin{cases}0, & \text{if } x\le a\\ (x-a)/(b-a), & \text{if } a<x\le b\\ 1, & \text{if } x>b;\end{cases}$$

and the second common uncertainty distribution is called zigzag, which is

$$\Phi(x)=\begin{cases}0, & \text{if } x\le a\\ (x-a)/[2(b-a)], & \text{if } a<x\le b\\ (x+c-2b)/[2(c-b)], & \text{if } b<x\le c\\ 1, & \text{if } x>c.\end{cases}$$

If ξ is an uncertain variable with regular uncertainty distribution $\Phi(x)$, the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ [24].

Liu [24] brought forward the theorem of calculation so as to build the inverse distribution of a strictly monotonous function of independent uncertain variables with regular uncertainty distributions as follows:

Theorem 2.2. [24] *Assume that $\Phi_1,\Phi_2,\cdots,\Phi_n$ represent regular uncertainty distributions of independent uncertain variables $\xi_1,\xi_2,\cdots,\xi_n$, respectively. If f is strictly increasing in respect of $\xi_1,\xi_2,\cdots,\xi_m$ $(m\le n)$ and strictly decreasing in respect of $\xi_{m+1},\xi_{m+2},\cdots,\xi_n$, then $\xi=f(\xi_1,\xi_2,\cdots,\xi_n)$ is an uncertain variable with an inverse uncertainty distribution*

$$\Phi^{-1}(\alpha)=f(\Phi_1^{-1}(\alpha),\cdots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\cdots,\Phi_n^{-1}(1-\alpha)).$$

The expected value is the mean value of the uncertain variable in the sense of the uncertain measure. Under various circumstances, there is an obligation to consider the expected value of ξ.

Definition 2.3. [21] *The expected value of uncertain variable ξ is defined by*

$$E[\xi]=\int_0^{+\infty}\mathcal{M}\{\xi\ge x\}\mathrm{d}x-\int_{-\infty}^0\mathcal{M}\{\xi\le x\}\mathrm{d}x,$$

provided that at least one of the two integrals is finite.

Suppose that ξ is an uncertain variable which accords with uncertainty distribution Φ. Then Liu [21] gave the formulas about the expected values of ξ as follows:

$$E[\xi]=\int_0^{+\infty}(1-\Phi(x))\mathrm{d}x-\int_{-\infty}^0\Phi(x)\mathrm{d}x.$$

Theorem 2.4. [26] *Assume that $\Phi_1,\Phi_2,\cdots,\Phi_n$ represent regular uncertainty distributions of independent uncertain variables $\xi_1,\xi_2,\cdots,\xi_n$, respectively. If f is strictly increasing in respect of $\xi_1,\xi_2,\cdots,\xi_m$ $(m\le n)$ and strictly decreasing in respect of $\xi_{m+1},\xi_{m+2},\cdots,\xi_n$, then the expected value of $\xi=f(\xi_1,\xi_2,\cdots,\xi_n)$ is as follows:*

$$E[\xi]=\int_0^1 f(\Phi_1^{-1}(\alpha),\cdots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\cdots,\Phi_n^{-1}(1-\alpha))\mathrm{d}\alpha.$$

3 Uncertain DEA models for Bi-echelon supply chain efficiency evaluation

In this section, assumption, as well as model formulation, will be presented. The bi-echelon supply chain consists of two members of a supply chain. This model is represented in a supply-chain context and can be easily stretched to match the realistic scene by means of a distinct analysis of the primary two members. Knowing the outcomes of the models, we would evaluate the supply chain more effectively and find the leader in the supply chain. According to the different statuses, both upstream members and the downstream participants can be regarded as the core firm depending on the real condition. Therefore, this study will discuss the models from two aspects. In the first part of this section, we discuss the overall efficiency and efficiency for the members of the two-stage supply chain when the upstream firm is the leader (e.g., Qualcomm.) and the downstream firm is the follower. Subsequently, in the second part, we will transform the perspective to the scenario when the downstream firm is the leader (e.g., Carrefour.) and the upstream firm is the follower. The basic symbols of uncertain DEA models are given as follows:

DMU$_i$: the ith DMU, $i = 1, 2, \ldots, n$
DMU$_o$: the target DMU
$\tilde{\boldsymbol{X}}_{Ai} = (\tilde{X}_{Ai1}, \tilde{X}_{Ai2}, \ldots, \tilde{X}_{Air})$: The uncertain inputs vector of DMU$_i$ in the first stage.
$\tilde{\boldsymbol{X}}_{Ao} = (\tilde{X}_{Ao1}, \tilde{X}_{Ao2}, \ldots, \tilde{X}_{Aor})$: The uncertain inputs vector of DMU$_o$ in the first stage.
$\tilde{\boldsymbol{Y}}_{Ai} = (\tilde{Y}_{Ai1}, \tilde{Y}_{Ai2}, \ldots, \tilde{Y}_{Ais})$: The uncertain outputs vector of DMU$_i$ in the first stage.
$\tilde{\boldsymbol{Y}}_{Ao} = (\tilde{Y}_{Ao1}, \tilde{Y}_{Ao2}, \ldots, \tilde{Y}_{Aos})$: The uncertain outputs vector of DMU$_o$ in the first stage.
$\tilde{\boldsymbol{X}}_{Bi} = (\tilde{X}_{B1}, \tilde{X}_{B2}, \ldots, \tilde{X}_{Br})$: The uncertain inputs vector of DMU$_i$ in the second stage.
$\tilde{\boldsymbol{Y}}_{Bi} = (\tilde{Y}_{Bi1}, \tilde{Y}_{Bi2}, \ldots, \tilde{Y}_{Bis})$: The uncertain outputs vector of DMU$_i$ in the second stage.
$\tilde{\boldsymbol{Y}}_{Bo} = (\tilde{Y}_{Bo1}, \tilde{Y}_{Bo2}, \ldots, \tilde{Y}_{Bos})$: The uncertain outputs vector of DMU$_o$ in the second stage.
$\boldsymbol{U} \in \Re^r$: The vector of output weights.
$\boldsymbol{V} \in \Re^s$: The vector of input weights.

3.1 Upstream firm is the leader and downstream firm is the follower

In this section, the following four models represent upstream-downstream firm interaction as a two-stage system with the upstream firm as the leader and the downstream participator as the follower. Figure 1 represents the scenario where the upstream firm leads the supply chain.

