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Original paper

Disturbance estimator based dynamic compensator design for fractional order fuzzy control systems

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Abstract

The robust stabilization problem for singular fractional order time delay T-S fuzzy systems with nonlinearities and unknown external disturbances is addressed in this paper. An equivalent-input-disturbance (EID) estimator is used to estimate the impact of external disturbances and nonlinearities on the system output. Based on this EID approach, a dynamic compensator is designed to solve the stabilization problem of the considered system. Moreover, by considering a relevant Lyapunov-Krasovskii functional candidate and by using Lyapunov technique, the stability conditions in terms of LMIs are acquired for the considered closed-loop system. At last, to validate the effectiveness of the proposed result, two numerical examples are provided.

Keywords: Dynamic output feedback design, disturbance estimator, unknown external disturbances, fractional order control systems.

1 Introduction

Recently, the study of the fractional order differential systems have received considerable progress because of its accurate demonstration and vital approaches in numerous fields such as in motor control, distributed parameter systems, eternal magnet synchronous generators, mobile robots, control of autonomous vehicles and power electronic converters [13, 14, 16]. Precisely, fractional order differential systems can be applied to wide number of areas when compared to integer order systems by providing exact description and offering deeper approach to physical procedures of dynamical control systems more exactly. On the other hand, singular dynamical systems also called implicit dynamical systems have been extensively applied to some real model systems such as in networked analysis, electrical circuit networks, biological systems and so on [19, 31]. Also, singular systems are complex when compared to normal systems due to its outstanding representation [18, 27]. There are some inconveniences named as vagueness and uncertainty in the true operations. In particular, the opposite of exactness otherwise called vagueness is unavoidable in the human world. So, fuzzy theory is considered as an appropriate one to tackle vagueness for extensive detailed descriptions of vague notations [5, 11]. Now a days, infinite number of results have been delivered on Takagi-Sugeno fuzzy model approach [1, 3, 29, 30, 34] to describe and approximate the nonlinear complex systems. The main advantage of this T-S fuzzy approach is that, it can transform the nonlinear dynamical system into linear dynamical systems by a set of considered system rules. Many control synthesis papers with the aid of Lyapunov stability approach and LMI approaches for T-S fuzzy dynamical systems have been presented in [10, 17, 32].

Moreover, nonlinearities and time-varying delays are normally present in almost all real world processes which is a complicate issue [4, 7]. It is clear from the results of existing literature that nonlinearities are unavoidable and that these nonlinear systems are better applicable in engineering field [21, 22]. The conditions to prove regularity for singular nonlinear dynamical systems in the presence of time varying delay is more adverse than the normal dynamical systems[24, 33]. To deal with the problems on time delay and nonlinearities, an appropriate control design method is

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required. Up to now, only a few methods were reported to overcome these two problems simultaneously. On the other hand, it should be emphasized that loss of robustness, poor performance and deterioration of stability may be due to the existence of uncertainties in the dynamics of systems. Therefore, there is an utmost need to discuss the stability issue of nonlinear singular dynamical systems in the presence of uncertainties, [13, 29, 31].

For a practical nonlinear time-delay system, it may be a difficult mission to obtain all the state variables. But most of the works in existing literature on nonlinear time-delay system are based on the state feedback approach. Nevertheless, in some practical cases, it is very difficult to collect full state information of an nonlinear system, while the relative output of the considered system is easy to acquire. In such situations, the output feedback control approach is proposed. In comparison with the state feedback control problem, the output feedback control approach is more challenging because of the limited information of state variables. It is well known that dynamic output feedback and static output feedback approaches have received lots of attention in various control systems. The dynamic output feedback control approach for different kinds of descriptor systems have been studied recently by many researchers using LMI approach [9, 28].

Also, it is important to deal with the disturbances in the control input channel to improve the control performance. Therefore to reject these disturbances, an EID approach [12] is presented, which can actively construct compensation signals for good disturbance rejection performance. Mainly, EID approach not only rejects but also estimates both mismatched and matched external disturbances [6, 20]. Motivated by this excellent quality, in this present study, we propose a dynamic output feedback compensator design which is based on the EID technique for obtaining the stabilization of the fuzzy singular fractional order differential system. In this paper, the stabilization issue through dynamical output feedback control approach for fuzzy singular fractional order control system has been focused. The significant contributions of this study can be precisely given as follows

- An EID based dynamic output feedback compensator design is considered for singular T-S fuzzy control systems.
- Sufficient conditions for the addressed system are proposed by using the Lyapunov-Krasovskii stability theory which further helps to retrieve the desired dynamic output feedback controller gain matrices in respect of linear matrix inequalities.
- Finally, the validity, feasibility and applicability of the proposed theoretical results are verified through two numerical examples.

2 Preliminaries

Consider the fractional-order singular T-S fuzzy control system with nonlinearities and external disturbances characterized as

Rule S_i: **IF** $\rho_1(\iota)$ is R_{i1}, ... and $\rho_p(\iota)$ is R_{ip} **THEN**

$$\begin{aligned} ED^{\wp}x(\iota) =& \quad \tilde{A}_i x(\iota) + \tilde{A}_{di} x(\iota - h(\iota)) + B_i u(\iota) + \ell(x(\iota)) + B_{\omega i}\omega(\iota), \\ y(\iota) =& \quad C_i x(\iota) \quad i = 1, \dots, k, \\ x(\iota) =& \quad \varphi(\iota), \quad \iota \in [-h, 0], \end{aligned} \tag{1}$$

where $D^{\wp}$ denotes the Riemann-Liouvilles fractional order differential derivative with order $\wp$ satisfying $0 < \wp < 1$; $x(\iota) \in R^n$ is state vector; $u(\iota) \in R^m$ is input vector; $\ell(x(\iota))$ is an unknown nonlinear function with $\ell(0) = 0$; $\omega(\iota)$ is the unknown external disturbance; $R_{ia}(a = 1, \dots, p)$ are the fuzzy sets, where $i = 1, \dots, k$, k is the number of IF-THEN fuzzy rules; $\rho_1(\iota)$, $\rho_2(\iota)$, ..., $\rho_p(\iota)$ are the fine variables; the time varying state delay, $h(\iota)$ is differentiable and is also assumed to satisfy $0 \leq h(\iota) \leq h < \infty$ and $\dot{h}(\iota) \leq h_1 < 1$; $y(\iota) \in R^p$ is output vector; $\varphi(\iota)$ is a vector-valued function which is continuous and differentiable. Let $\tilde{A}_i = A_i + \Delta A_i(\iota)$, $\tilde{A}_{di} = A_{di} + \Delta A_{di}(\iota)$; A_i, A_{di}, B_i, $B_{\omega i}$ and C_i are real constant system matrices with relevant dimensions and $\Delta A_i(\iota)$ and $\Delta A_{di}(\iota)$ are unknown uncertain matrices of the system that satisfies the admissible conditions: $\Delta A_i(\iota) = G_{1i}\theta_i(\iota)H_{1i}$, $\Delta A_{di}(\iota) = G_{2i}\theta_i(\iota)H_{2i}$ where G_{1i}, G_{2i}, H_{1i}, H_{2i} are known suitable dimensioned real constant matrices and $\theta_i(\iota) \in R^{n\times n}$ is an unknown time varying matrix function that satisfies $\theta_i^T(\iota)\theta_i(\iota) \leq I$, for all $\iota > 0$ and $0 < rank\{E\} = r < n$. The fuzzy system which is governed by the set of fuzzy rules is represented as

$$\begin{aligned} ED^{\wp}x(\iota) =& \quad \sum_{i=1}^{k} n_i(\rho(\iota))[\tilde{A}_i x(\iota) + \tilde{A}_{di} x(\iota - h(\iota)) + B_i u(\iota) + \ell(x(\iota)) + B_{\omega i}\omega(\iota)], \\ y(\iota) =& \quad \sum_{i=1}^{k} n_i(\rho(\iota)) C_i x(\iota), \end{aligned} \tag{2}$$

where $n_i(\rho(\iota))$ denotes the IF-THEN fuzzy rule normalized weight satisfying $n_i(\rho(\iota)) \geq 0$, $\sum_{i=1}^{k} n_i(\rho(\iota)) = 1$ and $n_i(\rho(\iota)) = \frac{m_i(\rho(\iota))}{\sum_{i=1}^{k} m_i(\rho(\iota))}$ with $m_i(\rho(\iota)) = \prod_{a=1}^{p} R_{ia}(\rho_a(\iota))$ and $R_{ia}(\rho_a(\iota))$ is the membership grade function of $\rho_a(\iota)$ in R_{ia}.

It is noted that uncertainties are nothing but the provoked mismatches between the original process and the identified system. These uncertainties and system nonlinearities are treated as additional load disturbance. Under these considerations, system (2) can be written as

$$\begin{aligned} ED^{\wp}x(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota))[A_i x(\iota) + A_{di}x(\iota - h(\iota)) + B_i u(\iota) \\ & + \{\Delta A_i x(\iota) + \Delta A_{di}x(\iota - h(\iota)) + \ell(x(\iota)) + B_{\omega i}\omega(\iota)\}], \\ y(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota)) C_i x(\iota). \end{aligned} \tag{3}$$

Further, to estimate the states responses of (2), we now construct a fractional order singular T-S fuzzy state observer which can be represented by

$$\begin{aligned} ED^{\wp}\hat{x}(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota))[A_i\hat{x}(\iota) + A_{di}\hat{x}(\iota - h(\iota)) + B_i u_F(\iota) + L_i(y(\iota) - \hat{y}(\iota))], \\ \hat{y}(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota)) C_i \hat{x}(\iota), \\ \hat{x}(\iota) = & \hat{\varphi}(\iota), \quad \iota \in [-h, 0], \end{aligned} \tag{4}$$

where L_i is the observer gain to be calculated; $\hat{x}(\iota)$ is the state observer of $x(\iota)$; $\hat{y}(\iota)$ denotes the observer output; $u_F(\iota)$ is the improved control input vector and $\hat{\varphi}(\iota)$ is the initial condition of the observer.

