Iranian Journal of Fuzzy Systems

Volume 18, Number 5, (2021), pp. 129-140

Original paper

Robust stabilization of uncertain rectangular singular fractional order T-S fuzzy systems with the fractional order $0 < \alpha < 1$

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Abstract

This paper presents a novel method to investigate the robust stabilization problem of uncertain rectangular singular fractional order Takagi-Sugeno (T-S) fuzzy systems with the fractional order $0 < \alpha < 1$. Firstly, the uncertain rectangular singular fractional order T-S fuzzy system is transformed into an augmented uncertain square singular fractional order T-S fuzzy system by designing a new T-S fuzzy dynamic compensator. Secondly, a sufficient condition in the form of linear matrix inequalities (LMI) is obtained for the robust stabilization of the uncertain rectangular singular fractional order T-S fuzzy system. Finally, a numerical example is given to verify the effectiveness of the results proposed.

Keywords: Fractional order systems, rectangular singular systems, T-S fuzzy systems, dynamic compensator, linear matrix inequalities.

1 Introduction

Fractional order systems (FOS) are an interesting class of systems, which can explain many physical phenomena having memory and genetic characteristics. More and more researchers pay attention to FOS since dynamic behaviors of most real-world systems can be described more precisely than integer order systems [4]. Therefore, FOS are more and more widely used in robotics systems [1], electrical circuit systems [9] and other fields [7]. Fruitful achievements on FOS have been published, such as stability analysis [2, 10, 21, 26, 27, 30], robust control [18, 19, 22], fuzzy control [11, 12].

Singular systems are also referred to as descriptor systems or implicit systems. Researchers have found that the study of singular systems is more sophisticated than that of normal systems. According to the relationship between the number of equations and the number of variables, singular systems are divided into square singular systems and rectangular singular systems. A great number of valuable research results on square singular FOS have been reported. The authors in [20] and [31] address the admissibility problem of singular FOS with the fractional order $0 < \alpha < 1$ and $1 \leq \alpha < 2$, respectively. On the basis of admissibility conditions, necessary and sufficient conditions are obtained for stabilizing singular FOS in [28]. The issue of observer-based stabilization for singular FOS is studied in [15]. However, the study of rectangular singular FOS has less achievements comparing to that of singular FOS, and many results are proposed in the case of integer order [6, 14, 29, 35]. In order to investigate rectangular singular systems, the dynamic compensator is presented to transform the rectangular singular system into an augmented square singular system in [35] for the first time. The design of the dynamic compensator has become a popular method to deal with problems of rectangular singular systems, since the closed-loop system based on the dynamic compensator can meet the requirements of regularity, impulse-free and stability. The authors in [17] propose two types of input signals of the dynamic compensator and discusses the stabilization of rectangular singular FOS. In [34], the robust stabilization for rectangular singular fractional order interval systems is considered by adopting the proportional and derivative type dynamic compensator to guarantee the closed-loop system normalized. In addition, the issue of the output feedback stabilization of uncertain rectangular singular FOS is studied in [36] by designing dynamic compensators and applying

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Received: October 2020; Revised: March 2021; Accepted: April 2021.

matrixs full rank decomposition approach. However, the above papers do not discuss the nonlinear uncertain rectangular singular FOS, which are limited in practical application.

T-S fuzzy model is a kind of nonlinear model, which can efficiently express the dynamic characteristics of complex systems. It is well known that most practical systems are nonlinear systems, such as telerobotic systems [3], car suspension systems [24]. The principle of T-S fuzzy model is to transform the global behavior of a nonlinear system into a number of linear time-invariant models, and then connects smoothly those linear models through nonlinear fuzzy membership functions. So far, the issue of the stabilization of T-S fuzzy systems has become a hot topic in the control field. For the sake of solving a general class of chaotic processes, a new approach is presented to study the stabilization of fuzzy logic control systems in [25]. The stabilization problems of fractional order T-S fuzzy systems are given in [5, 8, 16, 32]. The authors in [8] investigate the output feedback stabilization for uncertain fractional order T-S fuzzy systems and present intermediate matrices that satisfy the additional matrix equation condition to transform the matrix inequality with nonlinear term into LMI. However, the results only consider uncertain square fractional order T-S fuzzy systems. Since rectangular singular systems are more irregular than square singular systems, the stabilization of rectangular singular T-S fuzzy systems remains an open problem, let alone the robust stabilization of uncertain rectangular singular fractional order T-S fuzzy systems. Few conclusions have been drawn about rectangular singular fractional order T-S fuzzy systems. For rectangular singular integer order T-S fuzzy systems, the stabilization and normalization for rectangular singular T-S fuzzy systems are analyzed in [13]. In order to guarantee the closed-loop system normalized, the proportional and derivative type dynamic compensator is used to transform rectangular singular T-S fuzzy systems into augmented square singular T-S fuzzy systems in [13]. According to the method proposed in [13], the authors in [33] consider the normalization and stabilization for rectangular singular fractional order T-S fuzzy systems. However, it is worth noting that the approach in [33] is conservative due to the introduction of normalization conditions. In addition, the conclusion of [33] is expressed in terms of a set of bilinear matrix inequalities which cannot be straightforward dealt with any LMI toolbox and may cause a trouble in solving the conditions numerically.

