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Original paper

Novel distance measure between intuitionistic fuzzy sets and its application in pattern recognition

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Abstract

In this article, we propose a new distance measure between intuitionistic fuzzy sets(IFSs), which takes into account the membership degree, non-membership degree, and their difference between membership and non-membership degree of intuitionistic fuzzy sets, as well as the exponential distance measure to avoid information loss. Meanwhile we prove that it satisfies the axiomatic definition of distance measure, and do comparison analysis with some widely used distance measures. Finally, we apply our distance measure in pattern recognition, these results show that our distance measure can significantly overcome the drawback of information loss and have more widely application scope.

Keywords: Intuitionistic fuzzy set, distance measure, exponential distance measure, pattern recognition, decision making.

1 Introduction

As a generalization of Zadeh's traditional fuzzy set(FS) [30], intuitionistic fuzzy set(IFS) was introduced by Atanassov [1] in 1986, which its most significant feature is that it consists of both membership function and non-membership function, and can clearly express the evidence, hence it has the ability to handle the vagueness and hesitancy lying in imprecise information or knowledge. Being a very useful tool in modeling real life problems, IFS has attracted a large number of researchers to study the topic, and apply it in many fields such as decision making and evaluation system, image segmentation, clustering analysis, pattern recognition and so on. For example, Atanassov et al. [2, 3] investigated IFS and its some operations, Chaira et al. [4, 5] investigated intuitionistic fuzzy C-means clustering(IFCM) algorithm and applied in medical image segmentation and edge detection, Pedro et al. [19] investigated IFCM algorithm and applied in image segmentation, Xu et al. [29] investigated intuitionistic fuzzy clustering analysis based on transitive closure method, Hung et al. [14] investigated the divergence of IFS and applied in pattern recognition, Vlachos and Sergiadis [22] used intuitionistic fuzzy information and applied in pattern recognition, Wei [25] investigated gray relational analysis of IFS and applied group decision making, Chen et al. [6] proposed the score function of IFS and applied in multiple criteria decision making, Liu et al. [17] investigated multi-criteria decision making methods based on IFS environment, Wang et al. [24] investigated multi-criteria decision making method with incomplete information based on IFS environment, Xu [27] investigated intuitionistic preference relation and applied in group decision making.

Considering that the information measure is an important feature in describing fuzzy system and its system structure, generally speaking, information measure includes similarity measure, distance measure, entropy and inclusion measure, and it has extensively been applied in decision making, data mining, image segmentation and other artificial intelligence fields, hence information measure is extended to IFS theory. Many researchers have investigated the topic and obtained some meaningful achievement. For example, Chen et al. [8, 9] investigated the entropy measure of IFS to determine the objective weight, respectively, Xia et al. [26] investigated entropy and cross entropy of IFS and applied group decision

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making, Gou et al. [10, 11] proposed exponential operators under intuitionistic fuzzy numbers and interval-valued intuitionistic fuzzy numbers and gave their application in decision making, Hung et al. [13] investigated the similarity measure between IFSs based on L_p metric, Szmidt and Kacprzyk [21] used membership degree, non-membership degree and hesitancy degree of IFS to propose the distance measure between IFSs, Wang et al. [23] improved the formula to calculate the distance measure between IFSs and applied in pattern recognition, Zhang et al. [31] investigated the distance measure between IFSs and interval-valued fuzzy sets, Papakostas et al. [18] and Xu et al. [28] compared and analyzed distance measure and similarity measure between IFSs, and proposed some applications, respectively, Li et al. [15] investigated the relationship between similarity measure and entropy of intuitionistic fuzzy sets based on their axiomatic definitions, Huang et al. [12] proposed novel distance measure and score function under Pythagorean fuzzy environments, Lin et al. [16] proposed distance measure combining correlation coefficient measures, Chen and Deng [7] investigated novel distance measures under IFS and IVIFS, Rezaei [20] gave novel distance measures under hesitant soft environments.

However, aimed at the complexity in the real application, some existing distance measures for intuitionistic fuzzy sets have some shortcomings, we think that it is necessary to reconsider the distance measure between IFSs and investigate its properties. In this article, we introduce the exponential function to enlarge influence of membership degree, and propose a novel distance measure to overcome the shortcoming of existing distance measures in which the distance measure consists of two part. The first part can describe membership degree difference through exponential function and the second part concludes membership degree and non-membership degree. These two parts can describe intuitionistic fuzzy number comprehensively and focus on membership degree more. Meanwhile, we do comparison analysis with some existing distance measures between IFSs, and apply in pattern recognition, the experiment results show that our distance measure has good performance and can provide more objective in specific scenarios.