On the basis of the equivalent-input-disturbance concept, the uncertainties, nonlinearity $\ell(x(\iota))$ and the disturbance $\omega(\iota)$ can be treated as a state-dependent EID estimator $\omega_E(\iota)$ for all $\iota \geq 0$. Therefore the altered state space representation of the considered dynamical control system (1) can be given by

$$\begin{aligned} ED^{\wp}x(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota))[A_i x(\iota) + A_{di}x(\iota - h(\iota)) + B_i u(\iota) + B_i\omega_E(\iota)], \\ y_E(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota)) C_i x(\iota), \end{aligned} \tag{5}$$

where $y_E(\iota)$ is the output of the system which is equal to $y(\iota)$ when $u(\iota) = 0$. In order to maintain the system stability, it is important to deal with these external disturbances. On account of this, an equivalent-input-disturbance estimator in the following form is considered

$$\hat{\omega}(\iota) = \sum_{i=1}^{k} n_i(\rho(\iota)) B_i^{+} L_i(y(\iota) - \hat{y}(\iota)) + u_F(\iota) - u(\iota), \tag{6}$$

where $B_i^{+} = (B_i^T B_i)^{-1} B_i^T$ is the pseudo inverse of B_i. Now, the EID estimator and the fractional-order low pass filter $J(s)$ given by $J(s) = \frac{\omega_e}{s^{\wp} + \omega_e}$, where ω_e is the cut-off angular frequency are interconnected, in order to filter out high frequency noises. Further, the state-space representation of the fractional-order low-pass filter $J(s)$ can be expressed as

$$\begin{aligned} D^{\wp}x_F(\iota) = & A_F x_F(\iota) + B_F\hat{\omega}(\iota), \\ \tilde{\omega}(\iota) = & C_F x_F(\iota), \end{aligned} \tag{7}$$

where $x_F(\iota)$ is the state of the filter $J(s)$; A_F, B_F and C_F are known constants matrices with suitable dimensions and $\tilde{\omega}(\iota) > 0$ is the noise free estimated signal. Now, the control input in (2) can be modified as $u(\iota) = u_F(\iota)\tilde{\omega}(\iota)$.

In this study, we consider the following dynamic compensator to stabilize the considered fractional-order singular T-S fuzzy time-delay dynamical system (1).

Dynamic compensator rule j: IF $\rho_1(\iota)$ is R_{j1}, ... , and $\rho_p(\iota)$ is R_{jp} **THEN**

$$\begin{aligned} E_\delta D^{\wp}x_\delta(\iota) = & \sum_{j=1}^{k} n_j(\rho(\iota))[A_{\delta j}x_\delta(\iota) + B_{\delta j}y(\iota)], \\ u_F(\iota) = & \sum_{j=1}^{k} n_j(\rho(\iota))[C_{\delta j}x_\delta(\iota) + D_{\delta j}y(\iota)], \end{aligned} \tag{8}$$

where $x_\delta(\iota) \in R^{n_\delta}$ is the dynamic compensator state vector; $E_\delta \in R^{n_\delta \times m_\delta}$ is a singular matrix; $A_{\delta j}$, $B_{\delta j}$, $C_{\delta j}$ and $D_{\delta j}$ are gain matrices of the dynamic compensator with suitable dimensions. Let us denote the error between considered system (1) and the fractional order singular T-S fuzzy state observer (4) as $e(\iota) = x(\iota) - \hat{x}(\iota)$, then the dynamics of $e(\iota)$ can be characterized as

$$ED^{\wp}e(\iota) = \sum_{i=1}^{k} n_i(\rho(\iota))[A_i e(\iota) + \Delta A_i(\iota)x(\iota) + A_{di}e(\iota - h(\iota)) + \Delta A_{di}(\iota)x(\iota - h(\iota)) - B_i\tilde{\omega}(\iota) - L_iC_ie(\iota) + \ell(x(\iota)) + B_{\omega i}\omega(\iota)]. \tag{9}$$

Moreover, equation (7) can be rewritten as $D^{\wp}x_F(\iota) = A_F x_F(\iota) + B_F B_i^+ L_i C_i e(\iota) + B_F C_F x_F(\iota)$. Now, we are in a position to define a new augmented vector $\bar{x}(\iota) = [x^T(\iota) \quad e^T(\iota) \quad x_F^T(\iota) \quad x_\delta^T(\iota)]$ and frame a corresponding overall augmented singular fractional order T-S fuzzy closed-loop system as

$$\bar{E}D^{\wp}\bar{x}(\iota) = \sum_{i=1}^{k}\sum_{j=1}^{k} n_i(\rho(\iota))n_j(\rho(\iota))[\hat{A}_i\bar{x}(\iota) + \hat{A}_{di}\bar{x}(\iota - h(\iota)) + \bar{\omega}(\iota) + \ell(\bar{x}(\iota))], \tag{10}$$

where $\hat{A}_i = \bar{A}_i + \Delta\bar{A}_i(\iota)$, $\hat{A}_{di} = \bar{A}_{di} + \Delta\bar{A}_{di}(\iota)$, $\bar{E} = diag\{E, E, I, E_\delta\}$, $\bar{\omega}(\iota) = \hat{A}_{\omega i}\omega(\iota)$ and $\ell(\bar{x}(\iota)) = \hat{I}\ell(x(\iota))$, with

$$\bar{A}_i = \begin{bmatrix} A_i + B_iD_{\delta j}C_i & 0 & -B_iC_F & B_iC_{\delta j} \\ 0 & A_i - L_iC_i & -B_iC_F & 0 \\ 0 & B_FB_i^+L_iC_i & A_F + B_FC_F & 0 \\ B_{\delta j}C_i & 0 & 0 & A_{\delta j} \end{bmatrix}, \ \bar{A}_{di} = \begin{bmatrix} A_{di} & 0 & 0 & 0 \\ 0 & A_{di} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \hat{A}_{\omega i} = \begin{bmatrix} B_{\omega i} \\ B_{\omega i} \\ 0 \\ 0 \end{bmatrix},$$

$$\Delta\bar{A}_i(\iota) = \begin{bmatrix} \Delta A_i(\iota) & 0 & 0 & 0 \\ \Delta A_i(\iota) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \Delta\bar{A}_{di}(\iota) = \begin{bmatrix} \Delta A_{di}(\iota) & 0 & 0 & 0 \\ \Delta A_{di}(\iota) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } \hat{I} = \begin{bmatrix} I \\ I \\ 0 \\ 0 \end{bmatrix}.$$

For known real constant matrix U_1, we have $\|\ell(\bar{x}(\iota))\| = \|\ell(x(\iota))\| \leq \|U_1(\bar{x}(\iota))\| < \|U_1(x(\iota))\|$. Before we conclude this section, we consider the following assumption, definitions and lemma for proving our main results.

Assumption 2.1. *The disturbance function $\omega(\iota)$ in (1) is such that $\|\omega(\iota)\|_\infty < \omega_M$, where $\|.\|_\infty$ and ω_M are infinity norm and unknown real positive number respectively.*

Definition 2.2. [23] *The Riemann-Liouville fractional integral for a function $x(\iota)$ with order $\wp$ is defined as*

$$D^{-\wp}x(\iota) = \frac{1}{\Gamma(\wp)}\int_{\iota_0}^{\iota}(\iota - u)^{\wp-1}x(u)du, \quad \wp > 0.$$

Definition 2.3. [23] *The Riemann-Liouville fractional derivative for a function $x(\iota)$ with order $\wp$ is defined as*

$$D^{\wp}x(\iota) = \frac{1}{\Gamma(n-\wp)}\frac{d^n}{d\iota^n}\int_{\iota_0}^{\iota}\frac{x(u)}{(\iota - u)^{\wp+1-n}}du, \quad n - 1 \leq \wp < n,$$

where $\Gamma(.)$ is the Gamma function, $\Gamma(s) = \int_0^\infty \iota^{s-1}e^{-\iota}d\iota$.

Lemma 2.4. [15] *Let $x(\iota) \in R^n$ be a vector value differentiable function. Then the relationship $D^{\wp}(x^T(\iota)Px(\iota)) \leq 2x^T(\iota)PD^{\wp}x(\iota)$ holds, for all $\wp \in (0, 1)$, where $P \in R^{n\times n}$ is a constant, positive, semi-definite, square and symmetric matrix.*

Property 2.5. [8] *If $\wp_1 > \wp_2 > 0$, then $D^{\wp_2}(D^{\wp_1}x(\iota)) = D^{\wp_2\wp_1}x(\iota)$ holds for function $x(\iota)$. In particular, this relation holds if $x(\iota)$ is integrable.*

3 Main results

In this section, we initially present results for an unforced fractional-order singular T-S fuzzy model without disturbance and uncertainty and then proceed to obtain the conditions for stability of the considered fractional-order singular T-S fuzzy model (1). The unforced fractional-order singular T-S fuzzy model without disturbance and uncertainty can be given as

$$\begin{aligned} ED^{\wp}x(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota))[A_i x(\iota) + A_{di}x(\iota - h(\iota)) + \ell(x(\iota))], \\ y(\iota) = & \sum_{i=1}^{k} n_i(\rho(\iota))C_i x(\iota), \\ x(\iota) = & \varphi(\iota), \quad \iota \in [-h, 0]. \end{aligned} \tag{11}$$

Now, this section proceeds with discussing the problem of admissibility for the unforced fractional-order system (11) in the absence of uncertain term ($\Delta A_i(\iota)) = 0$ and $\Delta A_{di}(\iota)) = 0$). Then, it will be extended to the case with uncertain term and the explicit expression of dynamic output feedback control and observer gains will be presented for ensuring the robust asymptotic stability of the augmented system (10).