Based on the above discussions, we propose a new approach to address the robust stabilization for uncertain rectangular singular fractional order T-S fuzzy systems with fractional order $0 < \alpha < 1$ in this paper. The achievements of this paper are shown as follows:

- This paper considers uncertain rectangular singular fractional order T-S fuzzy systems with norm bounded for the first time and studies the robust stabilization problem for uncertain rectangular singular fractional order T-S fuzzy systems.
- Instead of designing the proportional and derivative type dynamic compensator in [33], a novel T-S fuzzy dynamic compensator is proposed to convert the uncertain rectangular singular fractional order T-S fuzzy systems into uncertain square singular fractional order T-S fuzzy systems.
- Compared with existing results, a new method is presented to solve the robust stabilization problem for augmented uncertain square singular fractional order T-S fuzzy systems straightway, which avoids normalizing augmented systems in [33] and [34].
- By introducing an intermediate matrix, the main result is obtained in terms of LMI, which avoid solving an NP-hard problem. This result is much more accurate and reliable, which can be easily solved by MATLAB LMI toolbox.

The remainder of this paper is organized as follows: Section 2 introduces some results and useful lemmas. Section 3 provides the main results of the paper. Section 4 proposes a numerical example to verify the effectiveness of results. Finally, conclusions are drawn in Section 5.

Notations: $\mathbb{R}^n$ denotes the n dimension column vector. $\mathbb{R}^{m\times n}$ represents the set of $m \times n$ real matrix. $X < 0$ and $X > 0$ denote negative definite matrix and positive definite matrix, respectively. $A^\top$ denotes the transpose of matrix A. $\mathrm{sym}(X)$ is used to denote $X + X^\top$. $\mathrm{asym}(X)$ is used to denote $X - X^\top$. The symbol $*$ denotes a block matrix inferred by symmetry. $a = \sin(\alpha\frac{\pi}{2})$, $b = \cos(\alpha\frac{\pi}{2})$.

2 Preliminaries

For the sake of getting main results, we introduce some definitions and lemmas which are helpful to obtain results.

Definition 2.1. [23]. *The Caputo fractional derivative with order* α *of function* $x(t)$ *is given by:*

$$D^{\alpha}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}x^{(n)}(\tau)d\tau,$$

where $\Gamma(\cdot)$ *is the Euler Gamma function, and* n *is an integer satisfying* $n-1<\alpha\leq n$.

For the unforced linear singular fractional order system

$$ED^{\alpha}x(t) = Ax(t), \tag{1}$$

we denote (1) with the pair (E, A). The definitions of the regularity, non-impulsiveness, stability and admissibility can be found in [31].

Lemma 2.2. [31]. *Assume that the pair* (E, A) *is regular and impulse free. Then there exist two invertible matrices* M *and* N *satisfying*

$$E = M\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}N, \quad A = M\begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}N.$$

Consider the uncertain rectangular singular fractional order T-S fuzzy system with q rules:
Rule i : **IF** $z_1(t)$ is H_{i1} and $\cdots$ and $z_s(t)$ is H_{is}.
THEN

$$\begin{aligned} ED^{\alpha}x(t) &= (A_i+\Delta A_i)x(t)+B_iu(t), \\ y(t) &= C_ix(t), \end{aligned} \qquad i=1,2,\cdots,q, \tag{2}$$

where α is the fractional derivative order. $0<\alpha<1$. $D^{\alpha}x(t)$ represents the Caputo fractional derivative. $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^k$ is the control input vector, $y(t)\in\mathbb{R}^p$ is the control output vector. The matrix $E\in\mathbb{R}^{m\times n}$, $\text{rank}(E)=r<n$, $0<r<\min\{m,n\}$. A_i, B_i and C_i are system matrix, input matrix and output matrix with appropriate dimensions for the ith subsystem, respectively. $z_j(x)$ and H_{ij} $(i=1,2,\cdots,q,\ j=1,2,\cdots,s)$ are the premise variables and the fuzzy sets, respectively. q is the IF-THEN rules number. ΔA_i are time-invariant uncertain matrices and

$$\Delta A_i = M_iF_i(\sigma)N_i, \tag{3}$$

where M_i and N_i are known real matrices with appropriate dimensions, and $F_i(\sigma)$ are matrices with norm bounded uncertainties satisfying

$$F_i^{\top}(\sigma)F_i(\sigma)\leq I, \tag{4}$$

where $\sigma\in\Theta$ and Θ is a compact set in $\mathbb{R}$.
Now, the uncertain rectangular singular fractional order T-S fuzzy system is given as

$$\begin{aligned} ED^{\alpha}x(t) &= \sum_{i=1}^{q}h_i(z)((A_i+\Delta A_i)x(t)+B_iu(t)), \\ y(t) &= \sum_{i=1}^{q}h_i(z)C_ix(t), \end{aligned} \tag{5}$$

where $z(t)=\begin{bmatrix} z_1(t) & z_2(t) & \cdots & z_q(t)\end{bmatrix}$, $h_i(z(t))=\frac{\omega_i(z(t))}{\sum_{i=1}^{q}\omega_i(z(t))}$, and $\omega_i(z)=\prod_{j=1}^{s}H_{ij}(z_j)$, $H_{ij}(z_j)$ is the grade of membership of z_j in H_{ij} satisfying

$$\begin{cases} \sum_{i=1}^{q}h_i(z)=1, \\ h_i(z)\geq 0,\ i=1,2,\cdots,q, \end{cases}$$

and

$$\begin{cases} \sum_{i=1}^{q}\omega_i(z)>0, \\ \omega_i(z)\geq 0,\ i=1,2,\cdots,q. \end{cases}$$