This article is organized as follows. In Section 2, some basic notions and some existing distance measures between IFSs are introduced. In Section 3, we propose a novel distance measure between IFSs, and do comparison analysis with some existing distance measures by using several data. In Section 4, we apply our novel distance measure into pattern recognition to illustrate its effectiveness. The conclusion is given in the last Section.

2 Preliminary

Throughout this study, let $X = \{x_1, x_2, \cdots, x_n\}$ be an nonempty set, we use IFS to denote the intuitionistic fuzzy set in X, and A, B and C are intuitionistic fuzzy subsets(IFSs) in X.

Definition 2.1. [1] *An intuitionistic fuzzy set(IFS) in X, A, is defined as follows:*

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\},$$

where $\mu_A(x) \to [0,1]$ and $\nu_A(x) \to [0,1]$ denote the membership degree and non-membership degree of an element $x \in X$ belonging to set A, respectively. And it satisfies the property: $0 \le \mu_A(x) + \nu_A(x) \le 1$. Here, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ denotes the hesitancy degree of an element $x \in X$ belonging to set A.

Specially, if $\mu_A(x) + \nu_A(x) = 1$, for all x in X, then the intuitionistic fuzzy set becomes the classical fuzzy set(FS).

Definition 2.2. [1] *Some operations of intuitionistic fuzzy sets were defined as follows.*
(1) $A \subseteq B$ *iff* $\forall X \in X$, $\mu_A(x) \le \mu_B(x)$ *and* $\nu_A(x) \ge \nu_B(x)$;
(2) $A = B$ *iff* $\forall X \in X$, $\mu_A(x) = \mu_B(x)$ *and* $\nu_A(x) = \nu_B(x)$;
(3) $A^c = \{< x, \nu_A(x), \mu_A(x) > | x \in X\}$, *where A^c is called as the complement of intuitionistic fuzzy set A.*

Definition 2.3. [22] *Let d be a mapping, d: IFSs×IFSs $\to [0,1]$. If $d(A,B)$ satisfies the following properties, then $d(A,B)$ is called as the distance measure between intuitionistic fuzzy sets A and B.*
(1) $0 \le d(A,B) \le 1$;
(2) $d(A,B) = d(B,A)$;
(3) $d(A,B) = 0$ *iff* $A = B$;
(4) *If* $A \subseteq B \subseteq C$, *then* $d(A,C) \ge d(A,B)$ *and* $d(A,C) \ge d(B,C)$.

It is well known that the distance measure between fuzzy sets is an important research topic in fuzzy set theory, which indicates the discrepancy between fuzzy sets. In this section, we will recall several widely used distance measures between intuitionistic fuzzy sets. For intuitionistic fuzzy sets

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\},\ B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\},$$

$d(A, B)$ is the distance measure between IFSs A and B. We will present some existing distance measures between IFSs in the following.

The distance measures of Szmidt and Kacprzyk [21]:

$$d_{hSK}(A,B) = \frac{\sum_{i=1}^{n}[|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|]}{2},$$

$$d_{nhSK}(A,B) = \frac{\sum_{i=1}^{n}[|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|]}{2n},$$

$$d_{eSK}(A,B) = \sqrt{\frac{\sum_{i=1}^{n}[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}{2}},$$

$$d_{neSK}(A,B) = \sqrt{\frac{\sum_{i=1}^{n}[|\mu_A(x_i) - \mu_B(x_i)|^2 + |\nu_A(x_i) - \nu_B(x_i)|^2 + (\pi_A(x_i) - \pi_B(x_i))^2]}{2n}},$$

The distance measure of Xu et al. [28]:

$$d_X(A,B) = \left[\frac{\sum_{i=1}^{n}|\mu_A(x_i) - \mu_B(x_i)|^\alpha + |\nu_A(x_i) - \nu_B(x_i)|^\alpha + |\pi_A(x_i) - \pi_B(x_i)|^\alpha}{2}\right]^{\frac{1}{\alpha}},$$

$$d_{nX}(A,B) = \left[\frac{\sum_{i=1}^{n}|\mu_A(x_i) - \mu_B(x_i)|^\alpha + |\nu_A(x_i) - \nu_B(x_i)|^\alpha + |\pi_A(x_i) - \pi_B(x_i)|^\alpha}{2n}\right]^{\frac{1}{\alpha}},$$

