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Identification of Switched ARX Systems Using an Iterative Weighted Least Squares Algorithm

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ABSTRACT

This paper presents a new algorithm for the identification of a specific class of hybrid systems. Hybrid system identification is a challenging problem since it involves the estimation of discrete and continuous states simultaneously. Using the method known as the product of errors, this problem can be formulated such that the identification of continuous state is independent of discrete state estimation. We propose a new iterative weighted least squares algorithm (IWLS) for the identification of switched autoregressive exogenous systems (SARX). In the method, the parameters of only one subsystem are updated at each iteration while the parameters of the other subsystems are assumed known. The proposed method estimates, all four main parameters of hybrid systems, namely subsystem degrees, number of subsystems, unknown parameters vector, and the switching signal. The simulation results show that our proposed method has a good performance in identifying the parameters of the subsystems and the switching signal. Also, the superiority of our algorithm is demonstrated by modeling two SARX systems.

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I. INTRODUCTION

Hybrid systems are dynamic models that consist of discrete and continuous states. These systems are used when there is an interaction between physical systems and logic devices such as digital computers and can model physical phenomena that exhibit discontinuous behaviors. A hybrid system is a combination of several continuous subsystems, only one of which is active at any time instance. Continuous subsystems are connected through a discrete state variable called switching signal or discrete state. When the switching signal changes, switching occurs between the subsystems.

Since the problem of hybrid system identification involves estimating sub-model parameters and how these sub-models relate to each other with respect to the switching signal, it cannot be solved by classical identification methods, so solving this problem has gained much attention among researchers. Due to the development of applications of hybrid systems, much research has been done in the field of hybrid system identiification. The proposed methods have mostly been developed around piecewise affine (PWA) and affine switched (SA) systems. Roll et al. [1] used mixed integer linear or quadratic programming for the identification of PWA autoregressive exogenous (PWARX) systems by focusing on hinging hyperplanes in

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which the convergence to a global optimum was guaranteed. Vidal et al. [2] proposed an algebraic method for the problem of switched linear system (SLS) identification using a new error function, which is called product of errors. The identification problem of hybrid systems was simplified in a way that the estimation of sub-model parameters became independent of the discrete state estimation. Then, by applying a technique known as algebraic method, the parameters of sub-models were estimated. Juloski et al. [3] used a probabilistic method for linear hybrid system identification. In this probabilistic method, at each iteration, using prior probability distribution of the parameters and the Bayesian rule, the posterior probability distribution of the system parameters is calculated.

A bounded-error approach for PWARX system identification is discussed in [4]. Minimality and also identifiability of SARX systems are presented in [5]. The problem of identifying SARX systems when measurement data is impregnated with large amounts of noise is discussed in [6]. The problem of identifying SARX models based on assigning measurement data to a suitable subsystem based on a new robust criterion is presented in [7]. Using the Bayesian system identification method, not only does it calculate a posterior distribution on the model parameters to indicate the level of uncertainty of the estimated values, but it also automatically determines the desired number of local models [8]. An algebraic geometric method (AG) is proposed in [9] to identify ARX systems when both process and measurement are noisy. A recursive identification method is proposed in [10] for piecewise ARX models, which uses a likelihood function that adaptively fines the complexity of the model. A novel incremental algorithm has been suggested, which is based on the genetic and LOLIMOT algorithms for identification and fault detection and is of high dimension systems [11].

In this paper, we propose a new IWLS algorithm for the identification problem of switched ARX (SARX) systems using the so-called hybrid decoupling constraint method and defining the error function as the error product. Identification of SARX systems is calculated so that the subsystem parameters are estimated independent of the switch signal so that the parameters of only one subsystem are updated at each iteration while the parameters of the other subsystems are assumed to be known.

The paper is organized as follows. Section II introduces different types of linear hybrid systems. Section III explains the identification problem of hybrid systems while, in Section IV, we reformulate this problem such that the identification of continuous state becomes independent of the estimation of discrete state. Section V suggests a new method based on iterative weighted least squares for the identification of hybrid systems. The validity and superiority of the proposed method is shown in Section VI through numerical examples and simulations.