The advantage of the dynamic compensator is that it can make the closed-loop system meet the requirement of regularity, but other types of feedback do not have the same advantage. Therefore, we use the following dynamic

compensator to compensate each rule of the fuzzy system.
Dynamic compensator rule i : **IF** $z_1(t)$ is H_{i1} and $\cdots$ and $z_s(t)$ is H_{is}.
THEN

$$\begin{aligned} E_c D^\alpha x_c(t) &= A_{ci}x_c(t) + B_{ci}y(t), \\ u(t) &= C_{ci}x_c(t) + D_{ci}y(t), \end{aligned} \tag{6}$$

where $x_c(t) \in \mathbb{R}_c^n$ is the state vector of the compensator. The matrix $E_c \in \mathbb{R}^{m_c \times n_c}$, $\text{rank}(E_c) = r_c < \min(m_c, n_c)$, $A_{ci},\ B_{ci},\ C_{ci},\ D_{ci}\ (i = 1, 2, \cdots, q)$ are dynamic compensator gain matrices with appropriate dimensions.
The overall dynamic compensator is given as follows:

$$\begin{aligned} E_c D^\alpha x_c(t) &= \sum_{i=1}^{q} h_i(z)(A_{ci}x_c(t) + B_{ci}y(t)), \\ u(t) &= \sum_{i=1}^{q} h_i(z)(C_{ci}x_c(t) + D_{ci}y(t)). \end{aligned} \tag{7}$$

Then, the following closed-loop system is derived by combining (5) and (7).

$$\begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix} \begin{bmatrix} D^\alpha x(t) \\ D^\alpha x_c(t) \end{bmatrix} = \begin{bmatrix} A(h) + \Delta A(h) & B(h)C_c(h) \\ B_c(h)C(h) & A_c(h) \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}, \tag{8}$$

where $A(h) = \sum_{i=1}^{q} h_i(z)A_i$, $B(h)$, $C(h)$, $D(h)$, $A_c(h)$, $B_c(h)$, $C_c(h)$, $D_c(h)$, are similar to $A(h)$. h_i denotes $h_i(z)$. Letting $\xi(t) = \begin{bmatrix} x(t) & x_c(t) \end{bmatrix}^\top$, then system (8) is rewritten as

$$\bar{E}D^\alpha \xi(t) = (\bar{A}(h) + \bar{B}(h)K(h)\bar{C}(h) + \Delta\bar{A}(h))\xi(t), \tag{9}$$

where

$$\bar{E} = \begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix},\ \bar{A}(h) = \begin{bmatrix} A(h) & 0 \\ 0 & 0 \end{bmatrix},\ \bar{B}(h) = \begin{bmatrix} B(h) & 0 \\ 0 & I \end{bmatrix}, \tag{10}$$

$$\bar{C}(h) = \begin{bmatrix} C(h) & 0 \\ 0 & I \end{bmatrix},\ \Delta\bar{A}(h) = \begin{bmatrix} \Delta A(h) & 0 \\ 0 & 0 \end{bmatrix}. \tag{11}$$

According to (3) and (4), one defines

$$\bar{M}(h) = \begin{bmatrix} M(h) & 0 \\ 0 & 0 \end{bmatrix},\ \bar{F}(\sigma) = \begin{bmatrix} F(\sigma) & 0 \\ 0 & 0 \end{bmatrix},\ \bar{N}(h) = \begin{bmatrix} N(h) & 0 \\ 0 & 0 \end{bmatrix},$$

then

$$\Delta\bar{A}(h) = \bar{M}(h)\bar{F}(\sigma)\bar{N}(h),\ \bar{F}(\sigma)\bar{F}^\top(\sigma) \leq I.$$

The control gain matrix in the dynamic compensator is expressed as

$$K = \begin{bmatrix} D_c(h) & C_c(h) \\ B_c(h) & A_c(h) \end{bmatrix}. \tag{12}$$

If the dimension of E_c satisfies

$$n + n_c = m + m_c,$$

system (9) is a square singular fractional order T-S fuzzy system. Next, we can study uncertain rectangular singular fractional order T-S fuzzy system (5) just by considering uncertain square singular fractional order T-S fuzzy system (9).

Lemma 2.3. [28]. *Let Φ, x, y be given matrices with appropriate dimensions, thus*

$$\begin{cases} \Phi < 0, \\ \Phi + \text{sym}(xy^\top) < 0, \end{cases}$$

holds if and only if there exists an appropriate dimension matrix G satisfying

$$\begin{bmatrix} \Phi & x + yG^\top \\ * & -G - G^\top \end{bmatrix} < 0. \tag{13}$$

Lemma 2.4. [31]. *For real symmetric matrix Π and two matrices Λ, Ξ,*

$$\Pi + \Lambda F(\sigma)\Xi + \Xi^\top F^\top(\sigma)\Lambda^\top < 0,$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Pi + \varepsilon\Lambda\Lambda^\top + \varepsilon^{-1}\Xi^\top\Xi < 0,$$

where $F^\top(\sigma)F(\sigma) \leq I$.