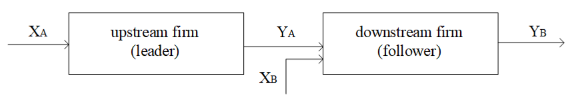

Figure 1: upstream-leader supply chain

We first use model (1) to evaluate the efficiency of the upstream company and then adopt the optimal value of it into model (3) to estimate the efficiency of the downstream member. Using the average number of the two models, we calculate the overall efficiency at the end of this part. Model (1) evaluates the efficiency of the upstream firm as the leader,

$$\begin{cases} \max\limits_{\boldsymbol{U},\boldsymbol{V}} & \varphi = E\left[\dfrac{\boldsymbol{U}_A^T\tilde{\boldsymbol{Y}}_{Ao}}{\boldsymbol{V}_A^T\tilde{\boldsymbol{X}}_{Ao}}\right] = E_{AA} \\ \text{s.t.} & E\left[\dfrac{\boldsymbol{U}_A^T\tilde{\boldsymbol{Y}}_{Aj}}{\boldsymbol{V}_A^T\tilde{\boldsymbol{X}}_{Aj}}\right] \le 1, \quad j = 1, 2, \ldots, n \\ & \boldsymbol{U}_A^T \ge 0 \\ & \boldsymbol{V}_A^T \ge 0. \end{cases} \tag{1}$$

Definition 3.1. *(Uncertain DEA Efficiency) DMU$_o$ is said to be efficient if the optimal value φ of model (1) can be 1.*

The uncertain DEA model above can be applied to assess the efficiency of the core firm. The equivalent form of model (1) is proved as follow:

Theorem 3.2. *Set regular uncertainty distributions $\Psi_{i1}, \Psi_{i2}, \cdots, \Psi_{ir}$ and Φ_{i1}, Φ_{i2}, $\cdots$,Φ_{kq} for independent uncertain input variables $\tilde{X}_{A1}, \tilde{X}_{A2}, \cdots, \tilde{X}_{Ar}$ and output variables $\tilde{Y}_{A1}, \tilde{Y}_{A2}, \cdots, \tilde{Y}_{As}$, respectively. Then model (1) can be transformed into the following model:*

$$\begin{cases} \max\limits_{\boldsymbol{U},\boldsymbol{V}} & \varphi = \displaystyle\int_0^1 \dfrac{\sum\limits_{k=1}^{s} U_{Ak}\Phi_{Aok}^{-1}(\alpha)}{\sum\limits_{l=1}^{r} V_{Al}\Psi_{Aol}^{-1}(1-\alpha)}\mathrm{d}\alpha \\ \text{s.t.} & \displaystyle\int_0^1 \dfrac{\sum\limits_{k=1}^{s} U_{Ak}\Phi_{Ajk}^{-1}(\alpha)}{\sum\limits_{l=1}^{r} V_{Al}\Psi_{Ajl}^{-1}(1-\alpha)}\mathrm{d}\alpha \le 1, \quad j = 1, 2, \ldots, n \\ & U_{Ak} = (U_{A1}, U_{A2}, \cdots, U_{As}) \ge 0 \\ & V_{Al} = (V_{A1}, V_{A2}, \cdots, V_{Ar}) \ge 0 \end{cases} \tag{2}$$

where $\Psi_{Ao1}, \Psi_{Ao2}, \cdots, \Psi_{Aor}$ *and* $\Phi_{Ao1}, \Phi_{Ao2}, \cdots, \Phi_{Aos}$ *are the regular uncertainty distributions of* $\tilde{X}_{A1}, \tilde{X}_{A2}, \cdots, \tilde{X}_{Ar}$ *and* $\tilde{Y}_{A1}, \tilde{Y}_{A2}, \cdots, \tilde{Y}_{As}$, *respectively.*

Proof. As the function $U_A^T\, \tilde{Y}_{Aj} / V_A^T\, \tilde{X}_{Aj}$ is strictly increasing in respect of $\tilde{Y}_{Aj}$ and strictly decreasing in respect of $\tilde{X}_{Aj}$ and $\tilde{Y}_{Aj}$ for each j, on the basis of Theorem 2.1, we can infer the inverse uncertainty distribution of $U_A^T\, \tilde{Y}_{Aj}$ / $V_A^T\, \tilde{X}_{Aj}$ is

$$L_{Aj}^{-1}(\alpha) = \frac{\sum\limits_{k=1}^{s} U_{Ak}\Phi_{Ajk}^{-1}(\alpha)}{\sum\limits_{l=1}^{r} V_{Al}\Psi_{Ajl}^{-1}(1-\alpha)}.$$

And according to Theorem 2.2, we can get

$$E\left[\frac{U_A^T\, \tilde{Y}_{Aj}}{V_A^T \tilde{\boldsymbol{X}}_{Aj}}\right] = \int_0^1 \frac{\sum\limits_{k=1}^{s} U_{Ak}\Phi_{Ajk}^{-1}(\alpha)}{\sum\limits_{l=1}^{r} V_{Al}\Psi_{Ajl}^{-1}(1-\alpha)} \mathrm{d}\alpha \le 1, \quad j = 1, 2, \ldots, n.$$

□

After gaining the optimal value of the first echelon, we will measure the efficiency of the second stage. It should be noticed that due to the properties of the supply chain such as the dynamics and complexity, the intermediate measures between the two stages vary according to the adjustment that the leader or follower makes in order to increase their own performance of the process. In other words, assume that the upstream enterprise is the leader who obtains its optimal efficiency without thinking the efficiency of the downstream member and has a certain output, yet the follower who is passive in the supply chain will not fully accept the leader's output on account of its own efficiency. Thus, we propose a parameter D called attrition rate to depict this phenomenon. Another exploring spot is that in the second stage of the supply chain, the inputs denoted by a variable X_B include not only intermediate products from the first stage but also other inputs which is the same kind of stuff as the first stage such as labor, capacity, etc. We have to consider X_B because the fixed cost in the operation of a company is indispensable. The model (3) evaluates the efficiency of the downstream firm as the follower, using the optimal value of the first stage into the model (3) to estimate the efficiency of the downstream member,