Xu et al. [28] also considered the weighted distance measures, which take into account the weight of every element $x_i \in X$, w_i, as follows:

$$d_{wX}(A,B) = \left[\frac{\sum_{i=1}^{n} w_i(|\mu_A(x_i) - \mu_B(x_i)|^\alpha + |\nu_A(x_i) - \nu_B(x_i)|^\alpha + |\pi_A(x_i) - \pi_B(x_i)|^\alpha)}{2}\right]^{\frac{1}{\alpha}},$$

$$d_{nwX}(A,B) = \left[\frac{\sum_{i=1}^{n} w_i(|\mu_A(x_i) - \mu_B(x_i)|^\alpha + |\nu_A(x_i) - \nu_B(x_i)|^\alpha + |\pi_A(x_i) - \pi_B(x_i)|^\alpha)}{2n}\right]^{\frac{1}{\alpha}},$$

The distance measure of Wang and Xin [23]:

$$d_{WX_1}(A,B) = \sum_{i=1}^{n} w_i \left[\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4} + \frac{max(|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|)}{2}\right],$$

$$d_{WX_2}(A,B) = \frac{\sum_{i=1}^{n}[|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|]}{2n}.$$

The distance measure of Zhang and Yu [31]:

Firstly, intuitionistic fuzzy sets A' and B' are transformed into the symmetric triangular fuzzy numbers $A = (\mu_A, m_A, 1 - \nu_A)$ and $B = (\mu_B, m_B, 1 - \nu_B)$, respectively, where $m_A = (\mu_A + 1 - \nu_A)/2$ and $m_B = (\mu_B + 1 - \nu_B)/2$. Without loss of generality, suppose that $m_A \le m_B$.

Secondly, let $\mu_A(t)$ and $\mu_B(t)$ be the membership functions of symmetric triangular fuzzy number A and symmetric triangular fuzzy number B, respectively, and be given as follows:

$$\mu_A(t) = \begin{cases} (t - \mu_A)/(m_A - \mu_A), & \mu_A \le t \le m_A \\ (1 - \nu_A - t)/(1 - \nu_A - m_A), & m_A \le t \le \nu_A \\ 0, & \text{Otherwise} \end{cases}$$

$$\mu_B(t) = \begin{cases} (t - \mu_B)/(m_B - \mu_B), & \mu_B \le t \le m_B \\ (1 - \nu_B - t)/(1 - \nu_B - m_B), & m_B \le t \le \nu_B \\ 0, & \text{Otherwise} \end{cases}$$

Hence, the distance measure between IFSs A' and B' is defined as follows:

$$d_{ZY}(A', B') = U - I,$$

where $U = \int_0^{m_A} max(\mu_A(t), \mu_B(t))dt + |m_B - m_A| + \int_{m_B}^1 max(\mu_A(t), \mu_A(t))dt$ and $I = \int_0^1 min(\mu_A(t), \mu_B(t))dt$.

The distance measures of Hung and Yang [14]:

$$d_{HY_1}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \frac{min(\mu_A(x_i), \mu_B(x_i)) + min(\nu_A(x_i), \nu_B(x_i))}{max(\mu_A(x_i), \mu_B(x_i)) + max(\nu_A(x_i), \nu_B(x_i))},$$

$$d_{HY_2}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \frac{1 - (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|)}{2},$$

$$d_{HY_3}(A, B) = \frac{\sum_{i=1}^{n}(min(\mu_A(x_i), \mu_B(x_i)) + min(\nu_A(x_i), \nu_B(x_i)))}{\sum_{i=1}^{n}(max(\mu_A(x_i), \mu_B(x_i)) + max(\nu_A(x_i), \nu_B(x_i)))},$$

$$d_{HY_4}(A, B) = 1 - \frac{max(|\mu_A(x_i) - \mu_B(x_i)| + max(|\nu_A(x_i) - \nu_B(x_i)|)}{2},$$

$$d_{HY_5}(A, B) = 1 - \frac{\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|)}{\sum_{i=1}^{n}(|\mu_A(x_i) + \mu_B(x_i)| + |\nu_A(x_i) + \nu_B(x_i)|)}.$$

Vlachos and Sergiadis [22] introduced the discrimination degree between IFSs, which is denoted by $I_{IFS}(A, B)$ and can also be called the discrimination information of IFSs as follows:

$$I_{IFS}(A, B) = \sum_{i=1}^{n} \Big[\mu_A(x_i) ln \frac{2\mu_A(x_i)}{\mu_A(x_i) + \mu_B(x_i)} + \nu_A(x_i) ln \frac{2\nu_A(x_i)}{\nu_A(x_i) + \nu_B(x_i)}\Big].$$

Therefore, the distance measure between IFSs A and B was defined as follows:

$$d_{VS}(A, B) = I_{IFS}(A, B) + I_{IFS}(B, A).$$

3 Novel distance measure between intuitionistic fuzzy sets

In this section, we will present a new method to calculate the distance measure between intuitionistic fuzzy sets. The proposed method will combine some information of intuitionistic fuzzy set including the membership degree, non-membership degree and their difference between membership and non-membership degree of intuitionistic fuzzy sets, as well as the exponential distance to avoid information loss.

