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Original paper

A novel method for ranking generalized fuzzy numbers with two different heights and its application in fuzzy risk analysis

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Abstract

Due to the large use of fuzzy numbers, the ranking of these numbers is very important. In this paper, we propose a new method for ranking generalized fuzzy numbers with different left and right heights. The proposed method, at first obtains the centers of gravity of fuzzy numbers and left and right side crisp numbers; then by computing left and right areas associated with them, ranks the fuzzy numbers. The proposed method can overcome the flaws and defects of some ranking methods, and the provided examples are evidence of this. Finally this method is applied to the fuzzy risk analysis problem.

Keywords: Generalized fuzzy numbers, ranking method, center of gravity, fuzzy risk analysis.

1 Introduction

In various sciences such as medicine, engineering, industry, natural sciences, psychology, etc., we deal with cases that are not precise and definite. Zadeh [24] presented the concept of fuzzy sets to encounter such issues. D. Dubois and H. Prade [10] defined the fuzzy number as a fuzzy subset in the real numbers line.

One of the important issues related to fuzzy numbers is their ranking, which has many applications in data analysis, optimization, approximate reasoning, economic, social systems, and so on. R. Jain presented the first method for ranking fuzzy numbers [13]. There are several methods like [1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 14, 16, 17, 19, 20, 21, 22, 23] which ranked fuzzy numbers based on specific features and suitable in certain situations.

T.C. Chu and C.T. Tsao [9] ranked the generalized fuzzy numbers by using the area between the centroid point and Y.J. Wang and H.Sh. Lee [19] revised Chu and Tsao's method. S. Abbasbandy and T. Hajjari [1] ranked the normal trapezoidal fuzzy numbers based on left and right $\alpha-$level of fuzzy numbers and also Y.M. Wang and Y. Luo [20] ranked them based on positive and negative ideal points and two risks indices. S.M. Chen and J.H. Chen [5] calculated the score of generalized trapezoidal fuzzy numbers based on defuzzified value, heights and spreads. But if the defuzzified value be equal to zero then the score will be zero, and as a result, the height and dispersion will be ineffective. S.M. Chen and K. Sanguansat [7], based on areas on the positive side, the areas on the negative side and the heights, has ranked the generalized trapezoidal fuzzy numbers. However, as in Chen and Chen's method, if defuzzified value be equal to zero, then the height will be ineffective. T. Hajjari [12] by presenting the new magnitude ($MagN$) ranked the generalized trapezoidal fuzzy numbers. S.M. Chen, et al. [6] based on the areas of the positive and negative side and centroid value, and W. Jiang [14] based on the areas of the positive side, the areas of the negative side and spread value ranked the generalized fuzzy numbers with different left and right heights. In these methods, like methods presented in [5] and [7], score of generalized trapezoidal fuzzy numbers with zero defuzzified value is equal zero. D. Wu [21] based on ordered weighted averaging (OWA) operator and consideration of the different importance of the three scoring factors defuzzified value, height and spread ranked the generalized fuzzy numbers with different left and right heights. In this method, by changing α, the weights also change. In [21], $\alpha = 0.7$ is considered and may not provide appropriate ranking in some situations.

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In this paper, for ranking generalized fuzzy numbers with different left and right heights, the centers of gravity of the generalized fuzzy numbers and center of gravity of left and right side crisp numbers obtained; then the fuzzy numbers are ranked by using the left and right areas associated with them. The purpose of this article is to provide a method that, in addition to the standards defined in the ranking fuzzy numbers, covers the defects and shortcomings of some above mentioned methods.

The paper has been organized as follows: Section 2 presents some definitions of generalized fuzzy numbers. In Section 3, the proposed method for ranking generalized fuzzy numbers is discussed. In Section 4, a comparison between some ranking methods with the proposed method is presented. In Section 5, the proposed ranking method for fuzzy risk analysis problem has been used and the conclusion is given in Section 6.

2 Preliminaries

Generalized fuzzy numbers concept with different left and right heights ($w_L, w_R \in [0,1]$ respectively), was presented in [6, 25]. This fuzzy number indicated as $\tilde{A} = (v_1, v_2, v_3, v_4; w_L; w_R)$ that can be seen in Fig. 1. If $0 \leq v_1 \leq v_2 \leq v_3 \leq v_4 \leq 1$, then we say $\tilde{A}$ is a standard generalized fuzzy number. The membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} f_L(x) & v_1 \leq x \leq v_2, \\ f_C(x) & v_2 \leq x \leq v_3, \\ f_R(x) & v_3 \leq x \leq v_4, \\ 0 & otherwise, \end{cases}$$

where $f_L(x) = \frac{w_L(x-v_1)}{v_2-v_1}$, $f_C(x) = \frac{w_L(v_3-v_2)+(w_R-w_L)(x-v_2)}{v_3-v_2}$ and $f_R(x) = \frac{w_R(x-v_4)}{v_3-v_4}$.