Lemma 2.5. [31]. *Unforced system (1) with order $0 < \alpha < 1$ is admissible if and only if there exist some matrices $X, Y \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{(n-m)\times n}$ satisfying*

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix} > 0, \tag{14}$$

$$\operatorname{sym}(A(aXE^\top - bYE^\top + E_0 Q)) < 0, \tag{15}$$

where $E_0 \in \mathbb{R}^{n\times(n-m)}$ is any matrix with full column rank and satisfies $EE_0 = 0$.

3 Main results

Theorem 3.1. *Unforced system (1) with order $0 < \alpha < 1$ is admissible if and only if there exist some matrices $X, Y \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{n\times(n-m)}$ satisfying (14) and*

$$\operatorname{sym}((aE^\top X + bE^\top Y + QE_0)A) < 0, \tag{16}$$

where $E_0 \in \mathbb{R}^{(n-m)\times n}$ is any matrix with full column rank and satisfies $E_0 E = 0$.

Proof. It is obvious that

$$\det(s^\alpha E - A) = \det(s^\alpha E^\top - A^\top).$$

Therefore, the pair (E, A) is regular and impulse free if and only if the pair $(E^\top, A^\top)$ is regular and impulse free. By applying Lemma 2.2, we have

$$E = M^{-1}\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} N^{-1}, \quad A = M^{-1}\begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix} N^{-1},$$

$$E^\top = N^{-\top}\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} M^{-\top}, \quad A^\top = N^{-\top}\begin{bmatrix} A_1^\top & 0 \\ 0 & I_{n-r} \end{bmatrix} M^{-\top}.$$

It is easy to see that the stability of the pair (E, A) and that of the pair $(E^\top, A^\top)$ depend entirely on A_1 and $A_1^\top$, respectively. The pair (E, A) is stable if and only if the pair $(E^\top, A^\top)$ is stable. Hence, the admissibility of the pair (E, A) is equivalent to that of the pair $(E^\top, A^\top)$. Consequently, this completes the proof. □ □

Theorem 3.2. *Unforced system (1) with order $0 < \alpha < 1$ is admissible if and only if there exist some matrices $\hat{X} \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{n\times(n-m)}$ satisfying*

$$\begin{bmatrix} b\cdot\operatorname{sym}(\hat{X}) & a\cdot\operatorname{asym}(\hat{X}) \\ * & b\cdot\operatorname{sym}(\hat{X}) \end{bmatrix} > 0, \tag{17}$$

$$\operatorname{sym}((E^\top\hat{X} + QE_0)A) < 0, \tag{18}$$

where $E_0 \in \mathbb{R}^{(n-m)\times n}$ is any matrix with full column rank and satisfies $E_0 E = 0$.

Proof. According to Lemma 2.5, setting $\hat{X} = aX + bY$, one has

$$\mathrm{sym}(\hat{X}) = \hat{X} + \hat{X}^\top = aX + bY + aX - bY = 2aX,$$

$$\mathrm{asym}(\hat{X}) = \hat{X} - \hat{X}^\top = aX + bY - aX + bY = 2bY.$$

It is obvious to point out

$$X = \frac{1}{2a}\mathrm{sym}(\hat{X}), \quad Y = \frac{1}{2b}\mathrm{asym}(\hat{X}). \tag{19}$$

Substituting (19) into (14) and (16), it yields

$$\begin{bmatrix} \frac{\mathrm{sym}(\hat{X})}{a} & \frac{\mathrm{asym}(\hat{X})}{b} \\ * & \frac{\mathrm{sym}(\hat{X})}{a} \end{bmatrix} > 0, \tag{20}$$

$$\mathrm{sym}((E^\top \hat{X} + QE_0)A) < 0. \tag{21}$$

Since $0 < \alpha < 1$, $a = \sin(\alpha\frac{\pi}{2}) > 0$, $b = \cos(\alpha\frac{\pi}{2}) > 0$, it can be seen that (20) is equivalent to (17). To this end, the matrices $\hat{X}$ and Q satisfy (14) and (16). This completes the proof. □ □

Theorem 3.3. *The uncertain rectangular singular fractional order T-S fuzzy system* (5) *with* $0 < \alpha < 1$ *is robust stabilization via the dynamic compensator* (7) *if there exist matrices* $\tilde{X} \in \mathbb{R}^{(n+n_c)\times(n+n_c)}$, $P_i \in \mathbb{R}^{(n+n_c)\times(n+n_r-r-r_c)}$, $G_i \in \mathbb{R}^{(k+n_c)\times(k+n_c)}$, $H_i \in \mathbb{R}^{(k+n_c)\times(p+n_c)}$ *and real scalars* $\zeta_{1i} > 0$, $\zeta_{2i} > 0$ $(i = 1, 2, \cdots, q)$ *such that* (17) *holds and*

$$\begin{bmatrix} \Omega_{i11} & \Omega_{i12} & \Omega_{i13} \\ * & -\zeta_{1i}I & 0 \\ * & * & -\zeta_{2i}I \end{bmatrix} < 0. \tag{22}$$