Suppose that $A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > | x \in X\}$ are intuitionistic fuzzy sets in X, then the novel distance measure between IFSs A and B is defined as follows.

$$d(A, B) = \frac{\sum_{i=1}^{n} d_{e_i}(A, B) \times d_{\mu\nu_i}(A, B)}{n},$$

where

$$d_{e_i}(A, B) = e^{|\mu_A(x_i) - \mu_B(x_i)| - 1}$$

$$d_{\mu\nu_i}(A, B) = \frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{2}.$$

Here, $d_{e_i}(A, B)$ denotes the exponential distance measure, which is determined by the difference of the ith membership degree between A and B, $d_{\mu\nu_i}(A, B)$ denotes the average including absolute value of ith membership degree and non-membership degree between A and B.

Theorem 3.1. *$d(A, B)$ is the distance measure between intuitionistic fuzzy sets A and B.*

Proof. (1) $d(A,B) \in [0,1]$;

Because $\mu_A(x_i), \mu_B(x_i), \nu_A(x_i)$ and $\nu_B(x_i) \in [0,1]$, thus, we have

1)

$$\begin{aligned} |\mu_A(x_i) - \mu_B(x_i)| \in [0,1] &\Longrightarrow |\mu_A(x_i) - \mu_B(x_i)| - 1 \in [-1,0] \\ &\Longrightarrow e^{|\mu_A(x_i) - \mu_B(x_i)|-1} \in [e^{-1},1] \subset [0,1] \\ &\Longrightarrow 0 < d_{e_i}(A,B) \leq 1. \end{aligned}$$

2)

$$\begin{aligned} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| \in [0,2] &\Longrightarrow \frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{2} \in [0,1] \\ &\Longrightarrow 0 \leq d_{\mu\nu_i}(A,B) \leq 1. \end{aligned}$$

Hence,

$$d_{e_i}(A,B) \times d_{\mu\nu_i}(A,B) \in [0,1] \Longrightarrow \frac{\sum_{i=1}^{n} d_{e_i}(A,B) \times d_{\mu\nu_i}(A,B)}{n} \in [0,1].$$

(2) $d(A,B) = d(B,A)$ is obvious;

(3) $d(A,B) = 0$ iff $A = B$;

1) Sufficiency: If $d(A,B) = 0$, then for every x_i, we have $d_{\mu\nu_i}(A,B) = 0$, which means

$$\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{2} = 0 \Longrightarrow |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| = 0.$$

Namely,

$$\begin{cases} |\mu_A(x_i) - \mu_B(x_i)| = 0 \\ |\nu_A(x_i) - \nu_B(x_i)| = 0 \end{cases}$$

Hence, we can have $\mu_A(x_i) = \mu_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$, for every x_i. That means $A = B$.

2) Necessity: If $A = B$, then it means that for every x_i, we have $\mu_A(x_i) = \mu_B(x_i), \nu_A(x_i) = \nu_B(x_i)$, thus, That is,

$$\begin{aligned} d_{\mu\nu_i}(A,B) &= \frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{2} = 0 \\ \Longrightarrow d(A,B) &= \frac{\sum_{i=1}^{n} d_{e_i}(A,B) \times d_{\mu\nu_i}(A,B)}{n} = 0. \end{aligned}$$

(4) For $A \subseteq B \subseteq C$, then we have $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$.

If $A \subseteq B \subseteq C$, known by Definition 2.2, we have

$$\begin{cases} \mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i) \\ \nu_A(x_i) \geq \nu_B(x_i) \geq \nu_C(x_i) \end{cases}$$

Then, we can get

$$\begin{cases} |\mu_A(x_i) - \mu_C(x_i)| \geq |\mu_A(x_i) - \mu_B(x_i)| \\ |\mu_A(x_i) - \mu_C(x_i)| \geq |\mu_B(x_i) - \mu_C(x_i)| \\ |\nu_A(x_i) - \nu_C(x_i)| \geq |\nu_A(x_i) - \nu_B(x_i)| \\ |\nu_A(x_i) - \nu_C(x_i)| \geq |\nu_B(x_i) - \nu_C(x_i)| \end{cases}$$

Further, we can get

$$\begin{cases} e^{|\mu_A(x_i) - \mu_C(x_i)|-1} \geq e^{|\mu_A(x_i) - \mu_B(x_i)|-1} \\ \frac{|\mu_A(x_i) - \mu_C(x_i)| + |\nu_A(x_i) - \nu_C(x_i)|}{2} \geq \frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{2} \end{cases}$$

That is,

$$\begin{cases} d_{e_i}(A,C) \geq d_{e_i}(A,B) \\ d_{\mu\nu_i}(A,C) \geq d_{\mu\nu_i}(A,B) \end{cases}$$

Namely, we have $d(A,C) \geq d(A,B)$.