II. LINEAR HYBRID SYSTEMS

In this paper, we deal with the problem of identifying a particular class of hybrid systems. Piecewise affine systems (PWA) are a special class of hybrid systems that combine a number of affine subsystems in such a way that only one subsystem is active at a time. A discrete-time PWA system with s subsystems can be represented in the form of state space as follow:

$$\begin{aligned} x(t+1) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + b_{\sigma(t)} \\ y(t) &= C_{\sigma(t)}x(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the continuous-state trajectory, $\sigma(t) \in \{1,2,\ldots,s\}$ is a piecewise constant function called switching signal, $A_i \in R^{n\times n}$, $B_i \in R^{n\times p}$, $b_i \in R^{n\times 1}$ are the affine section of PWA system, and $C_i \in R^{q\times n}$ is the state-space matrices corresponding to the i^{th} subsystem. Switching among subsystems occurs when the switching signal changes. When the switching signal changes, switching occurs between the subsystems. Switching signal changes can be determined in different ways. If the switching signal is deterministic and independent of continuous states, the system is called switched affine (SA). While in piecewise affine (PWA) systems, the discrete state is determined according to the continuous states and input variable as follows:

$$\sigma(t) = \mathrm{i} \quad \text{iff} \quad \begin{bmatrix} x(t) \\ \mathrm{u}(t) \end{bmatrix} \in \Omega_i \qquad i = 1,2,\ldots,s \tag{2}$$

where $\{\Omega_i\}_{i=1}^{s}$ is complete partitioning of state-input space. PWA and SA systems can also be shown in the input-output form with autoregressive exogenous input (ARX) models:

$$y(t) = \sum_{j=1}^{n_a} a_j(\sigma)y(t-j) + \sum_{j=1}^{n_b} b_j(\sigma)u(t-j) + c(\sigma) \tag{3}$$

where n_a is the system degree, n_b is the input, and c is affine part in the ARX model. When the switching signal is deterministic and independent of continuous states, system (3) is called switched ARX (SARX) and if the switching signal is determined according to the continuous states and input variable, the system is called piecewise ARX (PWARX). Eq. (3) can also be presented in the form of linear regression:

$$y(t)=\varphi^{T}(t)\theta_{\sigma(t)} \tag{4}$$

where $\varphi(t)$ is the extended regression vector and is defined as below:

$$\varphi(t)=[y(t-1)\ \cdots\ y(t-n_a)\ u(t-1)\ \cdots\ u(t-n_b)\ 1]^T \tag{5}$$

and θ_i is the vector of parameters for the i[th] subsystem.

III. Hybrid System Identification Problem

In the previous sections, different linear hybrid systems were introduced. Since SARX systems are more common than other linear hybrid systems, we focus on identifying SARX systems. The general problem of identifying SARX systems can be summarized as follows [2], [6].

Given the input-output data pairs, $\{(u(t),y(t))\}_{t=1}^{N}$ estimates:

1. Subsystem degrees, n_a and n_b
2. Number of subsystems s .
3. Unknown parameters vector, θ_{i} for each subsystem $i=1,2,\ldots,s$.
4. Discrete state or switching signal $\sigma(t)$ for $t>\max\{n_a,n_b\}$.

IV. Error Product Method

As mentioned, in the problem of identifying hybrid systems, not only the number of subsystems and the degree of the subsystems but also the switching signal and subsystem parameters must be estimated. As a result, it must be determined that each input/output data pair belongs to which subsystem and the system parameters are changed to approximate the data behavior. Conventional methods start using the clustering algorithm in all regression vector columns, and a linear model is embedded for each data cluster. The identification error in SARX systems is as follows:

$$\varepsilon(t)=\min_i\left\{y(t)-\varphi^{T}(t)\theta_i\right\} \quad i=1,2,3,\ldots,s \tag{6}$$

The cost function could be presented as the sum of errors as follows:

$$\begin{aligned} \mathrm{V}_N\left(\theta,Z^N\right)&=\sum_{t=1}^{\mathrm{N}}\ell(\varepsilon(t)) \\ &=\sum_{t=1}^{\mathrm{N}}\ell\left(\min_i\left\{y(t)-\varphi^{T}(t)\theta_i\right\}\right) \end{aligned} \tag{7}$$

where $\ell(\cdot)$ is a real positive function. For example, $\ell(\cdot)$ could be a quadratic or absolute value function. In order to minimize the cost function (7), the parameters of s hyperplanes should be determined such that each data pair is near to, at least, one hyperplane. We define the error function for all sub-models as (8).