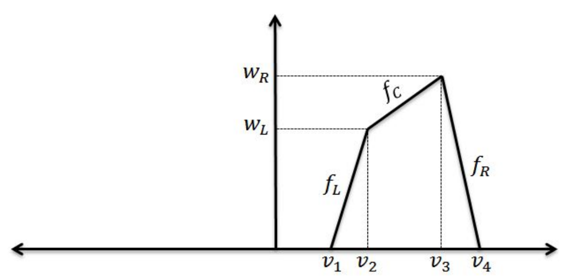

Figure 1: Generalized fuzzy number with different left and right heights

If $w_L = w_R = w$, then $\tilde{A}$ is a generalized trapezoidal fuzzy number and membership function as:

$$\mu_{\tilde{A}}(x) = \begin{cases} w\frac{x-v_1}{v_2-v_1} & v_1 \leq x \leq v_2, \\ w & v_2 \leq x \leq v_3, \\ w\frac{v_4-x}{v_4-v_3} & v_3 \leq x \leq v_4, \\ 0 & otherwise. \end{cases}$$

If $w_L = w_R = w$ and $v_2 = v_3$, then $\tilde{A}$ is a triangular fuzzy number. In mentioned states, if $w_L = w_R = 1$, then the fuzzy number is normal; and in the end if $v_1 = v_2 = v_3 = v_4$ and $w_L = w_R$, then we have crisp number.

3 A proposed method for ranking generalized fuzzy numbers

In this section, firstly some definitions of the centers of gravity of triangle and quadrilateral are presented, then the novel method for ranking generalized fuzzy numbers based on center of gravity of fuzzy numbers and areas associated with them has been introduced.

Definition 3.1. [15] *The centroid of a triangle is the point of intersection of its medians where it denoted by G in Fig. 2.*

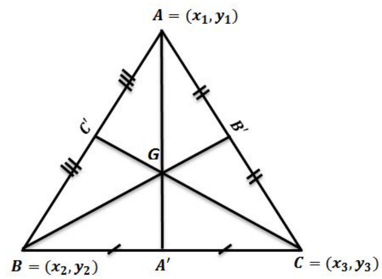

Figure 2: The center of gravity of triangle ABC.

Lemma 3.2. *The Cartesian coordinate of G in Fig. 2 is* $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$.

Proof. Cartesian coordinates of points A', B', and C' are $(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2})$, $(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2})$ and $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ respectively. Thus, by obtaining the equation of the lines passing through the pair of points $(A, A'; B, B'; C, C')$ and finds the point of intersection of the two lines of these three lines, we can easily attain the Cartesian coordinate of G. □

Definition 3.3. [15] *In Fig. 3, suppose that u, v, w and x are the centers of triangles $B_0B_1B_2$, $B_0B_2B_3$, $B_0B_1B_3$, and $B_1B_2B_3$ respectively, then the location of the intersection of uv and wx lines is the center of gravity of quadrilateral $B_0B_1B_2B_3$ where it is marked with G.*

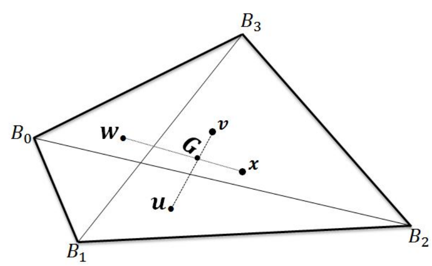

Figure 3: The center of gravity of quadrilateral $B_0B_1B_2B_3$.

Now suppose that n fuzzy numbers $\tilde{A}_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i}; w_{Li}; w_{Ri})$, $i = 1, 2, 3, ..., n$ to be ranked. The steps of the proposed method are as follows:

Step 1: Convert generalized fuzzy numbers to standard generalized fuzzy numbers:
Standard generalized fuzzy number Ai is obtained as follow:

$$A_i = (\frac{a_{1i}}{m}, \frac{a_{2i}}{m}, \frac{a_{3i}}{m}, \frac{a_{4i}}{m}; w_{Li}; w_{Ri}) = (v_{1i}, v_{1i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$$

where $m = \max\{\lceil|a_{1i}|\rceil, \lceil|a_{2i}|\rceil, \lceil|a_{3i}|\rceil, \lceil|a_{4i}|\rceil, 1\}$.