Similarly, we have $d(A,C) \geq d(B,C)$.

Hence, we complete the proof of Theorem 3.1. □

Theorem 3.2. *For IFSs A and B, suppose that w_i is the weight of element x_i, $w_i \in [0,1], \sum_i^n w_i = 1$, then $d_w(A,B)$ is the distance measure between intuitionistic fuzzy sets A and B.*

$$d_w(A,B) = \frac{\sum_{i=1}^n d_{we_i}(A,B) \times d_{w\mu\nu_i}(A,B)}{n},$$

where

$$d_{we_i}(A,B) = e^{w_i|\mu_A(x_i)-\mu_B(x_i)|-1}$$

$$d_{w\mu\nu_i}(A,B) = \frac{w_i|\mu_A(x_i)-\mu_B(x_i)| + w_i|\nu_A(x_i)-\nu_B(x_i)|}{2}.$$

The proof is similar to that of Theorem 3.1.

Furthermore, we will make comparison analysis with some existing distance measures between IFSs by the following 7 group data which are adapted from Zhang and Yu [31], and which is listed in Table 1 and Table 2.

Table 1: Pattern recognition data of two elements

	A_1	A_2	B
1	{(0.1, 0.2), (0.1, 0.7)}	{(0.3, 0.4), (0.75, 0.15)}	{(0.4, 0.5), (0.4, 0.4)}
2	{(0.1, 0.3), (0.3, 0.3)}	{(0.2, 0.2), (0.4, 0.2)}	{(0.4, 0.4), (0.6, 0.4)}
3	{(0.4, 0.5), (0.3, 0.4)}	{(0.5, 0.4), (0.4, 0.3)}	{(0.1, 0.1), (0.5, 0.5)}

Table 2: Pattern recognition data of three elements

	A_1	A_2	A_3	B
1	{(0.1, 0.1), (0.1, 0.3), (0.1, 0.9)}	{(0.7, 0.2), (0.1, 0.8), (0.4, 0.4)}		{(0.4, 0.4), (0.6, 0.2), (0, 0.8)}
2	{(0.4, 0.5), (0.7, 0.1), (0.3, 0.3)}	{(0.5, 0.4), (0.7, 0.2), (0.4, 0.3)}	{(0.5, 0.4), (0.7, 0.1), (0.4, 0.3)}	{(0.1, 0.1), (1, 0), (0, 1)}
3	{(0.2, 0.3), (0.1, 0.4), (0.2, 0.6)}	{(0.3, 0.2), (0.4, 0.1), (0.5, 0.3)}	{(0.2, 0.3), (0.4, 0.1), (0.5, 0.3)}	{(0.1, 0.2), (0.4, 0.5), (0, 0)}
4	{(0.2, 0.4), (0.1, 0.4), (0.1, 0.5)}	{(0.3, 0.3), (0.2, 0.3), (0.4, 0.3)}		{(0, 0), (0, 0), (0, 0)}

Table 3 and Table 4 represent the distance measures by different calculation methods based on Table 1 and Table 2, respectively, where $d_i(A_k,B), i = 1,2,3,4, k = 1,2,3$, represents the distance measure based on ith group data in Table 1 or Table 2.