$$\varepsilon_{\mathrm{i}}(t)=y(t)-\varphi^{T}(t)\theta_i \qquad i=1,2,\ldots,s \tag{8}$$

where $\varepsilon_{\mathrm{i}}(t)$ shows the prediction error of the t[th] data pair with the i[th] sub-model. If the output prediction errors defined in (8) are multiplied by each other, an error function called the error product is obtained as (9).

$$\begin{aligned} \varepsilon(t)&=\prod_{\mathrm{i}=1}^{\mathrm{s}}\ell_{\mathrm{i}}(\varepsilon_{\mathrm{i}}(t)) \\ &=\prod_{\mathrm{i}=1}^{\mathrm{s}}\ell_{\mathrm{i}}\left(y(t)-\varphi^{T}(t)\theta_i\right) \end{aligned} \tag{9}$$

where $\ell_i(\cdot)$ is a norm function related to the i[th] subsystem [2],[6]. According to (9), different norms can be defined for the error of each subsystem. When describing the identification method, the advantage of being able to choose different norms will be discussed in Section V. The most important result of the error function (9) is that the cost function for the problem of identifying SARX systems can be defined as follows:

$$V_N\left(\theta,Z^N\right)=\frac{1}{N}\sum_{t=1}^{N}\prod_{i=1}^{s}\ell_i\left(y(t)-\varphi^{T}(t)\theta_i\right) \tag{10}$$

According to (10), it can be concluded that the identification of SARX systems is equivalent to the determination of s hyperplanes where each hyperplane represents a subsystem. To minimize the cost function (10), the output $y(t)$ should only be close to one of the hyperplanes. In this case, the distance between $y(t)$ and the hyperplane is zero (or a small amount), and multiplying it by other distances reduces the error e(t). The most important advantage of the cost function (10) rather than (7) is that there is no need to cluster the data and determine which subsystem is responsible for producing the t[th] data while by minimizing cost function (10), unknown parameters θ_i can be estimated independently of the switching signal.

V. PROPOSED METHOD FOR PWARX IDENTIFICATION

In this section, a new method is proposed for identifying SARX systems. The advantage of this method over other methods is its simplicity in solving and low volume of calculations. We rewrite the cost function as follows:

$$V_N\left(\theta, Z^N\right)= \left\| \begin{matrix} \ell_1\left(y(1)-\varphi^T(1)\theta_1\right)\times \ell_2\left(y(1)-\varphi^T(1)\theta_2\right)\times\cdots\times \ell_s\left(y(1)-\varphi^T(1)\theta_s\right) \\ \ell_1\left(y(2)-\varphi^T(2)\theta_1\right)\times \ell_2\left(y(2)-\varphi^T(2)\theta_2\right)\times\cdots\times \ell_s\left(y(2)-\varphi^T(2)\theta_s\right) \\ \vdots \\ \ell_1\left(y(N)-\varphi^T(N)\theta_1\right)\times \ell_2\left(y(N)-\varphi^T(N)\theta_2\right)\times\cdots\times \ell_s\left(y(N)-\varphi^T(N)\theta_s\right) \end{matrix} \right\| \tag{11}$$

Minimizing the cost function (10) is an easy problem, assuming that only the i[th] hyperplane is unknown. The above cost function with the vector of parameters θ_j for $j \neq i$ is as follows:

$$V_N^i\left(\theta, Z^N\right)= \left\| \begin{matrix} W_i(1)\times \ell_i\left(y(1)-\varphi^T(1)\theta_i\right) \\ W_i(2)\times \ell_i\left(y(2)-\varphi^T(2)\theta_i\right) \\ \vdots \\ W_i(N)\times \ell_i\left(y(N)-\varphi^T(N)\theta_i\right) \end{matrix} \right\| \tag{12}$$

where $V_N^i\left(\theta, Z^N\right)$ is the cost function that specifies only unknown parameters when other parameters are known, and $W_i(t)$ determines the weight of the t[th] data and is defined as:

$$W_i(t)=\prod_{\substack{j=1\\ j\neq i}}^{N} \ell_j\left(e_j(t)\right)=\prod_{\substack{j=1\\ j\neq i}}^{N} \ell_j\left(y(t)-\varphi^T(t)\theta_j\right) \tag{13}$$