Step 2: Determine the geometric center of gravity of standard generalized fuzzy numbers:
If $A_i = (v_{1i}, v_{2i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$ be a triangle fuzzy number in which $v_{2i} = v_{3i} = {v_i}^*$ and $w_{Li} = w_{Ri} = w_i$ then, by employing Lemma 3.2 the center of gravity of A_i is $G_i = (G_{xi}, G_{yi}) = (\frac{v_{1i}+{v_i}^*+v_{4i}}{3}, \frac{w_i}{3})$.
If $A_i = (v_{1i}, v_{2i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$ be generalized fuzzy number in which $v_{1i} < v_{2i} < v_{3i} < v_{4i}$, then by employing Definition 3.3 the center of gravity of A_i is obtained by solving the following system

$$\begin{cases} G_{yi} - \frac{w_{Ri}}{v_{3i}-v_{1i}}(G_{xi} - \frac{v_{1i}+v_{2i}+v_{4i}}{3}) = \frac{w_{Li}}{3} \\ G_{yi} - \frac{w_{Li}}{v_{2i}-v_{4i}}(G_{xi} - \frac{v_{1i}+v_{2i}+v_{3i}}{3}) = \frac{w_{Li}+w_{Ri}}{3}. \end{cases}$$

If $A_i = (v_{1i}, v_{2i}, v_{3i}, v_{4i}; w_{Li}; w_{Ri})$ be a fuzzy number in which $v_{1i} = v_{2i} = v_{3i} = v_{4i} = v_i$ and $w_{Li} = w_{Ri} = w_i$ then it is obvious that the center of gravity of A_i is $G_i = (G_{xi}, G_{yi}) = (v_i, \frac{w_i}{2})$

Step 3: Obtain the left and right side areas of standard generalized fuzzy numbers:
If $G_i = (G_{xi}, G_{yi})$, $G_a = (-1, \frac{1}{2})$ and $G_b = (1, \frac{1}{2})$ be the centers of gravity of A_i, $(-1, -1, -1, -1; 1; 1)$ and $(1, 1, 1, 1; 1; 1)$ respectively and also g_{1i} and g_{2i} be lines joining G_a, G_i and G_b, G_i respectively, then the left and right side areas are as fallow:

$$S_{1i} = \int_{-1}^{G_{xi}} g_{1i}(x)dx, \quad S_{2i} = \int_{G_{xi}}^{1} g_{2i}(x)dx$$

, where S_{1i} is the left area and S_{2i} is the right area and that are shown in Fig. 4.

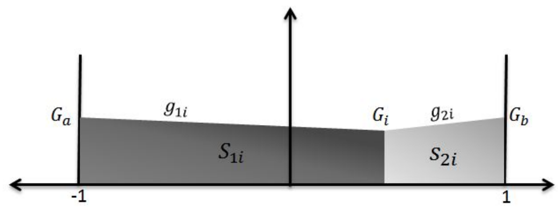

Figure 4: The areas S_{1i}, S_{2i} for standard fuzzy number A_i.

Step 4: Calculate $score(A_i)$ and ranked standard generalized fuzzy numbers:
If we set $score(A_i) = S_{1i} - S_{2i}$, then the ranking of fuzzy numbers A_i is determined as follow:
if $score(A_i) < score(A_j)$, then $A_i \prec A_j$,
if $score(A_i) > score(A_j)$, then $A_i \succ A_j$,
if $score(A_i) = score(A_j) = 0$ and $G_{yi} < G_{yj}$, then $A_i \prec A_j$,
if $score(A_i) = score(A_j) \neq 0$, then $A_i \approx A_j$.

In the following, some properties of proposed ranking method are presented:
Property 1. Suppose that $A = (v_1, v_2, v_3, v_4; w_L; w_R)$, $B = (v_1 + a, v_2 + a, v_3 + a, v_4 + a; w_L; w_R)$ standard generalized fuzzy numbers and $a \in \mathbb{R}$. If $a > 0$, then $B \succ A$, and if $a < 0$, then $B \prec A$.

Proof. Because the fuzzy number B is transmitted to the fuzzy number A in the direction of horizontal axis, we have $G_{xB} = a + G_{xA}$ and $G_{yA} = G_{yB}$. At first we obtain the $score(A)$:

$$g_{1A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} + 1}(x + 1) + \frac{1}{2}, \quad g_{2A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} - 1}(x - 1) + \frac{1}{2},$$

and so

$$S_{1A} = \int_{-1}^{G_{xA}} g_{1A}(x)dx = \frac{1}{2}(G_{xA} + 1)(G_{yA} + \frac{1}{2}) \tag{1}$$

$$S_{2A} = \int_{G_{xA}}^{1} g_{2A}(x)dx = \frac{1}{2}(1 - G_{xA})(G_{yA} + \frac{1}{2}). \tag{2}$$

Therefore,

$$score(A) = S_{1A} - S_{2A} = G_{xA}(G_{yA} + \frac{1}{2}), \tag{3}$$

and

$$score(B) = S_{1B} - S_{2B} = (a + G_{xA})(G_{yA} + \frac{1}{2}). \tag{4}$$

From relations (3), (4) we can conclude:
if $a > 0$ then $B \succ A$, and if $a < 0$ then $B \prec A$.
Property 2. For $A = (1, 1, 1, 1; 1; 1)$, $B = (-1, -1, -1, -1; 1; 1)$ we have $score(A) = 1$ and $score(B) = -1$.