$$\begin{cases} \max\limits_{\boldsymbol{U},\boldsymbol{V},\boldsymbol{D},\boldsymbol{\mu}} & \varsigma = E\left[\dfrac{\boldsymbol{U}_B^T \tilde{\boldsymbol{Y}}_{Bo}}{\boldsymbol{V}_B^T \tilde{\boldsymbol{X}}_{Bo} + D \times \boldsymbol{\mu}_A^T \tilde{\boldsymbol{Y}}_{Ao}}\right] = E_{AB} \\ \text{s.t.} & E\left[\dfrac{\boldsymbol{U}_B^T \tilde{\boldsymbol{Y}}_{Bj}}{\boldsymbol{V}_B^T \tilde{\boldsymbol{X}}_{Bj} + D \times \boldsymbol{\mu}_A^T \tilde{\boldsymbol{Y}}_{Aj}}\right] \le 1 \\ & E\left[\dfrac{\boldsymbol{\mu}_A^T \tilde{\boldsymbol{Y}}_{Aj}}{\boldsymbol{\omega}_A^T \tilde{\boldsymbol{X}}_{Aj}}\right] \le 1, \quad j = 1, 2, \ldots, n \\ & E\left[\dfrac{\boldsymbol{\mu}_A^T \tilde{\boldsymbol{Y}}_{Ao}}{\boldsymbol{\omega}_A^T \tilde{\boldsymbol{X}}_{Ao}}\right] = E_{AA}^* \\ & \boldsymbol{U}_B^T \ge 0 \\ & \boldsymbol{V}_B^T \ge 0 \\ & \boldsymbol{\mu}_A^T \ge 0 \\ & \boldsymbol{\omega}_A^T \ge 0 \\ & \mathrm{D} \ge 0. \end{cases} \tag{3}$$

Definition 3.3. *(Uncertain DEA Efficiency) DMU*$_o$ *is said to be efficient if the optimal value* ς *of model (3) can be 1.*

The uncertain DEA model above can be applied to assess the efficiency of the non-core firm. The equivalent form of model (3) is proved as follow:

Theorem 3.4. *Set regular uncertainty distributions* $\Psi_{i1}, \Psi_{i2}, \cdots, \Psi_{ir}$ *and* Φ_{i1}, Φ_{i2}, $\cdots$, Φ_{kq} *for independent uncertain input variables* $\tilde{X}_{B1}, \tilde{X}_{B2}, \cdots, \tilde{X}_{Bp}$, $\tilde{Y}_{A1}, \tilde{Y}_{A2}, \cdots, \tilde{Y}_{As}$, *and output variables* $\tilde{Y}_{B1}, \tilde{Y}_{B2}, \cdots$, $\tilde{Y}_{Bn}$, *respectively. Then model (3) can be transformed into its equivalent model:*

$$\begin{cases} \max\limits_{\boldsymbol{U},\boldsymbol{V},\boldsymbol{D},\boldsymbol{\mu}} \varsigma = \displaystyle\int_0^1 \frac{\sum\limits_{k=1}^{n} U_{Bk}\Phi_{Bok}^{-1}(\alpha)}{\sum\limits_{l=1}^{p} V_{Bl}\Psi_{Bol}^{-1}(1-\alpha) + D\times\sum\limits_{k=1}^{s}\mu_A^T\Phi_{Aok}^{-1}(1-\alpha)}\mathrm{d}\alpha \\ \text{s.t.} \quad \displaystyle\int_0^1 \frac{\sum\limits_{k=1}^{n} U_{Bk}\Phi_{Bjk}^{-1}(\alpha)}{\sum\limits_{l=1}^{p} V_{Bl}\Psi_{Bjl}^{-1}(1-\alpha) + D\times\sum\limits_{k=1}^{s}\mu_A^T\Phi_{Ajk}^{-1}(1-\alpha)}\mathrm{d}\alpha \le 1 \\ \qquad \displaystyle\int_0^1 \frac{\sum\limits_{k=1}^{s}\mu_A^T\Phi_{Aj}^{-1}(\alpha)}{\sum\limits_{l=1}^{r}\omega_A^T\Psi_{Al}^{-1}(1-\alpha)}\mathrm{d}\alpha \le 1, \quad j=1,2,\ldots,n \\ \qquad \displaystyle\int_0^1 \frac{\sum\limits_{k=1}^{s}\mu_A^T\Phi_{Ao}^{-1}(\alpha)}{\sum\limits_{l=1}^{r}\omega_A^T\Psi_{Ao}^{-1}(1-\alpha)}\mathrm{d}\alpha = E_{AA}^* \\ \qquad 0 \le \mathrm{D} \le 1 \\ \qquad U_{Bk} = (U_{B1}, U_{B2}, \cdots, U_{Bn}) \ge 0 \\ \qquad V_{Bl} = (V_{B1}, V_{B2}, \cdots, V_{Bp}) \ge 0 \\ \qquad \mu_A^T \ge 0 \\ \qquad \omega_A^T \ge 0 \end{cases} \tag{4}$$

where $\Psi_{Bo1}, \Psi_{Bo2}, \cdots, \Psi_{Bor}$, $\Phi_{Ao1}, \Phi_{Ao2}, \cdots, \Phi_{Aos}$ *and* $\Phi_{Bo1}, \Phi_{Bo2}, \cdots, \Phi_{Bos}$ *are the regular uncertainty distributions of* $\tilde{X}_{B1}, \tilde{X}_{B2}, \cdots, \tilde{X}_{Bp}$, $\tilde{Y}_{A1}, \tilde{Y}_{A2}, \cdots, \tilde{Y}_{As}$ *and* $\tilde{Y}_{B1}, \tilde{Y}_{B2}, \cdots, \tilde{Y}_{Bn}$, *respectively.*

Proof. As the function $U_B^T\, \tilde{Y}_{Bj}/(\, V_B^T\, \tilde{X}_{Bj} + D \times\, \mu_A^T\, \tilde{Y}_{Aj})$ is strictly increasing in respect of $\tilde{Y}_{Bj}$ and strictly decreasing in respect of $\tilde{X}_{Bj}$ and $\tilde{Y}_{Aj}$ for each j, they comply with Theorem 2.1 and then the inverse uncertainty distribution of $U_B^T\, \tilde{Y}_{Bj}/(\, V_B^T\, \tilde{X}_{Bj} + D \times\, \mu_A^T\, \tilde{Y}_{Aj})$ is

$$L_{Bj}^{-1}(\alpha) = \frac{\sum\limits_{k=1}^{n} U_{Bk}\Phi_{Bjk}^{-1}(\alpha)}{\sum\limits_{l=1}^{p} V_{Bl}\Psi_{Bjl}^{-1}(1-\alpha) + D\times\sum\limits_{k=1}^{s}\mu_A^T\Phi_{Ajk}^{-1}(1-\alpha)}.$$

On the basis of Theorem 2.2, we have

$$E\left[\frac{U_B^T\,\tilde{Y}_{Bj}}{V_B^T\,\tilde{X}_{Bj} + D\times\,\mu_A^T\,\tilde{Y}_{Aj}}\right] = \int_0^1 \frac{\sum\limits_{k=1}^{n} U_{Bk}\Phi_{Bjk}^{-1}(\alpha)}{\sum\limits_{l=1}^{p} V_{Bl}\Psi_{Bjl}^{-1}(1-\alpha) + D\times\sum\limits_{k=1}^{s}\mu_A^T\Phi_{Ajk}^{-1}(1-\alpha)}\mathrm{d}\alpha \le 1, \quad j=1,2,\ldots,n.$$

□

Taking all the above mention into account, we can delineate the overall efficiency of the bi-echelon supply chain as the mean value of the two optimal values considering the upstream firm as the leader which enjoys the priority in the supply chain,

$$e_{AB} = \frac{1}{2}(E_{AA}^* + E_{AB}^*). \tag{5}$$

3.2 Downstream firm is the leader and upstream firm is the follower

Similarly, it is also feasible to invent a procedure for the scenario when the downstream firm is operating as the leader and the upstream is the follower. This circumstance is also quite usual in reality because of the overwhelming resources of capital, information and goods of the downstream company like some big chain retailers. Then we can get another four models. Figure 2 represents the scenario where the downstream firm leads the supply chain.