What is interesting about cost function (12) is that if all weight values $W_i(t)$ are equal, the identification problem will be a prediction error method (PEM) in which case $W_i(t)$ weights will not be equal and the problem is converted to weighted PEM (WPEM). In fact, while only one sub-model is unknown and the other is known, the product of the errors is small and the corresponding weights are small for those data determined by known sub-models, so it has little effect on unknown parameters estimation. On the other hand, data not generated by known sub-models have the greatest impact on the optimization problem because the corresponding weights have significant values. The advantage of being able to choose different norms $\ell_i(\cdot)$ is that by choosing 2-norm, the problem becomes the problem of weighted least squares (WLS), so it can be easily solved by conventional analytical methods. The analytical solution of the cost function (12) is obtained as:

$$\hat{\theta}_i=\left[R(N)\right]^{-1} f(N) \tag{14}$$

$$R(N)=\frac{1}{N}\sum_{t=1}^{N} W_i(t)\varphi(t)\varphi^T(t) \tag{15}$$

$$f(N)=\frac{1}{N}\sum_{t=1}^{N} W_i(t)\varphi(t)y(t) \tag{16}$$

In this condition, other norms $\ell_j(\cdot)$ for $j \neq i$ can have different choices depending on the type of problem. The suitable choice of norms $\ell_j(\cdot)$ for $j \neq i$ can affect the rate of algorithm convergence and stability of system [6].

VI. SUGGESTED ALGORITHM

By choosing an appropriate norm $\ell_i(\cdot)$ for $j \neq i$ and acceptable value for prediction error ε and by assuming that the number of subsystems and their degrees are known, the following algorithm is proposed for SARX system identification. The degrees of all subsystems are assumed equal.

1- Let
$t=1:N$ and $i=1:s$, $\varepsilon_i(t)=1$ and $k=0$

2- Let
$k=k+1$ and i is the reminder of k divided by s

3- Update $W_i(t)$ using Eq. (13).

4- Update θ_i according to Eq. (14-16).

5- Update error vector $\varepsilon_i(t)$ by Eq. (8).

6- Repeat steps 2-5 until condition (17) is met:

$$\frac{1}{N}\sum_{t=1}^{N}\left\|\min_i\left\{y(t)-\varphi^T(t)\theta_i\right\}\right\| \leq \lambda \tag{17}$$

$\lambda = 0.01$, After determining the sub-model parameters, the switching signal is calculated by solving the following equation.

$$\sigma(t)=\arg\min_i\left\{\left|y(t)-\varphi^T(t)\theta_i\right|\right\} \quad i=1,2,\ldots,s \tag{18}$$

VII. Numerical Examples and Simulations

Two different systems have been used to evaluate the accuracy and efficiency of the proposed method.

A. *Example* 1

Assume that the input-output data pair $\{(x(t), y(t))\}_{t=1}^{71}$ is generated by the system [4,8].

$$y(t)=\begin{cases}\begin{bmatrix}1 & 0.5\end{bmatrix}\begin{bmatrix}x(t)\\ 1\end{bmatrix}+\upsilon_1, & \text{if } x\in[-5,-2)\\ \begin{bmatrix}-0.5 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ 1\end{bmatrix}+\upsilon_2, & \text{if } x\in[-2,2)\end{cases} \tag{19}$$

in which 30 points on the interval $[-5,-2)$ and 41 points on the interval $[-2,2)$ with uniform distribution are generated. The outputs are computed using model (19) and υ_1 and υ_2 indicate noise with the zero mean and uniform distribution on intervals $[-0.8,0.8]$ and z, respectively. The generated data from system (19) is illustrated in Fig. 1.