Proof.

$$S_{1A} = \int_{-1}^{1} \frac{1}{2} dx = 1, \quad S_{2A} = 0, \quad score(A) = 1 - 0 = 1$$

$$S_{1B} = 0, \quad S_{2B} = \int_{-1}^{1} \frac{1}{2} dx = 1, \quad score(B) = 0 - 1 = -1.$$

Property 3. Suppose $A = (v_1, v_2, v_3, v_4; w_L; w_R)$ is a standard generalized fuzzy numbers and $B = (-v_4, -v_3, -v_2, -v_1; w_R; w_L)$, then $score(B) = -score(A)$.

Proof. Fuzzy numbers A and B are symmetric relative to the vertex line, so $G_{xB} = -G_{xA},\ G_{yA} = G_{yB}$. Since

$$g_{1A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} + 1}(x+1) + \frac{1}{2}, \quad g_{2A}(x) = \frac{G_{yA} - \frac{1}{2}}{G_{xA} - 1}(x-1) + \frac{1}{2},$$

$$S_{1A} = \int_{-1}^{G_{xA}} g_{1A}(x)dx = \frac{1}{2}(G_{xA} + 1)(G_{yA} + \frac{1}{2}) \text{ and } S_{2A} = \int_{G_{xA}}^{1} g_{2A}(x)dx = \frac{1}{2}(1 - G_{xA})(G_{yA} + \frac{1}{2}),$$

$$score(A) = S_{1A} - S_{2A} = G_{xA}(G_{yA} + \frac{1}{2}), \tag{5}$$

we can write:

$$score(B) = S_{1B} - S_{2B} = -G_{xA}(G_{yA} + \frac{1}{2}). \tag{6}$$

Equations (5), (6) gives the result that $score(B) = -score(A)$.

Property 4. If $G_A = (G_{xA}, G_{yA}) = (0, G_{yA})$ is the center of gravity A, then $score(A) = 0$.

Proof.

$$S_{1A} = \int_{-1}^{0} (G_{yA} - \frac{1}{2})(x+1) + \frac{1}{2}dx = \frac{1}{2}G_{yA} + \frac{1}{4} \text{ and } S_{2A} = \int_{0}^{1} (G_{yA} - \frac{1}{2})(-x+1) + \frac{1}{2}dx = \frac{1}{2}G_{yA} + \frac{1}{4},$$

hence $score(A) = S_{1A} - S_{2A} = 0$.

4 The comparison of some ranking methods with the proposed method

In this section, we compare our scheme with the methods presented in [19, 1, 20, 5, 7, 12, 6, 14, 21] by using eight sets of fuzzy numbers represented in Fig. 5 and 6 . The results are shown in Tables 1 and 2.

For set 1 in Fig. 5, methods presented in [1, 20] only rank normal trapezoidal fuzzy numbers and method presented in [12] shows $A \approx B$ where there is no logical result but other methods and the proposed method get $A \prec B$.

For set 2 in Fig. 5, all methods presented in [19, 1, 20, 5, 7, 12, 6, 14, 21] and the proposed method get the same ranking order: $A \succ B$.

For set 3 in Fig. 5, methods presented in [19, 1, 12, 7, 6] show $A \approx B$ that ranking cannot be correct. The methods presented in [5, 14, 21] show $A \prec B$ because dispersion is a negative factor in the ranking of these methods. Method presented in [20] and proposed method get $A \succ B$ that is a logical consequence.

For set 4 in Fig. 5, the methods presented in [5, 7, 14], and [20] ($\alpha = 0.5$) get $A \approx B$ and, [1, 12, 6] get $A \prec B$ which are not true by intuition. Method presented in [19, 21],[20] ($\alpha = 1$) and the proposed method get $A \succ B$ that match the observations.

For set 5 in Fig. 6, methods presented in [19, 1, 20] are not able to rank crisp numbers A, B and C. By using methods presented in [5, 7, 12, 6, 14] we have $A \approx B \prec C$ and Wu, et al.'s method [21] get $A \prec C \prec B$. That are wrong ranking between A, B and C. But in proposed method we have $A \prec B \prec C$ that is a logical consequence.

For set 6 in Fig. 6, methods presented in [1, 20] are not able to calculate score for non normal fuzzy number A. Chen and Sanguansat's method [7] get $A \approx B$ which is incorrect ranking. methods presented in [12, 6] show $A \succ B$ that do not match the observations but the presented methods in [19, 5, 14, 21] and the proposed method show $A \prec B$ that is a logical result.