In the following part, we will estimate the downstream company and use it to evaluate the efficiency of the upstream player. After that we can get the overall efficiency which is the average number of the two optimal values. The model

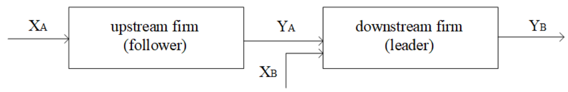

Figure 2: downstream-leader supply chain

(6) evaluates the efficiency of the downstream firm as the leader,

$$\begin{cases} \max\limits_{\boldsymbol{U},\boldsymbol{V}} & \varphi = E\left[\dfrac{\boldsymbol{U}_B^T\tilde{\boldsymbol{Y}}_{Bo}}{\boldsymbol{V}_B^T\tilde{\boldsymbol{X}}_{Bo}+\boldsymbol{V}^T\tilde{\boldsymbol{Y}}_{Ao}}\right] = E_{BB} \\ \text{s.t.} & E\left[\dfrac{\boldsymbol{U}_B^T\tilde{\boldsymbol{Y}}_{Bj}}{\boldsymbol{V}_B^T\tilde{\boldsymbol{X}}_{Bj}+\boldsymbol{V}^T\tilde{\boldsymbol{Y}}_{Aj}}\right] \le 1, \quad j=1,2,\ldots,n \\ & \boldsymbol{U}_B^T \ge 0 \\ & \boldsymbol{V}_B^T \ge 0 \\ & \boldsymbol{V}^T \ge 0. \end{cases} \tag{6}$$

Definition 3.5. *(Uncertain DEA Efficiency) DMU$_o$ is said to be efficient if the optimal value φ of model (6) can be 1.*

The uncertain DEA model above can be applied to assess the efficiency of the core firm. The equivalent form of model (6) is proved as follow:

Theorem 3.6. *Set regular uncertainty distributions $\Psi_{i1},\Psi_{i2},\cdots,\Psi_{ir}$ and Φ_{i1}, Φ_{i2}, $\cdots$, Φ_{kq} for independent input variables $\tilde{X}_{B1},\tilde{X}_{B2},\cdots,\tilde{X}_{Bp}$, $\tilde{Y}_{A1},\tilde{Y}_{A2},\cdots,\tilde{Y}_{As}$ and output variables $\tilde{Y}_{B1},\tilde{Y}_{B2},\cdots,\tilde{Y}_{Bn}$, respectively. Then model (6) has an equivalent form as follow:*

$$\begin{cases} \max\limits_{\boldsymbol{U},\boldsymbol{V}} & \varphi = \displaystyle\int_0^1 \frac{\sum\limits_{k=1}^{s} U_{Bk}\Phi_{Bok}^{-1}(\alpha)}{\sum\limits_{k=1}^{r} V_{Bk}\Psi_{Bok}^{-1}(1-\alpha)+\sum\limits_{k=1}^{r} V_k\Phi_{Aok}^{-1}(1-\alpha)}\mathrm{d}\alpha \\ \text{s.t.} & \displaystyle\int_0^1 \frac{\sum\limits_{k=1}^{s} U_{Bk}\Phi_{Bjk}^{-1}(\alpha)}{\sum\limits_{k=1}^{r} V_{Bk}\Psi_{Bjk}^{-1}(1-\alpha)+\sum\limits_{k=1}^{s} V_k\Phi_{Ajk}^{-1}(1-\alpha)}\mathrm{d}\alpha \le 1, \quad j=1,2,\ldots,n \\ & U_{Bk}=(U_{B1},U_{B2},\cdots,U_{Bs}) \ge 0 \\ & V_{Bk}=(V_{B1},V_{B2},\cdots,V_{Br}) \ge 0 \\ & V_k=(V_1,V_2,\cdots,V_r) \ge 0 \end{cases} \tag{7}$$

where $\Psi_{Bo1},\Psi_{Bo2},\cdots,\Psi_{Bor},\Phi_{Ao1},\Phi_{Ao2},\cdots,\Phi_{Aos}$, and $\Phi_{Bo1},\Phi_{Bo2},\cdots,\Phi_{Bos}$ are the regular uncertainty distributions of $\tilde{X}_{B1},\tilde{X}_{B2},\cdots,\tilde{X}_{Bp}$, $\tilde{Y}_{A1}$, $\tilde{Y}_{A2}$, $\cdots$, $\tilde{Y}_{As}$, $\tilde{Y}_{B1},\tilde{Y}_{B2},\cdots,\tilde{Y}_{Bn}$, respectively.

Proof. As the function $U_B^T\,\tilde{Y}_{Bj}/(V_B^T\,\tilde{X}_{Bj}+\,V^T\tilde{Y}_{Aj})$ is strictly increasing in respect of $\tilde{Y}_{Bj}$ and strictly decreasing in respect of $\tilde{X}_{Bj}$, $\tilde{Y}_{Aj}$ for each j, in accordance with Theorem 2.1, the inverse uncertainty distribution of $U_B^T\,\tilde{Y}_{Bj}/(V_B^T\,\tilde{X}_{Bj}+\,V^T\tilde{Y}_{Aj})$ is

$$L_{Bj}^{-1}(\alpha) = \frac{\sum\limits_{k=1}^{s} U_{Bk}\Phi_{Bjk}^{-1}(\alpha)}{\sum\limits_{k=1}^{r} V_{Bk}\Psi_{Bjk}^{-1}(1-\alpha)+\sum\limits_{k=1}^{s} V_k\Phi_{Ajk}^{-1}(1-\alpha)}.$$