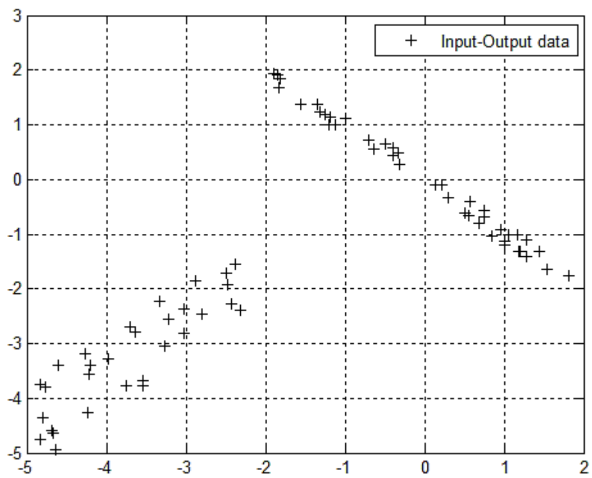

Fig. 1. Input/output data.

The sub-models obtained in the first iteration after applying the proposed algorithm to the data are shown in Fig. 2, which uses the Euclidean norm as the selected norm.

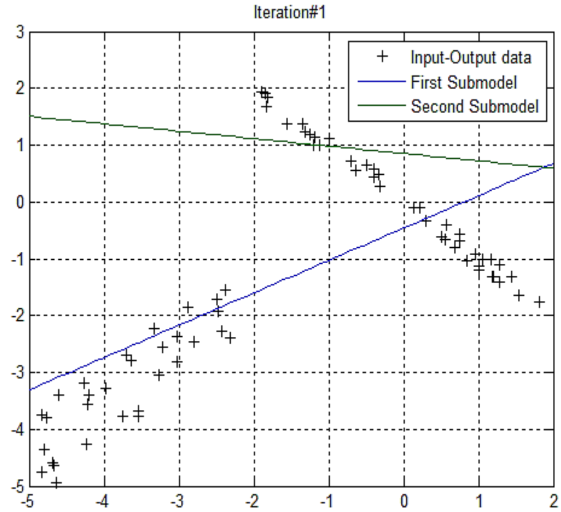

Fig. 2. The models obtained after the first iteration

Because of the weighting, the values are initially equal to one, the first sub-model tries to model all the data as shown in Fig. 2. After the first model is determined, the parameters of the second model are determined. Since the weight values have changed, the data that are far from the first model have a higher weight and more importance. Figs. 3 and 4 show how these sub-models have changed in the second and third iterations of the algorithm, and only after three iterations, the values of the parameters tend to their actual values. The model can predict the output as shown in Fig. 4. The values estimated for the switching signal are shown in Fig. 5.

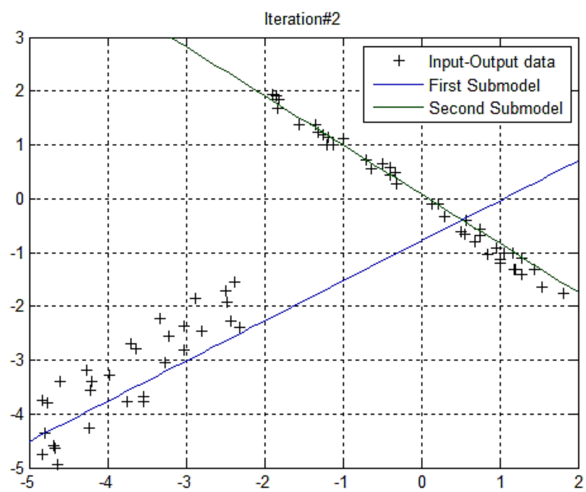

Fig. 3. Subsystem models after the second iteration.

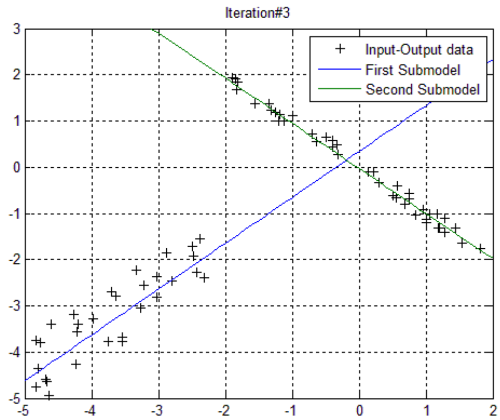

Fig. 4. Subsystem models after the third iteration.