Theorem 3.1. *For given constants $h_1 > 0$, $\wp > 0$, $\varsigma > 0$ and given matrix $U_1 > 0$, the unforced fractional-order singular T-S fuzzy dynmaical model* (11) *is admissible, if there exist symmetric matrices $\bar{P}$, $\bar{Q} > 0$ and $\bar{R} > 0$ such that the following relationships hold*

$$E^T \bar{P} = \bar{P}^T E \geq 0, \tag{12}$$

$$\Pi_{ij} < 0, \tag{13}$$

$$\frac{1}{k-1}\Pi_{ii} + \frac{1}{2}(\Pi_{ij} + \Pi_{ji}) < 0, \quad 1 \leq i \neq j \leq k, \tag{14}$$

where $\Pi_{ij} = [\pi]_{5\times 5}$ and the elements of Π_{ij} are given as $\pi_{1,1} = \bar{P}^T A_i + A_i^T \bar{P} + \bar{Q} + \bar{R}$, $\pi_{1,2} = \bar{P}A_{di}$, $\pi_{1,4} = \bar{P}\hat{I}$, $\pi_{1,5} = U_1$, $\pi_{2,2} = -(1-h_1)\bar{Q}$, $\pi_{3,3} = -\bar{R}$, $\pi_{4,4} = -\varsigma I$, $\pi_{5,5} = -\varsigma I$ and the remaining elements of Π_{ij} are zero.

Proof. To prove the admissibility of considered nominal singular T-S fuzzy system (11), we first prove that the system with $u(\iota) = 0$ is regular and impulse free. It follows from (13) and (14) that

$$\begin{bmatrix} \psi_{1,1} & \psi_{1,2} \\ * & \psi_{2,2} \end{bmatrix} < 0, \tag{15}$$

where $\psi_{1,1} = \bar{P}^T \sum_{i=1}^{k} n_i(\rho(\iota))A_i + \bar{Q}$, $\psi_{1,2} = \bar{P}^T \sum_{i=1}^{k} n_i(\rho(\iota))A_{di}$ and $\psi_{2,2} = -(1-h_1)\bar{Q}$.
Letting $G = \begin{bmatrix} I & A_i \\ 0 & A_{di} \end{bmatrix}$ and pre- and post- multiplying (15) by G and G^T, respectively, we get $\pi = \begin{bmatrix} \pi_1 & \pi_2 \\ * & \pi_3 \end{bmatrix}$, where $\pi_1 = \psi_{1,1} + \sum_{i=1}^{k} n_i(\rho(\iota))A_i\psi_{1,2}^T \sum_{i=1}^{k} n_i(\rho(\iota))A_i^T$, $\pi_2 = \psi_{1,2}\sum_{i=1}^{k} n_i(\rho(\iota))A_{di}^T + \sum_{i=1}^{k} n_i(\rho(\iota))A_i\psi_{2,2}\sum_{i=1}^{k} n_i(\rho(\iota))A_{di}^T$, $\pi_3 = A_{di}\psi_{2,2}A_{di}^T$.
Since $rank(E) = r < n$, there exist two nonsingular matrices M and N such that $MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. Also, for $MA(n)N = \begin{bmatrix} A_{11}(n) & A_{12}(n) \\ A_{13}(n) & A_{14}(n) \end{bmatrix}$, where $A(n) = \sum_{i=1}^{k} n_i(\rho(\iota))A_i$, $A_{11}(n)$, $A_{12}(n)$, $A_{13}(n)$, $A_{14}(n)$ are similar to $A(n)$, $G^{-T}\bar{P}M = \begin{bmatrix} P_{11} & P_{12} \\ P_{13} & P_{14} \end{bmatrix}$, $\bar{M} = M^T \begin{bmatrix} 0 \\ \phi \end{bmatrix}$, with $\phi \in R^{(n-r)\times(n-r)}$ is any non singular matrix and $\bar{N} = N^T F = \begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix}$. Now, pre- and post- multiplying both sides of π by M^T and M, we can easily obtain that $\hat{\pi} = \begin{bmatrix} \hat{\pi}_1 & \hat{\pi}_2 \\ \hat{\pi}_3 & \hat{\pi}_4 \end{bmatrix}$ with $\hat{\pi}_4 = A_{14}^T(n)\phi F_{12}^T + F_{12}\phi^T A_{14}(n)$. At this juncture, let us assume that (13) and (14) holds for $\bar{Q} > 0$ and $\bar{R} > 0$. Hence it can be easily verified that $\hat{\pi}_4 < 0$ which implies that $A_{14}(n)$ is nonsingular. Therefore, it can be readily obtained that $det(sMEN - \sum_{i=1}^{k} n_i(\rho(\iota))MA_iN) = det(sE - \sum_{i=1}^{k} n_i(\rho(\iota))A_i)$ which implies that $det(sE - \sum_{i=1}^{k} n_i(\rho(\iota))A_i)$ is not identically zero and $det(sE - \sum_{i=1}^{k} n_i(\rho(\iota))A_i) = r = rank(E)$. Therefore, $(E, \sum_{i=1}^{k} n_i(\rho(\iota))A_i)$ can be obtained to be regular and impulse free. In conjunction, to prove the stability of the system (11), the following Lyapunov-Krasovskii functional candidate is considered for system (11)

$$V(\iota, x(\iota)) = D^{\wp-1}(x^T(\iota)\bar{P}^T Ex(\iota)) + \int_{\iota-h(\iota)}^{\iota} x^T(s)\bar{Q}x(s)ds + \int_{\iota-h_1}^{\iota} x^T(s)\bar{R}x(s)ds. \tag{16}$$

Further, based on Lemma 2.4 and Property 2.5, we get the derivative of $V(\iota, x(\iota))$ along the trajectories of fractional-order singular T-S fuzzy system (11) as

$$\begin{aligned}
\dot{V}(\iota, x(\iota)) &= \sum_{i=1}^{k} n_i(\rho(\iota)) D^{\wp}(x^T(\iota)\bar{P}^T E x(\iota)) + x^T(\iota)\bar{Q}x(\iota) - (1-\dot{h}(\iota))x^T(\iota - h(\iota))\bar{Q}x(\iota - h(\iota)) \\
&\quad + x^T(\iota)\bar{R}x(\iota) - x^T(\iota - h_1)\bar{R}x(\iota - h_1) \\
&\leq \sum_{i=1}^{k} n_i(\rho(\iota)) 2x^T(\iota)\bar{P}^T E D^{\wp}(x(\iota)) + x^T(\iota)\bar{Q}x(\iota) - (1-h_1)x^T(\iota - h(\iota))\bar{Q}x(\iota - h(\iota)) \\
&\quad + x^T(\iota)\bar{R}x(\iota) - x^T(\iota - h_1)\bar{R}x(\iota - h_1) \\
&= \sum_{i=1}^{k} n_i(\rho(\iota)) 2x^T(\iota)\bar{P}^T\{A_i x(\iota) + A_{di}x(\iota - h(\iota)) + \ell_i(x(\iota))\} + x^T(\iota)(\bar{Q} + \bar{R})x(\iota) \\
&\quad - (1-h_1)x^T(\iota - h(\iota))\bar{Q}x(\iota - h(\iota)) - x^T(\iota - h_1)\bar{R}x(\iota - h_1) \\
&= \sum_{i=1}^{k} n_i(\rho(\iota)) x^T(\iota)\{\bar{P}^T A_i + A_i^T\bar{P} + \bar{Q} + \bar{R}\}x(\iota) + \sum_{i=1}^{k} n_i(\rho(\iota)) 2x^T(\iota)\bar{P}^T A_{di}x(\iota - h(\iota)) \\
&\quad + \sum_{i=1}^{k} n_i(\rho(\iota)) 2x^T(\iota)\bar{P}^T \ell(x(\iota)) - (1-h_1)x^T(\iota - h(\iota))\bar{Q}x(\iota - h(\iota)) - x^T(\iota - h_1)\bar{R}x(\iota - h_1) \\
&\quad + \varsigma^{-1}x^T(\iota)U_1^T U_1 x(\iota) - \varsigma^{-1}\ell^T(x(\iota))\ell(x(\iota)).
\end{aligned} \tag{17}$$