The control gain matrices in the dynamic compensator are given by

$$K_i = G_i^{-1}H_i, \tag{23}$$

where $E_1 \in \mathbb{R}^{(n+n_c-r-r_c)\times(n+n_c)}$, $\mathrm{rank}(E_1) = n + n_c - r - r_c$ *and* $E_1\bar{E} = 0$.

$$\Omega_{i11} = \begin{bmatrix} \Psi_{i11} & \Psi_{i12} \\ * & \Psi_{i22} \end{bmatrix}, \ \Omega_{i12} = \begin{bmatrix} \bar{E}^\top \tilde{X}\bar{M}_i \\ 0 \end{bmatrix}, \ \Omega_{i13} = \begin{bmatrix} P_iE_1\bar{M}_i \\ 0 \end{bmatrix},$$
$$\Psi_{i11} = \mathrm{sym}((\bar{E}^\top \tilde{X} + P_iE_1)(\bar{A}_i + \bar{B}_iL_i)) + (\zeta_{1i} + \zeta_{2i})\bar{N}_i^\top \bar{N}_i,$$
$$\Psi_{i12} = \mathrm{sym}((\bar{E}^\top \tilde{X} + P_iE_1)\bar{B}_i) + \bar{C}_i^\top H_i^\top - L_i^\top G_i^\top,$$
$$\Psi_{i22} = -G_i - G_i^\top.$$

The matrices L_i *are intermediate matrices derived from*

$$L_i = Z_i(\bar{E}^\top \tilde{X} + Q_iE_2)^{-1}. \tag{24}$$

The matrices $\bar{X}$, Q_i *and* $\varepsilon_{1i} > 0, \varepsilon_{2i} > 0$ *satisfy* (17) *and*

$$\begin{bmatrix} \Psi_i & \bar{E}^\top \bar{X}\bar{M}_i & Q_iE_2\bar{M}_i \\ * & -\varepsilon_{1i}I & 0 \\ * & * & -\varepsilon_{2i}I \end{bmatrix} < 0, \tag{25}$$

where $E_2 \in \mathbb{R}^{(n+n_c-r-r_c)\times(n+n_c)}$, $\mathrm{rank}(E_2) = n + n_c - r - r_c$ *and* $E_2\bar{E} = 0$.

$$\Psi_i = \mathrm{sym}(\bar{A}_i(\bar{E}^\top \bar{X} + Q_iE_2) + \bar{B}_iZ_i) + (\varepsilon_{1i} + \varepsilon_{2i})\bar{N}_i^\top \bar{N}_i.$$

Proof. Firstly, a virtual fuzzy dynamic compensator is designed.
Rule i: **IF** $z_1(t)$ is H_{i1} and $\cdots$ and $z_s(t)$ is H_{is}.
Then

$$\begin{aligned} E_cD^\alpha x_c(t) &= \bar{A}_{ci}x_c(t) + \bar{B}_{ci}x(t), \\ u(t) &= \bar{C}_{ci}x_c(t) + \bar{D}_{ci}x(t), \end{aligned} \tag{26}$$

where $\bar{A}_{ci},\ \bar{B}_{ci},\ \bar{C}_{ci},\ \bar{D}_{ci}$ $(i = 1, 2, \cdots, q)$ are compensator gain matrices with appropriate dimensions. Then the overall compensator can be inferred as

$$\begin{aligned} E_c D^\alpha x_c(t) &= \sum_{i=1}^{q} h_i(z)(\bar{A}_{ci} x_c(t) + \bar{B}_{ci} x(t)), \\ u(t) &= \sum_{i=1}^{q} h_i(z)(\bar{C}_{ci} x_c(t) + \tilde{D}_{ci} x(t)). \end{aligned} \tag{27}$$

In this case, the closed-loop control system is given by

$$\bar{E} D^\alpha \eta(t) = (\bar{A}(h) + \bar{B}(h)L(h) + \Delta\bar{A}(h))\eta(t), \tag{28}$$

where the definitions of $\bar{A}(h)$, $\bar{B}(h)$, $\Delta\bar{A}(h)$ are the same as the previous definitions of $\bar{A}(h)$, $\bar{B}(h)$, $\Delta\bar{A}(h)$ in (10) and (11)

$$\eta(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}, \quad L = \begin{bmatrix} \tilde{D}_c(h) & \bar{C}_c(h) \\ \bar{B}_c(h) & \bar{A}_c(h) \end{bmatrix}.$$

It follows from Theorem 3.2 that closed-loop control system (9) is robust stable if there exist matrices $\bar{X}$, $Q_i \in \mathbb{R}^{(n+n_c)\times(n+n_c-m)}$ such that (17) holds and

$$\Phi = \sum_{i=1}^{r} h_i(z(t))\mathrm{sym}((\bar{E}^\top \bar{X} + Q_i E_2)(\bar{A}_i + \bar{B}_i L_i + \Delta\bar{A}_i)) < 0, \tag{29}$$