Complying with Theorem 2.2, we can get

$$E\left[\frac{U_B^T\,\tilde{Y}_{Bj}}{V_B^T\,\tilde{X}_{Bj}+\,V^T\tilde{Y}_{Aj}}\right] = \int_0^1 \frac{\sum\limits_{k=1}^{s} U_{Bk}\Phi_{Bjk}^{-1}(\alpha)}{\sum\limits_{k=1}^{r} V_{Bk}\Psi_{Bjk}^{-1}(1-\alpha)+\sum\limits_{k=1}^{s} V_k\Phi_{Ajk}^{-1}(1-\alpha)}\mathrm{d}\alpha \le 1, \quad j=1,2,\ldots,n.$$

□

On the other hand, the upstream participant has to fulfill the downstream member's needs. Here we use parameter G to reveal this phenomenon and call it fulfillment rate. Model (8) assesses the efficiency of the downstream player as the follower,

$$\begin{cases} \max\limits_{G,\boldsymbol{\mu},\boldsymbol{V}} \quad \varsigma = E\left[\dfrac{G\boldsymbol{\mu}^T\tilde{\boldsymbol{Y}}_{Ao}}{\boldsymbol{V}_A^T\tilde{\boldsymbol{X}}_{Ao}}\right] = E_{BA} \\ \text{s.t.} \quad E\left[\dfrac{G\boldsymbol{\mu}^T\tilde{\boldsymbol{Y}}_{Aj}}{\boldsymbol{V}_A^T\tilde{\boldsymbol{X}}_{Aj}}\right] \le 1, \quad j = 1,2,\ldots,n \\ \qquad E\left[\dfrac{\boldsymbol{\mu}_B^T\tilde{\boldsymbol{Y}}_{Bj}}{\boldsymbol{\omega}_B^T\tilde{\boldsymbol{X}}_{Bj} + \boldsymbol{\mu}^T\tilde{\boldsymbol{Y}}_{Aj}}\right] \le 1 \\ \qquad E\left[\dfrac{\boldsymbol{\mu}_B^T\tilde{\boldsymbol{Y}}_{Bo}}{\boldsymbol{\omega}_B^T\tilde{\boldsymbol{X}}_{Bo} + \boldsymbol{\mu}^T\tilde{\boldsymbol{Y}}_{Ao}}\right] = E_{BB}^* \\ \qquad \boldsymbol{V}_A^T \ge 0 \\ \qquad \boldsymbol{\mu}^T \ge 0 \\ \qquad \boldsymbol{\mu}_B^T \ge 0 \\ \qquad \boldsymbol{\omega}_B^T \ge 0 \\ \qquad G \ge 0. \end{cases} \tag{8}$$

Definition 3.7. *(Uncertain DEA Efficiency) DMU_o is said to be efficient if the optimal value ς of model (8) can be 1.*

The uncertain DEA model above can be applied to assess the efficiency of the non-core firm. The equivalent form of model (8) is proved as follow:

Theorem 3.8. *Set regular uncertainty distributions $\Psi_{i1}, \Psi_{i2}, \cdots, \Psi_{ir}$ and Φ_{i1}, Φ_{i2}, $\cdots$, Φ_{kq} for independent input variables $\tilde{X}_{A1}, \tilde{X}_{A2}, \cdots, \tilde{X}_{Ar}$ and output variables $\tilde{Y}_{A1}, \tilde{Y}_{A2}, \cdots, \tilde{Y}_{As}$, respectively. Then the equivalent form of model (8) is as follow:*

$$\begin{cases} \max\limits_{G,\boldsymbol{\mu},\boldsymbol{V}} \quad \varsigma = \displaystyle\int_0^1 \dfrac{G\sum\limits_{k=1}^{n} \mu^T\Phi_{Aok}^{-1}(\alpha)}{\sum\limits_{k=1}^{p} V_{Ak}\Psi_{Aok}^{-1}(1-\alpha)}\mathrm{d}\alpha \\ \text{s.t.} \quad \displaystyle\int_0^1 \dfrac{G\sum\limits_{k=1}^{n} \mu^T\Phi_{Ajk}^{-1}(\alpha)}{\sum\limits_{k=1}^{p} V_{Ak}\Psi_{Ajl}^{-1}(1-\alpha)}\mathrm{d}\alpha \le 1, \quad j = 1,2,\ldots,n \\ \qquad \displaystyle\int_0^1 \dfrac{\sum\limits_{k=1}^{s} \mu_B^T\Phi_{Bjk}^{-1}(\alpha)}{\sum\limits_{k=1}^{r} \omega_A^T\Psi_{Bjk}^{-1}(1-\alpha) + \sum\limits_{k=1}^{n} \mu^T\Psi_{Ajk}(1-\alpha)}\mathrm{d}\alpha \le 1 \\ \qquad \displaystyle\int_0^1 \dfrac{\sum\limits_{k=1}^{s} \mu_B^T\Phi_{Bok}^{-1}(\alpha)}{\sum\limits_{k=1}^{r}\omega_A^T\Psi_{Bok}^{-1}(1-\alpha) + \sum\limits_{k=1}^{n}\mu^T\Psi_{Aok}(1-\alpha)}\mathrm{d}\alpha = E_{BB}^* \\ \qquad \mathrm{G} \ge 0 \\ \qquad \mu^T \ge 0 \\ \qquad V_{Ak} = (V_{A1}, V_{A2}, \cdots, V_{Ap}) \ge 0 \end{cases} \tag{9}$$

where $\Psi_{Ao1}, \Psi_{Ao2}, \cdots, \Psi_{Aor}$ and $\Phi_{Ao1}, \Phi_{Ao2}, \cdots, \Phi_{Aos}$ are the regular uncertainty distributions of $\tilde{X}_{A1}, \tilde{X}_{A2}, \cdots, \tilde{X}_{Ar}$ and $\tilde{Y}_{A1}, \tilde{Y}_{A2}, \cdots, \tilde{Y}_{As}$, respectively.