Therefore $\dot{V}(\iota, x(\iota)) < \sum_{i=1}^{k} n_i(\rho(\iota))\zeta^T(\iota)\Pi_{ij}\zeta(\iota)$, where $\zeta(\iota) = \begin{bmatrix} x^T(\iota) & x^T(\iota - h(\iota)) & x^T(\iota - h_1) & \ell^T(x(\iota)) \end{bmatrix}^T$ and $\Pi_{ij} = \begin{bmatrix} \bar{P}^T A_i + A_i^T\bar{P} + \bar{Q} + \bar{R} + \varsigma^{-1}U_1^T U_1 & \bar{P}^T A_{di} & 0 & \bar{P}^T\hat{I} \\ * & -(1-h_1)\bar{Q} & 0 & 0 \\ * & * & -\bar{R} & 0 \\ * & * & * & -\varsigma I \end{bmatrix}$. It is clear from Schur complement that $\Pi_{ij} < 0$ is equivalent to

$$\begin{bmatrix} \bar{P}^T A_i + A_i^T\bar{P} + \bar{Q} + \bar{R} & \bar{P}^T A_{di} & 0 & \bar{P}^T\hat{I} & U_1 \\ * & -(1-h_1)\bar{Q} & 0 & 0 & 0 \\ * & * & -\bar{R} & 0 & 0 \\ * & * & * & -\varsigma I & 0 \\ * & * & * & * & -\varsigma I \end{bmatrix} < 0. \tag{18}$$

Therefore, if the LMIs (12)(14) hold then $\dot{V}(\iota, x(\iota)) < 0$. Hence, it is concluded that the fractional-order singular T-S fuzzy system (11) is admissible. □

Next, based on the above obtained stability criteria, we are going to derive some sufficient conditions which guarantees the admissibility of the singular fractional-order augmented T-S fuzzy system (10) without uncertain matrices of the system. Here, for representation convenience, we denote $\hat{Q} = diag\{\hat{Q}_1, \hat{Q}_2, \hat{Q}_3, \hat{Q}_4\}$ and $\hat{R} = diag\{\hat{R}_1, \hat{R}_2, \hat{R}_3, \hat{R}_4\}$.

Theorem 3.2. *For given positive matrices U_1, U_2 and positive constants $\wp, h_1, \alpha, \varsigma, \delta_p, \varrho, \rho_q, (q = 1, 2, 4)$, the fractional-order singular augmented T-S fuzzy system (10) in the absence of uncertainty is admissible if there exist non singular matrices X_1, X_2, X_3 and X_4; symmetric matrices $\hat{Q} > 0$ and $\hat{R} > 0$ such that the below LMIs are true*

$$X_1^T E^T = EX_1 \geq 0, \quad X_2^T E^T = EX_2 \geq 0, \quad X_4^T E^T = EX_4 \geq 0, \tag{19}$$

$$\bar{\Theta}_{ij} < 0, \tag{20}$$

$$\frac{1}{k-1}\bar{\Theta}_{ii} + \frac{1}{2}(\bar{\Theta}_{ij} + \bar{\Theta}_{ji}) < 0, \quad 1 \leq i \neq j \leq k, \tag{21}$$

$$\begin{bmatrix} -\delta_p I & * \\ C_i X_p - \bar{X}_p C_i & -I \end{bmatrix} < 0, \quad p = 1, 2. \tag{22}$$

where $\bar{\Theta}_{ij} = [\theta]_{21\times 21}$ and the elements of $\bar{\Theta}_{ij}$ are given as $\theta_{1,1} = A_i X_1 + B_i Y_{D\delta j} C_i + \hat{Q}_1 + \hat{R}_1$, $\theta_{1,3} = -B_i C_F X_3$, $\theta_{1,4} = B_i Y_{C\delta j} + Y_{B\delta j} C_i$, $\theta_{1,5} = A_{di} X_1$, $\theta_{1,13} = I$, $\theta_{1,14} = X_1 U_1$, $\theta_{1,18} = X_1$, $\theta_{2,2} = A_i \alpha X_2 - \alpha W_{Li} C_i + \alpha\hat{Q}_2 + \alpha\hat{R}_2$,

$\theta_{2,3} = -B_iC_FX_3 + \alpha C_i^T W_{Li}^T B_i^{+T} B_F^T$, $\theta_{2,6} = A_{di}\alpha X_2$, $\theta_{2,13} = I$, $\theta_{2,15} = \alpha X_2 U_1$, $\theta_{2,19} = \alpha X_2$, $\theta_{3,3} = A_F X_3 + B_F C_F X_3 + B_F C_F X_3 + \hat{Q}_3 + \hat{R}_3$, $\theta_{3,16} = X_3 U_1$, $\theta_{3,20} = X_3$, $\theta_{4,4} = Y_{A\delta j} + \hat{Q}_4 + \hat{R}_4$, $\theta_{4,17} = X_4 3 U_1$, $\theta_{4,21} = X_4$, $\theta_{5,5} = -(1-h_1)\hat{Q}_1$, $\theta_{6,6} = -\alpha(1-h_1)\hat{Q}_2$, $\theta_{7,7} = -(1-h_1)\hat{Q}_3$, $\theta_{8,8} = -(1-h_1)\hat{Q}_4$, $\theta_{9,9} = -\hat{R}_1$, $\theta_{10,10} = -\alpha\hat{R}_2$, $\theta_{11,11} = -\hat{R}_3$, $\theta_{12,12} = -\hat{R}_4$, $\theta_{13,13} = -\varsigma I$, $\theta_{14,14} = -\varsigma I$, $\theta_{15,15} = -\varsigma I$, $\theta_{16,16} = -\varsigma I$, $\theta_{17,17} = -\varsigma I$, $\theta_{18,18} = -U_2^{-1}$, $\theta_{19,19} = -U_2^{-1}$, $\theta_{20,20} = -U_2^{-1}$, $\theta_{21,21} = -U_2^{-1}$ *and the remaining elements of* $\bar{\Theta}_{ij}$ *are zero. Moreover, the dynamic output feedback compensator gain matrices are calculated by using the relation* $Y_{A\delta j} = A_{\delta j}X_4$, $Y_{B\delta j} = B_{\delta j}X_4$, $Y_{C\delta j} = C_{\delta j}X_4$, $Y_{D\delta j} = D_{\delta j}X_1$ *and the observer gain matrix is calculated using the relation* $W_{Li} = L_i X_2$.

Proof. Choose the Lyapunov-Krasovskii functional candidate for the augmented system (10) as

$$V(\iota, x(\iota)) = D^{\wp-1}(\bar{x}^T(\iota)P^T\bar{E}\bar{x}(\iota)) + \int_{\iota-h(\iota)}^{\iota} \bar{x}^T(s)Q\bar{x}(s)ds + \int_{\iota-h_1}^{\iota} \bar{x}^T(s)R\bar{x}(s)ds, \tag{23}$$

where $P = diag\{P_1, \frac{1}{\alpha}P_2, P_3, P_4\}$, $Q = diag\{Q_1, Q_2, Q_3, Q_4\}$ and $R = diag\{R_1, R_2, R_3, R_4\}$ are positive definite matrices. Based on Lemma 2.4 and Property 2.5 we can obtain $\dot{V}(\iota, x(\iota))$ along the trajectories of fractional-order singular augmented time delay T-S fuzzy system (10) as

$$\begin{aligned}
\dot{V}(\iota, x(\iota)) &= D^{\wp}(\bar{x}^T(\iota)P^T\bar{E}\bar{x}(\iota)) + \bar{x}^T(\iota)Q\bar{x}(\iota) - (1-\dot{h}(\iota))\bar{x}^T(\iota-h(\iota))Q\bar{x}(\iota-h(\iota)) + \bar{x}^T(\iota)R\bar{x}(\iota) \\
&\quad - \bar{x}^T(\iota-h_1)R\bar{x}(\iota-h_1) \\
&\le 2\bar{x}^T(\iota)P^T\bar{E}D^{\wp}(\bar{x}(\iota)) + \bar{x}^T(\iota)Q\bar{x}(\iota) - (1-h_1)\bar{x}^T(\iota-h(\iota))Q\bar{x}(\iota-h(\iota)) + \bar{x}^T(\iota)R\bar{x}(\iota) \\
&\quad - \bar{x}^T(\iota-h_1)R\bar{x}(\iota-h_1) \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} n_i(\rho(\iota))n_j(\rho(\iota))2\bar{x}^T(\iota)P^T\{\hat{A}_i\bar{x}(\iota) + \hat{A}_{di}\bar{x}(\iota-h(\iota)) + \bar{\omega}(\iota) + \ell(\bar{x}(\iota))\} + \bar{x}^T(\iota)(Q+R)\bar{x}(\iota) \\
&\quad - (1-h_1)\bar{x}^T(\iota-h(\iota))Q\bar{x}(\iota-h(\iota)) - \bar{x}^T(\iota-h_1)R\bar{x}(\iota-h_1) \\
&= \sum_{i=1}^{k}\sum_{j=1}^{k} n_i(\rho(\iota))n_j(\rho(\iota))\bar{x}^T(\iota)\{P^T\hat{A}_i + \hat{A}_i^TP + Q + R\}\bar{x}(\iota) + 2\bar{x}^T(\iota)P^T\hat{A}_{di}\bar{x}(\iota-h(\iota)) \\
&\quad + 2\bar{x}^T(\iota)P^T\bar{\omega}(\iota) + 2\bar{x}^T(\iota)P^T\ell(\bar{x}(\iota)) - (1-h_1)\bar{x}^T(\iota-h(\iota))Q\bar{x}(\iota-h(\iota)) - \bar{x}^T(\iota-h_1)R\bar{x}(\iota-h_1) \\
&\quad + \varsigma^{-1}\bar{x}^T(\iota)U_1^TU_1\bar{x}(\iota) - \varsigma^{-1}\ell^T(\bar{x}(\iota))\ell(\bar{x}(\iota)).
\end{aligned}$$