For set 7 in Fig. 6, Abbasbandy's method [1] and Hajjari's method [12] get $A \prec B \prec C$ and $B \prec C \prec A$ respectively. By using methods presented in [5, 7, 6, 14] we have $A \approx B \prec C$. Methods presented in [19, 20, 21] and the proposed method show $B \prec A \prec C$ which is true by intuition.
For set 8 in Fig. 6, the methods presented in [19, 1, 20, 5, 7, 12] are not able to calculate score for fuzzy numbers B and C. By using method presented in [6], [14] we have $C \prec A \prec B$. Wu et al.'s method [21] and the proposed method show $C \prec B \prec A$ which is true by intuition.

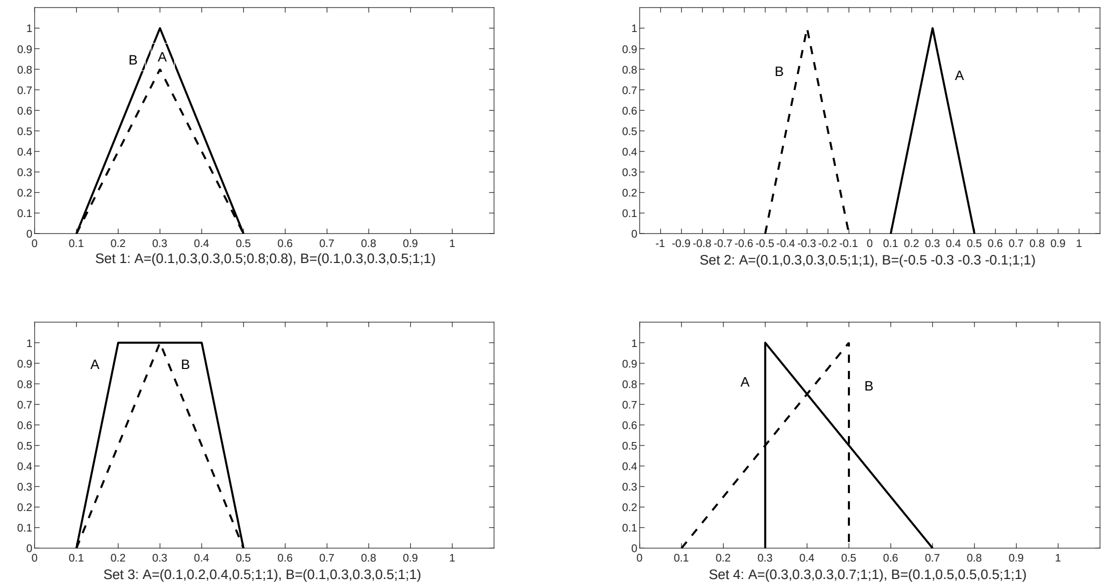

Figure 5: Sets of fuzzy numbers

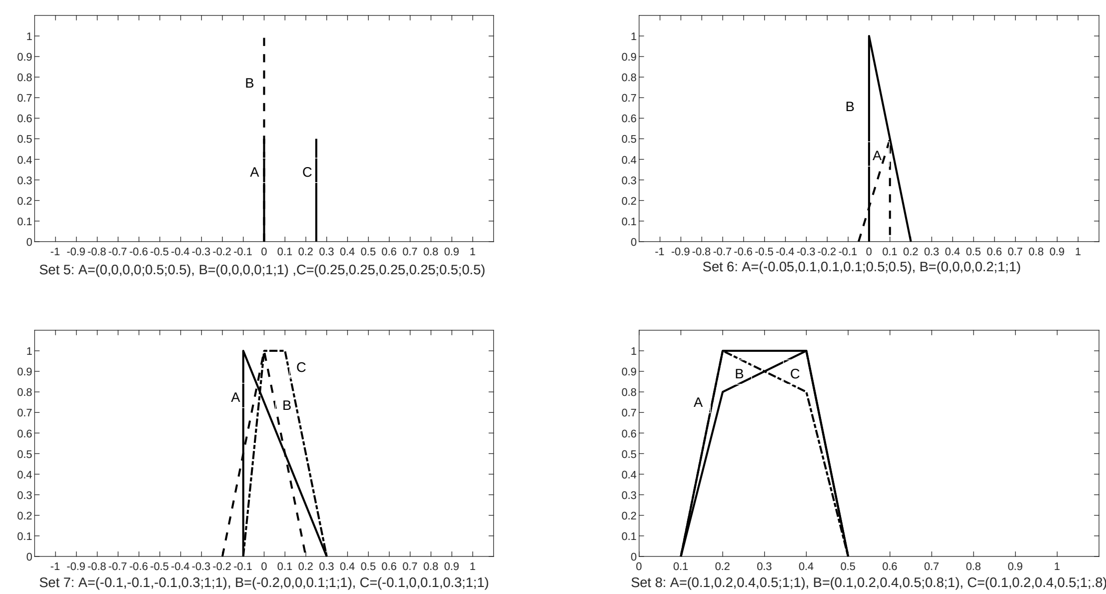

Figure 6: Sets of fuzzy numbers

Table 1: The comparison between existing methods of ranking the fuzzy numbers shown in Fig. 5