Proof. As the function $G\ \mu^T\ \tilde{Y}_{Aj}/\ V^T\ \tilde{X}_{Aj}$ is strictly increasing in respect of $\tilde{Y}_{Aj}$ and strictly decreasing in respect of $\tilde{X}_{Aj}$ for each j, it complies with Theorem 2.1 that we can get the inverse uncertainty distribution of $G\mu^T\ \tilde{Y}_{Aj}/\ V^T\ \tilde{X}_{Aj}$ is

$$L_{Aj}^{-1}(\alpha) = \frac{G\sum\limits_{k=1}^{n} \mu^T\Phi_{Ajk}^{-1}(\alpha)}{\sum\limits_{k=1}^{p} V_{Ak}\Psi_{Ajl}^{-1}(1-\alpha)}.$$

According to Theorem 2.2, we can get

$$E\left[\frac{G\mu^T \tilde{Y}_{Aj}}{V^T\ \tilde{X}_{Aj}}\right] = \int_0^1 \frac{G\sum_{k=1}^{n} \mu^T \Phi_{Ajk}^{-1}(\alpha)}{\sum_{k=1}^{p} V_{Ak} \Psi_{Ajl}^{-1}(1-\alpha)} \mathrm{d}\alpha \le 1, \quad j = 1, 2, \ldots, n.$$

□

Now we can use model (10) to calculate the overall efficiency of the bi-echelon supply chain where the downstream participant is the leader:

$$e_{BA} = \frac{1}{2}(E_{BB}^* + E_{BA}^*). \tag{10}$$

4 A numerical example

In this section, goals of evaluating uncertain DEA models for the bi-echelon supply chain we are looking forward to achieving are as follows. To begin with, we will exam the present uncertain DEA models for supply chain efficiency with numerical examples to illustrate the uncertain models mentioned above with 38 original supply chain decision-making units shown in Table 1 and Table 2. The upstream firm has 3 inputs and 3 outputs: X_{A1}, X_{A2}, X_{A3}, and Y_{A1}, Y_{A2}, Y_{A3}. And the downstream firm has 4 inputs and 3 outputs: X_B, Y_{A1}, Y_{A2}, Y_{A3} and Y_{B1}, Y_{B2}, Y_{B3}.

Table 1: DMU_s with three inputs and three outputs in the first stage where $\mathcal{L}(a,b)$ represent linear uncertain variables and $\mathcal{Z}(a,b,c)$ represent zigzag uncertain variables

DMU_i	X_{A1}	X_{A2}	X_{A3}	Y_{A1}	Y_{A2}	Y_{A3}
1	$L(4,7)$	$L(5,8)$	$L(4,8)$	$Z(18,20,22)$	$Z(22,24,26)$	$Z(30,35,38)$
2	$L(2,5)$	$L(3,6)$	$L(1,3)$	$Z(5,7,9)$	$Z(8,11,12)$	$Z(8,9,10)$
3	$L(8,10)$	$L(9,11)$	$L(9,10)$	$Z(32,35,38)$	$Z(25,28,32)$	$Z(25,28,32)$
4	$L(6,7)$	$L(4,6)$	$L(5,6)$	$Z(12,14,18)$	$Z(15,17,18)$	$Z(20,22,23)$
5	$L(1,6)$	$L(1,3)$	$L(1,3)$	$Z(10,12,14)$	$Z(5,6,7)$	$Z(8,9,10)$
6	$L(5,7)$	$L(6,8)$	$L(6,7)$	$Z(19,22,25)$	$Z(22,24,26)$	$Z(20,23,29)$
7	$L(8,9)$	$L(7,11)$	$L(10,11)$	$Z(25,28,30)$	$Z(30,31,33)$	$Z(20,22,24)$
8	$L(10,12)$	$L(12,13)$	$L(11,13)$	$Z(44,46,50)$	$Z(30,32,35)$	$Z(41,43,44)$
9	$L(12,14)$	$L(13,14)$	$L(7,11)$	$Z(33,35,38)$	$Z(28,29,31)$	$Z(40,51,58)$
10	$L(6,9)$	$L(12,14)$	$L(16,18)$	$Z(22,25,28)$	$Z(26,28,30)$	$Z(36,38,40)$
11	$L(11,13)$	$L(14,16)$	$L(15,18)$	$Z(24,28,30)$	$Z(30,36,40)$	$Z(40,50,60)$
12	$L(15,17)$	$L(16,18)$	$L(11,13)$	$Z(50,63,68)$	$Z(50,53,61)$	$Z(76,82,85)$
13	$L(12,13)$	$L(15,17)$	$L(16,18)$	$Z(28,32,35)$	$Z(32,36,38)$	$Z(36,39,42)$
14	$L(15,18)$	$L(20,25)$	$L(17,20)$	$Z(40,42,44)$	$Z(50,52,55)$	$Z(60,65,68)$
15	$L(16,18)$	$L(20,24)$	$L(15,18)$	$Z(60,70,80)$	$Z(55,58,62)$	$Z(70,72,76)$
16	$L(16,18)$	$L(20,22)$	$L(15,17)$	$Z(55,58,60)$	$Z(66,72,80)$	$Z(77,82,91)$
17	$L(1,6)$	$L(1,3)$	$L(1,4)$	$Z(10,12,14)$	$Z(5,6,7)$	$Z(8,9,19)$
18	$L(1,5)$	$L(2,4)$	$L(2,4)$	$Z(12,14,16)$	$Z(10,11,12)$	$Z(15,17,19)$
19	$L(5,6)$	$L(7,8)$	$L(5,6)$	$Z(28,30,32)$	$Z(36,38,40)$	$Z(26,28,30)$
20	$L(11,13)$	$L(13,16)$	$L(7,9)$	$Z(36,38,39)$	$Z(26,28,30)$	$Z(30,32,36)$
21	$L(10,12)$	$L(13,15)$	$L(9,11)$	$Z(36,38,40)$	$Z(42,46,50)$	$Z(52,56,58)$
22	$L(7,9)$	$L(11,13)$	$L(13,15)$	$Z(22,25,28)$	$Z(26,30,32)$	$Z(32,36,38)$
23	$L(10,12)$	$L(11,13)$	$L(5,8)$	$Z(30,32,36)$	$Z(24,28,30)$	$Z(32,36,38)$
24	$L(13,15)$	$L(12,16)$	$L(9,13)$	$Z(44,50,53)$	$Z(56,58,60)$	$Z(62,66,68)$
25	$L(13,15)$	$L(11,13)$	$L(12,15)$	$Z(28,30,36)$	$Z(32,38,40)$	$Z(42,46,50)$
26	$L(12,14)$	$L(11,15)$	$L(13,15)$	$Z(32,36,40)$	$Z(36,38,46)$	$Z(52,56,60)$
27	$L(13,15)$	$L(15,17)$	$L(16,18)$	$Z(30,36,38)$	$Z(38,40,42)$	$Z(50,52,56)$
28	$L(11,12)$	$L(12,14)$	$L(6,9)$	$Z(32,34,35)$	$Z(22,29,32)$	$Z(34,38,42)$
29	$L(12,15)$	$L(13,16)$	$L(9,10)$	$Z(35,37,39)$	$Z(29,32,35)$	$Z(42,53,59)$
30	$L(8,9)$	$L(12,14)$	$L(12,16)$	$Z(20,23,25)$	$Z(28,30,33)$	$Z(33,38,41)$
31	$L(7,10)$	$L(13,16)$	$L(18,20)$	$Z(24,26,30)$	$Z(27,30,34)$	$Z(36,38,40)$
32	$L(9,11)$	$L(10,12)$	$L(8,12)$	$Z(26,29,32)$	$Z(28,32,34)$	$Z(32,34,36)$
33	$L(14,16)$	$L(12,13)$	$L(13,16)$	$Z(26,32,38)$	$Z(34,40,42)$	$Z(44,46,52)$
34	$L(12,16)$	$L(14,18)$	$L(14,16)$	$Z(33,34,36)$	$Z(36,40,44)$	$Z(50,54,58)$
35	$L(14,16)$	$L(12,15)$	$L(11,13)$	$Z(30,34,38)$	$Z(30,36,42)$	$Z(38,44,52)$
36	$L(6,8)$	$L(5,7)$	$L(5,8)$	$Z(14,18,20)$	$Z(16,18,19)$	$Z(22,24,25)$
37	$L(12,14)$	$L(13,15)$	$L(16,17)$	$Z(26,30,32)$	$Z(32,38,42)$	$Z(42,52,64)$
38	$L(13,14)$	$L(16,18)$	$L(17,19)$	$Z(34,38,42)$	$Z(38,39,43)$	$Z(52,56,58)$