For any $\varrho > 0$, $2\bar{x}^T(\iota)P^T\bar{\omega}(\iota) \le \varrho^{-1}\bar{x}^T(\iota)P^T\hat{A}_{\omega i}\hat{A}_{\omega i}^TP\bar{x}(\iota) + \varrho\omega^T(\iota)\omega(\iota)$.
Therefore, $\dot{V}(\iota, x(\iota)) < \sum_{i=1}^{k}\sum_{j=1}^{k} n_i(\bar{x}(\iota))n_j(\bar{x}(\iota)\zeta^T(\iota)\Theta_{ij}\zeta(\iota)$, where $\zeta(\iota) = \begin{bmatrix}\bar{x}^T(\iota) & \bar{x}^T(\iota-h(\iota)) & \bar{x}^T(\iota-h_1) & \ell^T(\bar{x}(\iota))\end{bmatrix}^T$ and

$$\Theta_{ij} = \begin{bmatrix} \Theta_{1,1} - U_2 & P^T\hat{A}_{di} & 0 & P^T\hat{I} & P^T\hat{A}_{\omega i} \\ * & -(1-h_1)Q & 0 & 0 & 0 \\ * & * & -R & 0 & 0 \\ * & * & * & -\varsigma I & 0 \\ * & * & * & 0 & -\varrho I \end{bmatrix}$$

with $\Theta_{1,1} = P^T\hat{A}_i + \hat{A}_i^TP + Q + R + \varsigma^{-1}U_1^TU_1 + U_2$. Note that

$$\begin{bmatrix} P^T\hat{A}_i + \hat{A}_i^TP + Q + R + \varsigma^{-1}U_1^TU_1 + U_2 & P^T\hat{A}_{di} & 0 & P^T\hat{I} & P^T\hat{A}_{\omega i} \\ * & -(1-h_1)Q & 0 & 0 & 0 \\ * & * & -R & 0 & 0 \\ * & * & * & -\varsigma I & 0 \\ * & * & * & 0 & -\varrho I \end{bmatrix} < 0$$

implies that

$$\dot{V}(\iota, x(\iota)) < -\bar{x}^T(\iota)U_2\bar{x}(\iota) + \varrho^{-1}\bar{x}^T(\iota)P^T\hat{A}_{\omega i}\hat{A}_{\omega i}^TP\bar{x}(\iota) + \varrho\omega^T(\iota)\omega(\iota) < -\lambda_{min}(U_2)\|\bar{x}(\iota)\|^2 + \varrho\omega_M^2 + \varrho^{-1}\lambda_{max}(P^T\hat{A}_{\omega i}\hat{A}_{\omega i}^TP)\|\bar{x}(\iota)\|^2.$$

It is clear that, for $\varrho > 0$, $\lambda_{max}(P^T\hat{A}_{\omega i}\hat{A}_{\omega i}^T P) < \lambda_{min}(U_2)$. Therefore, the closed-loop system (10) is bounded uniformly when subject to nonlinearity and unknown disturbance. It is clear from Schur complement, $\Theta_{ij} < 0$ is equivalent to

$$\begin{bmatrix} P^T\hat{A}+\hat{A}^TP+Q+R & P^T\hat{A}_{di} & 0 & P^T\hat{I} & U_1 & I \\ * & -(1-h_1)Q & 0 & 0 & 0 & 0 \\ * & * & -R & 0 & 0 & 0 \\ * & * & * & -\varsigma I & 0 & 0 \\ * & * & * & * & -\varsigma I & 0 \\ * & * & * & * & * & -U_2^{-1} \end{bmatrix} < 0. \tag{24}$$

Let $X = P^{-1} = diag\{X_1, \alpha X_2, X_3, X_4\}$, $\hat{Q} = XQX$ and $\hat{R} = XRX$. Furthermore, assume that $C_i\alpha X_2 = \alpha\bar{X}_2C_i$, where X_2 is any appropriate dimensioned symmetric matrix. Now, pre- and post-multiplying Θ_{ij} by $diag\{X, X, X, I, I, I\}$, we can easily obtain LMI (20).

It is noted that the assumptions $C_iX_1 = \bar{X}_1C_i$ and $C_i\alpha X_2 = \alpha\bar{X}_2C_i$ are not strict LMIs and therefore it becomes quite difficult to solve the left hand side of LMI (22) via MATLAB LMI toolbox. To get over such a difficult situation, we apply the optimization technique for the assumptions $C_iX_1 = \bar{X}_1C_i$ and $C_i\alpha X_2 = \alpha\bar{X}_2C_i$, which are equivalently written as trace $[(C_iX_1-\bar{X}_1C_i)^T(C_iX_1-\bar{X}_1C_i)] = 0$ and $[(C_i\alpha X_2-\alpha\bar{X}_2C_i)^T(C_i\alpha X_2-\alpha\bar{X}_2C_i)] = 0$ respectively. Consecutively, by the aid of Schur complement and for any $\delta_p > 0$, $[(C_iX_1-\bar{X}_1C_i)^T(C_iX_1-\bar{X}_1C_i)] = 0$ and $[(C_i\alpha X_2-\alpha\bar{X}_2C_i)^T(C_i\alpha X_2-\alpha\bar{X}_2C_i)] = 0$ takes the form (22). Thus, if the LMIs (19)-(22) are true, then we get that $\dot{V}(\iota, x(\iota)) \leq 0$ is also true. □

Now, we are in a position to present the derivation for the dynamic output feedback controller design for the uncertain fractional-order singular time delayed T-S fuzzy control system. To be in particular, based on EID performance analysis, a novel control strategy is formulated and the LMI based sufficient conditions are obtained in the subsequent theorem.

Theorem 3.3. *For given positive matrices U_1, U_2 and positive constants $\wp, h_1, \alpha, \varsigma, \delta_p, \varrho, \rho_q, (q = 1, 2, 4)$, the uncertain fractional-order singular T-S fuzzy control system (1) is admissible if there exist non singular matrices X_1, X_2, X_3 and X_4; scalars ϵ_1, ϵ_2 and symmetric matrices $\hat{Q} > 0$, $\hat{R} > 0$ such that the below LMIs are true*

$$X_1^TE^T = EX_1 \geq 0, \quad X_2^TE^T = EX_2 \geq 0, \quad X_4^TE^T = EX_4 \geq 0, \tag{25}$$

$$\begin{bmatrix} \bar{\Theta}_{ij} & \bar{G}_{1i} & \bar{H}_{1i} & \bar{G}_{2i} & \bar{H}_{2i} \\ * & \epsilon_1 I & 0 & 0 & 0 \\ * & * & \epsilon_1 I & 0 & 0 \\ * & * & * & \epsilon_2 I & 0 \\ * & * & 0 & * & \epsilon_2 I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, k, \tag{26}$$

$$\frac{1}{k-1}\begin{bmatrix} \bar{\Theta}_{ii} & \bar{G}_{1i} & \bar{H}_{1i} & \bar{G}_{2i} & \bar{H}_{2i} \\ * & \epsilon_1 I & 0 & 0 & 0 \\ * & * & \epsilon_1 I & 0 & 0 \\ * & * & * & \epsilon_2 I & 0 \\ * & * & 0 & * & \epsilon_2 I \end{bmatrix} + \frac{1}{2}\begin{bmatrix} (\bar{\Theta}_{ij}+\bar{\Theta}_{ji}) & \bar{G}_{1i} & \bar{H}_{1i} & \bar{G}_{2i} & \bar{H}_{2i} \\ * & \epsilon_1 I & 0 & 0 & 0 \\ * & * & \epsilon_1 I & 0 & 0 \\ * & * & * & \epsilon_2 I & 0 \\ * & * & 0 & * & \epsilon_2 I \end{bmatrix} < 0, \quad 1 \leq i \neq j \leq k, \tag{27}$$

$$\begin{bmatrix} -\rho I & * \\ C_iX_{pi}-\bar{X}_{pi}C_i & -I \end{bmatrix} < 0, \tag{28}$$

where $\bar{G}_{1i} = [\epsilon_1G_{1i} \quad \epsilon_1G_{1i} \quad 0_{19n} \quad \epsilon_1 I]$, $\bar{H}_{1i} = [X_1H_{1i} \quad 0 \quad 0_{20n} \quad \epsilon_1 I]$, $\bar{G}_{2i} = [\epsilon_2G_{2i} \quad \epsilon_2G_{2i} \quad 0_{21n} \quad \epsilon_2 I]$ and $\bar{H}_{2i} = [X_1H_{2i} \quad 0 \quad 0_{22n} \quad \epsilon_2 I]$. Furthermore the control gain matrices can be estimated by using the relations $A_{\delta j} = Y_{A\delta j}X_4^{-1}$, $B_{\delta j} = Y_{B\delta j}X_4^{-1}$, $C_{\delta j} = Y_{C\delta j}X_4^{-1}$, $D_{\delta j} = Y_{D\delta j}\bar{X}_1^{-1}$ and $L_i = W_{Li}\bar{X}_2^{-1}$.