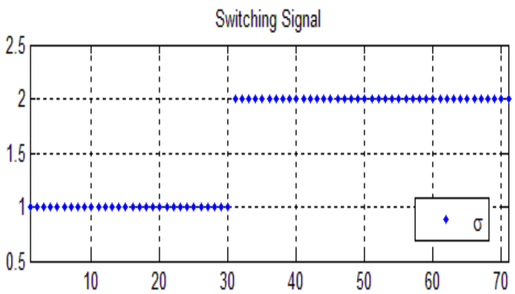

Fig. 5. The values estimated for the discrete signal.

B. Example 2

To compare the method proposed in this paper with other methods in identifying the hybrid system, the simulation results are compared with the method presented in [2]. The reason for choosing a reference [2] for comparison is that the method used in this reference is based on the product of errors and polynomial factors. Proposing a simple iterative least-squares weight algorithm to estimate the parameter of SARX systems is a novelty of our method in this paper compared to the reference method [2]. In addition, the cost function used in [2] is the same as Eq. (10) with the choice of 1-norm ($\ell_i(\varepsilon)=|\varepsilon|$) as the selective norm, while in the method proposed in this paper, the selective norm can be adjusted according to different conditions. Proper selection of the selected norm leads to a robust algorithm against outliers. Each linear system is defined as follows:

$$\begin{aligned} y(t) &= a_1(\sigma(t-1))y(t-1)+a_2(\sigma(t-1))y(t-2) \\ &\quad +c_1(\sigma(t-1))u(t-1)+e(t) \end{aligned} \tag{20}$$

where $\sigma(t)\in\{1,2,3\}$ is the switching signal and is determined after 1000 iterations as below:

$$\sigma(t)=\begin{cases} 1 & 1\le t\le 30 \\ 2 & 31\le t\le 60 \\ 3 & 61\le t\le 100 \end{cases} \tag{21}$$

Parameters were randomly selected and simulations were performed on 1000 SARX systems. The parameters a_1 and a_2 are selected in each experiment for each of the three sub-models such that the complex digital poles are evenly distributed on the wall $0.8\le\|z\|\le 1$. Fig. 6 shows the location of complex poles in the ring.

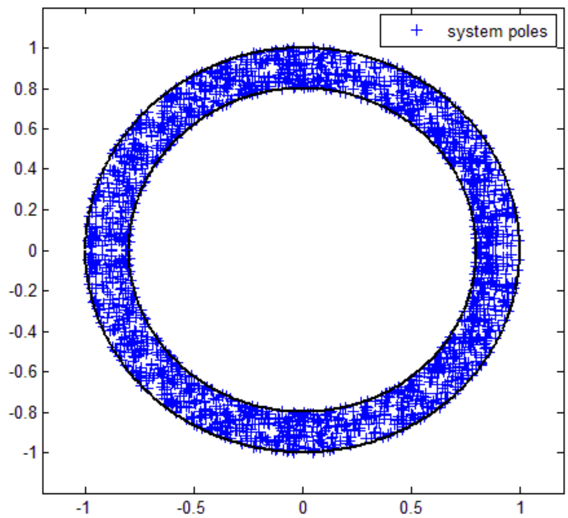

Fig. 6. The location of the system poles.

Parameter c_1 is also determined randomly in each trial and for each of the three linear sub-models with zero mean, unit variance, Gaussian distribution. The initial values of the continuous states are randomly chosen with zero mean Gaussian distribution and variance $\sum = I_2$ for each trial. The measurement noise $e(t)$ is a white noise with zero mean and variance σ_e. To investigate the effect of noise on the identification method, the problem is repeated five times with different values of σ_e. The results of the comparison between the method presented in this paper (IWLS) with the method presented in [2], which is a polynomial factorization algorithm (PFA) and a polynomial differentiation algorithm (PDA), are shown in Fig. 7-9. At each trial, and for each sub system, the error between real parameters of the system (a_1,a_2,c_1) and estimated parameters $(\hat{a}_1,\hat{a}_2,\hat{c}_1)$ is determined as $\|(a_1,a_2,c_1)-(\hat{a}_1,\hat{a}_2,\hat{c}_1)\|_2$, and the mean error is obtained with averaging on 1000 trials and 3 sub-systems. Fig. 7 illustrates the mean error of sub model parameters. In each experiment and for each subsystem, the error between the actual system parameters (a_1, a_2, c_1) and the estimated parameters $(\hat{a}_1, \hat{a}_2, \hat{c}_1)$ is determined as $\|(a_1, a_2, c_1) - (\hat{a}_1, \hat{a}_2, \hat{c}_1)\|_2$ and the mean of the error is obtained by averaging over 1000 experiments and three subsystems. Fig. 7 shows the mean error of the submodel parameters.