Authors	Fuzzy numbers	Set 1	Set 2	Set 3	Set 4
Wang, Lee [19]	A				
	B				
Results		$A \prec B$	$A \succ B$	$A \approx B$	$A \succ B$
Abbasbandy, Hajjari [1]	A	$*$	0.3000	0.3000	0.3333
	B	0.3000	-0.3000	0.3000	0.4667
Results		N	$A \succ B$	$A \approx B$	$A \prec B$
Wang, Luo [20]	A				
	B				
Results ($\alpha = 0.5, \alpha = 1$)		N	$(A \succ B, A \succ B)$	$(A \succ B, A \succ B)$	$(A \approx B, A \succ B)$
Chen, Chen [5]	A	0.2063	0.2537	0.2537	0.3333
	B	0.2579	-0.2597	0.2579	0.3333
Results		$A \prec B$	$A \succ B$	$A \prec B$	$A \approx B$
Chen, Sanguansat [7]	A	0.2824	0.3000	0.3000	0.4000
	B	0.3000	-0.3000	0.3000	0.4000
Results		$A \prec B$	$A \succ B$	$A \approx B$	$A \approx B$
Hajjari [12]	A	0.6000	1.2000	0.6000	0.8667
	B	0.6000	-1.2000	0.6000	1.1333
Results		$A \approx B$	$A \succ B$	$A \approx B$	$A \prec B$
Chen, et al. [6]	A	0.2462	0.2553	0.2553	0.3810
	B	0.2553	-0.2553	0.2553	0.3934
Results		$A \prec B$	$A \succ B$	$A \approx B$	$A \prec B$
Jiang [14]	A	0.2306	0.2882	0.2869	0.5714
	B	0.2882	-0.2882	0.2882	0.5714
Results		$A \prec B$	$A \succ B$	$A \prec B$	$A \approx B$
Wu, et al. [21]	A	0.5322	0.5906	0.5884	0.6604
	B	0.5906	-0.5906	0.5906	0.6235
Results		$A \prec B$	$A \succ B$	$A \prec B$	$A \succ B$
The proposed method	A	0.2300	0.2500	0.2833	0.3611
	B	0.2500	-0.2500	0.2500	0.3056
Results		$A \prec B$	$A \succ B$	$A \succ B$	$A \succ B$

5 An application of ranking of fuzzy number

In this section, we apply the proposed ranking method for fuzzy risk analysis.

Example 5.1. *In Table 3, the effects of some factors (Fac) on type 2 diabetes with linguistic terms (LingT) are presented that include: age, body mass index (BMI), family history (FHis), blood triglyceride level (BTL) and hypertension that causes high blood pressure and identified by systolic pressure (SP) and diastolic pressure (DP) [18]. Linguistic terms for five persons* S_1, S_2, S_3, S_4 *and* S_5 *associated with mentioned factors and Physical weakness (PW) due to factors given in Table 4. Notice that in this example we use the fuzzy linguistic terms in [3]. The goal is to determine which persons,* S_1, S_2, S_3, S_4 *and* S_5*, have a highest risk of developing diabetes. We have also used the following abbreviations: Very Low (VL), Low (L), Fairly Low (FL), Medium (M), Fairly High (FH), High (H) and Very High (VH).*

To solve this problem, we first use the Schumacher formula, then by proposed ranking method identify the person with the highest risk.

$$\tilde{R_i} = \sum_{j=1}^{j=n} \frac{\tilde{W_{ij}} \otimes \tilde{R_{ij}}}{\tilde{W_{ij}}}$$

in which $\tilde{R_i}$ is the risk of affected diabetes for ith person and $\tilde{R_{ij}}$ and $\tilde{W_{ij}}$ indicate the probability of being affected and the PW due to jth factor for ith person respectively.

$$\tilde{R_1} = (0.0757, 0.1282, 0.2202, 0.2703; 0.8; 0.8),$$

$$\tilde{R_2} = (0.2083, 0.2608, 0.4010, 0.4587; 0.7; 0.7),$$

$$\tilde{R_3} = (0.3834, 0.4288, 0.5148, 0.5535; 0.8; 0.8),$$

$$\tilde{R_4} = (0.1629, 0.2273, 0.3498, 0.404; 0.7; 0.7),$$

$$\tilde{R_5} = (0.4959, 0.5571, 0.6868, 0.7395; 0.8; 0.8).$$

Now we calculate the scores of generalized fuzzy numbers $\tilde{R_1}, \tilde{R_2}, \tilde{R_3}, \tilde{R_4}$ and $\tilde{R_5}$.