Proof. Replacing the constant matrices A_i and A_{di} by $A_i + G_{1i}\theta_i(\iota)H_{1i}$ and $A_{di} + G_{2i}\theta_i(\iota)H_{2i}$ respectively in LMI (20), we get

$$\bar{\Theta}_{ij} + sym(G_{1i}\theta_i(\iota)H_{1i}) + sym(G_{2i}\theta_i(\iota)H_{2i}) < 0. \tag{29}$$

By using Lemma 2.5 given in [25], for any scalars $\epsilon_1 > 0$ and $\epsilon_2 > 0$, (29) can be rewritten as

$$\bar{\Theta}_{ij} + \epsilon_1^{-1}\bar{G}_{1i}^T\bar{G}_{1i} + \epsilon_1\bar{H}_{1i}^T\bar{H}_{1i} + \epsilon_2^{-1}\bar{G}_{2i}^T\bar{G}_{2i} + \epsilon_1\bar{H}_{2i}^T\bar{H}_{2i} < 0. \tag{30}$$

Further, by applying Schur complement [2], it is obvious that (30) is equivalent to the LMIs (26) and (27). Thus, the proof is completed. Now, similar to the previous theorem, we use optimization technique to convert (27) as LMIs. For this purpose we define a positive scalar ρ_q such that $[(EX_q - X_q^T E^T)^T (EX_q - X_q^T E^T)] \leq \rho_q, (q = 1, 2, 4)$. Further, by Schur complement, we can obtain

$$\begin{bmatrix} -\rho_q I & * \\ EX_q - X_q^T E^T & -I \end{bmatrix} < 0 \text{ for every } q. \tag{31}$$

□

Remark 3.4. *It should be noted that most of the existing works on the stabilization problem for singular fractional-order T-S fuzzy systems are assumed to be delay independent or dependent. An extensive list of these works can be found in [18, 1, 26] and references therein. However, in most of the practical engineering systems, the delay varies with respect to time. In order to express the above situation, in this paper, the delay is assumed to be time-varying and bounded.*

Remark 3.5. *Fractional-order singular T-S fuzzy system is a generalization of the traditional (integer-order) singular T-S fuzzy system in which the order of derivatives and integrals can be any real or complex number. It is worth mentioning that many real-world problems can be modeled using fractional-order systems since it effectively reduces the modeling error and has bigger stability regions than integer-order systems. It should be pointed out that if* $\wp = 1$*, then the considered system* (1) *becomes an integer-order singular time delayed T-S fuzzy system.*

4 Numerical example

In this section of the paper, two numerical examples are presented to illustrate the effectiveness and validity of the proposed EID-based fractional-order fuzzy dynamic output feedback compensator design.

Example 4.1. *An uncertain fractional-order singular augmented T-S fuzzy system* (10) *with two fuzzy rules with the below parameters are considered to validate the proposed results.*
$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A_1 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$, $A_{d1} = \begin{bmatrix} -2 & 0 \\ -0.5 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} -4 & 0.1 \\ -1.8 & -2 \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B_{\omega 1} = \begin{bmatrix} -0.1 \\ -1 \end{bmatrix}$, $B_{\omega 2} = \begin{bmatrix} -0.2 \\ -0.6 \end{bmatrix}$, $C_1 = C_2 = \begin{bmatrix} 1 & 2 \end{bmatrix}$, $G_{11} = \begin{bmatrix} -0.03 & 0.1 \\ 0.2 & -0.1 \end{bmatrix}$, $G_{12} = \begin{bmatrix} -0.02 & 0.1 \\ 0.2 & -0.1 \end{bmatrix}$, $G_{21} = \begin{bmatrix} -0.01 & 0.1 \\ 0.3 & -0.1 \end{bmatrix}$, $G_{22} = \begin{bmatrix} -0.01 & 0.1 \\ 0.3 & -0.1 \end{bmatrix}$, $H_{11} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}$, $H_{12} = \begin{bmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \end{bmatrix}$, $H_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.3 \end{bmatrix}$ *and* $H_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.2 \end{bmatrix}$.

The parameters of the EID estimator are taken as $A_F = -101$, $B_F = 100$ and $C_F = 1$. Also, let us assume $n_1(\rho(\iota)) = sin(x_1(\iota))/x_1(\iota)$ and $n_2(\rho(\iota)) = 1 - n_1(\rho(\iota))$ as the fuzzy membership grade functions. The other constant parameters involved in the proposed theoretical results are selected as $\wp = 0.9$, $h_1 = 0.8$, $\alpha = 5$, $\varsigma = 1$, $\delta_1 = 263.3991$, $\delta_2 = 260.6860$, $\rho_1 = 0.001$, $\rho_2 = 0.001$ and $\rho_4 = 0.001$. Further, the disturbance $w(\iota) = 3cos(5\iota) - 0.1sinh(\iota - 15) + 0.1cosh(\iota - 10)$ is utilized to assess the validity of the designed EID based controller. The parameters related to time-varying delay, nonlinearity and boundedness are taken as $h(\iota) = 0.5 + 0.5sin(0.5\iota)$, $\ell(\iota) = \begin{bmatrix} 0.1 + 0.05sin(x_1(\iota)) \\ 0.1 + 0.05sin(x_2(\iota)) \end{bmatrix}$, $U_1 = diag\{0.1, 0.1\}$ and $U_2 = 0.0121$. In order to stabilize the concerned uncertain fractional-order singular augmented T-S fuzzy system (10), we have designed a more generalized proportional dynamic output feedback controller with $E_\delta = E$ as mentioned in (8). In this connection, by solving the sufficient conditions obtained in Theorem 3.3 it is quite easy to get the feasible solution by using the Matlab LMI toolbox. Further, the proportional dynamic output feedback gain and observer gain matrices can be obtained as $A_{\delta 1} = \begin{bmatrix} -16.8472 & -0.7694 \\ -0.7694 & -18.0013 \end{bmatrix}$, $A_{\delta 2} = \begin{bmatrix} -16.4332 & 0.0586 \\ 0.0586 & -16.3453 \end{bmatrix}$, $B_{\delta 1} = \begin{bmatrix} 13.4190 \\ 6.1391 \end{bmatrix}$, $B_{\delta 2} = \begin{bmatrix} 65.1715 \\ 66.5422 \end{bmatrix}$, $C_{\delta 1} = \begin{bmatrix} -10.1717 & -20.3435 \end{bmatrix}$, $C_{\delta 2} = \begin{bmatrix} -65.6148 & -131.2297 \end{bmatrix}$, $D_{\delta 1} = \begin{bmatrix} -0.3820 \end{bmatrix}$, $D_{\delta 2} = \begin{bmatrix} -1.1307 \end{bmatrix}$, $L_1 = \begin{bmatrix} 9.7022 \\ -5.6828 \end{bmatrix}$ and $L_2 = \begin{bmatrix} -4.8849 \\ 9.1252 \end{bmatrix}$. With the aforementioned proportional dynamic output feedback gain matrices and observer gain matrices together with the initial condition of the state, observer and dynamic compensator state vector as $x(0) = [2 \quad 2]^T, \hat{x}(0) = [3 \quad 2]^T, x_c(0) = [1 \quad 1]^T$, the associated graphical results are shown in figures Fig.1-Fig.4. For the considered dynamic output feedback controller design, Fig. 1 illustrates the true time responses of the actual state and their estimations, which shows that the estimated state traces the actual state after some time. Fig. 2 (a) shows the corresponding error state response and Fig. 2(b) shows the dynamic output feedback control response curves. From Fig. 2(a), we can observe that the estimated error $e(\iota) \to 0$ as time increases, which shows that the states are being estimated perfectly as time increases. In Fig. 3(a) the outputs of the considered system and the observer is depicted and in Fig. 3(b) the trajectories of the exact disturbance and its corresponding EID based estimation is depicted. From Fig. 3(b), it can be firmly concluded that the EID estimator effectively compensates for the external disturbances, uncertainties and nonlinearities occurring in the system model. Further, Fig. 4 is provided to reveal the state response of the considered system (1) in the presence and absence of EID estimator. It is quite easy to observe from Fig. 4 that the proposed EID based dynamic output compensator can effectively estimate the external disturbance without any steady-state error. Further, due to gain matrices $A_{\delta i}$, $B_{\delta i}$, $C_{\delta i}$ and $D_{\delta i}$, the EID based controller design improves the stability performance besides rejecting the external disturbances and the nonlinearities in the system.

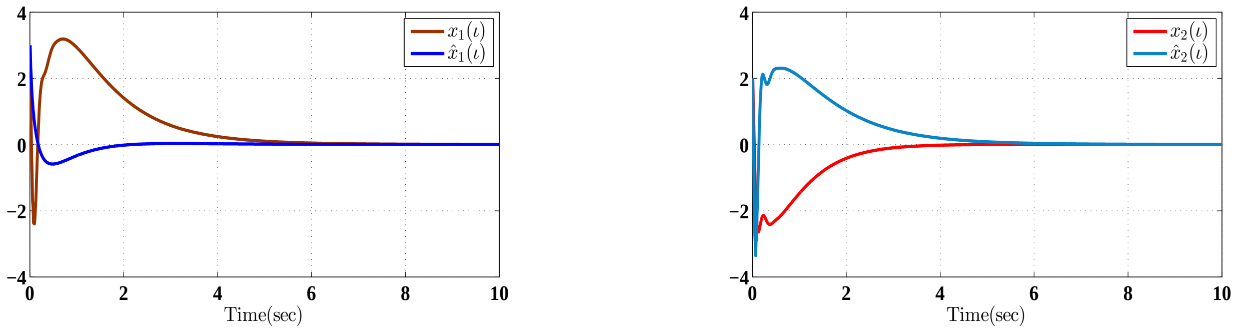
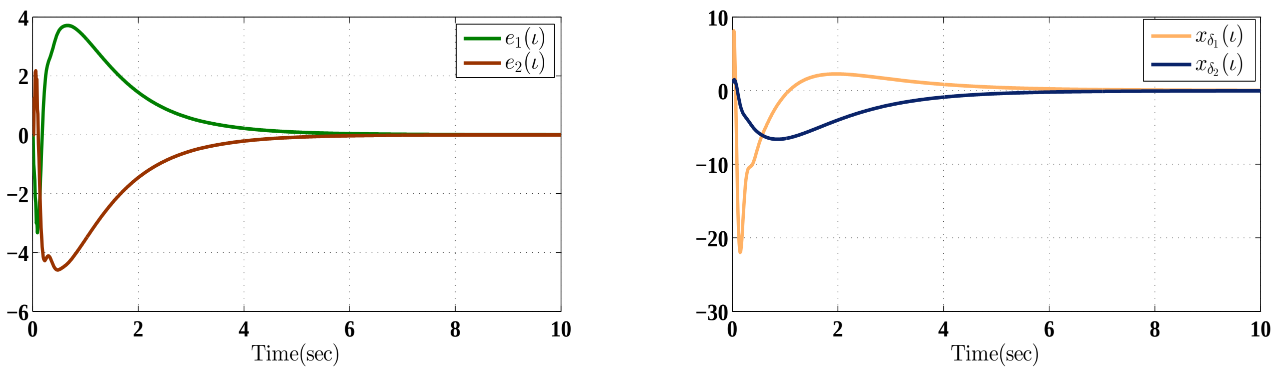