Noting $F_i^\top(\sigma)F_i(\sigma) \le I$ and applying Lemma 2.4, it can be seen that there exists any real scalar ε_{1i} such that

$$\mathrm{sym}(\bar{E}^\top \bar{X}\Delta\bar{A}_i) = \mathrm{sym}(\bar{E}^\top \bar{X}\bar{M}_i\bar{F}_i(\sigma)\bar{N}_i) \le \varepsilon_{1i}^{-1}\bar{E}^\top \bar{X}\bar{M}_i(\bar{E}^\top \bar{X}\bar{M}_i)^\top + \varepsilon_{1i}\bar{N}_i^\top \bar{N}_i. \tag{30}$$

Similarly, for any real scalars ε_{2i}, one has

$$\mathrm{sym}(Q_i E_2 \Delta\bar{A}_i) = \mathrm{sym}(Q_i E_2 \bar{M}_i\bar{F}_i(\sigma)\bar{N}_i) \le \varepsilon_{2i}^{-1} Q_i E_2 \bar{M}_i(Q_i E_2 \bar{M}_i)^\top + \varepsilon_{2i}\bar{N}_i^T \bar{N}_i. \tag{31}$$

Substituting (30) and (31) into (29), it yields

$$\begin{aligned} \Phi \le \sum_{i=1}^{r} & h_i(z(t))\mathrm{sym}((\bar{E}^\top \bar{X} + Q_i E_2)(\bar{A}_i + \bar{B}_i L_i)) + (\varepsilon_{1i} + \varepsilon_{2i})\bar{N}_i^\top \bar{N}_i \\ & + \varepsilon_{1i}^{-1}\bar{E}^\top \bar{X}\bar{M}_i(\bar{E}^\top \bar{X}\bar{M}_i)^\top + \varepsilon_{2i}^{-1} Q_i E_2 \bar{M}_i(Q_i E_2 \bar{M}_i)^\top. \end{aligned} \tag{32}$$

By Schur complement, one gets

$$\mathrm{sym}((\bar{E}^\top \bar{X} + Q_i E_2)(\bar{A}_i + \bar{B}_i L_i)) + (\varepsilon_{1i} + \varepsilon_{2i})\bar{N}_i^\top \bar{N}_i + \varepsilon_{1i}^{-1}\bar{E}^\top \bar{X}\bar{M}_i(\bar{E}^\top \bar{X}\bar{M}_i)^\top + \varepsilon_{2i}^{-1} Q_i E_2 \bar{M}_i(Q_i E_2 \bar{M}_i)^\top < 0, \tag{33}$$

which is equivalent to LMI (25). Taking $0 \le h_i(z(t)) \le 1$ and $\sum_{i=1}^{r} h_i(z(t)) = 1$ into consideration, we obtain $\Phi < 0$. Thus by Theorem 3.2, closed-loop control system (28) with fractional order $0 < \alpha < 1$ is robust stable.

Next, considering dynamic compensator (7), closed-loop control system (9) is robust stable if there exist matrices $\hat{X} \in \mathbb{R}^{(n+n_c)\times(n+n_c)}$, $P_i \in \mathbb{R}^{(n+n_c)\times(n+n_c-m)}$ satisfying (17) and

$$\Psi = \sum_{i=1}^{r} h_i(z(t))\mathrm{sym}((\bar{E}^\top \hat{X} + P_i E_1)(\bar{A}_i + \bar{B}_i K_i \bar{C}_i + \Delta\bar{A}_i)) < 0. \tag{34}$$

By (34), it is shown that

$$\begin{aligned} \mathrm{sym}((\bar{E}^\top \hat{X} + P_i E_1)(\bar{A}_i + \bar{B}_i K_i \bar{C}_i + \Delta\bar{A}_i)) &= \mathrm{sym}((\bar{E}^\top \hat{X} + P_i E_1)(\bar{A}_i + \bar{B}_i L_i + \Delta\bar{A}_i)) \\ &+ \mathrm{sym}((\bar{E}^\top \hat{X} + P_i E_1)\bar{B}_i(K_i \bar{C}_i - L_i)) < 0. \end{aligned} \tag{35}$$

According to Lemma 2.3, there exist matrices G_i satisfying

$$\Omega_i = \begin{bmatrix} \Omega_{i11} & \Omega_{i12} \\ * & -G_i - G_i^\top \end{bmatrix} < 0, \tag{36}$$

where

$$\Omega_{i11} = \mathrm{sym}((\bar{E}^\top \hat{X} + P_i E_1)(\bar{A}_i + \bar{B}_i L_i + \Delta \bar{A}_i))$$
$$\Omega_{i12} = (\bar{E}^\top \hat{X} + P_i E_1)\bar{B}_i + (\bar{C}_i^\top K_i^\top - L_i^\top) G_i^\top.$$