$$G_{\tilde{R_1}} = (0.1735, 0.3523),\ S_{1\tilde{R_1}} = 0.5001,\ S_{2\tilde{R_1}} = 0.3522,\ score(\tilde{R_1}) = 0.1479,$$

Table 2: The comparison between existing methods of ranking the fuzzy numbers shown in Fig. 6

Authors	Fuzzy numbers	Set 5	Set 6	Set 7	Set 8
Wang, Lee [19]	A				
	B				
	C		....		
Results		N	$A \prec B$	$B \prec A \prec C$	N
Abbasbandy, Hajjari [1]	A	$*$	$*$	-0.0667	0.3000
	B	0	0.0167	-0.0083	$*$
	C	$*$	....	0.0583	$*$
Results		N	N	$A \prec B \prec C$	N
Wang, Luo [20]	A				
	B				
	C		....		
Results ($\alpha = 0.5, \alpha = 1$)		N	N	$(B \prec A \prec C, B \prec A \prec C)$	N
Chen, Chen [5]	A	0	0.0297	0	0.2537
	B	0	0.4546	0	$*$
	C	0.1250	....	0.6410	$*$
Results		$A \approx B \prec C$	$A \prec B$	$A \approx B \prec C$	N
Chen, Sanguansat [7]	A	0	0.0500	0	0.3
	B	0	0.0500	0	$*$
	C	0.2000	....	0.0750	$*$
Results		$A \approx B \prec C$	$A \approx B$	$A \approx B \prec C$	N
Hajjari [12]	A	0	0.3250	0.2333	0.6000
	B	0	0.0833	-0.2167	$*$
	C	0.7500	....	0.2167	$*$
Results		$A \approx B \prec C$	$A \succ B$	$B \prec C \prec A$	N
Chen, et al. [6]	A	0	0.0431	0	0.2533
	B	0	0.0400	0	0.2687
	C	0.1111	....	0.6060	0.2420
Results		$A \approx B \prec C$	$A \succ B$	$A \approx B \prec C$	$C \prec A \prec B$
Jiang [14]	A	0	0.0305	0	0.2869
	B	0	0.0488	0	0.3012
	C	0.1250	....	0.0719	0.2726
Results		$A \approx B \prec C$	$A \prec B$	$A \approx B \prec C$	$C \prec A \prec B$
Wu, et al. [21]	A	0.0300	0.3170	0.4388	0.5884
	B	0.4460	0.4689	0.4244	0.5662
	C	0.4385	......	0.4679	0.5454
Results		$A \prec C \prec B$	$A \prec B$	$B \prec A \prec C$	$C \prec B \prec A$
The proposed method	A	0_1	0.0330	0.0278	0.2833
	B	0_2	0.5560	0	0.2774
	C	0.1875	....	0.0720	0.2641
Results		$A \prec B \prec C$	$A \prec B$	$B \prec A \prec C$	$C \prec B \prec A$

"$*$"and "N" denote the method cannot campute score and cannot rank the fuzzy numbers respectively.

$$G_{\tilde{R}_2} = (0.3323, 0.3171),\ S_{1\tilde{R}_2} = 0.5443,\ S_{2\tilde{R}_2} = 0.2728,\ score(\tilde{R}_2) = 0.2715,$$

$$G_{\tilde{R}_3} = (0.4699, 0.3562),\ S_{1\tilde{R}_3} = 0.6293,\ S_{2\tilde{R}_3} = 0.2269,\ score(\tilde{R}_3) = 0.4024,$$

$$G_{\tilde{R}_4} = (0.2858, 0.3119),\ S_{1\tilde{R}_4} = 0.5220,\ S_{2\tilde{R}_4} = 0.2899,\ score(\tilde{R}_4) = 0.2321,$$

$$G_{\tilde{R}_5} = (0.6196, 0.3593),\ S_{1\tilde{R}_5} = 0.6959,\ S_{2\tilde{R}_5} = 0.1634,\ score(\tilde{R}_5) = 0.5324.$$

Since $\tilde{R}_1 \prec \tilde{R}_4 \prec \tilde{R}_2 \prec \tilde{R}_3 \prec \tilde{R}_5$ hence S_5 person has the highest probabbility for type 2 diabetes followed by $\tilde{R}_3, \tilde{R}_2, \tilde{R}_4$ and $\tilde{R}_1$.

Table 3: The effects of factors on type 2 diabetes with linguistic terms in Example 5.1.

Fac\LingT	VL	L	FL	M	FH	H	VH
Age	under 30	30-45 and over 75		45-65			
BMI: Kg/m^2	under 27			27-30		over 30	
FHis		none			yes		
BTL: mg/dL		under 150	150-250			250-499	500
BP: SP		$100 - 140$		$140 - 160$		$over 160$	
BP: DP		$70 - 90$		$90 - 100$		$over 100$	

Table 4: The fuzzy linguistic terms for S_1, S_2, S_3, S_4 and S_5 in Example 5.1.