Table 3: Distance measures by different calculation methods based on Table 1

	$d_1(A_1,B)$	$d_1(A_2,B)$	$d_2(A_1,B)$	$d_2(A_2,B)$	$d_3(A_1,B)$	$d_3(A_2,B)$
d_{hSK}	0.900	0.550	0.800	0.800	1.000	1.000
d_{nhSK}	0.450	0.275	0.400	0.400	0.500	0.500
d_{eSK}	0.600	0.357	0.510	0.490	0.663	0.663
d_{neSK}	0.424	0.252	0.361	0.346	0.469	0.469
$d_X(\alpha=3)$	0.545	0.326	0.451	0.431	0.617	0.617
$d_X(\alpha=4)$	0.533	0.319	0.429	0.412	0.614	0.614
$d_{nX}(\alpha=3)$	0.433	0.259	0.358	0.342	0.490	0.490
$d_{nX}(\alpha=4)$	0.449	0.269	0.361	0.346	0.516	0.516
$d_{wX}(\alpha=3)$	0.273	0.163	0.226	0.215	0.309	0.309
$d_{wX}(\alpha=4)$	0.267	0.160	0.214	0.206	0.307	0.307
$d_{nwX}(\alpha=3)$	0.216	0.130	0.179	0.171	0.245	0.245
$d_{nwX}(\alpha=4)$	0.224	0.134	0.180	0.173	0.258	0.258
d_{WX_1}	0.300	0.213	0.250	0.200	0.275	0.275
d_{WX_2}	0.300	0.200	0.200	0.200	0.250	0.250
d_{ZY}	0.350	0.238	0.231	0.200	0.256	0.256
d_{HY_1}	0.394	0.628	0.550	0.550	0.461	0.461
d_{HY_2}	0.200	0.300	0.300	0.300	0.250	0.250
d_{HY_3}	0.400	0.610	0.556	0.556	0.474	0.474
d_{HY_4}	0.700	0.700	0.800	0.800	0.650	0.650
d_{HY_5}	0.571	0.758	0.714	0.714	0.643	0.643
d_{VS}	0.301	0.126	0.162	0.122	0.273	0.273
d	0.149	0.099	0.099	0.090	0.121	0.127

Table 4: Distance measures by different calculation methods based on Table 2

	$d_1(A_1,B)$	$d_1(A_2,B)$	$d_2(A_1,B)$	$d_2(A_2,B)$	$d_2(A_3,B)$	$d_3(A_1,B)$	$d_3(A_2,B)$	$d_3(A_3,B)$	$d_4(A_1,B)$	$d_4(A_2,B)$
d_{hSK}	1.300	1.300	1.700	1.700	1.700	1.400	1.400	1.400	1.700	1.800
d_{nhSK}	0.433	0.433	0.567	0.567	0.567	0.467	0.467	0.467	0.567	0.600
d_{eSK}	0.714	0.735	0.900	0.900	0.900	0.825	0.831	0.825	0.894	0.911
d_{neSK}	0.412	0.424	0.520	0.520	0.520	0.476	0.480	0.476	0.516	0.526
$d_X(\alpha=3)$	0.617	0.632	0.767	0.767	0.767	0.748	0.739	0.737	0.743	0.756
$d_X(\alpha=4)$	0.586	0.596	0.727	0.727	0.727	0.733	0.719	0.719	0.684	0.704
$d_{nX}(\alpha=3)$	0.428	0.439	0.532	0.532	0.532	0.519	0.513	0.511	0.515	0.524
$d_{nX}(\alpha=4)$	0.445	0.453	0.552	0.552	0.552	0.557	0.546	0.546	0.520	0.535
$d_{wX}(\alpha=3)$	0.206	0.211	0.256	0.256	0.256	0.249	0.246	0.246	0.248	0.252
$d_{wX}(\alpha=4)$	0.195	0.199	0.242	0.242	0.242	0.244	0.240	0.240	0.228	0.235
$d_{nwX}(\alpha=3)$	0.143	0.146	0.177	0.177	0.177	0.173	0.171	0.170	0.172	0.175
$d_{nwX}(\alpha=4)$	0.148	0.151	0.184	0.184	0.184	0.186	0.182	0.182	0.173	0.178
d_{WX_1}	0.267	0.417	0.408	0.425	0.417	0.283	0.300	0.283	0.358	0.317
d_{WX_2}	0.233	0.400	0.350	0.383	0.367	0.233	0.233	0.233	0.283	0.300
d_{ZY}	0.296	0.483	0.401	0.418	0.410	0.267	0.314	0.283	0.349	0.305
d_{HY_1}	0.461	0.364	0.363	0.340	0.358	0.385	0.385	0.385	0.000	0.000
d_{HY_2}	0.267	0.100	0.150	0.117	0.133	0.267	0.267	0.267	0.217	0.200
d_{HY_3}	0.481	0.351	0.364	0.343	0.353	0.364	0.364	0.364	0.000	0.000
d_{HY_4}	0.600	0.450	0.500	0.450	0.450	0.550	0.550	0.550	0.650	0.650
d_{HY_5}	0.650	0.520	0.533	0.511	0.522	0.533	0.533	0.533	0.000	0.000
d_{VS}	0.473	0.812	0.745	0.883	0.814	0.684	0.752	0.727	1.178	1.248
d	0.124	0.226	0.174	0.206	0.192	0.107	0.120	0.119	0.120	0.151

Remark 3.3. *Known from Table 3 and Table 4, our proposed distance has good judgment and recognition ability.*

4 Pattern recognition applications

In this section, we will use our proposed distance measure and apply in the pattern recognition field.