By considering $\Delta \bar{A}_i$ in (36) and by Lemma 2.4, we have

$$\begin{aligned}\Omega_i =& \begin{bmatrix} \mathrm{sym}((\bar{E}^\top \hat{X} + P_i E_1)(\bar{A}_i + \bar{B}_i L_i)) & (\bar{E}^\top \hat{X} + P_i E_1)\bar{B}_i + (\bar{C}_i^\top K_i^\top - L_i^\top) G_i^\top \\ * & -G_i - G_i^\top \end{bmatrix} \\ &+ \mathrm{sym}(\begin{bmatrix} \bar{E}^\top \hat{X} \bar{M}_i \\ 0 \end{bmatrix} F_i(\sigma) \begin{bmatrix} \bar{N}_i & 0 \end{bmatrix}) + \mathrm{sym}(\begin{bmatrix} P_i E_i \bar{M}_i \\ 0 \end{bmatrix} F_i(\sigma) \begin{bmatrix} \bar{N}_i & 0 \end{bmatrix}) \\ \leq& \begin{bmatrix} \mathrm{sym}((\bar{E}^\top \hat{X} + P_i E_1)(\bar{A}_i + \bar{B}_i L_i)) & (\bar{E}^\top \hat{X} + P_i E_1)\bar{B}_i + (\bar{C}_i^\top K_i^\top - L_i^\top) G_i^\top \\ * & -G_i - G_i^\top \end{bmatrix} \\ &+ \beta_{1i}^{-1} \begin{bmatrix} \bar{E}^\top \hat{X} \bar{M}_i \\ 0 \end{bmatrix} \begin{bmatrix} (\bar{E}^\top \hat{X} \bar{M}_i)^\top & 0 \end{bmatrix} + \beta_{2i}^{-1} \begin{bmatrix} P_i E_1 \bar{M}_i \\ 0 \end{bmatrix} \begin{bmatrix} (P_i E_1 \bar{M}_i)^\top & 0 \end{bmatrix} \\ &+ (\beta_{1i} + \beta_{2i}) \begin{bmatrix} \bar{N}_i^\top \bar{N}_i & 0 \\ 0 & 0 \end{bmatrix} < 0. \end{aligned} \tag{37}$$

By Schur complement, we get inequality (37) is equivalent to LMI (22). Taking $0 \leq h_i(z(t)) \leq 1$ and $\sum_{i=1}^r h_i(z(t)) = 1$ into consideration, we obtain $\Psi < 0$. Thus by Theorem 3.2, closed-loop control system (9) with fractional-order $0 < \alpha < 1$ is robust stable. This completes the proof. □ □

Remark 3.4. *When the number of rules q turns into one, uncertain rectangular singular fractional order T-S fuzzy system* (5) *reduces to the uncertain rectangular singular linear FOS. When $\Delta A_i = 0$, uncertain rectangular singular fractional order T-S fuzzy system* (5) *reduces to certain rectangular singular fractional order T-S fuzzy systems. Hence, Theorem* 3.3 *can be regarded as an extension of the stabilization theory from rectangular singular linear FOS to uncertain rectangular singular nonlinear FOS.*

Remark 3.5. *It is worth noting that the T-S fuzzy modeling approach is a reliable method to analyze nonlinear FOS. The advantage of the robust control approach is that control systems can maintain some performance characteristics under perturbations. Combining these two methods, this paper comes up with an effective way to study the robust stabilization for rectangular singular nonlinear FOS with disturbances.*

4 Numerical simulations

In this section, we provide an example to demonstrate the validity of the controller design method.

Example 4.1. *Consider system* (5) *with $\alpha = 0.8$ and the following parameters*

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1.2 & 1 \\ -1.5 & -1 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.2 & 2 \\ -2.5 & -2 \\ 1 & 1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.1 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}, \quad N_2 \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},$$
$$F_1(\sigma) = \sin(\sigma), \quad F_2(\sigma) = \cos(\sigma).$$

The following membership functions are used for Rules 1 and 2:

$$H_1(z(t)) = (\sin(z(t)))^2, \quad H_2(z(t)) = 1 - H_1(z(t)).$$

It is well known that rectangular singular systems are not regular since the number of variables and the number of equations are different. So, rectangular singular fractional order T-S fuzzy system (5) *is not admissible. Different from the method of* [33], *we design dynamic compensator* (7) *to transform rectangular singular fractional order T-S fuzzy system* (5) *into augmented square singular fractional order T-S fuzzy system* (9) *and investigate the robust stabilization problem of system* (9) *directly instead of normalizing system* (9)*. Moreover, gain matrices of the dynamic compensator can be solved by an equivalent static output feedback. Similar to* [8], *we obtain gain matrices of the dynamic compensator* K_i *by introducing intermediate matrices* L_i $(i = 1, 2)$.

Firstly, we propose a virtual fuzzy dynamic compensator (27) *to convert rectangular singular fractional order T-S fuzzy system* (5) *to square singular fractional order T-S fuzzy system* (28) *and study the stabilization of system* (28)*. According to* $E_1\bar{E} = 0$*, The null function is used to get* $E_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$*. Let* $E_c = \begin{bmatrix} 0 & 1 \end{bmatrix}$*, a feasible solution of LMIs* (17) *and* (25) *in Theorem* 3.3 *is solved by using MATLAB LMI toolbox as*

$$\bar{X} = \begin{bmatrix} 40.8148 & 1.8393 & -0.6068 & 0 \\ 5.1887 & 22.6842 & -0.7590 & 0 \\ -0.4409 & -0.5515 & 41.4328 & 0 \\ 0 & 0 & 0 & 41.4328 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 21.7603 \\ 34.9964 \\ 0 \\ 0 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 19.1504 \\ -13.4774 \\ 0 \\ 0 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 21.7656 & 46.2629 & -77.9105 & 0 \\ 0 & 0 & 0 & -21.7825 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 50.6484 & 23.8658 & -27.4864 & 0 \\ 0 & 0 & 0 & -21.7825 \end{bmatrix},$$

$$\varepsilon_{11} = \varepsilon_{22} = 43.4824,$$

$$\varepsilon_{21} = \varepsilon_{22} = 43.6131.$$

Thus, by Theorem 3.3*, it follows from* (24) *that intermediate matrices are obtained as*

$$L_1 = \begin{bmatrix} -1.1004 & -1.5420 & -114.5616 & 0 \\ 0 & 0 & 0 & -0.5257 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 4.7964 & 8.8549 & 265.9922 & 0 \\ 0 & 0 & 0 & -0.5257 \end{bmatrix}.$$