Table 3 is quite revealing in several ways. First of all, as can be seen clearly from DMU_{17}, in the first stage, the overall efficiency is 1.0000, and in the second stage, the overall efficiency is 1.0000, too. That indicates that only all the components of a DMU achieve 1.0000 in both stages, and the whole system can be regarded as an efficient system. Then we can conclude that the DMU is efficient if and only if all the subsystems of it are efficient. This reflects Wei's definition for efficient DMUs in network DEA. We name it as *efficient type*. Besides, there are some other intriguing findings lying in the rest of the results.

The *efficient type* is not as common as the second one which is called *regular type*. It depicts that only one member of the whole system is efficient in a single pattern. This type can be reflected by DMU_5, the efficiency of the upstream firm is 1.0000, and it is considered to be efficient when its role is leader. Then the overall efficiency is 0.7774. However, when its role converts to follower, its efficiency is 0.7813, which means that overall efficiency declines with its own efficiency reducing. As the result, the leader is upstream firm. The same goes for DMU_2, DMU_{13}, DMU_{22}, DMU_{27}, DMU_{30}, DMU_{32}, DMU_{34}, DMU_{35}, and DMU_{38}. Compare these conditions, it is concluded that identifying the leader is significant to improve the efficiency of the whole system. A supply chain is a system that focuses more on the overall efficiency, which makes it become even more critical to identify the different power.

Table 2: DMU_s with a new input and three outputs in the second stage where $\mathcal{L}(a, b)$ represent linear uncertain variables and $\mathcal{Z}(a, b, c)$ represent zigzag uncertain variables

DMU_i	X_B	Y_{B1}	Y_{B2}	Y_{B3}
1	$L(10, 12)$	$L(12, 13)$	$L(9, 12)$	$Z(26, 30, 32)$
2	$L(7, 9)$	$L(9, 11)$	$L(7, 8)$	$Z(6, 9, 10)$
3	$L(12, 14)$	$L(8, 11)$	$L(10, 11)$	$Z(37, 39, 41)$
4	$L(7, 9)$	$L(4, 5)$	$L(7, 9)$	$Z(23, 24, 27)$
5	$L(9, 12)$	$L(7, 9)$	$L(12, 14)$	$Z(12, 15, 17)$
6	$L(8, 9)$	$L(7, 10)$	$L(10, 11)$	$Z(35, 39, 41)$
7	$L(9, 11)$	$L(9, 10)$	$L(12, 14)$	$Z(41, 43, 44)$
8	$L(9, 12)$	$L(10, 13)$	$L(11, 14)$	$Z(44, 50, 53)$
9	$L(11, 13)$	$L(12, 15)$	$L(12, 15)$	$Z(52, 56, 58)$
10	$L(12, 15)$	$L(13, 16)$	$L(18, 20)$	$Z(42, 52, 60)$
11	$L(16, 17)$	$L(12, 15)$	$L(16, 18)$	$Z(55, 58, 60)$
12	$L(14, 16)$	$L(13, 17)$	$L(17, 19)$	$Z(82, 85, 89)$
13	$L(13, 16)$	$L(14, 16)$	$L(13, 14)$	$Z(55, 58, 63)$
14	$L(12, 16)$	$L(18, 20)$	$L(14, 16)$	$Z(62, 66, 68)$
15	$L(17, 19)$	$L(12, 18)$	$L(15, 18)$	$Z(82, 85, 89)$
16	$L(16, 18)$	$L(13, 17)$	$L(16, 19)$	$Z(88, 92, 95)$
17	$L(9, 12)$	$L(15, 20)$	$L(22, 26)$	$Z(18, 19, 20)$
18	$L(10, 12)$	$L(16, 18)$	$L(18, 20)$	$Z(23, 24, 25)$
19	$L(11, 13)$	$L(18, 20)$	$L(20, 24)$	$Z(44, 50, 58)$
20	$L(14, 17)$	$L(13, 15)$	$L(12, 15)$	$Z(60, 62, 63)$
21	$L(12, 15)$	$L(11, 13)$	$L(13, 14)$	$Z(59, 62, 65)$
22	$L(13, 16)$	$L(14, 16)$	$L(11, 15)$	$Z(67, 71, 75)$
23	$L(7, 9)$	$L(13, 15)$	$L(13, 17)$	$Z(63, 69, 72)$
24	$L(11, 12)$	$L(12, 13)$	$L(16, 19)$	$Z(71, 75, 80)$
25	$L(17, 19)$	$L(12, 15)$	$L(15, 17)$	$Z(55, 63, 70)$
26	$L(15, 16)$	$L(14, 18)$	$L(16, 19)$	$Z(61, 67, 69)$
27	$L(12, 16)$	$L(16, 20)$	$L(18, 22)$	$Z(61, 65, 68)$
28	$L(8, 11)$	$L(12, 16)$	$L(13, 17)$	$Z(65, 70, 73)$
29	$L(12, 13)$	$L(13, 15)$	$L(13, 16)$	$Z(53, 58, 62)$
30	$L(14, 16)$	$L(15, 16)$	$L(12, 17)$	$Z(68, 73, 77)$
31	$L(12, 16)$	$L(14, 17)$	$L(18, 12)$	$Z(44, 52, 62)$
32	$L(11, 14)$	$L(18, 19)$	$L(18, 20)$	$Z(32, 34, 36)$
33	$L(18, 20)$	$L(13, 16)$	$L(16, 19)$	$Z(58, 65, 72)$
34	$L(12, 14)$	$L(18, 20)$	$L(16, 20)$	$Z(63, 66, 70)$
35	$L(15, 17)$	$L(13, 14)$	$L(16, 17)$	$Z(58, 68, 72)$
36	$L(8, 10)$	$L(10, 12)$	$L(9, 11)$	$Z(24, 26, 27)$
37	$L(17, 18)$	$L(13, 15)$	$L(13, 17)$	$Z(41, 43, 44)$
38	$L(14, 16)$	$L(18, 22)$	$L(20, 24)$	$Z(60, 66, 68)$