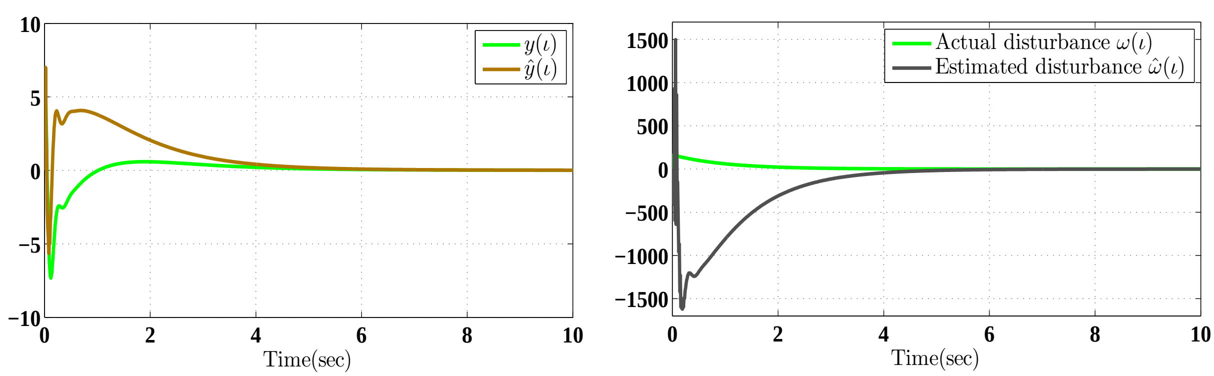
Figure 1: State $x(\iota)$ and its estimation $\hat{x}(\iota)$ under the proposed controller (8)

Example 4.2. *To show the effectiveness and applicability of the proposed design, we consider a nonlinear electric*



(a) (b)

Figure 2: (a) Error $e(\iota)$ (b) Dynamic output feedback control response



(a) (b)

Figure 3: (a) Output responses (b) Actual disturbance and the estimated disturbance

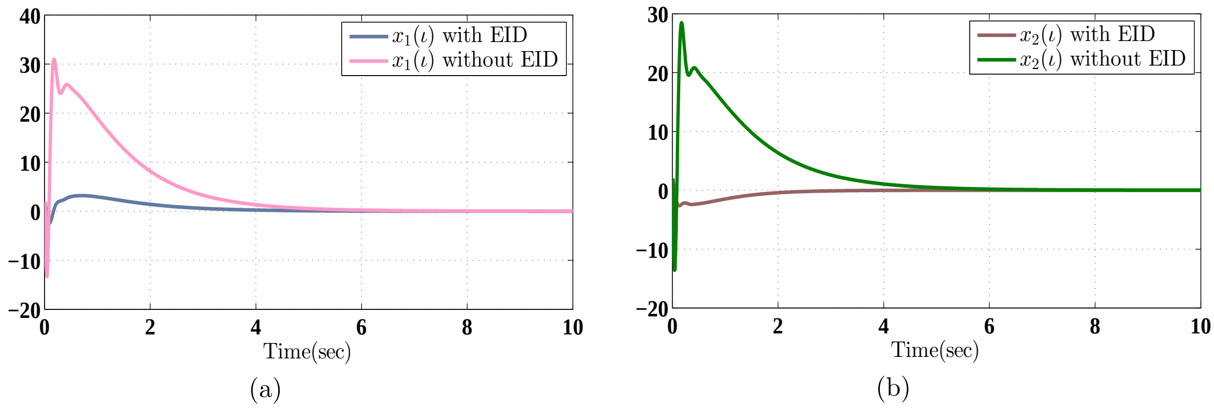


Figure 4: State response of the closed-loop system (2) with and without EID

circuit containing parastic capacitor and non linear resistor [26], which is described by the following expressions

$$L\dot{I}_L(\iota) = -I_L(\iota)R - v_c + u(\iota), \tag{32}$$
$$C\dot{v}_c(\iota) = I_L(\iota) - 0.2(v_c^3(\iota) - v_c) + \rho u(\iota), \tag{33}$$

where $C = 1$, $\rho = 0.5$, $R = 2\Omega$ *and* $L = 0.1H$. *Taking* $LI_L(\iota) = x_1(\iota)$, $v_c(\iota) = x_2(\iota)$, *the external disturbance* $\omega(\iota)$, *nonlinearity* $\ell(x(\iota))$ *and uncertainty, the overall T-S fuzzy singular time delayed system can be written as*

$$\begin{aligned} E\dot{x}(\iota) =& \sum_{i=1}^{k} n_i(\rho(\iota))[\tilde{A}_i x(\iota) + \tilde{A}_{di}x(\iota - h(\iota)) + B_i u(\iota) + \ell(x(\iota)) + B_{\omega i}\omega(\iota)], \\ y(\iota) =& \sum_{i=1}^{k} n_i(\rho(\iota))C_i x(\iota), \end{aligned} \tag{34}$$

where $E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} -20 & -1 \\ 10 & 0.2 \end{bmatrix}$, $A_{d1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $A_2 = \begin{bmatrix} -20 & -1 \\ 10 & -1.6 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} -2 & 0.1 \\ 0 & -2 \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$, $B_{\omega 1} = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}$, $B_{\omega 2} = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}$, $C_1 = C_2 = \begin{bmatrix} 0.2 & 0.6 \end{bmatrix}$, $G_{11} = \begin{bmatrix} -0.02 & 0.2 \\ 0.02 & -0.1 \end{bmatrix}$, $G_{12} = \begin{bmatrix} -0.03 & 0.01 \\ 0.3 & -0.05 \end{bmatrix}$, $G_{21} = \begin{bmatrix} -0.1 & 0.01 \\ 0.25 & -0.08 \end{bmatrix}$, $G_{22} = \begin{bmatrix} -0.1 & 0.09 \\ 0.2 & -0.2 \end{bmatrix}$, $H_{11} = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}$, $H_{12} = \begin{bmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \end{bmatrix}$, $H_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.3 \end{bmatrix}$ *and* $H_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.2 \end{bmatrix}$.

The time varying delay, external disturbance and the nonlinearity are considered as in [26] by $h(\iota) = 0.5 + 0.5sin(0.5\iota)$, $w(\iota) = \frac{sin(\iota)}{1+\iota^2}$ and $\ell(\iota) = \begin{bmatrix} 0.1 + 0.05sin(x_1(\iota)) \\ 0.1 + 0.05sin(x_2(\iota)) \end{bmatrix}$ respectively. Meanwhile, the membership functions of fuzzy sets and the other parameters which are considered in Theorem 3.3 are taken as $n_1(\rho(\iota)) = 1 - x_2^2(\iota)/9$, $n_2(\rho(\iota)) = 1 - n_1(\rho(\iota))$, $h_1 = 0.8$, $\alpha = 5$, $\varsigma = 1$, $\delta_1 = 263.3991$, $\delta_2 = 260.6860$, $\rho_1 = 0.001$, $\rho_2 = 0.001$, $\rho_4 = 0.001$ and $U_1 = U_2 = I$. Further, the EID estimator parameters are taken as $A_F = -11$, $B_F = 10$ and $C_F = 1$. Now, by using the MATLAB LMI toolbox we solve the sufficient conditions (25) (28) of Theorem 3.3 with $\wp = 1$, the dynamic output feedback gain matrices is obtained as $A_{\delta 1} = \begin{bmatrix} -3.2278 & -0.0579 \\ -0.0579 & -3.3823 \end{bmatrix}$, $A_{\delta 2} = \begin{bmatrix} -3.2094 & -0.0029 \\ -0.0029 & -3.2171 \end{bmatrix}$, $B_{\delta 1} = \begin{bmatrix} 7.6012 \\ 1.4439 \end{bmatrix}$, $B_{\delta 2} = \begin{bmatrix} 15.9549 \\ 9.3558 \end{bmatrix}$, $C_{\delta 1} = \begin{bmatrix} -1.1205 & -3.3616 \end{bmatrix}$, $C_{\delta 2} = \begin{bmatrix} -3.4240 & -10.2719 \end{bmatrix}$, $D_{\delta 1} = \begin{bmatrix} -4.9159 \end{bmatrix}$ and $D_{\delta 2} = \begin{bmatrix} -4.5720 \end{bmatrix}$. Moreover the observer gain matrices are also obtained as $L_1 = \begin{bmatrix} -12.4771 \\ 25.8211 \end{bmatrix}$ and $L_2 = \begin{bmatrix} -11.8287 \\ 24.5313 \end{bmatrix}$. Under the above mentioned dynamic output feedback control gain values with the initial conditions $x(0) = [2 \quad 2]^T, \hat{x}(0) = [3 \quad 2]^T, x_\delta(0) = [1 \quad 2]^T$, the graphical results are displayed in figures Fig.5- Fig.(8). Fig. 5 displays the state $x(\iota)$ and observer $\hat{x}(\iota)$ responses of the nonlinear electric circuit (34). It can be clearly recognized from Fig. 5 that the states of the nonlinear electric circuit can be robustly stabilized by using the proposed disturbance estimator based dynamic output feedback controller design. Further, Fig. 6(a) reveals that the estimated error $e(\iota)$ approaches zero and that the states are being estimated accurately with the proposed controller design. Moreover, Fig. 6(b) is plotted to show the state responses of the dynamic output feedback compensator $x_\delta(\iota)$. In Fig. 7(a) the output responses of the original and the estimated outputs are shown, which coincides with each other as time increases. Hence the estimation is almost perfectly done as time increases by using the proposed controller. Further, Fig. (7)(b) shows the responses of the actual disturbance and the equivalent input disturbance. It can be observed from Fig. (7)(b) that $\omega_E(\iota)$ effectively compensates for $\omega(\iota)$, $\ell(\iota)$ and system uncertainties. Further Fig. (8) reveals the state responses in the presence and absence of EID estimator which shows the advantage of the proposed method. Apparently, the EID based approach provided much better disturbance rejection than that of the conventional approach.