Fac, PW	S_1	S_2	S_3	S_4	S_5
Age	L	L	M	L	M
PW	VL	VL	FH($w_L = w_R = 0.8$)	VL	FH($w_L = w_R = 0.8$)
BMI	VL	M	VL	M	H
PW	L	FL($w_L = w_R = 0.7$)	L	FH($w_L = w_R = 0.7$)	H($w_L = w_R = 0.95$)
FHis	L	L	L	L	FH
PW	L($w_L = w_R = 0.8$)	L($w_L = w_R = 0.8$)	L($w_L = w_R = 0.8$)	L($w_L = w_R = 0.8$)	H($w_L = w_R = 0.95$)
BTL	FL	FL	H	FL	H
PW	M($w_L = w_R = 0.9$)	M($w_L = w_R = 0.9$)	VH	M($w_L = w_R = 0.9$)	VH
BP	L	M	L	L	L
PW	H($w_L = w_R = 0.95$)	VL	H($w_L = w_R = 0.95$)	H($w_L = w_R = 0.95$)	H($w_L = w_R = 0.95$)

Example 5.2. *An example of fuzzy risk analysis was presented in [5], [7] and with changes in [6]. Here, we focus our attention on solving that example in the final step, namely, ranking, using our proposed ranking method. Now the assumption of the probability of failure of each component A_i generated by manufactory C_i is equal $\tilde{R}_i$ for $i = 1, 2, 3$.*

$$\tilde{R}_1 = (0.1765, 0.2860, 0.7244, 1.0574; 0.5; 0.6),$$

$$\tilde{R}_2 = (0.3221, 0.4949, 1.1392, 1.6373; 0.4; 0.5),$$

$$\tilde{R}_3 = (0.3290, 0.4890, 1.1737, 1.7787; 0.5; 0.6).$$

We first convert $\tilde{R}_1$, $\tilde{R}_2$ and $\tilde{R}_3$ to standard numbers R_1, R_2 and R_3 shown in Fig. 7.

$$R_1 = (\frac{0.1765}{2}, \frac{0.2860}{2}, \frac{0.7244}{2}, \frac{1.0574}{2}; 0.5; 0.6) = (0.0883, 0.1430, 0.3622, 0.5287; 0.5; 0.6),$$

$$R_2 = (\frac{0.3221}{2}, \frac{0.4949}{2}, \frac{1.1392}{2}, \frac{1.6373}{2}; 0.4; 0.5) = (0.1611, 0.2475, 0.5696, 0.8187; 0.4; 0.5),$$

$$R_3 = (\frac{0.3290}{2}, \frac{0.4890}{2}, \frac{1.1737}{2}, \frac{1.7787}{2}; 0.5; 0.6) = (0.1645, 0.2445, 0.5869, 0.8894; 0.5; 0.6).$$

We will have

$$G_{R_1} = (0.2901, 0.2471),\ G_{R_2} = (0.4655, 0.2024),\ G_{R_3} = (0.4883, 0.2455)$$

and

$$S_{1R_1} = 0.4819,\ S_{2R_1} = 0.2652,\ score(R_1) = 0.2167$$

$$S_{1R_2} = 0.5147,\ S_{2R_2} = 0.1877,\ score(R_2) = 0.3269$$

$$S_{1R_3} = 0.5547,\ S_{2R_2} = 0.1907,\ score(R_3) = 0.3640.$$

Therefore $R_1 \prec R_2 \prec R_3$; that is, the ranking order of the risk of manufactories C_1, C_2 and C_3 is $C_1 \prec C_2 \prec C_3$; that is the component A_3 generated by manufactory C_3 has a highest probability of failure, then C_2, C_1 respectively.

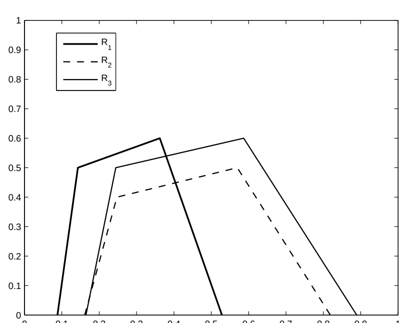

Figure 7: Set of standard fuzzy numbers R_1, R_2, R_3

6 Conclusion

This work introduces the new method for ranking generalized fuzzy numbers with different left and right heights. By using the centers of gravity of the fuzzy numbers, centers of gravity of left and right side crisp numbers and left and right areas associated with them, this method ranked the fuzzy numbers. The proposed method having standards of the fuzzy ranking numbers, can cover the defects and shortcomings of previously mentioned methods . It is hoped that this method will provide a more useful and more general future for ranking fuzzy numbers and its applications.

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