Example 4.1. (Data from Wang et al. [23]) *The pattern recognition of building material. Here, there exist four types of building materials which are represented intuitionistic fuzzy sets A_1, A_2, A_3 and A_4 in the feature set $X = \{x_1, x_2, x_3, x_4, x_5, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}\}$, respectively, and the data was given in Table 5, where IFS B is a kind of unknown building material, we try to justify which type of building material the IFS B should belong to.*

Table 5: Building material data

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
$u_{A_1}(x)$	0.173	0.102	0.530	0.965	0.420	0.008	0.331	1.000	0.215	0.432	0.750	0.432
$v_{A_1}(x)$	0.524	0.818	0.326	0.008	0.351	0.956	0.512	0.000	0.625	0.534	0.126	0.430
$u_{A_2}(x)$	0.510	0.627	1.000	0.125	0.026	0.732	0.556	0.650	1.000	0.145	0.047	0.760
$v_{A_2}(x)$	0.365	0.125	0.000	0.648	0.823	0.153	0.303	0.267	0.000	0.762	0.923	0.231
$u_{A_3}(x)$	0.495	0.603	0.987	0.073	0.037	0.690	0.147	0.213	0.501	1.000	0.324	0.045
$v_{A_3}(x)$	0.387	0.298	0.006	0.849	0.923	0.268	0.812	0.653	0.284	0.000	0.483	0.912
$u_{A_4}(x)$	1.000	1.000	0.857	0.734	0.021	0.076	0.152	0.113	0.489	1.000	0.386	0.028
$v_{A_4}(x)$	0.000	0.000	0.123	0.158	0.896	0.912	0.712	0.756	0.389	0.000	0.485	0.912
$u_B(x)$	0.978	0.980	0.798	0.693	0.051	0.123	0.152	0.113	0.494	0.987	0.376	0.012
$v_B(x)$	0.003	0.012	0.132	0.213	0.876	0.756	0.721	0.732	0.368	0.000	0.423	0.897

Then we have the following calculation results in Table 6.

Table 6: Distance measure between IFSs A_i and $B, i = 1, 2, 3, 4$ by difference calculation methods

	$d(A_1, B)$	$d(A_2, B)$	$d(A_3, B)$	$d(A_4, B)$
d_{hSK}	5.717	5.829	2.695	0.512
d_{nhSK}	0.476	0.486	0.225	0.043
d_{eSK}	1.724	1.709	1.023	0.181
d_{neSK}	0.498	0.493	0.295	0.052
$d_X(\alpha = 3)$	1.247	1.197	0.808	0.148
$d_X(\alpha = 4)$	1.091	1.023	0.733	0.142
$d_{nX}(\alpha = 3)$	0.545	0.523	0.353	0.065
$d_{nX}(\alpha = 4)$	0.586	0.550	0.394	0.076
$d_{wX}(\alpha = 3)$	0.104	0.100	0.067	0.012
$d_{wX}(\alpha = 4)$	0.104	0.085	0.061	0.012
$d_{nwX}(\alpha = 3)$	0.045	0.044	0.029	0.005
$d_{nwX}(\alpha = 4)$	0.049	0.046	0.033	0.006
d_{WX_1}	0.454	0.460	0.211	0.034
d_{WX_2}	0.431	0.436	0.198	0.027
d_{ZY}	5.808	5.765	2.754	0.492
d_{HY_1}	0.387	0.384	0.698	0.944
d_{HY_2}	0.069	0.064	0.302	0.473
d_{HY_3}	0.349	0.349	0.643	0.943
d_{HY_4}	0.153	0.198	0.372	0.893
d_{HY_5}	0.518	0.517	0.783	0.971
d_{VS}	4.167	3.982	1.499	0.042
d	**0.295**	**0.287**	**0.116**	**0.010**

Remark 4.2. *In the engineering application field, recognition of building materials is a practical problem. Hence, we choose this problem to make comparison experiments between proposed method and existing methods. Known from the calculation results in Table 6, we can find that A_4 should approach B. Obviously, this result is the same as Wang and Xin's.*

Example 4.3. (Data from Wang et al. [23]) *The pattern recognition of mineral. Here, there exist five kinds of hybrid minerals which are represented intuitionistic fuzzy sets A_1, A_2, A_3, A_4 and A_5 in the feature set $X = \{x_1, x_2, x_3, x_4, x_5, x_5, x_6\}$, respectively, and the data was given in Table 7, where IFS B is another kind of unknown hybrid mineral, now we want to justify which kind of mineral the IFS B should belong to.*