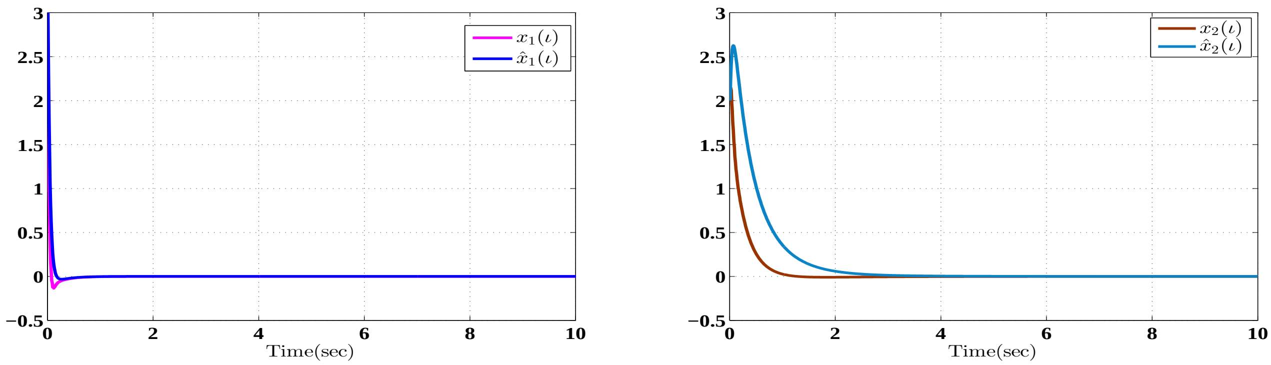
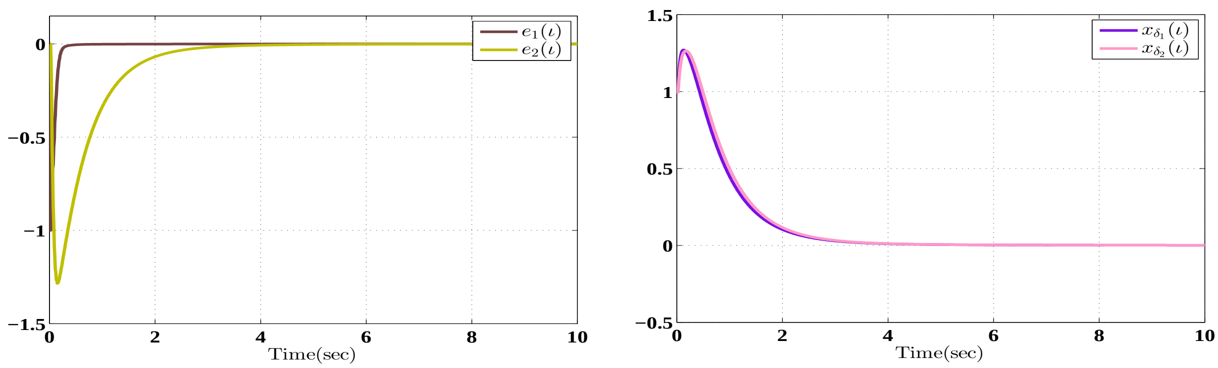
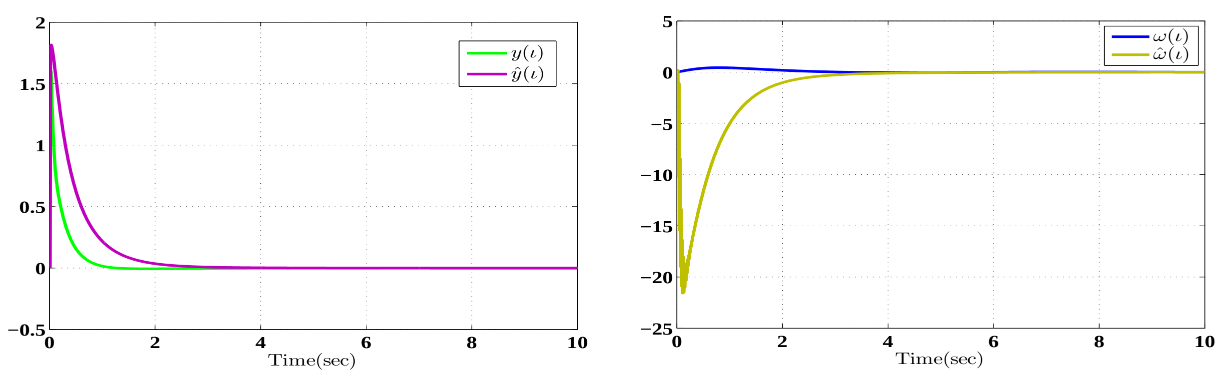


Figure 5: State $x(\iota)$ and its estimation $\hat{x}(\iota)$ under the proposed controller (8)



(a) (b)

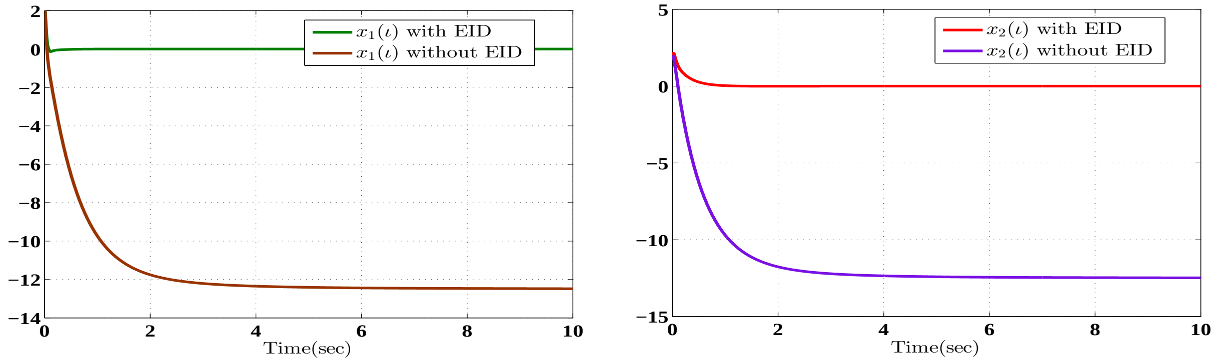
Figure 6: (a) Estimated error $e(\iota)$ (b)Dynamic output feedback control response



(a) (b)

Figure 7: (a) Output responses (b) Actual disturbance and the estimated disturbance

Remark 4.3. *The authors in [26] studied the stability of nonlinear electric circuit system using sliding mode approach. Whereas, in this paper, we have stabilized the nonlinear electric circuit system using EID based dynamic compensator*



(a) (b)

Figure 8: State response of the closed-loop system (2) with and without EID

in the presence of uncertainties. It should be noted that the state trajectories of the nonlinear electric circuit system under the proposed EID based controller (8) *converges much faster than the sliding mode based conventional controller proposed in [26]. Hence we can infer that the proposed EID based controller effectively handles the impact of uncertainties, nonlinearities, disturbances and quickly ensures the stability of the considered system than the sliding mode based controller.*

5 Conclusion

In this paper, a dynamic compensator design has been proposed for stabilization of fractional-order time-delay singular T-S fuzzy systems. In particular, an equivalent-input-disturbance estimator to compensate for the disturbances is incorporated in the controller design. Then, the EID based dynamic compensator has been utilized to compose an augmented fractional-order closed-loop singular T-S fuzzy time-delay system. Sufficient conditions which guarantee the admissibility of the augmented fractional-order closed-loop system have been derived. By means of these conditions, the corresponding control design problem has been transformed into a convex optimization problem and consequently, the desired dynamic compensator gain matrices can be easily obtained by solving the LMIs. Before concluding, the correctness and validity of the derived conditions have been verified through two numerical examples. In future, it could be very interesting to extend the proposed method for fractional-order interval type-2 fuzzy singular systems with quantization.

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