Based on intermediate matrices L_i $(i = 1, 2)$*, by using LMI toolbox, we can get that inequalities* (17) *and* (22) *in Theorem* 3.3 *are feasible, which implies that system* (5) *in Example* 4.1 *is robust stabilization by the dynamic compensator* (7)*. The following feasible solutions are obtained as follows:*

$$\hat{X} = \begin{bmatrix} 6.2925 & -3.1833 & -0.1941 & 0 \\ -1.3594 & 6.4792 & -0.1282 & 0 \\ -0.1410 & -0.0932 & 17.0356 & 0 \\ 0 & 0 & 0 & 17.0360 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} -6.0987 \\ 1.4871 \\ 0.0782 \\ 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} -6.0976 \\ 1.4903 \\ -0.0337 \\ 0 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 4.0919 & 0 \\ 0 & 8.9562 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 1.7503 & 0 \\ 0 & 8.9562 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -4.5025 & -468.8503 & 0 \\ 0 & 0 & -21.7443 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 11.9454 & 465.6064 & 0 \\ 0 & 0 & -21.7443 \end{bmatrix},$$

$$\zeta_{11}=\zeta_{12}=17.7701,\ \zeta_{21}=\zeta_{22}=17.9925.$$

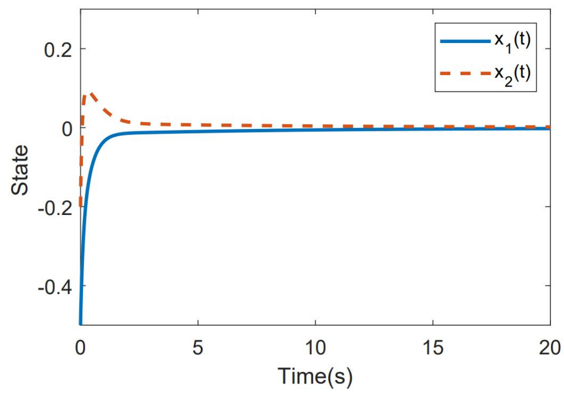

Figure 1: State responses of the uncertain rectangular descriptor fractional order T-S fuzzy system in Example 4.1

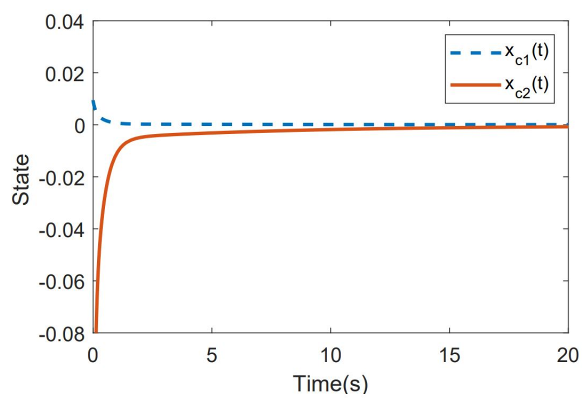

Figure 2: State responses of the dynamic compensator in Example 4.1

Therefore, by calculating (23)*, compensator parameter matrrices are obtained as*

$$K_1=\begin{bmatrix}-1.1004 & -114.5807 & 0\\ 0 & 0 & -2.4279\end{bmatrix},$$

$$K_2=\begin{bmatrix}6.8247 & 266.0115 & 0\\ 0 & 0 & -2.4279\end{bmatrix}.$$

Under the above gain matrices, take the initial condition

$$x_0=\begin{bmatrix}-0.5 & -0.2 & -0.1 & -0.2\end{bmatrix}^\top.$$

The state trajectories of closed-loop system (9) *and the dynamic compensator* (7) *are depicted in Figs.* 1 *and* 2*, respectively. It is worth noting that the system states converge to zero in Figs.* 1 *and* 2*. Therefore, dynamic compensator* (7) *makes the uncertain rectangular singular fractional order T-S fuzzy system* (5) *robust stabilization. To sum up, Example* 4.1 *shows the validity of the controller design method.*

5 Conclusions

In this paper, a new method is proposed to study the problem of the robust stabilization for uncertain rectangular singular fractional order T-S fuzzy systems with the fractional order $0<\alpha<1$. Dynamic compensators are designed to convert rectangular singular fractional order T-S fuzzy systems to square singular fractional order T-S fuzzy systems. By adopting intermediate matrices, the criterion in terms of LMI is proposed for the robust stabilization for uncertain rectangular singular fractional order T-S fuzzy systems. In the end, an example is presented to demonstrate the validity of the conclusion.

Acknowledgement

The authors wish to express their appreciation for several excellent suggestions for improvements in this paper made by the referees.

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