The third conclusion hides in DMU_{18}, which shows the fact that the upstream firm keeps efficient regardless that it is the leader or follower. We define it as *invariant type*. In DMU_{18}, the efficiency of the upstream firm is always 1.0000. Moreover, when the upstream firm is the leader, the overall efficiency is 0.9552, which is lower than the other situation where the downstream firm is the leader. In this case, to make the whole system efficient, we can make a decision on the basis of who is the leader. Therefore, the firm that can make the whole system get higher efficiency should be the dominant player in this kind of case because it can make the whole system get a higher overall efficiency.

The fourth one is *balance type*. In DMU_{23}, when the upstream firm is the leader, the efficiency of each subsystem is 0.8786 and 1.0000, respectively, and the overall efficiency is 0.9393. Moreover, when the downstream firm is the leader, the subsystem's efficiency is still 1.0000 and 0.8786, which do not change. Thus the overall efficiency does not change either. As a result, it is apparent from DMU_{23} that no matter who is the leader, the overall and individual efficiency do not change. It indicates that they have the same power. Thus, they are balanced in the supply chain. To ameliorate the overall efficiency, decision-makers need to perfect both upstream and down firm's performance to obtain advantages in competition with other supply chain players.

Table 3: Results of efficiency evaluation for DMUs where e_{AB} shows the efficiency when the upstream firm is the leader and e_{BA} shows the efficiency when the downstream firm is the leader

DMU_i	$E_{AA}(seller)$	$E_{AB}(buyer)$	e_{AB}	$E_{BB}(buyer)$	$E_{BA}(seller)$	e_{BA}
1	0.9976	0.6680	0.8328	0.7164	0.6802	0.6983
2	0.8864	0.7151	0.8008	1.0000	0.5954	0.7977
3	0.7168	0.5061	0.6115	0.6897	0.4892	0.5895
4	0.6945	0.5181	0.6063	0.6768	0.4733	0.5751
5	1.0000	0.5548	0.7774	0.7813	0.7357	0.7585
6	0.7154	0.6643	0.6899	0.8336	0.6101	0.7219
7	0.6815	0.7192	0.7004	0.9717	0.4121	0.6919
8	0.7667	0.6765	0.7216	0.6321	0.5929	0.6125
9	0.8305	0.6950	0.7628	0.7484	0.5074	0.6279
10	0.7681	0.8358	0.8020	0.8675	0.5832	0.7254
11	0.6177	0.8316	0.7247	0.7700	0.3948	0.5824
12	0.9860	0.8135	0.8998	0.6666	0.8082	0.7374
13	0.5083	1.0000	0.7542	0.7655	0.4415	0.6035
14	0.6150	0.7319	0.6735	0.7883	0.4514	0.6199
15	0.7903	0.9524	0.8714	0.6066	0.7006	0.6536
16	0.8451	0.9733	0.9092	0.7101	0.6424	0.6763
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	0.9103	0.9552	0.9364	1.0000	0.9682
19	1.0000	0.9464	0.9732	0.9239	0.7417	0.8328
20	0.7577	0.7290	0.7434	0.9446	0.5948	0.7697
21	0.8742	0.6658	0.7700	0.6851	0.6668	0.6760
22	0.6987	0.9060	0.8024	1.0000	0.6917	0.8459
23	0.8786	1.0000	0.9393	1.0000	0.8786	0.9393
24	0.8788	0.8857	0.8832	0.7548	0.8788	0.8186
25	0.6081	0.8664	0.7373	0.7230	0.4507	0.5869
26	0.7116	0.8325	0.7721	0.7567	0.5280	0.6424
27	0.5886	1.0000	0.7943	0.8401	0.4450	0.6426
28	0.7960	0.9611	0.8786	0.9992	0.5518	0.7755
29	0.7932	0.7059	0.7496	0.7199	0.5127	0.6163
30	0.6824	0.9395	0.8110	1.0000	0.6485	0.8243
31	0.6963	0.9026	0.7995	0.8551	0.5432	0.6992
32	0.5937	1.0000	0.7969	0.8657	0.5487	0.7072
33	0.5909	0.9412	0.7661	0.7319	0.4969	0.6144
34	0.6062	1.0000	0.8031	0.9057	0.4473	0.6765
35	0.5501	1.0000	0.7751	0.7726	0.5457	0.6592
36	0.6327	0.7459	0.6893	0.7471	0.5408	0.6440
37	0.6324	0.6903	0.6614	0.5855	0.4160	0.5008
38	0.6074	1.0000	0.8037	0.8177	0.4830	0.6504

5 Conclusions

It has always been vital to evaluate the efficiency of a supply chain for decision-makers [15]. This paper measures the efficiency of the overall system and members by developing four models in a bi-echelon DEA supply chain using uncertainty theory because the two-stage structure is a fundamental structure that can be easily stretched to other multiple structures and imprecise data can be coped with uncertainty theory effectively. We construct a leader-follower pattern in a bi-echelon supply chain context where the leading firm is evaluated firstly. After that, the evaluation of the following firm is based on the optimal value that the leader gained. A simple numerical example has been established to illustrate these models.

The aims of this paper are as follows. To begin with, the proposed models can be applied to many circumstances with inaccurate statistics so that they can be adapted for a wider range in efficiency evaluation. With the help of this paper, decision-makers can monitor the operations of their companies' supply chain in a more powerful and reasonable way because this paper provides a new horizon of correctly characterizing the power of members in a bi-echelon supply chain. What's more, this paper proposes four kinds of types to reveal some special cases when at least one member of the supply chain is efficient. In addition, this paper can help recognize the inefficient or weak parts of the whole system. Last but not least, uncertain DEA models for bi-echelon supply chain can be applied to the various supply chain with more than two echelons.

This paper discusses the efficiency under the leader-follower pattern where members do not cooperate. The results might be different if the members operate in a cooperative pattern. Furthermore, this paper uses the mean value of substage to evaluate the overall efficiency. It is feasible to assign a weight to each member of the supply chain, and then the outcome may change. In our future study, we will develop current models so that we can provide more advice on improvement techniques.

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