Table 7: Mineral data

	x_1	x_2	x_3	x_4	x_5	x_6
$u_{A_1}(x)$	0.739	0.033	0.188	0.492	0.020	0.739
$v_{A_1}(x)$	0.125	0.818	0.626	0.358	0.628	0.125
$u_{A_2}(x)$	0.124	0.030	0.048	0.136	0.019	0.393
$v_{A_2}(x)$	0.665	0.825	0.800	0.648	0.823	0.653
$u_{A_3}(x)$	0.449	0.662	1.000	1.000	1.000	1.000
$v_{A_3}(x)$	0.387	0.298	0.000	0.000	0.000	0.000
$u_{A_4}(x)$	0.280	0.521	0.470	0.295	0.188	0.735
$v_{A_4}(x)$	0.715	0.368	0.423	0.658	0.806	0.118
$u_{A_5}(x)$	0.326	1.000	0.182	0.156	0.049	0.675
$v_{A_5}(x)$	0.452	0.000	0.725	0.765	0.896	0.263
$u_B(x)$	0.629	0.524	0.210	0.218	0.069	0.658
$v_B(x)$	0.303	0.356	0.689	0.753	0.876	0.256

Then we can obtain the following calculation results in Table 8.

Known by the calculation results in Table 8, we can find that A_5 should approach B. Obviously, this result is the same as Wang and Xin's.

Remark 4.4. *In this example, we use a practical mineral task to compare different distance measures. Firstly, we collect the data described by intuitionistic fuzzy set; Secondly, we calculate these distance measures between the samples $A_1 - A_4$ and B. All the experiment results show that our method can provide the minimal distance measure in order to illustrate the most closeness A_4 and B. And among these distance measures, our method can provide more accurate*

Table 8: Distance measure between IFSs A_i and $B, i = 1, 2, 3, 4, 5$ by difference calculation methods

	$d(A_1, B)$	$d(A_2, B)$	$d(A_3, B)$	$d(A_4, B)$	$d(A_5, B)$
d_{hSK}	1.577	1.848	3.163	1.042	0.921
d_{nhSK}	0.263	0.308	0.527	0.174	0.154
d_{eSK}	0.685	0.783	1.448	0.500	0.508
d_{neSK}	0.280	0.320	0.591	0.204	0.207
$d_X(\alpha = 3)$	0.561	0.631	1.179	0.427	0.456
$d_X(\alpha = 4)$	0.521	0.578	1.075	0.405	0.443
$d_{nX}(\alpha = 3)$	0.309	0.347	0.649	0.235	0.251
$d_{nX}(\alpha = 4)$	0.333	0.369	0.687	0.259	0.283
$d_{wX}(\alpha = 3)$	0.094	0.105	0.196	0.071	0.076
$d_{wX}(\alpha = 4)$	0.087	0.096	0.179	0.068	0.074
$d_{nwX}(\alpha = 3)$	0.051	0.058	0.108	0.039	0.042
$d_{nwX}(\alpha = 4)$	0.055	0.062	0.114	0.043	0.047
d_{WX_1}	0.230	0.270	0.509	0.165	0.138
d_{WX_2}	0.209	0.255	0.490	0.156	0.124
d_{ZY}	1.521	1.738	3.105	1.109	0.915
d_{HY_1}	0.642	0.590	0.393	0.734	0.793
d_{HY_2}	0.291	0.245	0.010	0.343	0.376
d_{HY_3}	0.613	0.556	0.317	0.711	0.763
d_{HY_4}	0.524	0.513	0.097	0.620	0.584
d_{HY_5}	0.760	0.715	0.481	0.831	0.865
d_{VS}	0.585	0.829	2.888	0.305	0.395
d	**0.104**	**0.138**	**0.386**	**0.074**	**0.066**

results. For example, when two samples are very close according many distance measures, our proposed method can provide more accurate level of closeness.

5 Conclusion

Aimed at the intuitionistic fuzzy information in real life, we propose a novel distance measure between intuitionistic fuzzy sets and prove that our proposed distance measure satisfy the axiomatic definition of distance measure. In the article, we make a comparative analysis with some existing distance measures between intuitionistic fuzzy sets and apply into pattern recognition field, these results show the effectiveness of our proposed distance measure.

It needs to point out that we have some work to do in the future. One is to continue the investigation of the structure of intuitionistic fuzzy set in-depth, present more and more characteristics to describe intuitionistic fuzzy set based on the different problem background, and generalize its application scope to some fields such as medical diagnosis, fault diagnosis, risk analysis, decision making and evaluation system, and so on. The other is to further research the information measure between intuitionistic fuzzy sets, and investigate their relationship based on their axiomatic definitions.

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