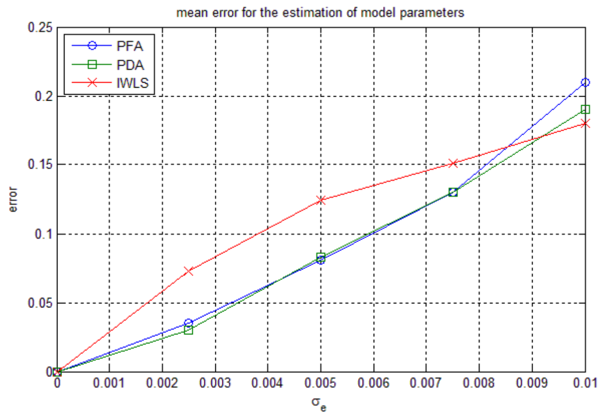

Fig. 7. The mean error of the estimated parameters.

Fig. 8 demonstrates the mean error of output prediction. At each trial, the output prediction error is determined as $\sum_{t=1}^{100} |y(t) - \hat{y}(t)|$, and the mean error is obtained by averaging on 1000 trials.

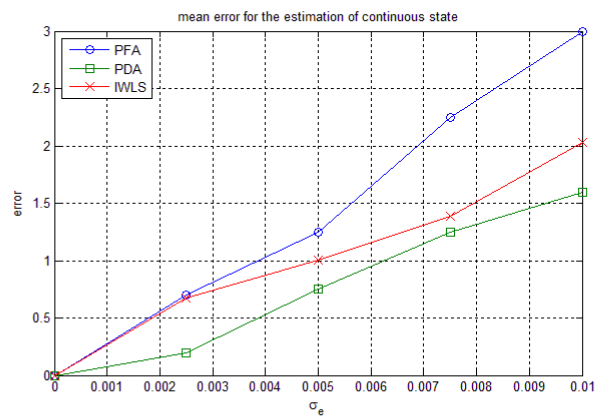

Fig. 8. The mean error for the output prediction.

The mean error of switching signal estimation is shown in Fig. 9. The mean error of switching signal estimation is calculated by dividing the number of cases in which the switching signal is incorrectly estimated in all cases. As shown in Fig. 7, the method proposed in this paper shows a little more error in estimating the subsystem parameters but has less error in output prediction compared to PDA and more than PFA as shown in Fig. 8. Finally as shown in Fig. 9, the error of the proposed method in estimating the switching signal is much less than PDA and PFA methods.

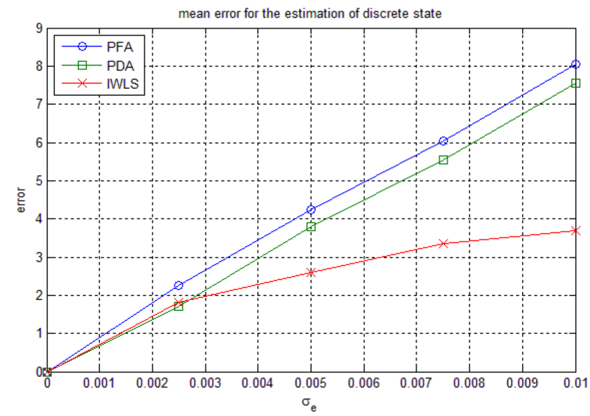

Fig. 9. The mean error of the estimation of switching signal.

CONCLUSION

This paper presented a new method for identifying SARX systems. Using a method called the product of errors, the cost function for identifying SARX systems is defined in such a way that the continuous state estimation is independent of the discrete state estimation. An iterative least squares weight method is also proposed to estimate the sub-model parameters, in which only one sub-model is unknown and the other models are assumed known in each iteration, so the cost function can be easily solved analytically. To show the effectiveness and superiority of the proposed method in identifying SARX systems, simulations and numerical examples were given.

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